

***Finite Element Analysis 5th edition Moaveni
Solutions Manual***

SKU: 9780135212103-SOLUTIONS

Chapter 1

1.1

(a) Using two elements

$$A(y) = 0.25 - 0.0125y$$

$$A_1 = 0.25 \text{ in}^2 \quad A_2 = 0.1875 \text{ in}^2$$

$$A_3 = 0.125 \text{ in}^2$$

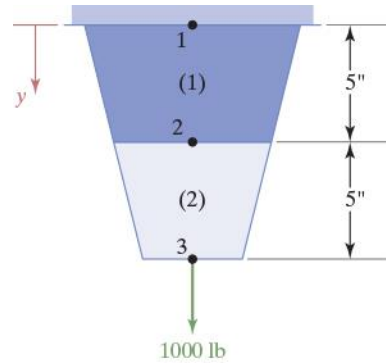
$$K_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

$$K_1 = \frac{(0.25 + 0.1875)(10.4 \times 10^6)}{2(5)} = 455,000 \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{(0.1875 + 0.125)(10.4 \times 10^6)}{2(5)} = 325,000 \frac{\text{lb}}{\text{in}}$$

$$10^3 \begin{bmatrix} \cancel{455} & \cancel{-455} & 0 \\ \cancel{-455} & 455 + 325 & -325 \\ 0 & -325 & 325 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$$u_1 = 0, \quad \underline{\underline{u_2 = 0.002197 \text{ in}}} \quad \underline{\underline{u_3 = 0.005274 \text{ in}}} \quad \leftarrow$$



(b)

$$A_1 = 0.25 \text{ in}^2$$

$$A_2 = 0.234375 \text{ in}^2$$

$$A_3 = 0.21875 \text{ in}^2$$

$$A_4 = 0.203125 \text{ in}^2$$

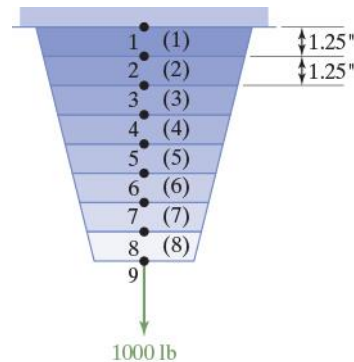
$$A_5 = 0.1875 \text{ in}^2$$

$$A_6 = 0.171875 \text{ in}^2$$

$$A_7 = 0.15625 \text{ in}^2$$

$$A_8 = 0.140625 \text{ in}^2$$

$$A_9 = 0.125 \text{ in}^2$$



1.1 Cont.

$$K_1 = 2015000 \frac{\text{lb}}{\text{in}}$$

$$K_2 = 1885000 \frac{\text{lb}}{\text{in}}$$

$$K_3 = 1755000 \frac{\text{lb}}{\text{in}}$$

$$K_4 = 1625000 \frac{\text{lb}}{\text{in}}$$

$$K_5 = 1495000 \frac{\text{lb}}{\text{in}}$$

$$K_6 = 1365000 \frac{\text{lb}}{\text{in}}$$

$$K_7 = 1235000 \frac{\text{lb}}{\text{in}}$$

$$K_8 = 1105000 \frac{\text{lb}}{\text{in}}$$

~~2015000~~ 1	~~-2015000~~ 0							
~~-2015000~~ 0	2015000 +1885000	-1885000						
	-1885000	1885000 +1755000	-1755000					
		-1755000	1755000 +1625000	-1625000				
			-1625000	1625000 +1495000	-1495000			
				-1495000	1495000 +1365000	-1365000		
					-1365000	1365000 +1235000	-1235000	
						-1235000	1235000 +1105000	-1105000
							-1105000	1105000

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$$u_1 = 0, u_2 = 0.00049628, u_3 = 0.0010268, u_4 = 0.0015965, u_5 = 0.0022119, \\ u_6 = 0.0028808, u_7 = 0.0036134, u_8 = 0.0044232, u_9 = 0.0053281 \text{ in.}$$

y (in)	exact defl.	Two-element	eight element
0	0	0	0
1.25	0.00049645		0.00049628
2.5	0.0010272		0.0010268
3.75	0.0015972		0.0015965
5.0	0.0022129	0.002197	0.0022119
6.25	0.0028822		0.0028808
7.5	0.0036154		0.0036134
8.75	0.0044259		0.0044232
10.0	0.0053319	0.005274	0.0053281

