

***Digital Communications 3rd edition Sklar
Solutions Manual***

SKU: 9780134588568-SOLUTIONS

Solutions Manual

Answers to End-of-Chapter Problems

Digital Communications

Third Edition

Bernard Sklar

fred harris

This Solutions Manual provides answers to the Problems that appear at the end of Chapters 1-17.

The pages contain a mix of typeset and handwritten text. The handwritten text is comprised of updates and solutions new to this edition.

Copyright © 2021 Pearson Education, Inc.

Chapter 1

Answers to Problems

The pages in Chapter 1 contain a mix of typeset and handwritten text. The handwritten text is comprised of updates and solutions new to this edition.

Chapter 1

1.1 (a) $x(t) = A \cos 2\pi f_0 t$: Power signal

$$\begin{aligned} P_x &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \frac{A^2}{2} dt = \frac{A^2}{2} \end{aligned}$$

(b)
$$x(t) = \begin{cases} A \cos 2\pi f_0 t & \text{for } -T_0/2 \leq t \leq T_0/2 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t dt = \frac{A^2 T_0}{2}$$

(c)
$$x(t) = \begin{cases} A \exp(-at) & \text{for } t > 0, a > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$\begin{aligned} E_x &= \int_{-\infty}^{\infty} x^2(t) dt = \int_0^{\infty} A^2 \exp(-2at) dt \\ &= \left[\frac{A^2 \exp(-2at)}{-2a} \right]_0^{\infty} = \frac{A^2}{2a} \end{aligned}$$

(d) $x(t) = \cos t + 5 \cos 2t$ for $-\infty < t < \infty$
 Power signal

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2 t + 25 \cos^2 2t dt; \quad \begin{matrix} 2\pi f_0 = 1 \\ T_0 = 2\pi \end{matrix}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{1}{2} + \frac{25}{2} \right) dt = \frac{1}{2\pi} (26\pi) = 13$$

1.2 $x(t) = \text{rect}(t/T)$

$$= \begin{cases} 1 & \text{for } -T/2 \leq t \leq T/2 \\ 0 & \text{elsewhere} \end{cases}$$

ESD $\Psi(f) = |X(f)|^2 = T^2 \text{sinc}^2(fT)$

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-T/2}^{T/2} dt = T$$

1.3 Using Equations (1.18) and (1.19)

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f) df = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0) df$$

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2$$

$$\underline{1.4} \quad P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt; \quad 2\pi f_0 = 10$$

$$f_0 = \frac{5}{\pi}$$

$$T_0 = \pi/5$$

$$P_x = \frac{5}{\pi} \int_{-\pi/10}^{\pi/10} 100 \cos^2 10t + 400 \cos^2 20t dt$$

$$= \frac{5}{2\pi} \int_{-\pi/10}^{\pi/10} 100 (1 + \cos 20t) + 400 (1 + \cos 40t) dt$$

$$= \frac{5}{2\pi} \left[100t + 400t \right]_{-\pi/10}^{\pi/10} = 250 \text{ W}$$

$$\underline{1.5} \quad G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - n f_0)$$

$$c_1 = c_{-1} = \frac{10}{2} = 5; \quad c_2 = c_{-2} = \frac{20}{2} = 10$$

$$c_n = 0 \quad \text{for } n = 0, \pm 3, \pm 4, \dots$$

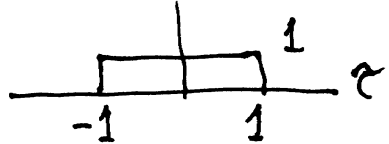
$$G_x(f) = (5)^2 \delta(f - \frac{5}{\pi}) + (5)^2 \delta(f + \frac{5}{\pi})$$

$$+ (10)^2 \delta(f - \frac{10}{\pi}) + (10)^2 \delta(f + \frac{10}{\pi})$$

$$P_x = \int_{-\infty}^{\infty} G_x(f) df = 25 + 25 + 100 + 100$$

$$= 250 \text{ W}$$

1.6 $\mathcal{F}\{R(\tau)\}$ must be a nonnegative function because $\mathcal{F}\{R(\tau)\} = G(f)$; and, the power spectral density, $G(f)$, must be a nonnegative function.

(a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \leq \tau \leq 1 \\ 0 & \text{otherwise} \end{cases}$ 

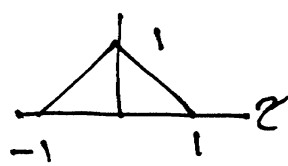
NO $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \geq x(\tau) \checkmark \\ 3. \mathcal{F}\{x(\tau)\} \text{ is a positive and negative going function.} \end{cases}$

(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

NO 1. $x(\tau) \neq x(-\tau) \times$

(c) $x(\tau) = \exp(i\tau)$

NO $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \neq x(\tau) \times \end{cases}$

(d) $x(\tau) = \begin{cases} -\tau+1 & \text{for } 0 \leq \tau \leq 1 \\ \tau+1 & \text{for } -1 \leq \tau \leq 0 \end{cases}$ 

YES $\begin{cases} 1. x(\tau) = x(-\tau) \checkmark \\ 2. x(0) \geq x(\tau) \checkmark \\ 3. \mathcal{F}\{x(\tau)\} = 2 \operatorname{sinc}^2 f \tau \end{cases}$

is a nonnegative function. $\checkmark$

1.7 (a) $X(f) = \delta(f) + \cos^2 2\pi f$

YES $\begin{cases} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) = P_x(f) \checkmark \end{cases}$

(b) $X(f) = 10 + \delta(f-10)$

No $\begin{cases} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) \neq P_x(f) \times \end{cases}$

(c) $X(f) = \exp(-2\pi|f-10|)$

No $\begin{cases} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) \neq P_x(f) \times \end{cases}$

(d) $X(f) = \exp[-2\pi(f^2-10)]$

YES $\begin{cases} 1. \text{ always real } \checkmark \\ 2. P_x(f) \geq 0 \checkmark \\ 3. P_x(-f) = P_x(f) \checkmark \end{cases}$

