

***Trigonometry 5th edition Dugopolski Solutions
Manual***

SKU: 9780135207338-SOLUTIONS

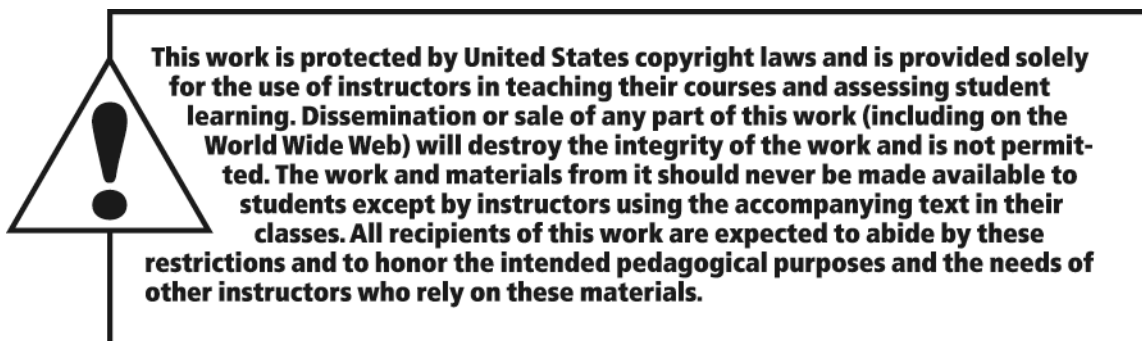
INSTRUCTOR'S SOLUTIONS MANUAL

EDGAR REYES
Southeastern Louisiana University

TRIGONOMETRY FIFTH EDITION

Mark Dugopolski
Southeastern Louisiana University





The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Reproduced by Pearson from electronic files supplied by the author.

Copyright © 2020, 2015, 2011 by Pearson Education, Inc. 221 River Street, Hoboken, NJ 07030. All rights reserved.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America.



ISBN-13: 978-0-13-520744-4

ISBN-10: 0-13-520744-4

Table of Contents

Chapter P	1
Chapter 1	32
Chapter 2	83
Chapter 3	137
Chapter 4	195
Chapter 5	245
Chapter 6	302

For Thought

- False, the point $(2, -3)$ is in Quadrant IV.
- False, the point $(4, 0)$ does not belong to any quadrant.
- False, since the distance is $\sqrt{(a-c)^2 + (b-d)^2}$.
- False, since $Ax + By = C$ is a linear equation.
- True
- False, since $\sqrt{7^2 + 9^2} = \sqrt{130} \approx 11.4$
- True
- True
- True
- False, since the radius is 3.

P.1 Exercises

- ordered
- Cartesian
- x -axis
- origin
- Pythagorean theorem
- circle
- linear equation
- y -intercept
- $(4, 1)$, Quadrant I
- $(-3, 2)$, Quadrant II
- $(1, 0)$, x -axis
- $(-1, -5)$, Quadrant III
- $(5, -1)$, Quadrant IV
- $(0, -3)$, y -axis
- $(-4, -2)$, Quadrant III
- $(-2, 0)$, x -axis
- $(-2, 4)$, Quadrant II
- $(1, 5)$, Quadrant I
- $c = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$
- Since $a^2 + a^2 = \sqrt{2}^2$, we get $2a^2 = 2$ or $a^2 = 1$. Then $a = 1$.
- Since $b^2 + 2^2 = 3^2$, we get $b^2 + 4 = 9$ or $b^2 = 5$. Then $b = \sqrt{5}$.
- Since $b^2 + \left(\frac{1}{2}\right)^2 = 1^2$, we get $b^2 + \frac{1}{4} = 1$ or $b^2 = \frac{3}{4}$. Thus, $b = \frac{\sqrt{3}}{2}$.
- Since $a^2 + 3^2 = 5^2$, we get $a^2 + 9 = 25$ or $a^2 = 16$. Then $a = 4$.
- $c = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$
- $\sqrt{4 \cdot 7} = 2\sqrt{7}$
- $\sqrt{25 \cdot 2} = 5\sqrt{2}$
- $\frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$
- $\frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$
- $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
- $\frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$
- $\frac{2\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$
- $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$
- $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- $\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
- $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
- $\frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$
- Distance is $\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$, midpoint is $(2.5, 5)$
- Distance is $\sqrt{144+25} = 13$, midpoint is $(3, 0.5)$
- Distance is $\sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$, midpoint is $(0, -1)$

40. Distance is $\sqrt{4+4} = 2\sqrt{2}$, midpoint is $(0, 1)$

41. Distance is $\sqrt{\left(\frac{\sqrt{2}}{2} - 0\right)^2 + \left(\frac{\sqrt{2}}{2} - 0\right)^2} =$
 $\sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$, midpoint is
 $\left(\frac{\sqrt{2}/2 + 0}{2}, \frac{\sqrt{2}/2 + 0}{2}\right) = \left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

42. Distance is $\sqrt{(\sqrt{3} - 0)^2 + (1 - 0)^2} =$
 $\sqrt{3+1} = 2$, midpoint is
 $\left(\frac{\sqrt{3} + 0}{2}, \frac{1 + 0}{2}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

43. Distance is $\sqrt{(\sqrt{18} - \sqrt{8})^2 + (\sqrt{12} - \sqrt{27})^2} =$
 $\sqrt{(3\sqrt{2} - 2\sqrt{2})^2 + (2\sqrt{3} - 3\sqrt{3})^2} =$
 $\sqrt{(\sqrt{2})^2 + (-\sqrt{3})^2} = \sqrt{5}$,
midpoint is $\left(\frac{\sqrt{18} + \sqrt{8}}{2}, \frac{\sqrt{12} + \sqrt{27}}{2}\right) =$
 $\left(\frac{3\sqrt{2} + 2\sqrt{2}}{2}, \frac{2\sqrt{3} + 3\sqrt{3}}{2}\right) = \left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{3}}{2}\right)$

44. Distance is $\sqrt{(\sqrt{72} - \sqrt{50})^2 + (\sqrt{45} - \sqrt{20})^2} =$
 $\sqrt{(6\sqrt{2} - 5\sqrt{2})^2 + (3\sqrt{5} - 2\sqrt{5})^2} =$
 $\sqrt{(\sqrt{2})^2 + (\sqrt{5})^2} = \sqrt{7}$,
midpoint is $\left(\frac{\sqrt{72} + \sqrt{50}}{2}, \frac{\sqrt{45} + \sqrt{20}}{2}\right) =$
 $\left(\frac{6\sqrt{2} + 5\sqrt{2}}{2}, \frac{3\sqrt{5} + 2\sqrt{5}}{2}\right) = \left(\frac{11\sqrt{2}}{2}, \frac{5\sqrt{5}}{2}\right)$

45. Distance is $\sqrt{(1.2 + 3.8)^2 + (4.4 + 2.2)^2} =$
 $\sqrt{25 + 49} = \sqrt{74}$, midpoint is $(-1.3, 1.3)$

46. Distance is $\sqrt{49 + 81} = \sqrt{130}$,
midpoint is $(1.2, -3)$

47. Distance is $\frac{\sqrt{\pi^2 + 4}}{2}$, midpoint is $\left(\frac{3\pi}{4}, \frac{1}{2}\right)$

48. Distance is $\frac{\sqrt{\pi^2 + 4}}{2}$, midpoint is $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

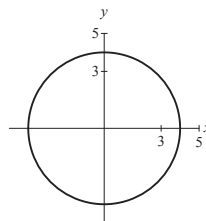
49. Distance is $\sqrt{(2\pi - \pi)^2 + (0 - 0)^2} = \sqrt{\pi^2} = \pi$,
midpoint is $\left(\frac{2\pi + \pi}{2}, \frac{0 + 0}{2}\right) = \left(\frac{3\pi}{2}, 0\right)$

50. Distance is $\sqrt{\left(\pi - \frac{\pi}{2}\right)^2 + (1 - 1)^2} = \sqrt{\frac{\pi^2}{4}} =$
 $\frac{\pi}{2}$, midpoint is $\left(\frac{\pi + \pi/2}{2}, \frac{1 + 1}{2}\right) = \left(\frac{3\pi}{4}, 1\right)$

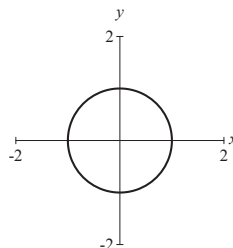
51. Distance is $\sqrt{\left(\frac{\pi}{2} - \frac{\pi}{3}\right)^2 + \left(-\frac{1}{3} - \frac{1}{2}\right)^2} =$
 $\sqrt{\frac{\pi^2}{36} + \frac{25}{36}} = \frac{\sqrt{\pi^2 + 25}}{6}$,
midpoint is $\left(\frac{\frac{\pi}{2} + \frac{\pi}{3}}{2}, \frac{\frac{1}{2} - \frac{1}{3}}{2}\right) = \left(\frac{5\pi}{12}, \frac{1}{12}\right)$

52. Distance is $\sqrt{\left(\pi - \frac{2\pi}{3}\right)^2 + \left(-1 + \frac{1}{2}\right)^2} =$
 $\sqrt{\frac{\pi^2}{9} + \frac{1}{4}} = \frac{\sqrt{4\pi^2 + 9}}{6}$,
midpoint is $\left(\frac{\frac{2\pi}{3} + \pi}{2}, \frac{-\frac{1}{2} - 1}{2}\right) = \left(\frac{5\pi}{6}, -\frac{3}{4}\right)$

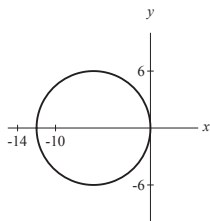
53. Center $(0, 0)$, radius 4



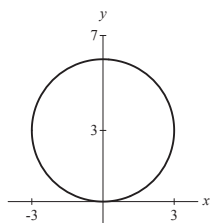
54. Center $(0, 0)$, radius 1



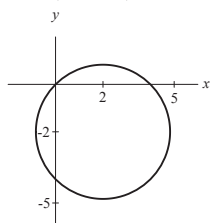
55. Center
- $(-6, 0)$
- , radius 6



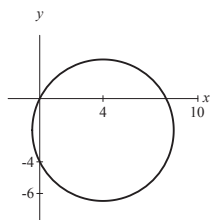
56. Center
- $(0, 3)$
- , radius 3



57. Center
- $(2, -2)$
- , radius
- $2\sqrt{2}$



58. Center
- $(4, -2)$
- , radius
- $2\sqrt{5}$



59. $x^2 + y^2 = 7$

60. $x^2 + y^2 = 12$ since $(2\sqrt{3})^2 = 12$

61. $(x + 2)^2 + (y - 5)^2 = 1/4$

62. $(x + 1)^2 + (y + 6)^2 = 1/9$

63. The distance between
- $(3, 5)$
- and the origin is
- $\sqrt{34}$
- which is the radius. The standard equation is
- $(x - 3)^2 + (y - 5)^2 = 34$
- .

64. The distance between
- $(-3, 9)$
- and the origin is
- $\sqrt{90}$
- which is the radius. The standard equation is
- $(x + 3)^2 + (y - 9)^2 = 90$
- .

65. Note, the distance between
- $(\sqrt{2}/2, \sqrt{2}/2)$
- and the origin is 1. Thus, the radius is 1. The standard equation is
- $x^2 + y^2 = 1$
- .

66. Note, the distance between
- $(\sqrt{3}/2, 1/2)$
- and the origin is 1. Thus, the radius is 1. The standard equation is
- $x^2 + y^2 = 1$
- .

67. The radius is
- $\sqrt{(-1 - 0)^2 + (2 - 0)^2} = \sqrt{5}$
- . The standard equation is
- $(x + 1)^2 + (y - 2)^2 = 5$
- .

68. Since the center is
- $(0, 0)$
- and the radius is 2, the standard equation is
- $x^2 + y^2 = 4$
- .

69. Note, the center is
- $(1, 3)$
- and the radius is 2. The standard equation is
- $(x - 1)^2 + (y - 3)^2 = 4$
- .

70. The radius is
- $\sqrt{(2 - 0)^2 + (2 - 0)^2} = \sqrt{8}$
- . The standard equation is
- $(x - 2)^2 + (y - 2)^2 = 8$
- .