1.2

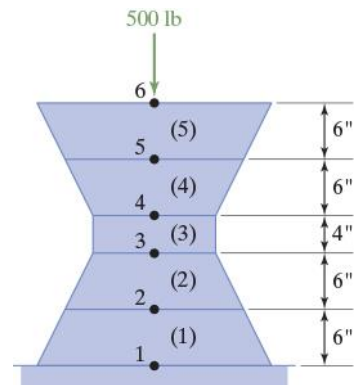
$$K = \frac{A_{\text{avg.}} E}{\ell}$$

$$K_1 = K_5 = \frac{\left(\frac{10.5 + 12}{2}\right)(3)(3.27 \times 10^6)}{6}$$

$$K_1 = K_5 = 18,393,750 \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_4 = \frac{\left(\frac{10.5 + 9}{2}\right)(3)(3.27 \times 10^6)}{6} = 15,941,250 \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{(9)(3)(3.27 \times 10^6)}{4} = 22,072,500 \frac{\text{lb}}{\text{in}}$$



1 ~~18393750~~	0 ~~-18393750~~				
~~-18393750~~ 0	18393750 +15941250	-15941250			
	-15941250	15941250 +22072500	-22072500		
		-22072500	22072500 +15941250	-15941250	
			-15941250	15941250 +18393750	-18393750
				-18393750	18393750

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}$$

$$\{u\} = \begin{Bmatrix} 0 \\ -2.71831 \times 10^{-5} \\ -5.85483 \times 10^{-5} \\ -8.12009 \times 10^{-5} \\ -1.12566 \times 10^{-4} \\ -1.39749 \times 10^{-4} \end{Bmatrix} \text{ in} \quad \leftarrow$$

$$\sigma^{(3)} = \frac{(-8.12009 \times 10^{-5} + 5.85483) \times 10^{-5} \times (3.27 \times 10^6)}{4}$$

$$\sigma^{(3)} = \underline{\underline{18.5 \text{ Psi C}}}$$

1.2 Cont.

as a check:

$$\sigma^{(3)} = \frac{F}{A} = \frac{500}{(9)(3)} = 18.5 \text{ Psi}$$

$$\sigma = \frac{(u_{i+1} - u_i)}{\ell} E$$

$$\sigma^{(1)} = \frac{(-2.71831 \times 10^{-5} - 0)(3.27 \times 10^6)}{6} = \underline{\underline{14.8 \text{ Psi C}}}$$

$$\sigma^{(5)} = \frac{(-1.39749 + 1.12566) \times 10^{-4} \times (3.27 \times 10^6)}{6} = \underline{\underline{14.8 \text{ Psi C}}}$$

as a check:

$$\sigma^{(1)} = \sigma^{(5)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5 + 12}{2}\right)(3)} = 14.8 \text{ Psi C}$$

$$\sigma^{(2)} = \frac{(-5.85483 + 2.71831) \times 10^{-5} \times (3.27 \times 10^6)}{6} = \underline{\underline{17.1 \text{ Psi C}}}$$

$$\sigma^{(4)} = \frac{(-1.12566 \times 10^{-4} + 8.12009 \times 10^{-5}) \times (3.27 \times 10^6)}{6} = \underline{\underline{17.1 \text{ Psi C}}}$$

as a check:

$$\sigma^{(2)} = \sigma^{(4)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5 + 9}{2}\right)(3)} = 17.1 \text{ Psi}$$

1.3

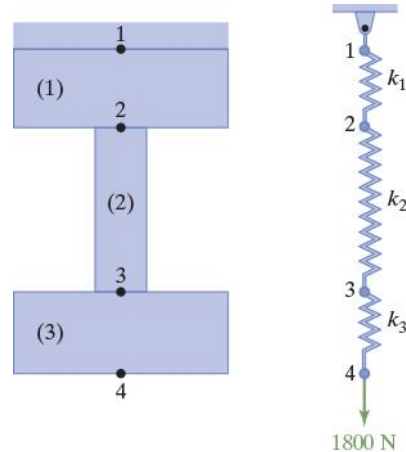
$$K = \frac{AE}{\ell}$$

$$K_1 = K_3 = \frac{(0.08)(0.006)(68.9 \times 10^9)}{0.025}$$

$$K_1 = K_3 = 1.32288 \times 10^9 \frac{\text{N}}{\text{m}}$$

$$K_2 = \frac{(0.02)(0.006)(68.9 \times 10^9)}{0.1}$$

$$K_2 = 0.08268 \times 10^9 \frac{\text{N}}{\text{m}}$$



$$10^9 \begin{bmatrix} \cancel{1.32288} & \cancel{-1.32288} & & \\ \cancel{-1.32288} & 1.32288 + 0.08268 & -0.08268 & \\ & -0.08268 & 0.08268 + 1.32288 & -1.32288 \\ & & -1.32288 & 1.32288 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1800 \end{Bmatrix}$$