1.8

$$R_x(\tau) = \left\langle A \cos(2\pi f_0 t + \phi) A \cos(2\pi f_0 t + 2\pi f_0 \tau + \phi) \right\rangle$$

where $\langle \cdot \rangle$ is the time averaging operator $\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} dt$

Upon expanding (see Appendix D),

$R_x(\tau)$ becomes:

$$R_x(\tau) = A^2 \left[\cos 2\pi f_0 \tau \left\langle \cos^2(2\pi f_0 t + \phi) \right\rangle - \sin 2\pi f_0 \tau \left\langle \cos(2\pi f_0 t + \phi) \sin(2\pi f_0 t + \phi) \right\rangle \right]$$

The negative term in the above expression goes to zero, and hence

$$R_x(\tau) = \frac{A^2}{2} \cos 2\pi f_0 \tau$$

$$P_x = R_x(0) = A^2/2$$

1.9 (a) $R_x(\tau) = \frac{100}{2} \cos 10\tau + \frac{400}{2} \cos 20\tau$

where $2\pi f_0 = 10$

(b) $P_x = R_x(0) = 50 + 200 = 250 \text{ W}$

1.10 (a) average value of $x(t)$

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t) dt = 1$$

(b) the ac power of $x(t)$

$$\langle x^2(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2 2\pi f_0 t dt = \frac{1}{2}$$

(c) the rms value of $x(t)$

$$\begin{aligned} \langle x^2(t) \rangle &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\pi f_0 t)^2 dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + 2\cos 2\pi f_0 t + \cos^2 2\pi f_0 t) dt = \frac{3}{2} \\ x_{rms} &= \sqrt{3/2} \end{aligned}$$

$$\frac{1.11}{(a)} \langle X(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cos(2\pi f_0 t + \phi) dt = 0$$

$$\begin{aligned} \langle X^2(t) \rangle &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} [A \cos(2\pi f_0 t + \phi)]^2 dt \\ &= \frac{A^2}{2} \end{aligned}$$

$$1.11 (b) E\{X\} = \int_{-\infty}^{\infty} X(\phi) p(\phi) d\phi$$

$p(\phi) = \frac{1}{2\pi}$ since ϕ is uniformly distributed over $(0, 2\pi)$

$$E\{X\} = \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)] \frac{1}{2\pi} d\phi = 0$$

$$\begin{aligned} E\{X^2\} &= \int_0^{2\pi} [A \cos(2\pi f_0 t + \phi)]^2 \frac{1}{2\pi} d\phi \\ &= A^2/2 \end{aligned}$$

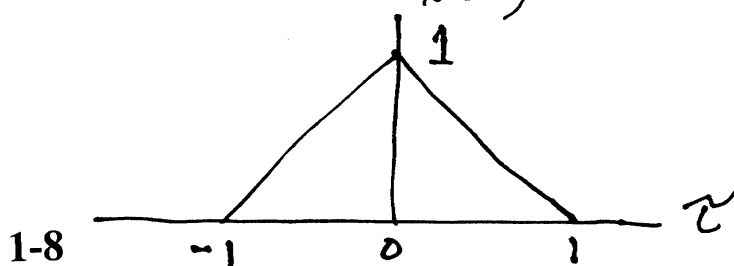
$$\underline{1.12} \quad X(f) = \text{sinc } f$$

$$\Psi_x(f) = |X(f)|^2 = \text{sinc}^2 f$$

$$R_x(\tau) = \mathcal{F}^{-1}\{\Psi_x(f)\}$$

From Table A.1, $R_x(\tau)$ is seen to be the following triangular function:

$$R_x(\tau) = \begin{cases} 1 - |\tau| & \text{for } |\tau| < 1 \\ 0 & \text{elsewhere} \end{cases}$$



$$\underline{1.13} \quad (a) \int_{-\infty}^{\infty} \cos 6t \delta(t-3) dt = \cos 18$$

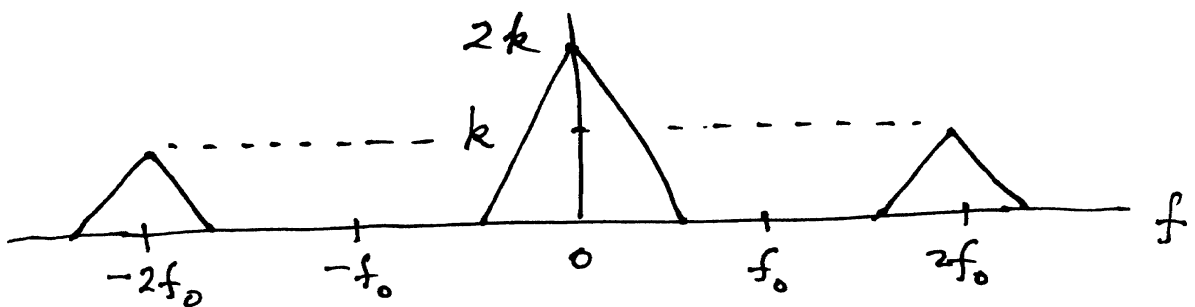
$$(b) \int_{-\infty}^{\infty} 10 \delta(t) (1+t)^{-1} dt = 10$$

$$(c) \int_{-\infty}^{\infty} \delta(t+4) (t^2+6t+1) dt = -7$$

$$(d) \int_{-\infty}^{\infty} \exp(-t^2) \delta(t-2) dt = 0.0183$$

$$\underline{1.14} \quad X_2(f) = k [\delta(f-f_0) + \delta(f+f_0)]$$

$$X_1(f) * X_2(f) = X_1(f) * k [\delta(f-f_0) + \delta(f+f_0)]$$



$$\underline{1.15} \quad (a) \quad P_x = 2 \int_0^{10 \text{ kHz}} G_x df = 2 \int_0^{10 \text{ kHz}} 10^{-6} f^2 df$$

$$= 2 \left[\frac{10^{-6} f^3}{3} \right]_0^{10^4} = 667 \text{ kW}$$

$$(b) \quad P_x = 2 \int_{5 \text{ kHz}}^{10 \text{ kHz}} 10^{-6} f^2 df = 2 \left[\frac{10^{-6} f^3}{3} \right]_{5000}^{10000}$$

$$= 583 \text{ kW}$$

$$\underline{1.16} \quad 10 \log_{10} \left[\frac{100 \times 2 \times \frac{1}{2}}{\frac{1}{2}} \right] = 23 \text{ dB}$$