71. We solve for
- a
- .

$$\begin{aligned} a^2 + \left(\frac{3}{5}\right)^2 &= 1 \\ a^2 &= 1 - \frac{9}{25} \\ a^2 &= \frac{16}{25} \\ a &= \pm \frac{4}{5} \end{aligned}$$

72. We solve for
- a
- .

$$\begin{aligned} a^2 + \left(-\frac{1}{2}\right)^2 &= 1 \\ a^2 &= 1 - \frac{1}{4} \\ a^2 &= \frac{3}{4} \\ a &= \pm \frac{\sqrt{3}}{2} \end{aligned}$$

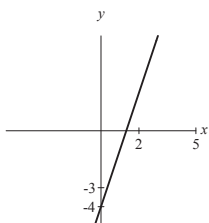
73. We solve for
- a
- .

$$\begin{aligned} \left(-\frac{2}{5}\right)^2 + a^2 &= 1 \\ a^2 &= 1 - \frac{4}{25} \\ a^2 &= \frac{21}{25} \\ a &= \pm \frac{\sqrt{21}}{5} \end{aligned}$$

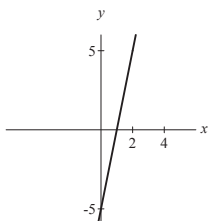
74. Solve for a :

$$\begin{aligned}\left(\frac{2}{3}\right)^2 + a^2 &= 1 \\ a^2 &= 1 - \frac{4}{9} \\ a^2 &= \frac{5}{9} \\ a &= \pm \frac{\sqrt{5}}{3}\end{aligned}$$

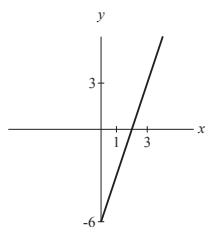
75. $y = 3x - 4$ goes through $(0, -4)$, $\left(\frac{4}{3}, 0\right)$.



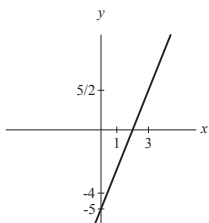
76. $y = 5x - 5$ goes through $(0, -5)$, $(1, 0)$.



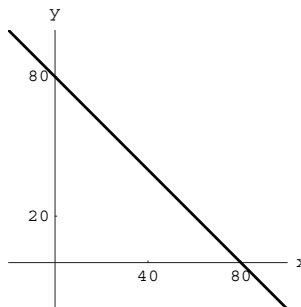
77. $3x - y = 6$ goes through $(0, -6)$, $(2, 0)$.



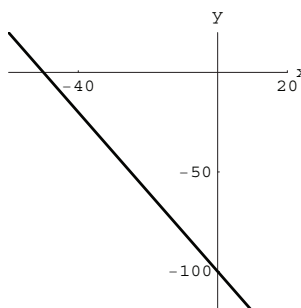
78. $5x - 2y = 10$ goes through $(0, -5)$, $(2, 0)$.



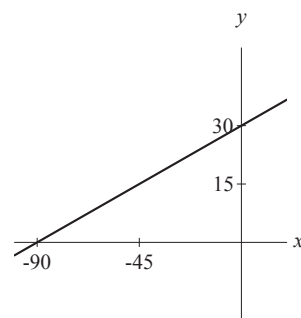
79. $x + y = 80$ goes through $(0, 80)$, $(80, 0)$.



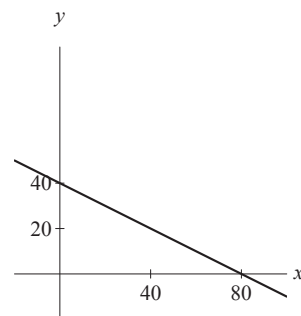
80. $2x + y = -100$ goes through $(0, -100)$, $(-50, 0)$.



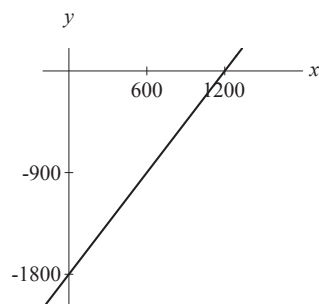
81. $x = 3y - 90$ goes through $(0, 30)$, $(-90, 0)$.



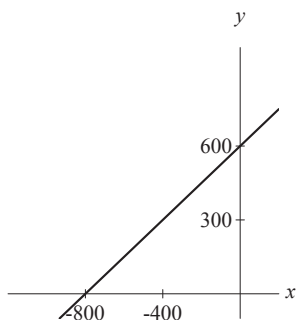
82. $x = 80 - 2y$ goes through $(0, 40)$, $(80, 0)$.



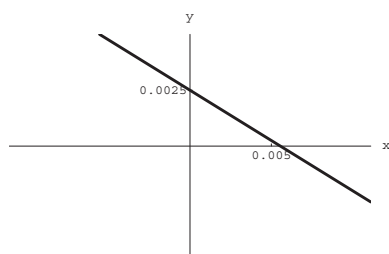
83. $\frac{1}{2}x - \frac{1}{3}y = 600$ goes through $(0, -1800)$, $(1200, 0)$.



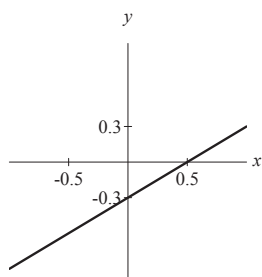
84. $\frac{2}{3}y - \frac{1}{2}x = 400$ goes through $(0, 600)$, $(-800, 0)$.



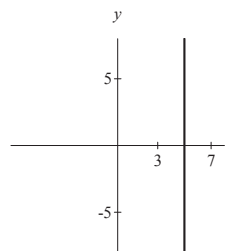
85. Intercepts are $(0, 0.0025)$, $(0.005, 0)$.



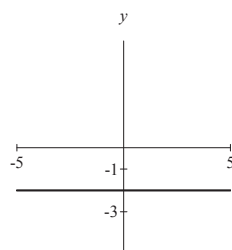
86. Intercepts are $(0, -0.3)$, $(0.5, 0)$.



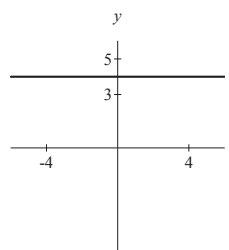
87. $x = 5$



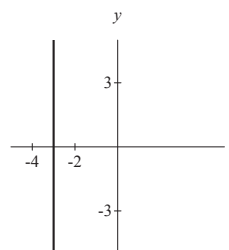
88. $y = -2$



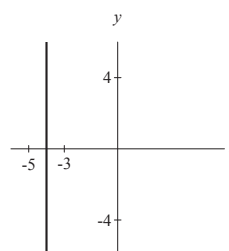
89. $y = 4$



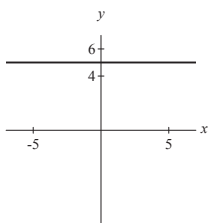
90. $x = -3$



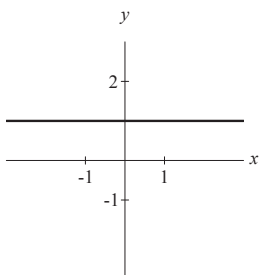
91. $x = -4$



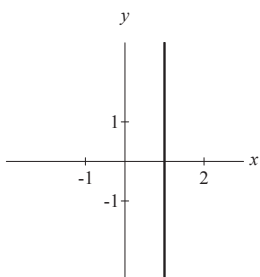
92. $y = 5$



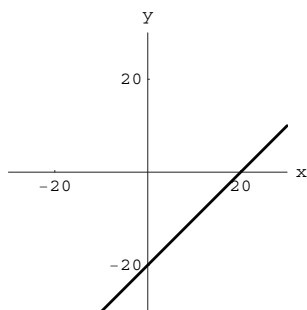
93. Solving for y , we have $y = 1$.



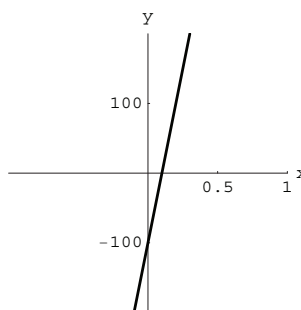
94. Solving for x , we get $x = 1$.



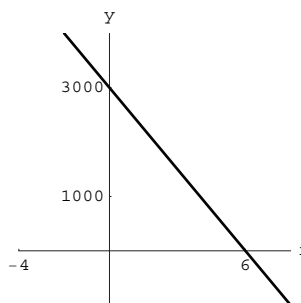
95. $y = x - 20$ goes through $(0, -20)$, $(20, 0)$.



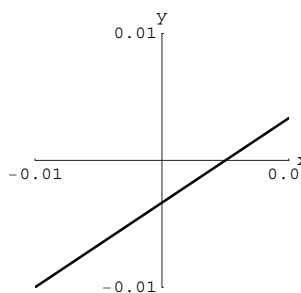
96. $y = 999x - 100$ goes through $(0, -100)$, $(100/999, 0)$.



97. $y = 3000 - 500x$ goes through $(0, 3000)$, $(6, 0)$.



98. $y = \frac{1}{300}(200x - 1)$ goes through $(0, -1/300)$, $(1/200, 0)$.



99. The hypotenuse is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

100. The other leg is $\sqrt{10^2 - 4^2} = \sqrt{84} = 2\sqrt{21}$ ft.

101. a) Let r be the radius of the smaller circle. Consider the right triangle with vertices at the origin, another vertex at the center of a smaller circle, and a third vertex at the center of the circle of radius 1. By the Pythagorean Theorem, we obtain

$$\begin{aligned} 1 + (2 - r)^2 &= (1 + r)^2 \\ 5 - 4r &= 1 + 2r \end{aligned}$$

$$\begin{aligned} 4 &= 6r \\ r &= \frac{2}{3}. \end{aligned}$$

The diameter of the smaller circle is $2r = \frac{4}{3}$.

- b) The smallest circles are centered at $(\pm r, 0)$ or $(\pm 4/3, 0)$. The equations of the circles are

$$\left(x - \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

and

$$\left(x + \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

- 102.** Draw a right triangle with vertices at the centers of the circles, and another vertex at a point of intersection of the two circles. The legs of the right triangle are 5 and 12. By the Pythagorean theorem, the hypotenuse is $\sqrt{5^2 + 12^2} = 13$.

- 103.** Let $C(h, k)$ and r be the center and radius of the smallest circle, respectively. Then $k = -r$. We consider two right triangles each of which has a vertex at C .

The right triangles have sides that are perpendicular to the coordinate axes. Also, one side of each right triangle passes through the center of a larger circle.

Applying the Pythagorean Theorem, we list a system of equations

$$(r + 1)^2 = h^2 + (1 - r)^2$$

$$(2 - r)^2 = h^2 + r^2.$$

The solutions are $r = 1/2$, $h = \sqrt{2}$, and $k = -r = -1/2$.

The equation of the smallest circle is

$$(x - \sqrt{2})^2 + (y + 1/2)^2 = 1/4.$$

- 104.** We apply symmetry to the centers of the remaining three circles. From the answer or equation in Exercise 103, the equations of the remaining circles are

$$(x - \sqrt{2})^2 + (y - 1/2)^2 = 1/4$$

$$(x + \sqrt{2})^2 + (y - 1/2)^2 = 1/4$$

$$(x + \sqrt{2})^2 + (y + 1/2)^2 = 1/4.$$

- 105.** The midpoint of $(0, 20.8)$ and $(48, 27.4)$ is

$$\left(\frac{0 + 48}{2}, \frac{20.8 + 27.4}{2}\right) = (24, 24.1).$$

In 1994 ($= 1970 + 24$), the median age at first marriage was 24.1 years.

- 106.** a) If $h = 0$, then $0 = 0.229n + 5.203$.

Then $n = -5.203/0.229 \approx -22.72$.

The n -intercept is

$$(-22.72, 0).$$

There were no unmarried couples in 1977 ($\approx 2000 - 22.7$). Nonsense.

- b) If $n = 0$, then $h = 0.229(0) + 5.203 = 5.203$. The h -intercept is $(0, 5.203)$. In 2000, there were 5,203,000 unmarried-couple households.

- 107.** The distance between $(10, 0)$ and $(0, 0)$ is 10.

The distance between $(1, 3)$ and the origin is $\sqrt{10}$. If two points have integer coordinates, then the distance between them is of the form $\sqrt{s^2 + t^2}$ where $s^2, t^2 \in \{0, 1, 2^2, 3^2, 4^2, \dots\} = \{0, 1, 4, 9, 16, \dots\}$.

Note, there are no numbers s^2 and t^2 in $\{0, 1, 4, 9, 16, \dots\}$ satisfying $s^2 + t^2 = 19$.

Thus, one cannot find two points with integer coordinates whose distance between them is $\sqrt{19}$.

- 108.** One can assume the vertices of the right triangle are $A(0, 0)$, $B(1, \sqrt{3})$, and $C(1, 0)$.

The midpoint of the hypotenuse AB is

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right).$$

The distance between the midpoint and C is $\sqrt{\left(\frac{1}{2} - 1\right)^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2} = 1$, which is also the distance from the midpoint to A , and the distance from the midpoint to B .