$$\begin{bmatrix} 1.40556 & -0.08268 & 0 \\ -0.08268 & 1.40556 & -1.32288 \\ 0 & -1.32288 & 1.32288 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1800 \times 10^{-9} \end{Bmatrix}$$

$$\underline{u_2 = 1.360668 \times 10^{-6} \text{ m}} \quad \underline{u_3 = 2.313135 \times 10^{-5} \text{ m}} \quad \underline{u_4 = 2.449202 \times 10^{-5} \text{ m}} \quad \Leftarrow$$

$$\sigma^{(1)} = E \frac{u_2 - u_1}{\ell} = (68.9 \times 10^9) \frac{(1.360668 \times 10^{-6} - 0)}{0.025} = 3750000 = \underline{\underline{3.75 \text{ MPa}}}$$

as a check: $\sigma^{(1)} = \frac{F}{A} = \frac{1800 \text{ N}}{(0.08)(0.006)} = \underline{\underline{3.75 \text{ MPa}}}$

$$\sigma^{(2)} = E \frac{u_3 - u_2}{\ell} = (68.9 \times 10^9) \frac{(2.313135 \times 10^{-5} - 1.360668 \times 10^{-6})}{0.1} = \underline{\underline{15 \text{ MPa}}} \quad \Leftarrow$$

as a check: $\sigma^{(2)} = \frac{1800 \text{ N}}{(0.02)(0.006)} = \underline{\underline{15 \text{ MPa}}}$

$$\sigma^{(3)} = E \frac{u_4 - u_3}{\ell} = (68.9 \times 10^9) \frac{(2.449202 - 2.313135) \times 10^{-5}}{0.025} = \underline{\underline{3.75 \text{ MPa}}}$$

1.4

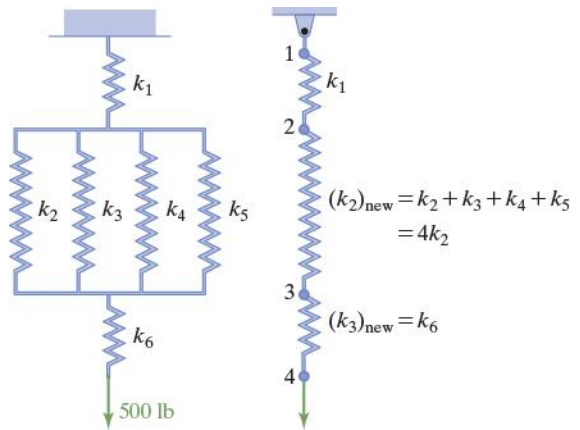
$$K_1 = K_6 = \frac{(4)(0.125)(28 \times 10^6)}{2}$$

$$K_1 = K_6 = 7 \times 10^6 \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_3 = K_4 = K_5 = \frac{(0.625)(0.125)(28 \times 10^6)}{8}$$

$$K_2 = 273438 \frac{\text{lb}}{\text{in}}$$

$$(K_2)_{\text{new}} = (4)(273438) = 1093750 \frac{\text{lb}}{\text{in}}$$



$$\begin{bmatrix} \cancel{7 \times 10^6} & \cancel{-7 \times 10^6} & & & \\ \cancel{-7 \times 10^6} & 7 \times 10^6 + 1093750 & -1093750 & & \\ & -1093750 & 1093750 + 7 \times 10^6 & -7 \times 10^6 & \\ & & -7 \times 10^6 & 7 \times 10^6 & \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$\begin{bmatrix} 8093750 & -1093750 & 0 \\ -1093750 & 8093750 & -7 \times 10^6 \\ 0 & -7 \times 10^6 & 7 \times 10^6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$u_2 = \underline{\underline{7.142857 \times 10^{-5} \text{ in}}} \quad u_3 = \underline{\underline{5.285714 \times 10^{-4} \text{ in}}} \quad u_4 = \underline{\underline{6 \times 10^{-4} \text{ in}}}$$

$$\sigma^{(1)} = E \left(\frac{u_2 - u_1}{\ell} \right) = 28 \times 10^6 \left(\frac{7.142857 \times 10^{-5} \text{ in}}{2} \right) = \underline{\underline{1000 \frac{\text{lb}}{\text{in}^2}}}$$