1.17 (a) Since $|H(f)|$ decreases monotonically with $|f|$, and $|H(0)| = 1$, we can write the following relationship in terms of the -1 dB frequency, f_1 .

$$10 \log_{10} |H(f_1)|^2 = -1 \text{ dB}$$

$$\log_{10} \left[\frac{1}{(1 + f_1/f_n)^{2n}} \right] = -\frac{1}{10}$$

$$\left[1 + f_1/f_n \right]^{2n} = 10^{1/10}$$

$$\left[f_1/f_n \right]^{2n} = 10^{1/10} - 1 = 0.2584$$

$$\therefore n \geq \frac{1}{2} \left[\frac{\log 0.2584}{\log (f_1/f_n)} \right]$$

$$\text{For } f_1/f_n = 0.9, \quad n \geq 6.4$$

$$\text{Thus, } n = 7$$

1.17 (b) In the limit, as $n \rightarrow \infty$

$$\left(\frac{f}{f_m}\right)^{2n} \rightarrow 0, \quad |H(f)| \rightarrow 1, \quad \text{for } \left|\frac{f}{f_m}\right| < 1$$

$$\left(\frac{f}{f_m}\right)^{2n} \rightarrow \infty, \quad |H(f)| \rightarrow 0, \quad \text{for } \left|\frac{f}{f_m}\right| > 1$$

Hence, $|H(f)|$ approaches the transfer characteristic of an ideal low-pass filter with a cut-off frequency at f_m , as n approaches infinity.

1.18 $y(t) = \delta(t) * h(t)$

$$Y(f) = 1 * H(f)$$

since $\mathcal{F}\{\delta(t)\} = 1$.

Hence, $y(t) = \mathcal{F}^{-1}\{Y(f)\} = \mathcal{F}^{-1}\{H(f)\}$
 $= h(t)$

1.19 Let $x(t) = \delta(t)$

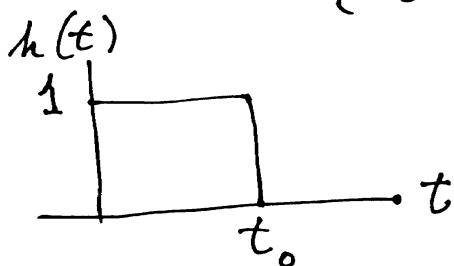
$$g(t) = \delta(t) - \delta(t - t_0)$$

$$h(t) = \int_{-\infty}^t [\delta(\tau) - \delta(\tau - t_0)] d\tau$$

$$= u(t) - u(t - t_0)$$

where $u(t)$ is the unit step function defined as follows:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{elsewhere} \end{cases}$$



1.20 (a) Half-power bandwidth is the bandwidth from half-power point to half-power point. $BW = 2f_0$

where $\frac{1}{2} = \left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right]^2$

$$0.707 = \frac{\sin x_0}{x_0}, \quad x_0 = \pi f_0 10^{-4}$$

$$x_0 \cong 1.4 \Rightarrow f_0 = 4.46 \text{ kHz} \Rightarrow BW \cong 9 \text{ kHz}$$

1.20 (b) Noise equivalent bandwidth

$$BW = 2 \int_0^{\infty} \left[\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}} \right]^2 df$$

$$= \frac{2 \times 10^4}{\pi} \int_0^{\infty} \left[\frac{\sin x}{x} \right]^2 dx$$

$\rightarrow \pi/2$

$$= 10 \text{ kHz}$$

(c) Null-to-null bandwidth: $BW = 2f_0$
where f_0 is the frequency where

$$\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} = 0$$

The minimum f_0 corresponding to the first null is found by:

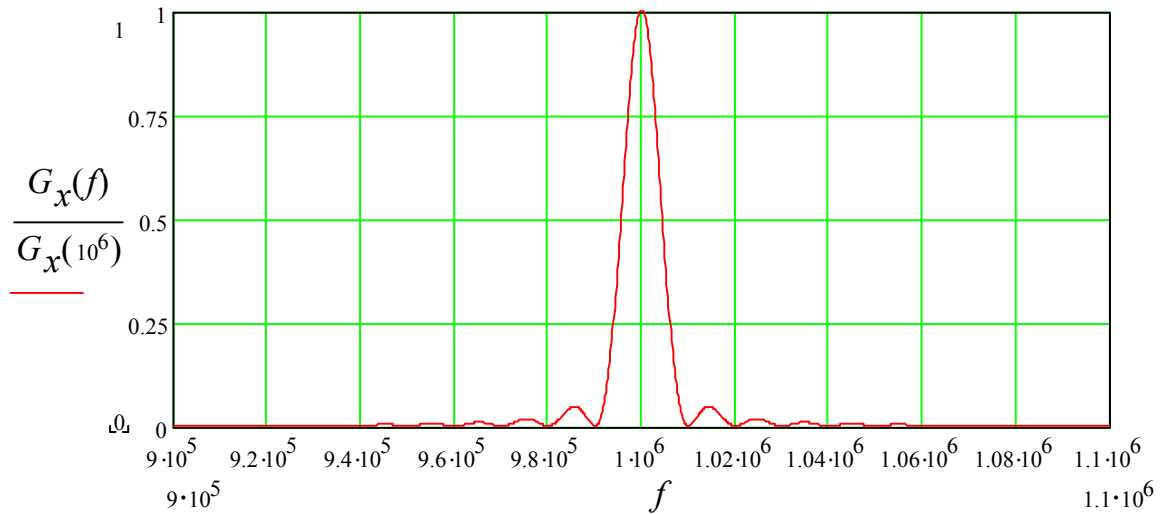
$$\pi f_0 10^{-4} = \pi$$

$$BW = 2f_0 = 20 \text{ kHz}$$

(d) 99% of power bandwidth, $BW = 2f_0$

Where $0.995 = \frac{10^{-4} \int_0^{f_0} \left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}} \right)^2 df}{10^{-4} \int_0^{\infty} \left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}} \right)^2 df}$