- 111.** On day 1, break off a 1-dollar piece and pay the gardener.

On day 2, break off a 2-dollar and pay the gardener. The gardener will give you back your change which is a 1-dollar piece.

On day 3, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 4, pay the gardener with the 4-dollar bar. The gardener will give you back your change which will consist of a 1-dollar piece and a 2-dollar piece.

On day 5, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 6, pay the gardener with the 2-dollar piece you received as change from day 4. The gardener will give you back your change which is a 1-dollar piece.

On day 7, pay the gardener with the 1-dollar piece you received as change from day 6.

- 112.** Let $\triangle ABC$ be a right triangle with vertices at $A(2, 7)$, $B(0, -3)$, and $C(6, 1)$. Notice, the midpoints of the sides of $\triangle ABC$ are $(3, -1)$, $(4, 4)$ and $(1, 2)$. The area of $\triangle ABC$ is

$$\begin{aligned}\frac{1}{2}\overline{AC} \times \overline{BC} &= \frac{1}{2}\sqrt{4^2 + 6^2}\sqrt{6^2 + 4^2} \\ &= \frac{1}{2}(52) = 26.\end{aligned}$$

P.1 Pop Quiz

1. The distance is $\sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$.

2. Center $(3, -5)$, radius 9

3. Completing the square, we find

$$\begin{aligned}(x^2 + 4x + 4) + (y^2 - 10y + 25) &= -28 + 4 + 25 \\ (x + 2)^2 + (y - 5)^2 &= 1.\end{aligned}$$

The center is $(-2, 5)$ and the radius is 1.

4. The distance between $(3, 4)$ and the origin is 5, which is the radius. The circle is given by $(x - 3)^2 + (y - 4)^2 = 25$.

5. By setting $x = 0$ and $y = 0$ in $2x - 3y = 12$ we find $-3y = 12$ and $2x = 12$, respectively. Since $y = -4$ and $x = 6$ are the solutions of the two equations, the intercepts are $(0, -4)$ and $(6, 0)$.

6. $(5, -1)$

For Thought

- True, since the number of gallons purchased is 20 divided by the price per gallon.
- False, since a student's exam grade is a function of the student's preparation. If two classmates had the same IQ and only one prepared then the one who prepared will most likely achieve a higher grade.
- False, since $\{(1, 2), (1, 3)\}$ is not a function.
- True
- True
- True
- False, the domain is the set of all real numbers.
- True
- True, since $f(0) = \frac{0 - 2}{0 + 2} = -1$.
- True, since if $a - 5 = 0$ then $a = 5$.

P.2 Exercises

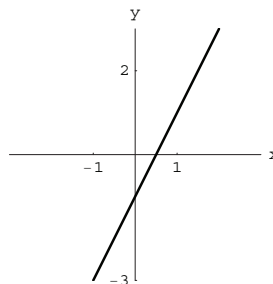
- function
- independent, dependent
- domain, range
- parabola
- function
- function
- Note, $b = 2\pi a$ is equivalent to $a = \frac{b}{2\pi}$. Then a is a function of b , and b is a function of a .
- Note, $b = 2(5 + a)$ is equivalent to $a = \frac{b - 10}{2}$. Then a is a function of b , and b is a function of a .
- a is a function of b since a given denomination has a unique length. Since a dollar bill and a five-dollar bill have the same length, then b is not a function of a .

10. Since different U.S. coins have different diameters, then a is a function of b , and b is a function of a .
11. Since an item has only one price, b is a function of a . Since two items may have the same price, a is not a function of b .
12. a is not a function of b since there may be two students with the same semester grades but different final exams scores. b is not a function of a since there may be identical final exam scores with different semester grades.
13. a is not a function of b since it is possible that two different students can obtain the same final exam score but the times spent on studying are different.
 b is not a function of a since it is possible that two different students can spend the same time studying but obtain different final exam scores.
14. a is not a function of b since it is possible that two adult males can have the same shoe size but have different ages.
 b is not a function of a since it is possible for two adults with the same age to have different shoe sizes.
15. Since 1 in ≈ 2.54 cm, a is a function of b and b is a function of a .
16. Since there is only one cost for mailing a first class letter, then a is a function of b . Since two letters with different weights each under 1/2-ounce cost 47 cents to mail first class, b is not a function of a .
17. Since $b = a^3$ and $a = \sqrt[3]{b}$, we get that b is a function of a , and a is a function of b .
18. Since $b = a^4$ and $a = \pm \sqrt[4]{b}$, we get that b is a function of a , but a is not a function of b .
19. Since $b = |a|$, we get b is a function of a . Since $a = \pm b$, we find a is not a function of b .
20. Note, $b = \sqrt{a}$ since $a \geq 0$, and $a = b^2$. Thus, b is a function of a , and a is a function of b .
21. $A = s^2$ 22. $s = \sqrt{A}$ 23. $s = \frac{\sqrt{2}d}{2}$

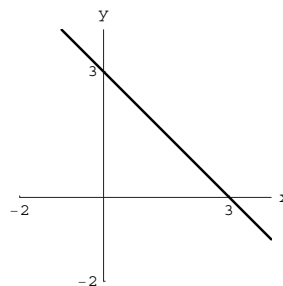
24. $d = s\sqrt{2}$ 25. $P = 4s$ 26. $s = P/4$

27. $A = P^2/16$ 28. $d = \sqrt{2A}$

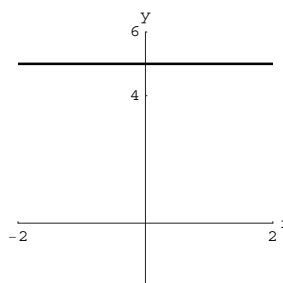
- 29.
- $y = 2x - 1$
- has domain
- $(-\infty, \infty)$
- and range
- $(-\infty, \infty)$
- , some points are
- $(0, -1)$
- and
- $(1, 1)$



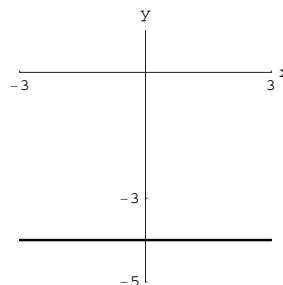
- 30.
- $y = -x + 3$
- has domain
- $(-\infty, \infty)$
- and range
- $(-\infty, \infty)$
- , some points are
- $(0, 3)$
- and
- $(3, 0)$



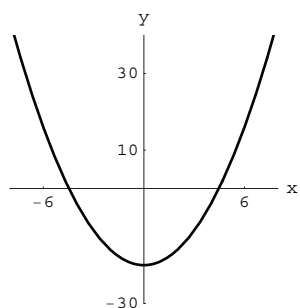
- 31.
- $y = 5$
- has domain
- $(-\infty, \infty)$
- and range
- $\{5\}$
- , some points are
- $(0, 5)$
- and
- $(1, 5)$



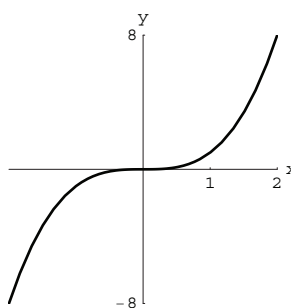
- 32.
- $y = -4$
- has domain
- $(-\infty, \infty)$
- and range
- $\{-4\}$
- , some points are
- $(0, -4)$
- and
- $(1, -4)$



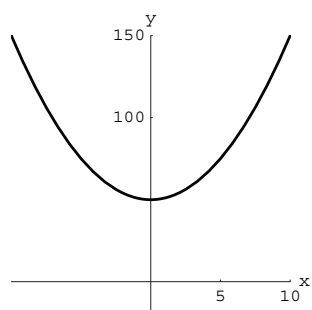
33. $y = x^2 - 20$ has domain $(-\infty, \infty)$ and range $[-20, \infty)$, some points are $(0, -20)$ and $(6, 16)$



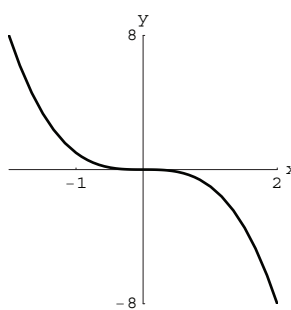
37. $y = x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, 0)$ and $(2, 8)$



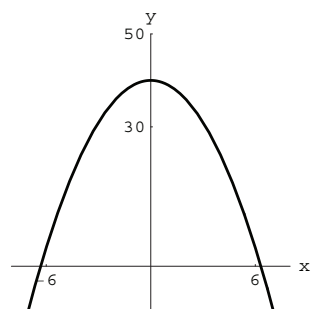
34. $y = x^2 + 50$ has domain $(-\infty, \infty)$ and range $[50, \infty)$, some points are $(0, 50)$ and $(5, 75)$



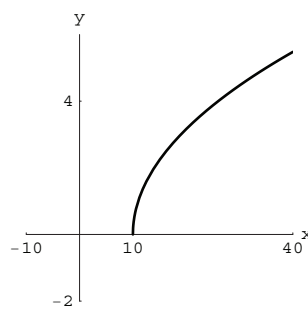
38. $y = -x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are $(0, 0)$ and $(0, -8)$



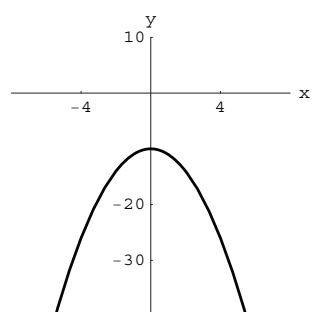
35. $y = 40 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 40]$, some points are $(0, 40)$ and $(6, 4)$



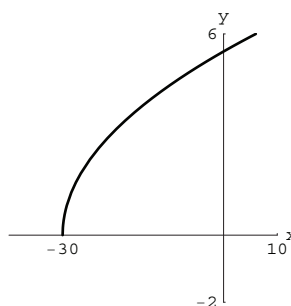
39. $y = \sqrt{x - 10}$ has domain $[10, \infty)$ and range $[0, \infty)$, some points are $(10, 0)$ and $(14, 2)$



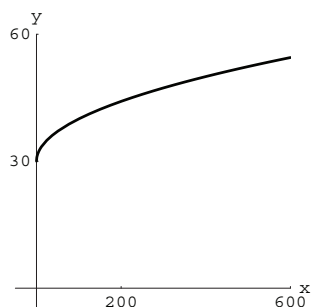
36. $y = -10 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, -10]$, some points are $(0, -10)$ and $(4, -26)$



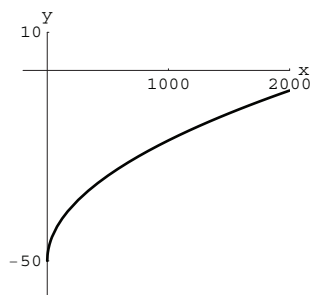
40. $y = \sqrt{x + 30}$ has domain $[-30, \infty)$ and range $[0, \infty)$, some points are $(-30, 0)$ and $(-26, 2)$



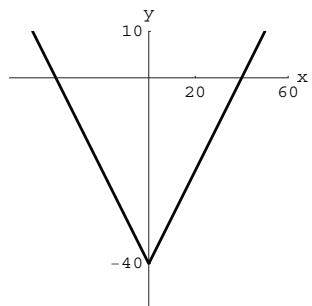
41. $y = \sqrt{x} + 30$ has domain $[0, \infty)$ and range $[30, \infty)$, some points are $(0, 30)$ and $(400, 50)$



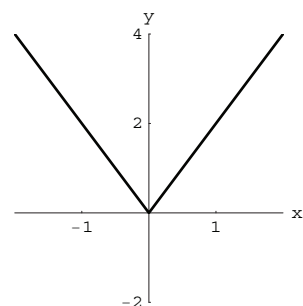
42. $y = \sqrt{x} - 50$ has domain $[0, \infty)$ and range $[-50, \infty)$, some points are $(0, -50)$ and $(900, -20)$



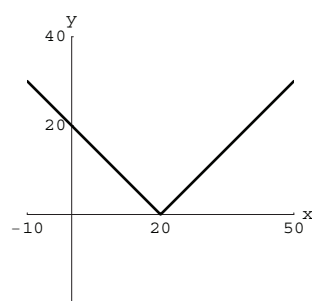
43. $y = |x| - 40$ has domain $(-\infty, \infty)$ and range $[-40, \infty)$, some points are $(0, -40)$ and $(40, 0)$



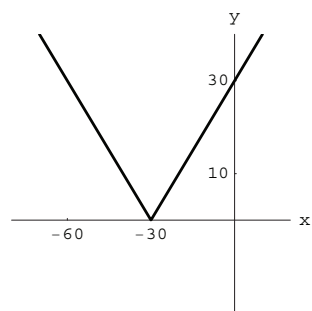
44. $y = 2|x|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 0)$ and $(1, 2)$