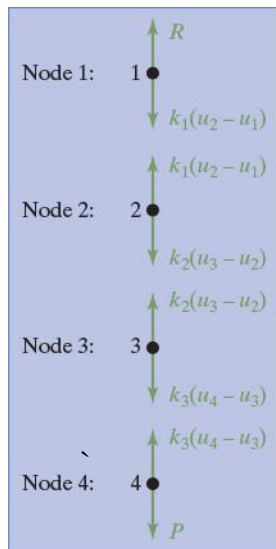
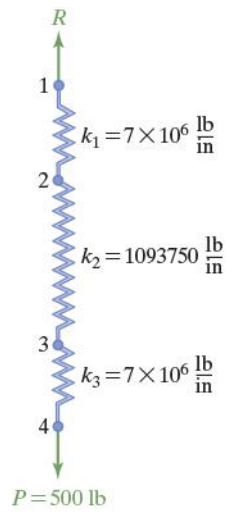
as a check: $\sigma^{(1)} = \frac{F}{A} = \frac{500}{(4)(0.125)} = 1000 \frac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = E \left(\frac{u_3 - u_2}{\ell} \right) = 28 \times 10^6 \left(\frac{5.285714 \times 10^{-4} - 7.142857 \times 10^{-5}}{8} \right) = \underline{\underline{1600 \frac{\text{lb}}{\text{in}^2}}} \quad \leftarrow$$

as a check: $\sigma^{(2)} = \frac{500}{(2.5)(0.125)} = 1600 \frac{\text{lb}}{\text{in}^2}$

$$\sigma^{(3)} = E \left(\frac{u_4 - u_3}{\ell} \right) = 28 \times 10^6 \left(\frac{6 \times 10^{-4} - 5.285714 \times 10^{-4}}{2} \right) = \underline{\underline{1000 \frac{\text{lb}}{\text{in}^2}}}$$

1.5



$$R - K_1(u_2 - u_1) = 0$$

$$K_1(u_2 - u_1) - K_2(u_3 - u_2) = 0$$

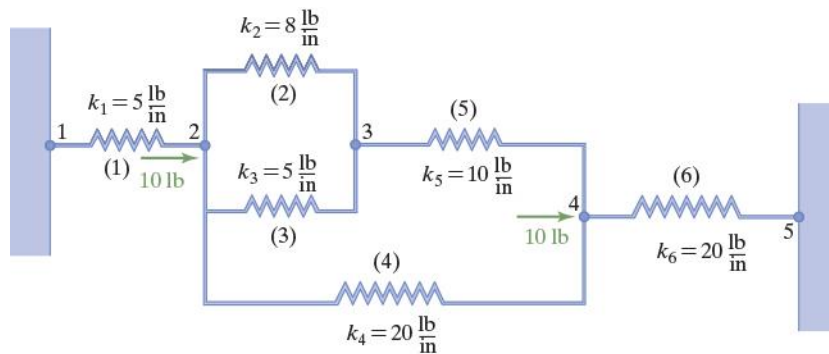
$$K_2(u_3 - u_2) - K_3(u_4 - u_3) = 0$$

$$K_3(u_4 - u_3) - P = 0$$

In matrix form:

$$\begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 + K_2 & -K_2 & \\ & -K_2 & K_2 + K_3 & -K_3 \\ & & K_3 & K_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \\ 0 \\ P \end{Bmatrix}$$

1.6



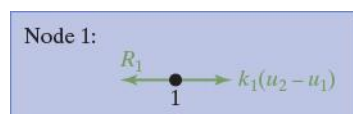
Size of the global matrix: 5×5

$$\begin{aligned}
 [K]^{(1)} &= \begin{bmatrix} \textcircled{1} & \textcircled{2} \\ 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix} & [K]^{(2)} &= \begin{bmatrix} \textcircled{2} & \textcircled{3} \\ 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{matrix} \textcircled{2} \\ \textcircled{3} \end{matrix} \\
 [K]^{(3)} &= \begin{bmatrix} \textcircled{2} & \textcircled{3} \\ 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{matrix} \textcircled{2} \\ \textcircled{3} \end{matrix} & [K]^{(4)} &= \begin{bmatrix} \textcircled{2} & \textcircled{4} \\ 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{matrix} \textcircled{2} \\ \textcircled{4} \end{matrix} \\
 [K]^{(5)} &= \begin{bmatrix} \textcircled{3} & \textcircled{4} \\ 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{matrix} \textcircled{3} \\ \textcircled{4} \end{matrix} & [K]^{(6)} &= \begin{bmatrix} \textcircled{4} & \textcircled{5} \\ 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{matrix} \textcircled{4} \\ \textcircled{5} \end{matrix} \\
 [K]^{(6)} &= \begin{bmatrix} 5 & -5 & & & \\ -5 & 5+8+5+20 & -8-5 & & -20 \\ & -8-5 & 8+5+10 & -10 & \\ & -20 & -10 & 20+10+20 & -20 \\ & & & -20 & 20 \end{bmatrix}
 \end{aligned}$$