1.20 (d) The normalized spectrum, $G_x(f)/G_x(10^6)$, appears as:



Applying numerical methods with Mathcad ®, the two-sided 99% bandwidth can be found by numerical integration as:

$$\frac{\int_{10^6-f_1}^{10^6+f_1} G_x(f) df}{\int_{-\infty}^{\infty} G_x(f) df} = 0.99$$

where f_1 is found to be equal to 103 kHz. Thus, the two-sided 99% bandwidth is equal to 206 kHz.

Since this bandwidth corresponds to the given spectrum (with signaling rate = 10,000 symbols/s), normalizing it relative to one symbol per second, yields the two-sided 99% bandwidth as $(206 \times 10^3 / 10^4) = 20.6$ Hz for one symbol per second, or in general the 99% bandwidth in terms of the signaling rate, R , is $20.6 \times R$ Hz.

1.20 (e)

35-dB Bandwidth:

$$35\text{-dB attenuation} \Rightarrow 10^{-3.5} = 3.16 \times 10^{-4}$$

Since $\sin^2 x$ is unity for $x = \frac{\pi}{2} (2k+1)$, $k=0, 1, \dots$, the lobe beyond which the attenuation criterion is guaranteed to be met is the minimum k for which

$$10^{-3.5} \geq \frac{1}{\left[\frac{\pi}{2} (2k+1)\right]^2}$$

$$3162.28 \leq \left[\frac{\pi}{2} (2k+1)\right]^2$$

$$56.23 \leq \frac{\pi}{2} (2k+1)$$

$$35.80 \leq 2k+1 \Rightarrow k = 18$$

Thus, the 18th sidelobe meets the 35-dB criterion, and the actual 35-dB point will be on the falling edge of the 17th sidelobe. $BW = 2f_0$, where f_0 is the minimum value satisfying:

$$\pi f_0 10^{-4} > \frac{\pi}{2} (35)$$

$$\text{and } \left[\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right]^2 = 10^{-3.5}$$

Solving iteratively, we get: $\pi f_0 10^{-4} = 55.171$

$$BW = 2f_0 = 351.2 \text{ kHz}$$

1.20 (f) The absolute bandwidth is infinite, since for any finite test-BW,

$$10^{-4} \left[\frac{\sin \pi (f-10^6) 10^{-4}}{\pi (f-10^6) 10^{-4}} \right]^2 \text{ will have positive measure beyond it.}$$

Chapter 2

Answers to Problems

The pages in Chapter 2 contain a mix of typeset and handwritten text. The handwritten text is comprised of updates and solutions new to this edition.

Chapter 2

2.1 (a) $\underbrace{00010010}_H \quad \underbrace{11110011}_O \quad \underbrace{11101011}_W$

(b) $\underbrace{000}_0 \quad \underbrace{100}_4 \quad \underbrace{101}_5 \quad \underbrace{111}_7 \quad \underbrace{001}_1 \quad \underbrace{111}_7 \quad \underbrace{101}_5 \quad \underbrace{011}_3$

8 symbols

(c) $24 \text{ bits} / 4 \text{ bits per symbol} = 6 \text{ symbols}$

(d) $24 \text{ bits} / 8 \text{ bits per symbol} = 3 \text{ symbols}$

2.2 (a) $800 \text{ char/s} \times 8 \text{ bits/char} = 6400 \text{ bits/s}$

(b) $\frac{6400 \text{ bits/s}}{4 \text{ bits/symbol}} = 1600 \text{ symbols/s.}$

2.3 (a) $100 \text{ char/2 s} \times 8 \text{ bits/char} = 400 \text{ bits/s.}$

$\frac{400 \text{ bits/s}}{5 \text{ bits/symbol}} = 80 \text{ symbols/s}$

(b) 16-level PCM: $400 \text{ bits/s}, 100 \text{ symbols/s.}$

8-level PCM: $400 \text{ bits/s}, 133.3 \text{ symbols/s.}$

4-level PCM: $400 \text{ bits/s}, 200 \text{ symbols/s.}$

2-level PCM: $400 \text{ bits/s}, 400 \text{ symbols/s.}$

$$\begin{aligned}
 \underline{2.4} \quad x_s(t) &= x(t) x_p(t) \\
 &= x(t) \left\{ \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t} \right\} \\
 &= x(t) \left\{ c_0 + 2 \sum_{n=1}^{\infty} c_n \cos 2\pi n f_s t \right\}
 \end{aligned}$$

$$\begin{aligned}
 x_1(t) &= x_s(t) \cos 2\pi m f_s t \\
 &= x(t) \left\{ \begin{aligned} &\cancel{c_0 \cos 2\pi m f_s t} \quad \text{LPF} \\ &+ 2 \sum_{\substack{n=1 \\ n \neq m}}^{\infty} \cancel{c_n \cos 2\pi n f_s t} \cos 2\pi m f_s t \quad \text{LPF} \\ &+ 2 c_m \cos^2 2\pi m f_s t \end{aligned} \right\}
 \end{aligned}$$

$$\begin{aligned}
 x_0(t) &= x(t) 2 c_m \left(\frac{1}{2} + \frac{1}{2} \cancel{\cos 4\pi m f_s t} \right) \quad \text{LPF} \\
 &= c_m x(t)
 \end{aligned}$$