45. $y = |x - 20|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 20)$ and $(20, 0)$



46. $y = |x + 30|$ has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are $(0, 30)$ and $(-30, 0)$



47. $3 \cdot 4 - 2 = 10$ 48. $3(16) + 4 = 52$

49. $-4 - 2 = -6$ 50. $-8 - 2 = -10$

51. $|8| = 8$ 52. $|-1| = 1$

53. $4 + (-6) = -2$ 54. $24 \cdot 6 = 144$

55. $80 - 2 = 78$ 56. $2/2 = 1$

57. $3a^2 - a$ 58. $4b - 2$

59. $f(-x) = 3(-x)^2 - (-x) = 3x^2 + x$

60. $g(-x) = 4(-x) - 2 = -4x - 2$

61. Factoring, we get $x(3x - 1) + 0$. So $x = 0, 1/3$.

62. Since $4x - 2 = 3$, we get $x = 5/4$.

63. Since $|a + 3| = 4$ is equivalent to $a + 3 = 4$ or $a + 3 = -4$, we have $a = 1, -7$.

64. Since $3t^2 - t - 10 = (t - 2)(3t + 5) = 0$, we find $t = 2, -5/3$.

65. $C = 353n$

66. $P = 580n$

67. $C = 35n + 50$

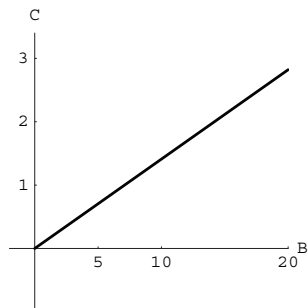
68. $C = 2.50 + 0.50n$

69. We find

$$C = \frac{4B}{\sqrt[3]{D}} = \frac{4(12 + 11/12)}{\sqrt[3]{22,800}} \approx 1.822$$

and a sketch of the graph of $C = \frac{4B}{\sqrt[3]{22,800}}$

is given below.



70. Solving for B , we get

$$\begin{aligned} \frac{4B}{\sqrt[3]{22,800}} &< 2 \\ B &< \frac{\sqrt[3]{22,800}}{2} \\ B &< 14 \text{ ft, } 2 \text{ in.} \end{aligned}$$

Then the maximum displacement is 14 ft, 2 in.

With D fixed, we get that C becomes larger as the beam B becomes larger. Thus, a boat is more likely to capsize as the beam gets larger.

71. Let $N = 2$, $B = 3.498$, and $S = 4.250$. Then

$$\begin{aligned} D &= \frac{\pi}{4} B^2 \cdot S \cdot N \\ &= 81.686 \text{ in.}^3 \end{aligned}$$

Then $D \approx 81.7 \text{ in.}^3$.

72. Let $N = 2$, $B = 3.518$, and $S = 4.250$. Then

$$\begin{aligned} D &= \frac{\pi}{4} B^2 \cdot S \cdot N \\ &= 82.622 \text{ in.}^3 \end{aligned}$$

Using the unrounded answer to Exercise 71, the difference in the displacement is

$$82.622 - 81.686 \approx 0.94 \text{ in.}^3.$$

73. Solving for B ,

$$D = \frac{\pi}{4} B^2 \cdot S \cdot N$$

$$\frac{4D}{\pi S \cdot N} = B^2$$

$$B = 2\sqrt{\frac{D}{\pi S \cdot N}}.$$

74. Solving for V ,

$$CR = 1 + \frac{\pi B^2 \cdot S}{4V}$$

$$CR - 1 = \frac{\pi B^2 \cdot S}{4V}$$

$$\frac{1}{CR - 1} = \frac{4V}{\pi B^2 \cdot S}$$

$$V = \frac{\pi B^2 \cdot S}{4(CR - 1)}.$$

75. Pythagorean, legs, hypotenuse

76. circle, radius, center

$$77. \sqrt{(2+3)^2 + (-4+6)^2} = \sqrt{29}$$

$$78. \left(\frac{4-6}{2}, \frac{-8+16}{2} \right) = (-1, 4)$$

79. If we replace $x = 0$ in $4x - 6y = 40$, then $-6y = 40$ or $y = -20/3$. The y -intercept is $(0, -\frac{20}{3})$.

If we replace $y = 0$ in $4x - 6y = 40$, then $4x = 40$ or $x = 10$. The x -intercept is $(10, 0)$.

$$80. \text{The diagonal is } \sqrt{3^2 + 7^2} = \sqrt{58} \text{ ft.}$$

81. Rewriting the equation, we find

$$\frac{1}{3^3} \cdot 3^{100} \cdot \frac{1}{3^4} \cdot 3^{2x} = \frac{1}{3} \cdot 3^x$$

$$\frac{1}{3^3} \cdot 3^{100} \cdot \frac{1}{3^4} \cdot 3^{2x} = 3^{x-1}$$

$$3^{2x+93} = 3^{x-1}$$

$$2x + 93 = x - 1$$

$$x = -94.$$

82. First, $9 = (a+b)^2 = (a^2 + b^2) + 2ab = 89 + 2ab$.
Then $ab = -40$. Thus,

$$\begin{aligned} a^3 + b^3 &= (a+b)(a^2 + b^2 - ab) \\ &= 3(89 + 40) \\ &= 387. \end{aligned}$$

P.2 Pop Quiz

1. Yes, since $r \geq 0$ and $r = \sqrt{A/\pi}$
2. Since $A = s^2$ and $s \geq 0$, we obtain $s = \sqrt{A}$.
3. No, since $b = \pm a$.
4. $[1, \infty)$
5. $[2, \infty)$
6. $f(3) = 3(3) + 6 = 15$
7. We find

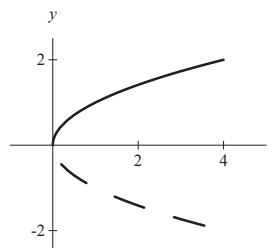
$$\begin{aligned} 2a - 4 &= 10 \\ 2a &= 14 \\ a &= 7. \end{aligned}$$

For Thought

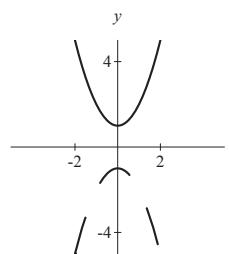
1. False, it is a reflection in the y-axis.
2. False, the graph of $y = x^2 - 4$ is shifted down 4 units from the graph of $y = x^2$.
3. False, rather it is a left translation.
4. True
5. True
6. False, the down shift should come after the reflection.
7. True
8. False, since the domains are different.
9. True
10. True, since $f(-x) = -f(x)$ where $f(x) = x^3$.

P.3 Exercises

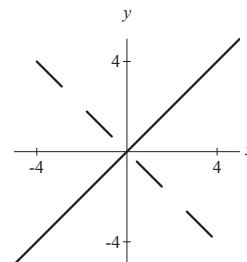
1. rigid
2. nonrigid
3. reflection
4. upward translation, downward translation
5. right, left
6. stretching, shrinking
7. odd
8. even
9. transformation
10. family
11. $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x}$



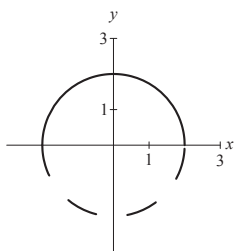
12. $f(x) = x^2 + 1$, $g(x) = -x^2 - 1$



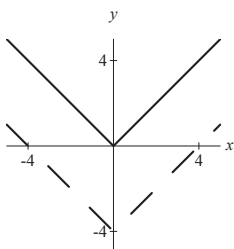
13. $y = x$, $y = -x$



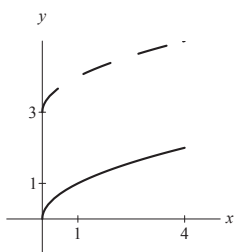
14. $y = \sqrt{4 - x^2}$, $y = -\sqrt{4 - x^2}$



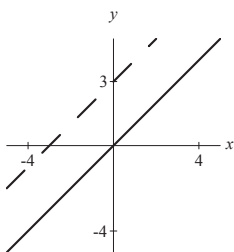
15. $f(x) = |x|$, $g(x) = |x| - 4$



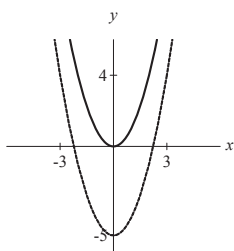
16. $f(x) = \sqrt{x}$, $g(x) = \sqrt{x} + 3$



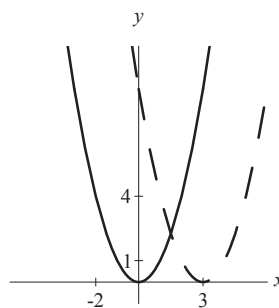
17. $f(x) = x$, $g(x) = x + 3$



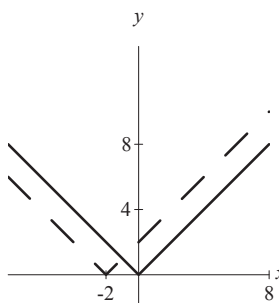
18. $f(x) = x^2$, $g(x) = x^2 - 5$



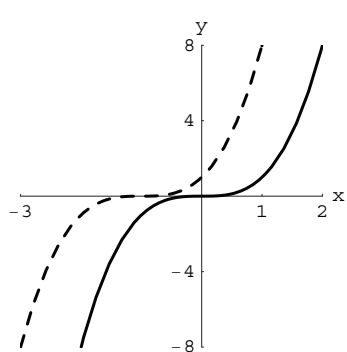
19. $y = x^2$, $y = (x - 3)^2$



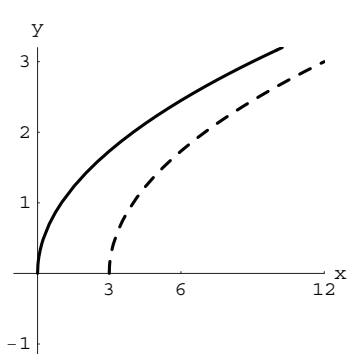
20. $y = |x|$, $y = |x + 2|$



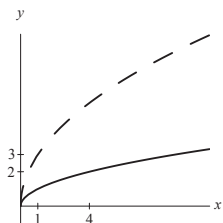
21. $f(x) = x^3$, $g(x) = (x + 1)^3$



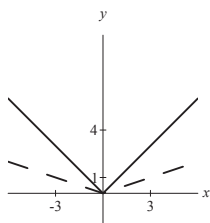
22. $f(x) = \sqrt{x}$, $g(x) = \sqrt{x - 3}$



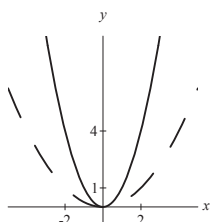
23. $y = \sqrt{x}, y = 3\sqrt{x}$



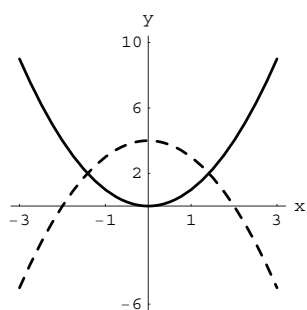
24. $y = |x|, y = \frac{1}{3}|x|$



25. $y = x^2, y = \frac{1}{4}x^2$



26. $y = x^2, y = 4 - x^2$



27. g 28. h 29. b 30. d

31. c 32. a 33. f 34. e

35. $y = (x - 10)^2 + 4$

36. $y = \sqrt{x + 5} - 12$

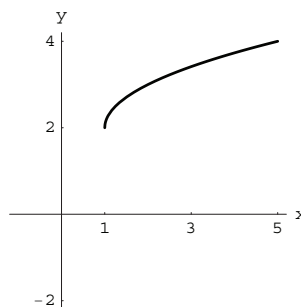
37. $y = -3|x - 7| + 9$

38. $y = -2(x + 6) - 8$ or $y = -2x - 20$

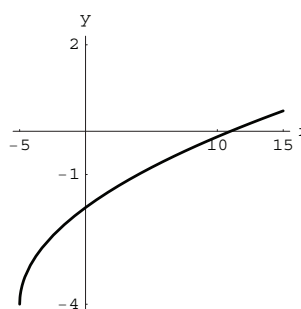
39. $y = -(3\sqrt{x} + 5)$ or $y = -3\sqrt{x} - 5$

40. $y = -((x - 13)^2 - 6)$ or $y = -(x - 13)^2 + 6$

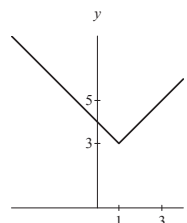
41. $y = \sqrt{x - 1} + 2$; right by 1, up by 2