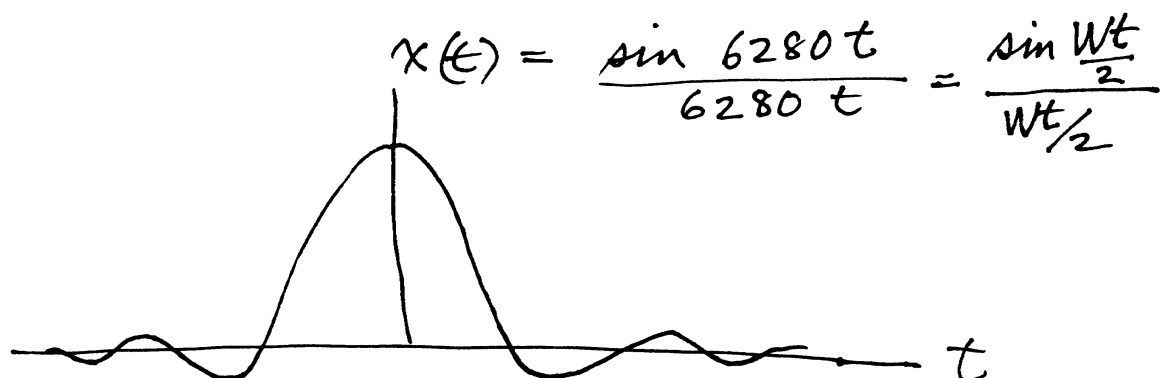
2.5 (a) L quantization levels requires a minimum of $\log_2 L$ bits. It is necessary to transmit at least $\log_2 L$ bits in T_s seconds. Thus, the time duration, T , for one bit is

$$T \leq \frac{T_s}{\log_2 L}$$

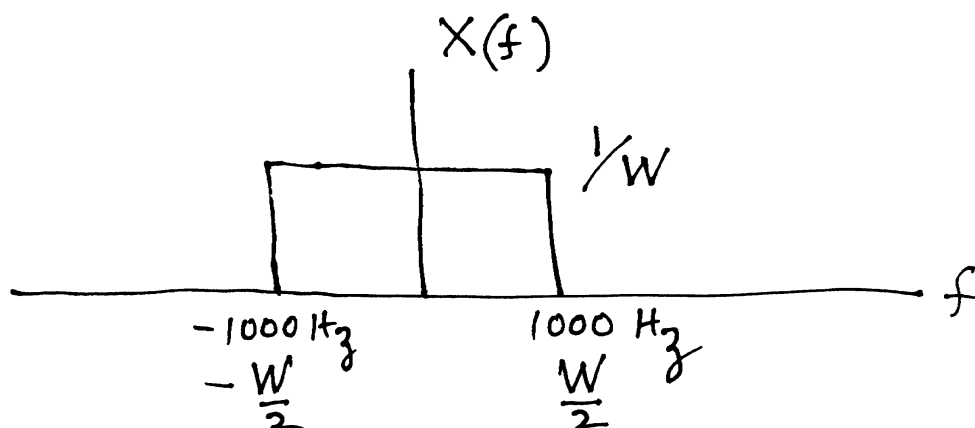
(b) The equality sign is valid if L is a power of 2.

2.6 (a) $2^5 = 32$ (b) $2^8 = 256$ (c) 2^x

2.7



where $\frac{W}{2} = 2\pi f = 6280$ radians
 $f = 1000$ Hz



$$X(f) = \begin{cases} 1/W & \text{for } |f| \leq 1000 \text{ Hz} \\ 0 & \text{elsewhere} \end{cases}$$

$$f_m = 1000 \text{ Hz}$$

Minimum sampling rate $f_s = 2f_m$
 $= 2000$
 samples/s.

$$\underline{2.8} \text{ (a)} \left(\frac{S}{N}\right)_q = 3L^2 \geq 30 \text{ dB}$$

$$10 \log_{10} (3L^2) \geq 30 \text{ dB}$$

$$L = \lceil 18.26 \rceil = 19$$

minimum number of quantization levels

$$l = \lceil \log_2 L \rceil = \lceil \log_2 19 \rceil = 5 \text{ bits/sample}$$

$$\text{(b)} T_b = \frac{T_s}{l} = \frac{1}{lf_s} = \frac{1}{5(8000)} = 25 \mu\text{s.}$$

where T_b is the time duration of a bit.

Required bandwidth, W , is

$$W = \frac{1}{T_b} = \frac{1}{25 \mu\text{s}} = 400 \text{ kHz.}$$

$$\underline{2.9} \text{ (a)} W_m = 2\pi f_m = 2000$$

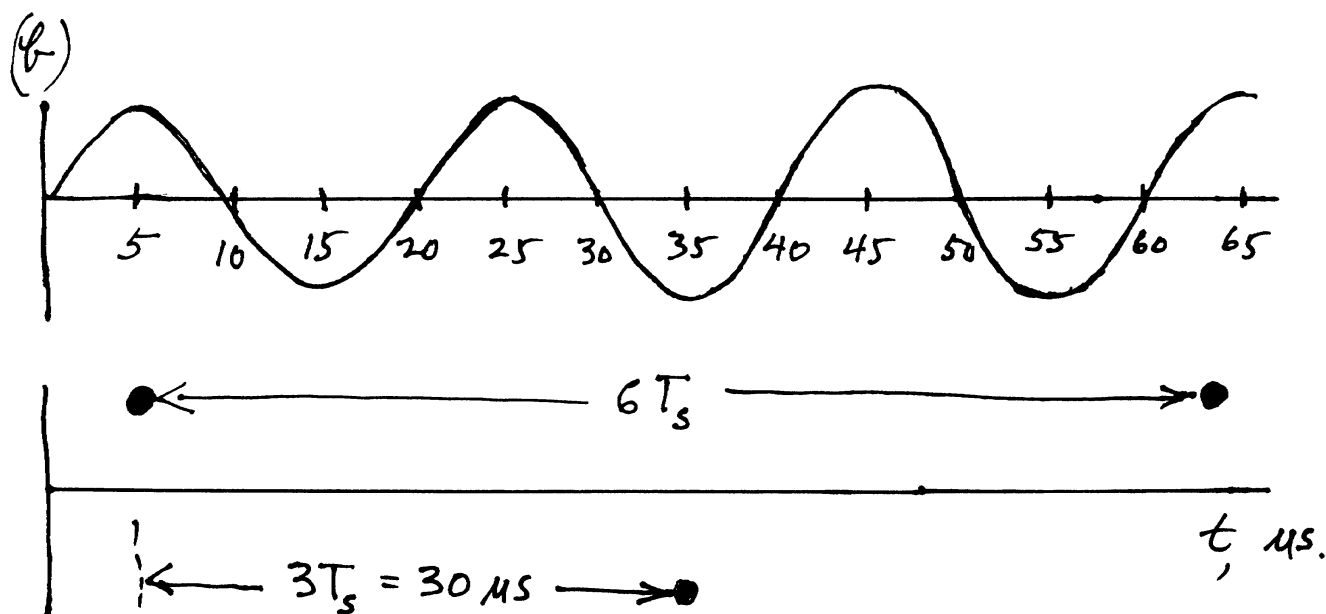
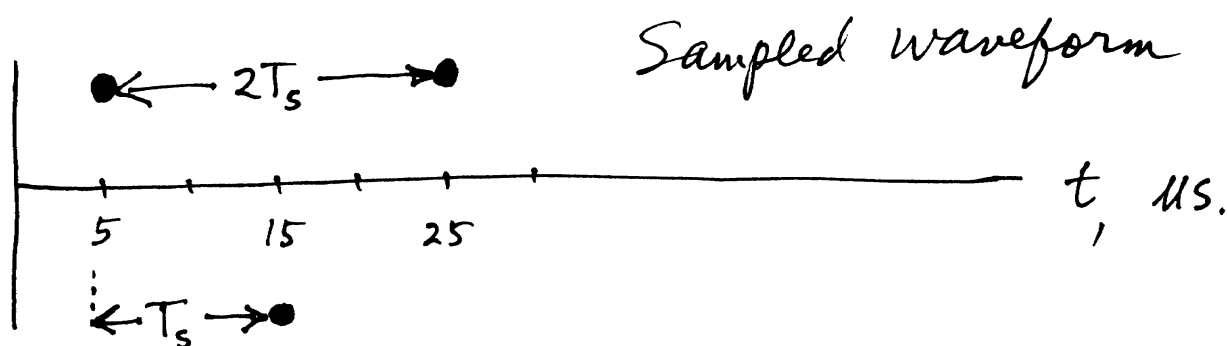
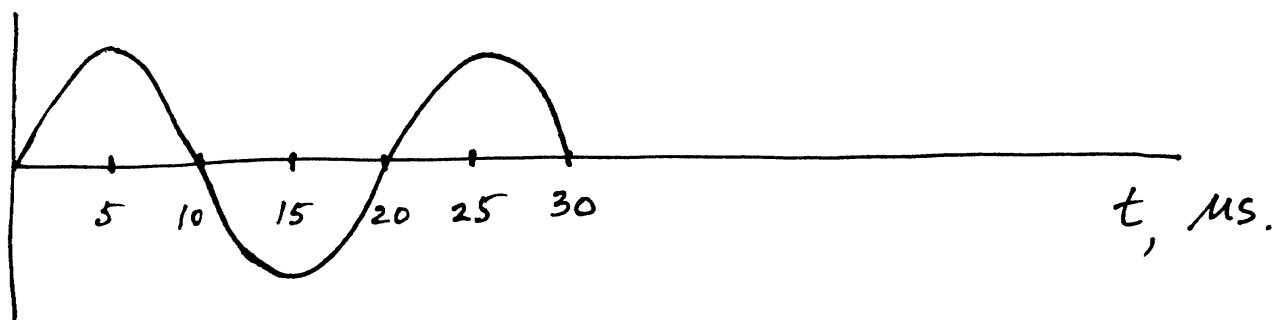
$$f_m = 2000/2\pi = 318.3 \text{ Hz}$$

$$f_s \geq 2f_m = 636.6 \text{ samples/s.}$$

$$T_s = \frac{1}{f_s} \leq 0.00157 \text{ s.}$$

$$\begin{aligned} \text{(b)} \quad & 636.6 \text{ samples/s} \times 3600 \text{ s} \\ &= 2.29 \times 10^6 \\ &\text{samples} \end{aligned}$$

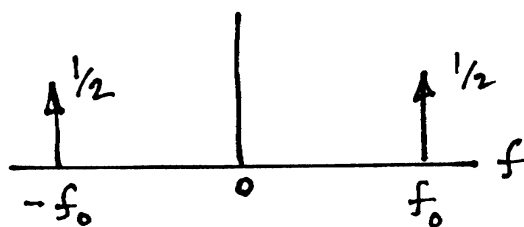
2.10 (a) $f_0 = 50 \text{ kHz}$; $T_s = \frac{1}{2f_0} = 10 \mu\text{s}$.



Period of reconstructed waveform $= 6T_s$
 $\therefore f = \frac{1}{6T_s} = 16.67 \text{ kHz} \neq 50 \text{ kHz}$.