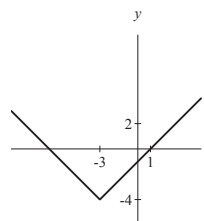
42. $y = \sqrt{x + 5} - 4$; left by 5, down by 4



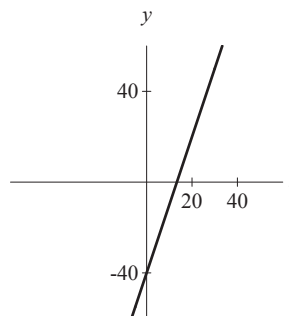
43. $y = |x - 1| + 3$; right by 1, up by 3



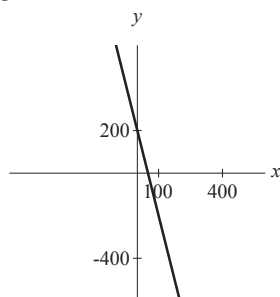
44. $y = |x + 3| - 4$; left by 3, down by 4



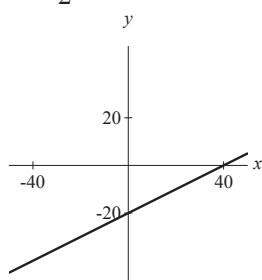
45. $y = 3x - 40$



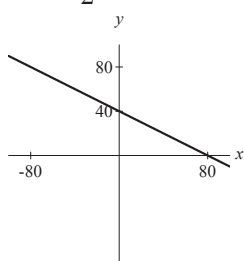
46. $y = -4x + 200$



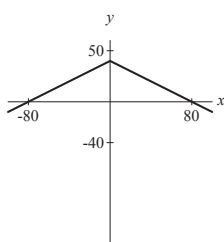
47. $y = \frac{1}{2}x - 20$



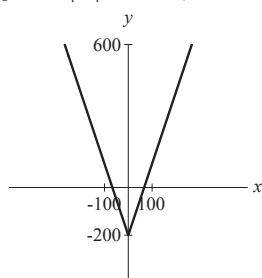
48. $y = -\frac{1}{2}x + 40$



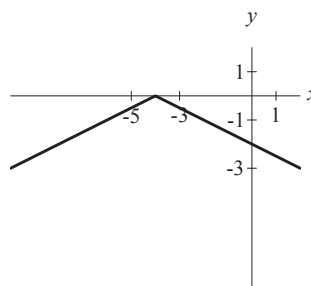
49. $y = -\frac{1}{2}|x| + 40$, shrink by 1/2,
reflect about x -axis, up by 40



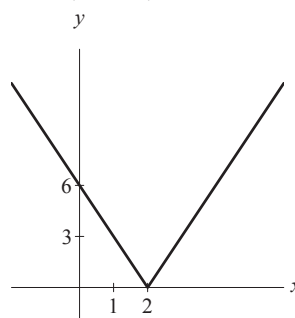
50. $y = 3|x| - 200$, stretch by 3, down by 200



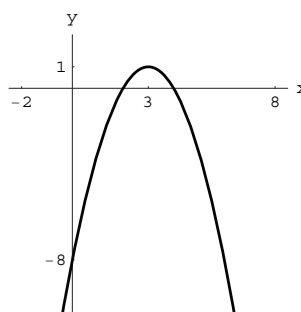
51. $y = -\frac{1}{2}|x + 4|$, left by 4,
reflect about x -axis, shrink by 1/2



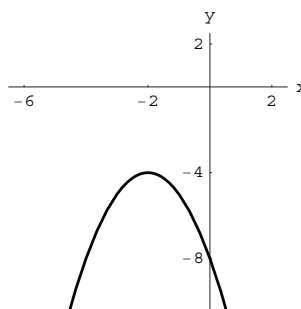
52. $y = 3|x - 2|$, right by 2, stretch by 3



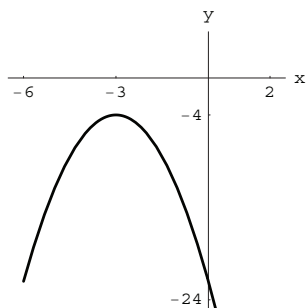
53. $y = -(x - 3)^2 + 1$; right by 3,
reflect about x -axis, up by 1



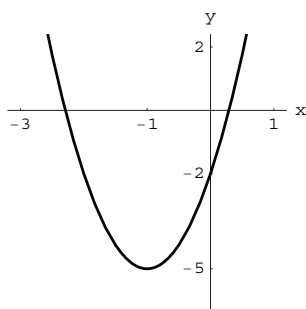
54. $y = -(x + 2)^2 - 4$; left by 2,
reflect about x -axis, down by 4



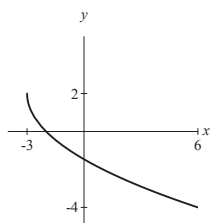
55. $y = -2(x + 3)^2 - 4$; left by 3,
stretch by 2, reflect about x -axis, down by 4



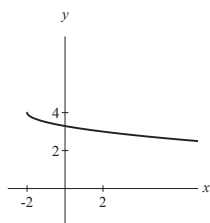
56. $y = 3(x + 1)^2 - 5$; left by 1,
stretch by 3, down by 5



57. $y = -2\sqrt{x + 3} + 2$, left by 3, stretch by 2,
reflect about x -axis, up by 2



58. $y = -\frac{1}{2}\sqrt{x + 2} + 4$, left by 2, shrink by $1/2$,
reflect about x -axis, up by 4



59. Symmetric about y -axis, even function
since $f(-x) = f(x)$

60. Symmetric about y -axis, even function
since $f(-x) = f(x)$

61. No symmetry, neither even nor odd
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

62. Symmetric about the origin, odd function
since $f(-x) = -f(x)$

63. Neither symmetry, neither even nor odd since
 $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

64. Neither symmetry, neither even nor odd
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

65. No symmetry, not an even or odd function
since $f(-x) = -f(x)$ and $f(-x) \neq -f(x)$

66. Symmetric about the y -axis, even function
since $f(-x) = f(x)$

67. Symmetric about the origin, odd function
since $f(-x) = -f(x)$

68. Symmetric about the origin, odd function
since $f(-x) = -f(x)$

69. No symmetry, not an even or odd function
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

70. No symmetry, not an even or odd function
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

71. No symmetry, not an even or odd function
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

72. Symmetric about the y -axis, even function
since $f(-x) = f(x)$

73. Neither symmetry, not an even or odd function
since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

74. Symmetric about the y -axis, even function
since $f(-x) = f(x)$

75. Symmetric about the y -axis, even function
since $f(-x) = f(x)$

76. Symmetric about the y -axis, even function
since $f(-x) = f(x)$

$$\text{since } f(-x) = f(x)$$

$$\text{since } f(-x) = f(x)$$

77. e 78. a 79. g 80. h

81. b 82. d 83. c 84. f

85. $N(x) = x + 2000$

86. $N(x) = 1.05x + 3000$. Yes, if the merit increase is followed by the cost of living raise then the new salary becomes higher and is
 $N'(x) = 1.05(x + 3000) = 1.05x + 3150$.

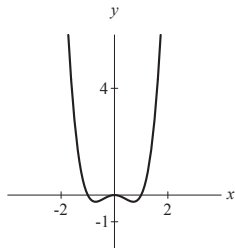
87. If inflation rate is less than 50%, then
 $1 - \sqrt{x} < \frac{1}{2}$. This simplifies to $\frac{1}{2} < \sqrt{x}$. After squaring we have $\frac{1}{4} < x$ and so $x > 25\%$.

88. If production is at least 28 windows, then $1.75\sqrt{x} \geq 28$. They need at least

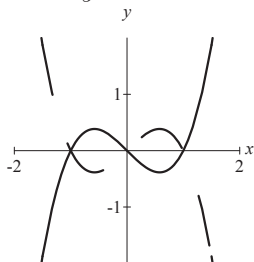
$$x = \left(\frac{28}{1.75}\right)^2 = 256 \text{ hrs.}$$

89.

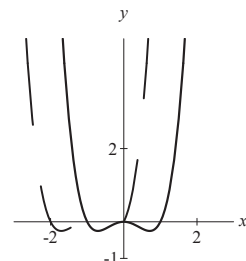
- (a) Both functions are symmetric about the y -axis, and the graphs are identical.



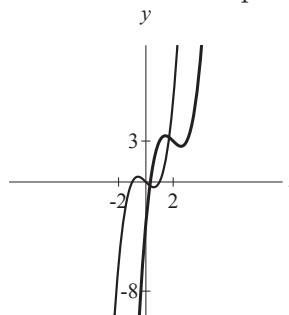
- (b) One graph is a reflection of the other about the y -axis, and both are symmetric about the y -axis.



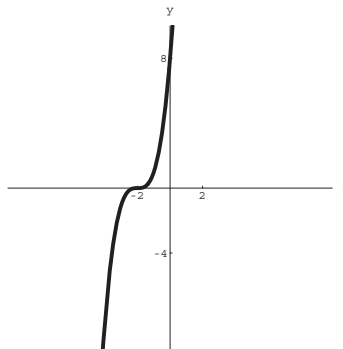
- (c) The second graph is obtained by translating the first one to the left by 1 unit.



- (d) The second graph is obtained by translating the first one to the right by 2 units and 3 units up.



90. The graph of $y = x^3 + 6x^2 + 12x + 8$ or equivalently $y = (x + 2)^3$ can be obtained by shifting the graph of $y = x^3$ to the left by 2 units.



91. $x^2 + y^2 = 1$

92. $y = 5$

93. $x = 4$

94. If A is the area and s is the length of a side, then $A = s^2$.

95. $f(6) = 2(36) - 3(6) = 72 - 18 = 54$
 $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

96. Solve for a :

$$\begin{aligned} 2a^2 + 1 &= 9 \\ 2a^2 &= 8 \\ a^2 &= 4 \\ a &= \pm 2. \end{aligned}$$

97. Draw the numbers in $\{1, 2, 3, \dots, 999\}$ that can be written in the form

$$(2n + 1) \cdot 2^{2m}$$

for $n, m \geq 0$. These means that you will have drawn 666 numbers. Then the 667th number you will draw has the form

$$(2n + 1) \cdot 2^{2m+1}$$

which must be twice one of the first 666 numbers.

98. Let $A(x_1, 2)$ and $B(x_2, 2)$ be the other two opposite vertices where $x_1 < x_2$. Since the sides at B are perpendicular, the slopes are negative reciprocals, i.e.,

$$\frac{3}{3 - x_2} = -\frac{x_2 - 1}{3}.$$

Solving, $x_2 = 2 + \sqrt{10}$. Since the distance AB is the same as the distance between $(1, -1)$ and $(3, 5)$, we obtain

$$\begin{aligned} x_2 - x_1 &= \sqrt{40} = 2\sqrt{10} \\ x_1 &= x_2 - 2\sqrt{10} \\ x_1 &= (2 + \sqrt{10}) - 2\sqrt{10} \\ x_1 &= 2 - \sqrt{10}. \end{aligned}$$

Then the other vertices are $(2 \pm \sqrt{10}, 2)$.