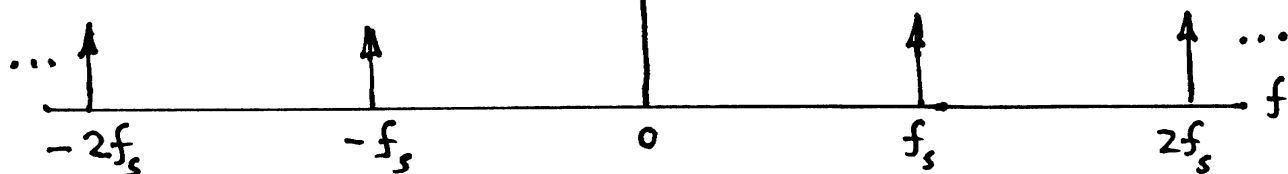
2.11

$$X(f) = \frac{1}{2} [\delta(f-f_0) + \delta(f+f_0)]$$

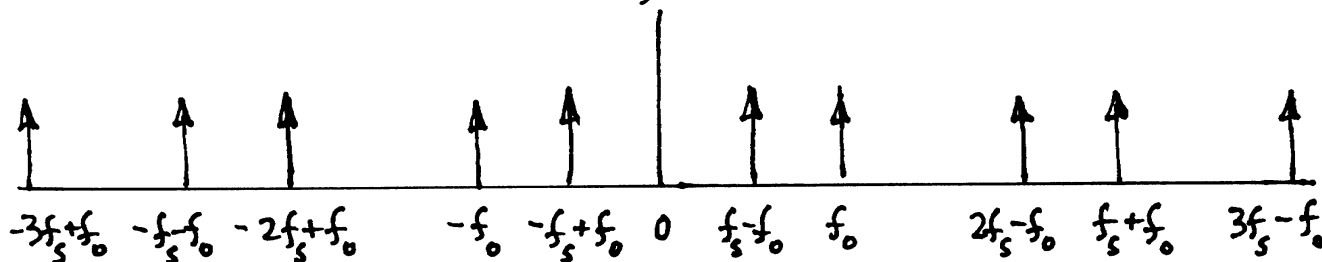


$$X_s(f)$$

$$f_s = \frac{3}{2} f_0$$



$$X(f) * X_s(f)$$



2.12 (a) Using Equation (1.65)

$$|H(f)|^2 = \frac{1}{1 + (f/f_m)^{2n}}$$

for a 6th order Butterworth filter and the given requirements, we write:

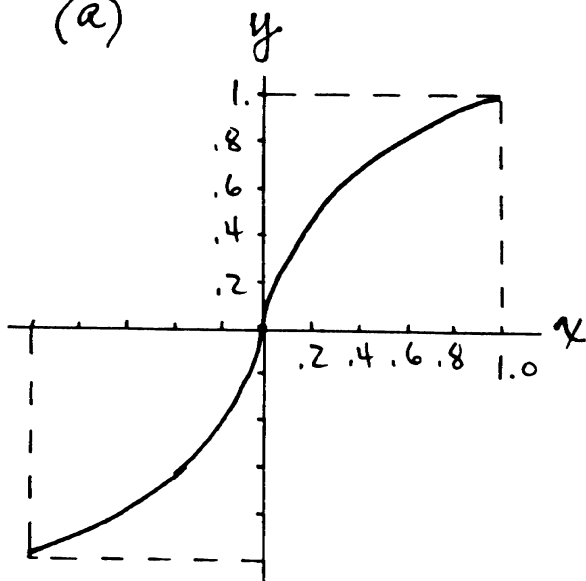
$$10^{-5} = \frac{1}{1 + (f/1000)^{12}}$$

This yields $f = 2610$ Hz. Thus, the sampling rate must be at least 5220 samples/s.

(b) Using Equation (1.65) for a 12th order filter and the same requirements yields $f = 1616$ Hz. Thus, the sampling rate must be at least 3232 samples/s.

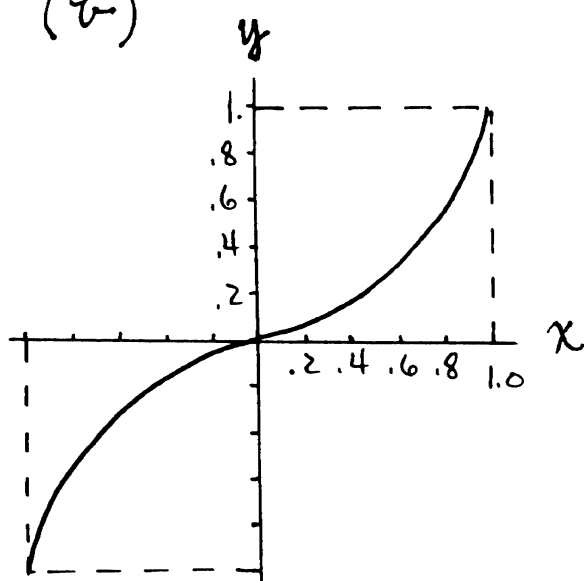
2.13

(a)



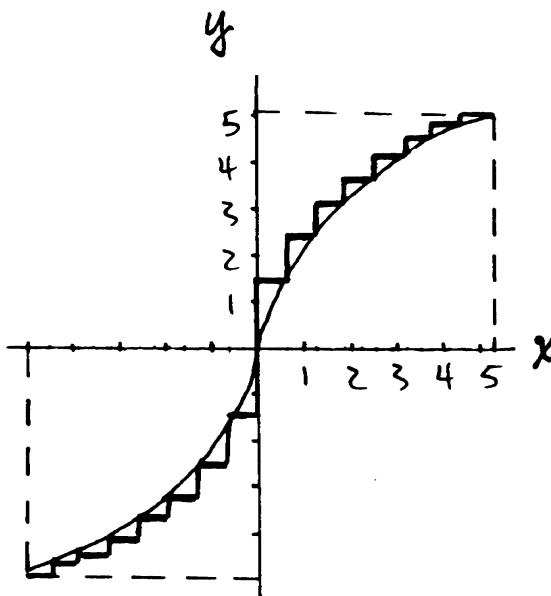
compression
characteristic $\mu=10$

(b)



expansion
characteristic $\mu=10$

(c)



16-level
nonuniform
quantizer
characteristic

$$\underline{2.14} \text{ (a)} \quad L \geq \frac{1}{2p} = \frac{1}{0.02} = 50 \text{ levels}$$

$$l = \lceil \log_2 50 \rceil = 6 \text{ bits/sample}$$

$$\text{(b)} \quad f_s = 2f_m = 2 \times 4000 = 8000 \text{ samples/s.}$$

$$\text{Bit rate: } R = 8000 \text{ samples/s} \times 6 \text{ bits/sample} \\ = 48,000 \text{ bits/s.}$$

$$\text{(c)} \quad 16\text{-level pulses: } 16 = M = 2^k$$

$$k = 4 \text{ bits/pulse}$$

$$\text{Symbol rate: } R_s = \frac{R}{\log_2 M} = \frac{48,000 \text{ bits/s}}{4 \text{ bits/symbol}} \\ = 12,000 \text{ symbols/s.}$$