P.3 Pop Quiz

1. $y = \sqrt{x} + 8$
2. $y = (x - 9)^2$
3. $y = (-x)^3$ or $y = -x^3$
4. Domain $[1, \infty)$, range $(-\infty, 5]$
5. $y = -3(x - 6)^2 + 4$
6. Even function

For Thought

1. True, since $A = P^2/16$.
2. False, rather $y = (x - 1)^2 = x^2 - 2x + 1$.
3. False, rather $(f \circ g)(x) = \sqrt{x - 2}$.
4. True
5. False, since $(h \circ g)(x) = x^2 - 9$.
6. False, rather $f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}$.
7. True
8. False; g^{-1} does not exist since the graph of g which is a parabola fails the horizontal line test.
9. False, since $y = x^2$ is a function which does not have an inverse function.
10. True

P.4 Exercises

1. composition
2. one-to-one
3. invertible
4. inverse
5. switch-and-solve
6. symmetric
7. $y = 2(3x + 1) - 3 = 6x - 1$
8. $y = -4(-3x - 2) - 1 = 12x + 7$
9. $y = (x^2 + 6x + 9) - 2 = x^2 + 6x + 7$
10. $y = 3(x^2 - 2x + 1) - 3 = 3x^2 - 6x$
11. $y = 3 \cdot \frac{x + 1}{3} - 1 = x + 1 - 1 = x$
12. $y = 2 \left(\frac{1}{2}x - \frac{5}{2} \right) + 5 = x - 5 + 5 = x$
13. $y = 2 \cdot \frac{x + 1}{2} - 1 = x + 1 - 1 = x$
14. $y = \frac{(5x - 1) + 1}{5} = \frac{5x}{5} = x$

15. $f(2) = 5$ 16. $g(-4) = 17$

17. $f(2) = 5$ 18. $h(-22) = -7$

19. $f(17) = 3(17) - 1 = 50$

20. $g\left(-\frac{8}{3}\right) = \frac{64}{9} + 1 = \frac{73}{9}$

21. $3(x^2 + 1) - 1 = 3x^2 + 2$

22. $(3x - 1)^2 + 1 = 9x^2 - 6x + 2$

23. $(3x - 1)^2 + 1 = 9x^2 - 6x + 2$

24. $3(x^2 + 1) - 1 = 3x^2 + 2$ 25. $\frac{x^2 + 2}{3}$

26. $\frac{x^2 + 2x + 1}{9} + 1 = \frac{1}{9}x^2 + \frac{2}{9}x + \frac{10}{9}$

27. $F = g \circ h$ 28. $G = g \circ f$

29. $H = h \circ g$ 30. $M = f \circ g$

31. $N = f \circ h$ 32. $R = h \circ f$

33. Interchange x and y then solve for y .

$$\begin{aligned} x &= 3y - 7 \\ \frac{x+7}{3} &= y \\ \frac{x+7}{3} &= f^{-1}(x) \end{aligned}$$

34. Interchange x and y then solve for y .

$$\begin{aligned} x &= -2y + 5 \\ 2y &= 5 - x \\ f^{-1}(x) &= \frac{5 - x}{2} \\ f^{-1}(x) &= \frac{x - 5}{-2} \end{aligned}$$

35. Interchange x and y then solve for y .

$$\begin{aligned} x &= 2 + \sqrt{y - 3} \quad \text{for } x \geq 2 \\ (x - 2)^2 &= y - 3 \quad \text{for } x \geq 2 \\ f^{-1}(x) &= (x - 2)^2 + 3 \quad \text{for } x \geq 2 \end{aligned}$$

36. Interchange x and y then solve for y .

$$\begin{aligned} x &= \sqrt{3y - 1} \quad \text{for } x \geq 0 \\ x^2 + 1 &= 3y \quad \text{for } x \geq 0 \\ f^{-1}(x) &= \frac{x^2 + 1}{3} \quad \text{for } x \geq 0 \end{aligned}$$

37. Interchange x and y then solve for y .

$$\begin{aligned} x &= -y - 9 \\ y &= -x - 9 \\ f^{-1}(x) &= -x - 9 \end{aligned}$$

38. Interchange x and y then solve for y .

$$\begin{aligned} x &= -y + 3 \\ y &= -x + 3 \\ f^{-1}(x) &= -x + 3 \end{aligned}$$

39. Interchange x and y then solve for y .

$$\begin{aligned} x &= -\frac{1}{y} \\ xy &= -1 \\ f^{-1}(x) &= -\frac{1}{x} \end{aligned}$$

40. Clearly $f^{-1}(x) = x$ 41. Interchange x and y then solve for y .

$$\begin{aligned} x &= \sqrt[3]{y - 9} + 5 \\ x - 5 &= \sqrt[3]{y - 9} \\ (x - 5)^3 &= y - 9 \\ f^{-1}(x) &= (x - 5)^3 + 9 \end{aligned}$$

42. Interchange x and y then solve for y .

$$\begin{aligned} x &= \sqrt[3]{\frac{y}{2}} + 5 \\ x - 5 &= \sqrt[3]{\frac{y}{2}} \\ (x - 5)^3 &= \frac{y}{2} \\ f^{-1}(x) &= 2(x - 5)^3 \end{aligned}$$

43. Interchange x and y then solve for y .

$$\begin{aligned} x &= (y - 2)^2 \quad x \geq 0 \\ \sqrt{x} &= y - 2 \\ f^{-1}(x) &= \sqrt{x} + 2 \end{aligned}$$

44. Interchange x and y then solve for y .

$$\begin{aligned} x &= (y + 3)^2 \quad x \geq 0 \\ \sqrt{x} &= y + 3 \\ f^{-1}(x) &= \sqrt{x} - 3 \end{aligned}$$

$$\begin{aligned}
 45. \quad (f^{-1} \circ f)(x) &= \frac{1}{2}(2x - 1) + \frac{1}{2} = x \text{ and} & &= \sqrt[5]{x^5 - 1 + 1} \\
 (f \circ f^{-1})(x) &= 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1 = x & &= \sqrt[5]{x^5} \\
 (f^{-1} \circ f)(x) &= x
 \end{aligned}$$

$$\begin{aligned}
 46. \quad (f^{-1} \circ f)(x) &= 3\left(\frac{x+1}{3}\right) - 1 = x \text{ and} & \text{and} \\
 (f \circ f^{-1})(x) &= \frac{(3x-1)+1}{3} = x & (f \circ f^{-1})(x) &= \sqrt[3]{\left(\sqrt[5]{x^3+1}\right)^5} - 1
 \end{aligned}$$

$$\begin{aligned}
 47. \quad (f^{-1} \circ f)(x) &= 0.25(4x + 4) - 1 = x \text{ and} & &= \sqrt[3]{x^3 + 1 - 1} \\
 (f \circ f^{-1})(x) &= 4(0.25x - 1) + 4 = x & &= \sqrt[3]{x^3}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad (f^{-1} \circ f)(x) &= -0.2(20 - 5x) + 4 = x \text{ and} & (f \circ f^{-1})(x) &= x. \\
 (f \circ f^{-1})(x) &= 20 - 5(-0.2x + 4) = x
 \end{aligned}$$

49. We obtain

$$\begin{aligned}
 (f^{-1} \circ f)(x) &= \frac{4 - \left(\sqrt[3]{4 - 3x}\right)^3}{3} \\
 &= \frac{4 - (4 - 3x)}{3} \\
 &= \frac{3x}{3} \\
 (f^{-1} \circ f)(x) &= x
 \end{aligned}$$

and

$$\begin{aligned}
 (f \circ f^{-1})(x) &= \sqrt[3]{4 - 3\left(\frac{4 - x^3}{3}\right)} \\
 &= \sqrt[3]{4 - (4 - x^3)} \\
 &= \sqrt[3]{x^3} \\
 (f \circ f^{-1})(x) &= x.
 \end{aligned}$$

50. We obtain

$$\begin{aligned}
 (f^{-1} \circ f)(x) &= \sqrt[3]{(x^3 + 5) - 5} \\
 &= \sqrt[3]{x^3} \\
 (f^{-1} \circ f)(x) &= x
 \end{aligned}$$

and

$$\begin{aligned}
 (f \circ f^{-1})(x) &= \left(\sqrt[3]{x - 5}\right)^3 + 5 \\
 &= (x - 5) + 5 \\
 (f \circ f^{-1})(x) &= x.
 \end{aligned}$$

51. We obtain

$$(f^{-1} \circ f)(x) = \sqrt[5]{\left(\sqrt[3]{x^5 - 1}\right)^3 + 1}$$

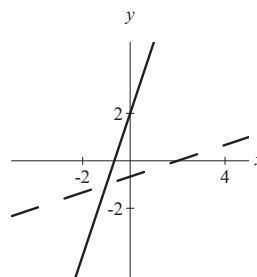
52. We find

$$\begin{aligned}
 (f^{-1} \circ f)(x) &= \left((x^{3/5} - 3) + 3\right)^{5/3} \\
 &= \left(x^{3/5}\right)^{5/3} \\
 (f^{-1} \circ f)(x) &= x
 \end{aligned}$$

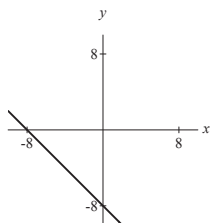
and

$$\begin{aligned}
 (f \circ f^{-1})(x) &= \left((x + 3)^{5/3}\right)^{3/5} - 3 \\
 &= (x + 3) - 3 \\
 (f \circ f^{-1})(x) &= x.
 \end{aligned}$$

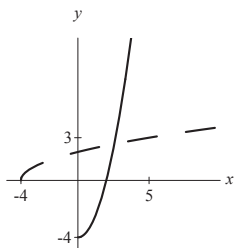
$$53. \quad f^{-1}(x) = \frac{x-2}{3}$$



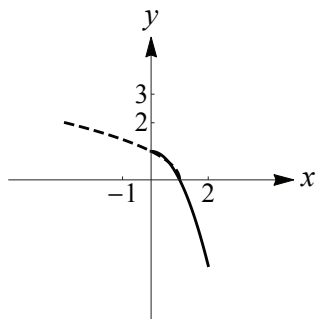
54. $f^{-1}(x) = -x - 8$



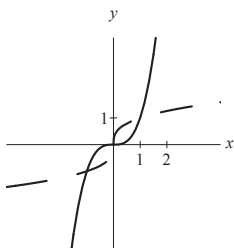
55. $f^{-1}(x) = \sqrt{x+4}$



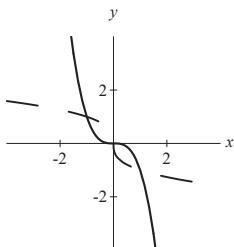
56. $f^{-1}(x) = \sqrt{1-x}$



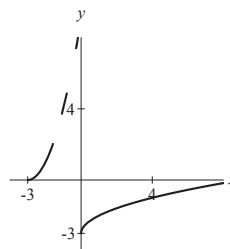
57. $f^{-1}(x) = \sqrt[3]{x}$



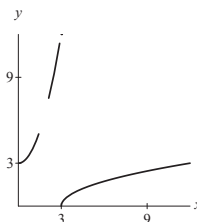
58. $f^{-1}(x) = -\sqrt[3]{x}$



59. $f^{-1}(x) = (x+3)^2$ for $x \geq -3$



60. $f^{-1}(x) = x^2 + 3$ for $x \geq 0$



61. The inverse is the composition of subtracting 2 and dividing by 5, i.e., $f^{-1}(x) = \frac{x-2}{5}$

62. The inverse is the composition of adding 1 and dividing by 0.5, i.e., $f^{-1}(x) = \frac{x+1}{0.5} = 2x+2$

63. The inverse is the composition of adding 88 and dividing by 2, i.e.,

$$f^{-1}(x) = \frac{x+88}{2} = \frac{1}{2}x + 44$$

64. The inverse is the composition of subtracting 99 and dividing by 3, i.e.,

$$f^{-1}(x) = \frac{x-99}{3} = \frac{1}{3}x - 33$$

65. The inverse is the composition of subtracting 4 and dividing by -3 , i.e.,

$$f^{-1}(x) = \frac{x-4}{-3} = -\frac{1}{3}x + \frac{4}{3}$$

66. The inverse is the composition of adding 5 and dividing by -2 , i.e.,

$$f^{-1}(x) = \frac{x-5}{-2} = -\frac{1}{2}x + \frac{5}{2}$$

67. The inverse is the composition of adding 9 and multiplying by 2, i.e.,

$$f^{-1}(x) = 2(x+9) = 2x+18$$

68. The inverse is the composition of subtracting 6 and multiplying by 3, i.e.,

$$f^{-1}(x) = 3(x-6) = 3x-18$$

- 69.** The inverse is the composition of subtracting 3 and taking the multiplicative inverse, i.e.,

$$f^{-1}(x) = \frac{1}{x-3}$$

- 70.** The inverse is the composition of taking the multiplicative inverse and subtracting 3, i.e.,

$$f^{-1}(x) = \frac{1}{x} - 3$$

- 71.** The inverse is the composition of adding 9 and raising an expression to the third power, i.e.,

$$f^{-1}(x) = (x+9)^3.$$

- 72.** The inverse is the composition of raising an expression to the third power and adding 9, i.e., $f^{-1}(x) = x^3 + 9$.

- 73.** The inverse is the composition of adding 7, dividing by 2, and taking the cube root, i.e.,

$$f^{-1}(x) = \sqrt[3]{\frac{x+7}{2}}.$$

- 74.** The inverse is the composition of subtracting by 4, multiplying by -1 , and taking the cube root, i.e., $f^{-1}(x) = \sqrt[3]{-(x-4)} = \sqrt[3]{4-x}$

75. $A = d^2/2$ **76.** $P = 4\sqrt{A}$

- 77.** Let x be the number of ice cream bars sold. Then $W(x) = (0.20)(0.40x+200) = 0.08x+40$.

- 78.** Let x be the number of radios manufactured. Then $P(x) = (0.80)(0.90x) = 0.72x$.

- 79.** $C = 1.08P$ expresses the total cost as a function of the purchase price; and $P = C/1.08$ is the purchase price as a function of the total cost.