2.15 Binary case:

$$R = 8000 \text{ samples/s} \times 6 \text{ bits/sample} = 48,000 \text{ bits/s}$$

$$W = \frac{1}{T_b} = R = 48,000 \text{ Hz.}$$

$$\left(\frac{S}{N}\right)_q = 3L^2 = 3(64)^2 = 12,288 \\ \approx 41 \text{ dB}$$

Four-level case:

$$R_s = \frac{48,000 \text{ bits/s}}{2 \text{ bits/symbol}} = 24,000 \text{ symbols/s.}$$

$$W = \frac{1}{T} = R_s = 24,000 \text{ Hz}$$

$$\left(\frac{S}{N}\right)_q = \text{the same as in the binary case} \approx 41 \text{ dB}$$

2.16 (a) Assume that the L quantization levels are equally spaced and symmetrical about zero. Then, the maximum possible quantization noise voltage equals $\frac{1}{2}$ the q volt interval between any two neighboring levels. Also, the peak quantization noise power, N_q , can be expressed as $(q/2)^2$.

The peak signal power, S , can be designated $(V_{pp}/2)^2$, where $V_{pp} = V_p - (-V_p)$ is the peak-to-peak signal voltage, and V_p is the peak voltage.

Since there are L quantization levels and $(L - 1)$ intervals (each interval corresponding to q volts), we can write:

$$\begin{aligned} \left(\frac{S}{N_q} \right)_{\text{peak}} &= \frac{(V_{pp}/2)^2}{(q/2)^2} = \frac{[q(L-1)/2]^2}{(q/2)^2} \\ &\approx \frac{q^2 L^2 / 4}{q^2 / 4} = L^2 \end{aligned}$$

Thus, we need to compute how many levels, L , will yield a $(S/N_q)_{\text{peak}} = 96$ dB. We therefore write:

$$\begin{aligned} 96 \text{ dB} &= 10 \log_{10} \left(\frac{S}{N_q} \right)_{\text{peak}} = 10 \log_{10} L^2 \\ &= 20 \log_{10} L \end{aligned}$$

$$L = 10^{96/20} = 63,096 \text{ levels}$$

(b) The number of bits that correspond to 63,096 levels is

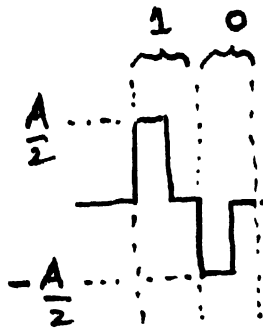
$$\ell = \lceil \log_2 L \rceil = \lceil \log_2 63,096 \rceil = 16 \text{ bits/sample}$$

(c) $R = 16 \text{ bits/sample} \times 44.1 \text{ ksamples/s} = 705,600 \text{ bits/s}$.

2.17 Let the peak-to-peak amplitude separation be A volts.

Bipolar RZ

See Figure 2.22

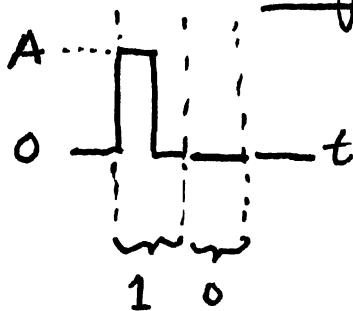


Average power =

$$\frac{1}{2} \left[\frac{1}{2} \left(\frac{A}{2} \right)^2 + \frac{1}{2} (0)^2 \right] + \frac{1}{2} \left[\frac{1}{2} \left(-\frac{A}{2} \right)^2 + \frac{1}{2} (0)^2 \right] = \frac{A^2}{8}$$

Unipolar RZ

See Figure 2.22



Average power =

$$\frac{1}{2} \left[\frac{1}{2} (A)^2 + \frac{1}{2} (0)^2 \right] + \frac{1}{2} \left[(0)^2 \right] = \frac{A^2}{4}$$

Bipolar signaling requires half the average power for the same separation between the binary one and zero. The disadvantage in using bipolar signaling is the need for 2 power supplies.

2.18 The data rate for T1 service is:

$$24 \text{ samples/frame} \times 8 \text{ bits/sample} \times 8000 \text{ frames/s} \\ + 1 \text{ alignment bit/frame} \\ = 193 \text{ bits/frame} \times 8000 \text{ frames/s} = 1,544 \times 10^6 \text{ bits/s}$$

Bandwidth efficiency is:

$$\frac{R}{W} = \frac{1.544 \times 10^6}{386 \times 10^3} = 4 \text{ bits/s/Hz}$$

2.19 (a) Using Equations (2.26) to (2.28)

$2^L = L \geq \frac{1}{2p}$ levels. Given that $p = 0.02$,

$$\text{then } L = \left\lceil \log_2 \frac{1}{0.04} \right\rceil = \left\lceil \log_2 25 \right\rceil$$

Thus, there must be at least 25 quantization levels, or 5 bits per sample, to meet the fidelity criterion. The data rate is:

$$8000 \text{ samples/s} \times 5 \text{ bits/sample} = 40,000 \text{ bits/s}$$

This data rate needs to be sent in a 4000 Hz bandwidth. Hence, the required bandwidth efficiency is:

$$\frac{R}{W} = \frac{40,000 \text{ bits/s}}{4000 \text{ Hz}} = 10 \text{ bits/s/Hz}$$

(b) When the analog signal has a 20 kHz bandwidth, the Nyquist sampling rate is 40 ksamples/s, and the bit rate is:

$$40,000 \text{ samples/s} \times 5 \text{ bits/sample} = 200,000 \text{ bits/s}$$

Hence the required bandwidth efficiency is:

$$\frac{R}{W} = \frac{200,000}{4,000} = 50 \text{ bits/s/Hz},$$

which is a challenging requirement!