80. $V(x) = x^3, S(x) = \sqrt[3]{x}$

81.

$$r = \sqrt{5.625 \times 10^{-5} - \frac{V}{500}}$$

where $0 \leq V \leq 0.028125$

- 82.** Solving for I in $E = 0.4I + 10,000$, we

obtain $\frac{E-10,000}{0.4} = I$ or equivalently

$$I = \frac{5}{2}E - 25,000.$$

If $E = I$, then $I = 0.4I + 10,000$ or $0.6I = 10,000$. Thus, if $E = I$ then $I = \frac{10,000}{0.6}$ or $I = \$16,666.67$.

- 83.** When $V = \$18,000$, the depreciation rate is

$$1 - \left(\frac{18,000}{50,000}\right)^{1/5} \approx 0.1848$$

or the depreciation rate is 18.48%.

Solving for V , we obtain

$$\begin{aligned} \left(\frac{V}{50,000}\right)^{1/5} &= 1 - r \\ \frac{V}{50,000} &= (1 - r)^5 \\ V &= 50,000(1 - r)^5. \end{aligned}$$

- 84.** When $P = \$23,580$, the annual growth rate is

$$\left(\frac{23,580}{10,000}\right)^{1/10} - 1 \approx 0.0896$$

or the annual growth rate is 8.96%.

Solving for P , we obtain

$$\begin{aligned} \left(\frac{P}{10,000}\right)^{1/10} &= r + 1 \\ \frac{P}{10,000} &= (r + 1)^{10} \\ P &= 10,000(r + 1)^{10}. \end{aligned}$$

- 85.** Since $g^{-1}(x) = \frac{x+5}{3}$ and $f^{-1}(x) = \frac{x-1}{2}$, we have

$$g^{-1} \circ f^{-1}(x) = \frac{\frac{x-1}{2} + 5}{3} = \frac{x+9}{6}.$$

Likewise, since $(f \circ g)(x) = 6x - 9$, we get

$$(f \circ g)^{-1}(x) = \frac{x+9}{6}.$$

Hence, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

86. Since $(f \circ g \circ g^{-1} \circ f^{-1})(x) = (f \circ (g \circ g^{-1}))(f^{-1}(x)) = f(f^{-1}(x)) = x$ and the range of one function is the domain of the other function, we have $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

87. $x^2 + (y - 1)^2 = 9$

88. $\left(\frac{\pi/3 + \pi/2}{2}, \frac{1+1}{2}\right) = \left(\frac{5\pi/6}{2}, 1\right) = \left(\frac{5\pi}{12}, 1\right)$

89. function

90. reflection

91. Domain $(-\infty, \infty)$, range $[1, \infty)$

92. Since $x - 2 \geq 0$, the domain $[2, \infty)$.

Note, the range of $-\sqrt{x}$ is $(-\infty, 0]$. Then the range of $y = -\sqrt{x-2} + 3$ is $(-\infty, 3]$.

93. Note, if $a > b > 0$ and $n > 0$, then

$$\frac{a}{n} + \frac{b}{n+1} > \frac{b}{n} + \frac{a}{n+1}.$$

Thus, the arrangement with the largest sum is

$$\frac{2025}{1} + \frac{2024}{2} + \dots + \frac{2}{2024} + \frac{1}{2025}.$$

94. If $f(x) = mx + b$, then

$$(f \circ f \circ f)(x) = m^3x + b(m^2 + m + 1).$$

Since

$$(f \circ f \circ f)(x) = 27x + 26$$

we obtain $m^3 = 27$ or $m = 3$.

Also, $b(m^2 + m + 1) = 26$. If we substitute $m = 3$, then $13b = 26$ or $m = 2$.

Thus, $f(x) = 3x + 2$. The y -intercept is $(0, 2)$.

P.4 Pop Quiz

1. $A = \pi r^2 = \pi(d/2)^2$ or $A = \frac{\pi d^2}{4}$

2. $y = 3(4x - 5) - 1 = 12x - 16$

3. $(f \circ g)(5) = f(3) = 9$

4. $M = g \circ h \circ f$

5. Interchange x and y then solve for y .

$$\begin{aligned} 2y - 1 &= x \\ y &= \frac{x+1}{2} \\ f^{-1}(x) &= \frac{x+1}{2} \end{aligned}$$

6. Interchange x and y then solve for y .

$$\begin{aligned} \sqrt[3]{y+1} - 4 &= x \\ y+1 &= (x+4)^3 \\ y &= (x+4)^3 - 1 \\ g^{-1}(x) &= (x+4)^3 - 1 \end{aligned}$$

Chapter P Review Exercises

1. $\sqrt{49 \cdot 2} = 7\sqrt{2}$

2. $\sqrt{100 \cdot 2} = 10\sqrt{2}$

3. $\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

4. $\frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$

5. $\frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$

6. $\frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{4}$

7. $\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

8. $\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

9. The distance is $\sqrt{(-3-2)^2 + (5-(-6))^2} = \sqrt{(-5)^2 + 11^2} = \sqrt{25 + 121} = \sqrt{146}$.

The midpoint is $\left(\frac{-3+2}{2}, \frac{5-6}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$.

10. The distance is

$$\sqrt{(-1-(-2))^2 + (1-(-3))^2} = \sqrt{1^2 + 4^2} = \sqrt{17}. \text{ The midpoint is } \left(\frac{-1-2}{2}, \frac{1-3}{2}\right) = \left(-\frac{3}{2}, -1\right).$$

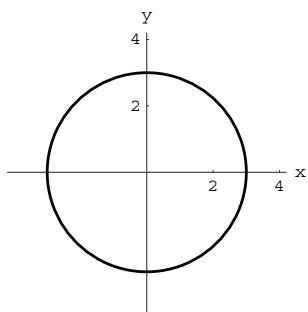
11. The distance is $\sqrt{\left(\pi - \frac{\pi}{2}\right)^2 + (1-1)^2} =$

$$\sqrt{\left(\frac{\pi}{2}\right)^2 + 0} = \frac{\pi}{2}. \text{ The midpoint is } \left(\frac{\pi/2 + \pi}{2}, \frac{1+1}{2}\right) = \left(\frac{3\pi}{4}, 1\right).$$

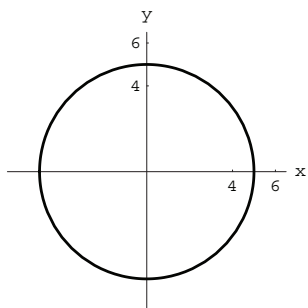
12. The distance is $\sqrt{\left(\frac{\pi}{2} - \frac{\pi}{3}\right)^2 + (2-2)^2} =$

$$\sqrt{\left(\frac{\pi}{6}\right)^2 + 0} = \frac{\pi}{6}. \text{ The midpoint is } \left(\frac{\pi/2 + \pi/3}{2}, \frac{2+2}{2}\right) = \left(\frac{5\pi}{12}, 2\right).$$

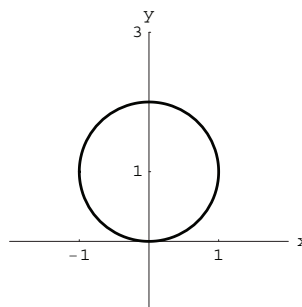
13. Circle with center at $(0, 0)$ and radius 3



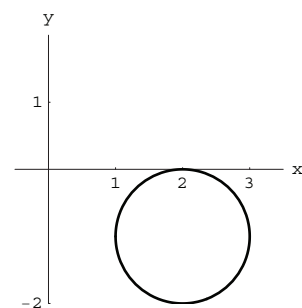
14. Circle with center at $(0, 0)$ and radius 5



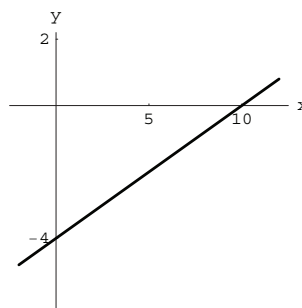
15. Circle with center at $(0, 1)$ and radius 1



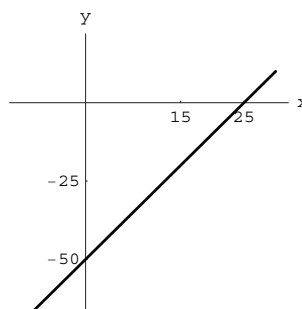
16. Circle with center at $(2, -1)$ and radius 1



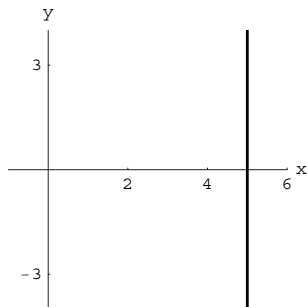
17. The line through the points $(10, 0)$ and $(0, -4)$.



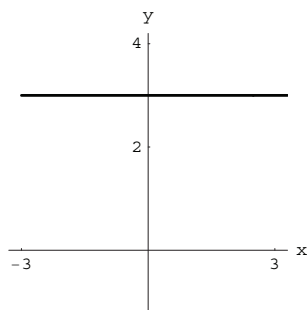
18. The line through the points $(25, 0)$ and $(0, -50)$.



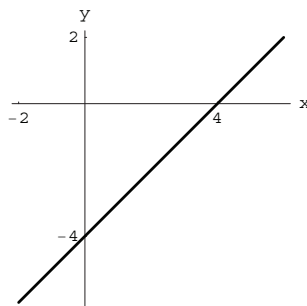
19. The vertical line through the point $(5, 0)$.



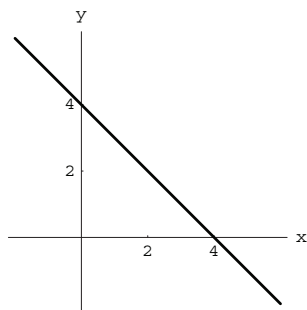
20. The horizontal line through the point $(0, 3)$.



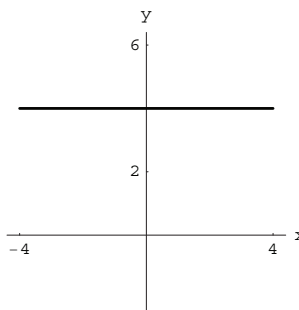
21. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



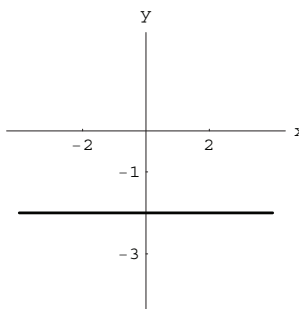
22. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



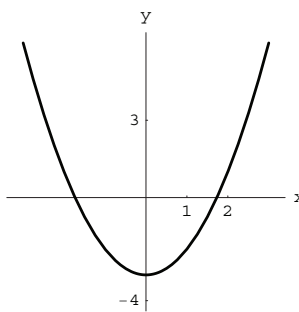
23. Domain $(-\infty, \infty)$ and range $\{4\}$. The horizontal line through the point $(0, 4)$.



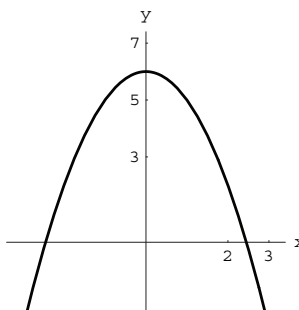
24. Domain $(-\infty, \infty)$ and range $\{-2\}$



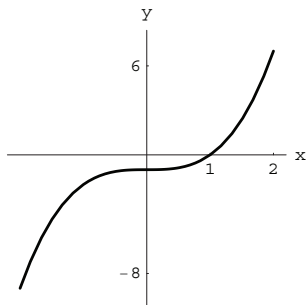
25. Domain $(-\infty, \infty)$ and range $[-3, \infty)$



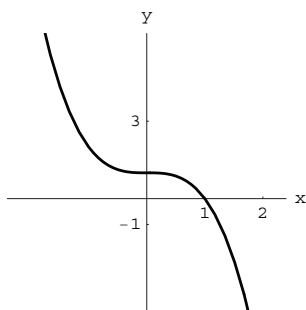
26. Domain $(-\infty, \infty)$ and range $(-\infty, 6]$



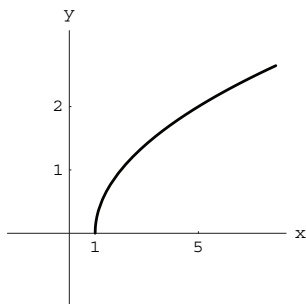
27. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



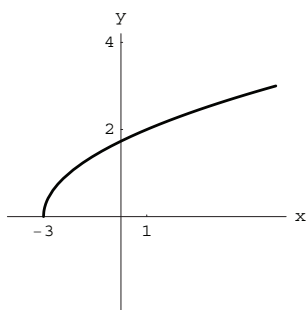
28. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



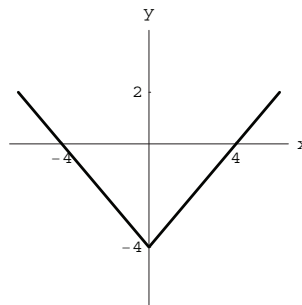
29. Domain $[1, \infty)$ and range $[0, \infty)$



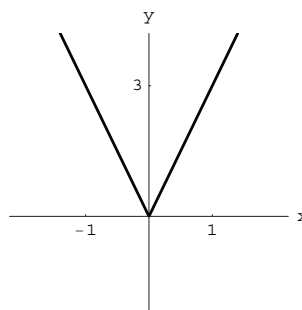
30. Domain $[-3, \infty)$ and range $[0, \infty)$



31. Domain $(-\infty, \infty)$ and range $[-4, \infty)$



32. Domain $(-\infty, \infty)$ and range $[0, \infty)$



33. $9 + 3 = 12$ 34. $6 - 7 = -1$

35. $24 - 7 = 17$ 36. $1 + 3 = 4$

37. $g(12) = 17$ 38. $f(-1) = 4$

39. $f(-3) = 12$ 40. $g(7) = 7$

41.

$$\begin{aligned} f(g(x)) &= f(2x - 7) \\ &= (2x - 7)^2 + 3 \\ &= 4x^2 - 28x + 52 \end{aligned}$$

42.

$$\begin{aligned} g(f(x)) &= g(x^2 + 3) \\ &= 2(x^2 + 3) - 7 \\ &= 2x^2 - 1 \end{aligned}$$

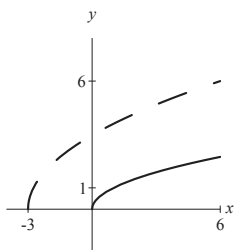
43. $g\left(\frac{x+7}{2}\right) = (x+7) - 7 = x$

44. $f(\sqrt{x-3}) = (x-3) + 3 = x$

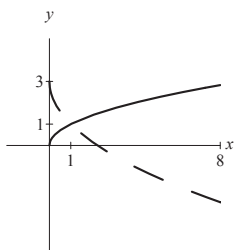
45. $g^{-1}(x) = \frac{x+7}{2}$

46. From No. 45, $g^{-1}(-3) = \frac{-3+7}{2} = 2$

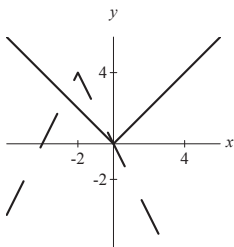
47. $f(x) = \sqrt{x}$, $g(x) = 2\sqrt{x+3}$; left by 3, stretch by 2



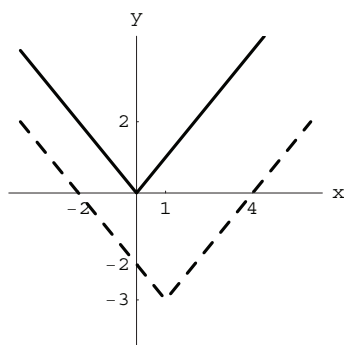
48. $f(x) = \sqrt{x}$, $g(x) = -2\sqrt{x} + 3$; stretch by 2, reflect about x -axis, up by 3



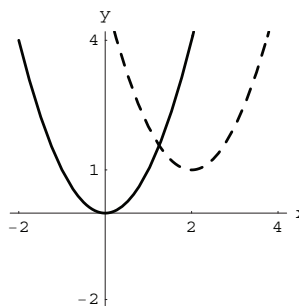
49. $f(x) = |x|$, $g(x) = -2|x+2| + 4$; left by 2, stretch by 2, reflect about x -axis, up by 4



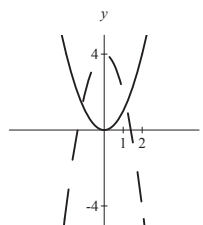
50. $f(x) = |x|$, $g(x) = |x-1| - 3$; right by 1 and down by 3



51. $f(x) = x^2$, $g(x) = (x-2)^2 + 1$; right by 2 and up by 1



52. $f(x) = x^2$, $g(x) = -2x^2 + 4$; stretch by 2, reflect about x -axis, up by 4



53. $N = h \circ j$ 54. $P = j \circ h$

55. $R = h \circ g$ 56. $Q = g \circ h$

57. $F = f \circ g$ 58. $G = g \circ f$

59. $H = f \circ g \circ j$ 60. $M = j \circ h \circ g$

61. $y = |x| - 3$, domain is $(-\infty, \infty)$, range is $[-3, \infty)$

62. $y = |x-3| - 1$, domain is $(-\infty, \infty)$, range is $[-1, \infty)$

63. $y = -2|x| + 4$, domain is $(-\infty, \infty)$, range is $(-\infty, 4]$

64. $y = -\frac{1}{2}|x| + 2$, domain is $(-\infty, \infty)$, range is $(-\infty, 2]$

65. $y = |x+2| + 1$, domain is $(-\infty, \infty)$, range is $[1, \infty)$

66. $y = -|x+1| - 2$, domain $(-\infty, \infty)$, range $(-\infty, -2]$

67. Symmetry: y -axis 68. Symmetry: y -axis

69. Symmetric about the origin

70. Symmetric about the origin

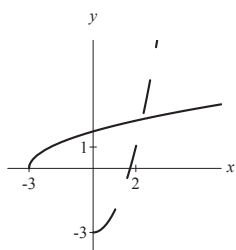
71. Neither symmetry 72. Neither symmetry

73. Symmetric about the y -axis

74. Symmetric about the y -axis

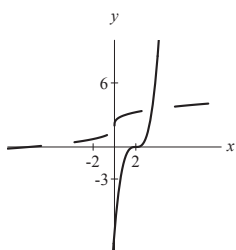
75. inverse functions,

$$f(x) = \sqrt{x+3}, g(x) = x^2 - 3 \text{ for } x \geq 0$$



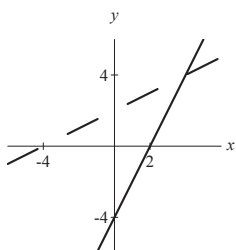
76. inverse functions,

$$f(x) = (x-2)^3, g(x) = \sqrt[3]{x} + 2$$



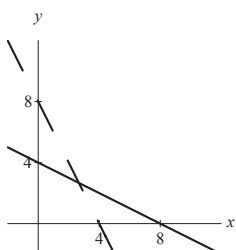
77. inverse functions,

$$f(x) = 2x - 4, g(x) = \frac{1}{2}x + 2$$



78. inverse functions,

$$f(x) = -\frac{1}{2}x + 4, g(x) = -2x + 8$$



79. $f^{-1}(x) = \frac{x}{5}$ 80. $f^{-1}(x) = x - 9$

81. $f^{-1}(x) = \frac{x+21}{3}$ or $f^{-1}(x) = \frac{1}{3}x + 7$

82. $f^{-1}(x) = \frac{x+5}{2}$ or $f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$

83. $f^{-1}(x) = \sqrt[3]{x+1}$

84. $f^{-1}(x) = x^3 - 5$

85. $f^{-1}(x) = \frac{1}{x} + 3$

86. $f^{-1}(x) = \frac{1}{x-5}$

87. $f^{-1}(x) = \sqrt{x}$

88. $f^{-1}(x) = x^2 + 3$ where $x \geq 0$

89. $\sqrt{(\sqrt{5})^2 + 2^2} = \sqrt{9} = 3$

90. $\sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

91. From $3x - 4(0) = 9$, we get that the x -intercept is $(3, 0)$. From $3(0) - 4y = 9$, we find that the y -intercept is $(0, -\frac{9}{4})$.

92. From $0 = 2x - 3$, we get that the x -intercept is $(\frac{3}{2}, 0)$. From $y = 2(0) - 3$, we find that the y -intercept is $(0, -3)$.

93. $(x+3)^2 + (y-5)^2 = 3$

94. The center is $(3, -1)$ and the radius is 4.

95. Since $C = 2\pi r$, we get $r = \frac{C}{2\pi}$.

96. Since $A = \pi \left(\frac{d}{2}\right)^2$, we get $4A = \pi d^2$.

Then $d = \sqrt{\frac{4A}{\pi}}$ or $d = 2\sqrt{\frac{A}{\pi}}$.

97. No, since the product of the numbers from 1 through 9 is

$$1 \cdot 2 \cdot 3 \cdots 9 = 362,880$$

$$\text{and } 71^3 = 357911.$$

98. The fourth point of the parallelogram is either $Q(15, a-30)$ or $R(25, a+30)$. Suppose the two unknown points are $P(20, a)$ and $Q(15, a-30)$. The line through the origin and point Q is

$$\frac{a-30}{15}x - y = 0.$$

We apply a known formula. The shortest distance between point (x_0, y_0) and a line $Ax + By + C = 0$ is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

Then the shortest distance h between $(5, 30)$ and the line through the origin and Q is

$$h = \frac{|\frac{a-30}{15}(5) - 30|}{\sqrt{(\frac{a-30}{15})^2 + 1}} = \frac{|\frac{a-120}{3}|}{\sqrt{(\frac{a-30}{15})^2 + 1}}.$$

The distance b between the origin and Q is

$$b = \sqrt{15^2 + (a-30)^2}.$$

The area of a parallelogram is the product of the base b and height h . Since the area is 580, we obtain through some cancelations that

$$\sqrt{15^2 + (a-30)^2} \frac{|\frac{a-120}{3}|}{\sqrt{(\frac{a-30}{15})^2 + 1}} = 580$$

$$15 \left| \frac{a-120}{3} \right| = 580$$

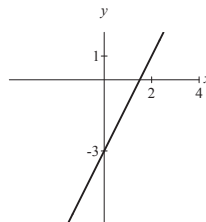
$$|a-120| = 116.$$

Hence, $a = 4$ or $a = 236$.

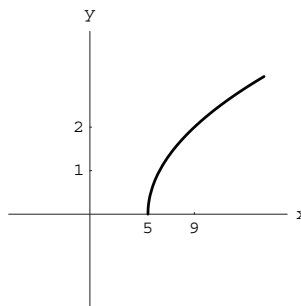
If the fourth point is $R(25, a+30)$, we still find $a = 4$ or $a = 236$. There are four parallelograms but only two values of a .

Chapter P Test

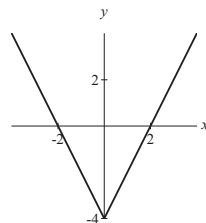
1. Domain $(-\infty, \infty)$, range $[0, \infty)$
2. Domain $[9, \infty)$, range $[0, \infty)$
3. Domain $(-\infty, \infty)$, range $(-\infty, 2]$
4. The graph of $y = 2x - 3$ includes the points $(3/2, 0), (0, -3)$



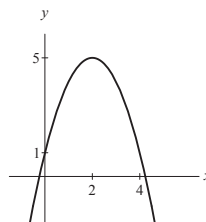
5. The graph of $f(x) = \sqrt{x-5}$ goes through $(5, 0), (9, 2)$.



6. The graph of $y = 2|x| - 4$ includes the points $(0, -4), (\pm 3, 2)$



7. The graph of $y = -(x-2)^2 + 5$ is a parabola with vertex $(2, 5)$



8. $\sqrt{9} = 3$
9. $3(2) - 1 = 5$

10. $f(5) = \sqrt{7}$

11. $g^{-1}(x) = \frac{x+1}{3}$ or $g^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

12. Using the answer from Exercise 11,

$$\text{we find } g^{-1}(20) = \frac{20+1}{3} = 7.$$

13. $\sqrt{72} = 6\sqrt{2}$

14. $\frac{\sqrt{25}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$

15. $\frac{20}{2\sqrt{2}} = 5\sqrt{2}$

16. $\left(\frac{3+(-3)}{2}, \frac{5+6}{2}\right) = \left(0, \frac{11}{2}\right)$

17. $\sqrt{(-2-3)^2 + (4-(-1))^2} = \sqrt{25+25} = 5\sqrt{2}.$

18. $(x+4)^2 + (y-1)^2 = 7$

19. Center $(4, -1)$, radius $\sqrt{6}$

20. Solving $3a - 9 = 6$, we obtain $a = 5$.

21. Symmetric about the y -axis

$$\text{since } f(-x) = f(x)$$

22. Interchange x and y then solve for y .

$$\begin{aligned}x &= 3 + \sqrt[3]{y-2} \\x-3 &= \sqrt[3]{y-2} \\(x-3)^3 &= y-2 \\(x-3)^3 + 2 &= y\end{aligned}$$

The inverse is $g^{-1}(x) = (x-3)^3 + 2$.

23. Since $A = \left(\frac{p}{4}\right)^2$, we get $A = \frac{p^2}{16}$.