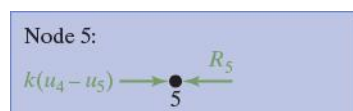
applying B.Cs and loads; $u_1 = u_5 = 0$ $F_2 = 10 \text{ lb}$, $F_4 = 10 \text{ lb}$

$$\begin{bmatrix} 38 & -13 & -20 \\ -13 & 23 & -10 \\ -20 & -10 & 50 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 10 \end{Bmatrix}$$

$$u_2 = \underline{\underline{0.962 \text{ in}}} \quad u_3 = \underline{\underline{0.874 \text{ in}}} \quad u_4 = \underline{\underline{0.760 \text{ in}}} \quad \leftarrow$$



$$R_1 = 5(0.962 - 0) = \underline{\underline{4.8 \text{ lb}}} \quad \leftarrow$$



$$R_5 = 20(0.760 - 0) = \underline{\underline{15.2 \text{ lb}}}$$

1.6 Cont.

$$K(u_4 - u_5)$$

note also as a check,

$$R_1 + R_5 = \sum F_{\text{external}}$$

$$4.8 + 15.2 = 10 + 10$$

1.7

$$150 \begin{bmatrix} \frac{1}{150} & 0 & & & & & & \\ \cancel{5.88} & \cancel{5.88} & & & & & & \\ -5.88 & 5.88 + 2.27 & -2.27 & & & & & \\ & -2.27 & 2.27 + 10 & -10 & & & & \\ & & -10 & 10 + 0.581 & -0.581 & & & \\ & & & -0.581 & 0.581 + 0.781 & -0.781 & & \\ & & & & -0.781 & 0.781 + 2.22 & -2.22 & \\ & & & & & -2.22 & 2.22 + 1.47 & -1.47 \\ & & & & & & \cancel{1.47} & \cancel{1.47} \\ & & & & & & 0 & \frac{1}{150} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 10 \\ 12.04 \\ 17.31 \\ 18.51 \\ 39.12 \\ 54.46 \\ 59.85 \\ 68 \end{Bmatrix} ^\circ\text{F} \quad \Longleftarrow$$

$$q = UA\Delta T$$

$$q = (1.47)(150)(68 - 59.85) = \underline{\underline{1800 \frac{\text{Btu}}{\text{hr}}}} \quad \Longleftarrow$$

or as another example:

$$q = (0.781)(150)(54.46 - 39.12) = 1800 \frac{\text{Btu}}{\text{hr}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{(150)(68 - 10)}{0.17 + 0.44 + 0.1 + 1.72 + 1.28 + 0.45 + 0.68} = 1800 \frac{\text{Btu}}{\text{hr}}$$

1.8

$$150 \begin{bmatrix} \frac{1}{150} & 0 & & & & & \\ \cancel{5.88} & \cancel{-5.88} & & & & & \\ -5.88 & 5.88 + 1.23 & -1.23 & & & & \\ & -1.23 & 1.23 + 0.76 & -0.76 & & & \\ & & -0.76 & 0.76 + 0.053 & -0.053 & & \\ & & & -0.053 & 0.053 + 2.22 & -2.22 & \\ & & & & -2.22 & 2.22 + 1.47 & -1.47 \\ & & & & & \cancel{-1.47} & \cancel{1.47} \\ & & & & & 0 & \frac{1}{150} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 20.37 \\ 22.12 \\ 24.95 \\ 65.57 \\ 66.54 \\ 68 \end{Bmatrix} ^\circ\text{F} \quad \leftarrow$$

$$q = UA\Delta T$$

as an example:

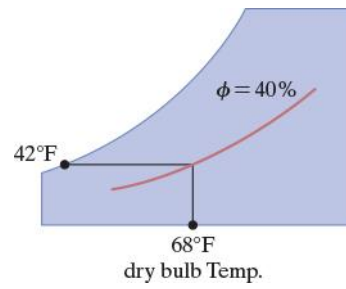
$$q = (1.47)(150)(68 - 66.54) = \underline{\underline{320 \frac{\text{Btu}}{\text{hr}}}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{(150)(68 - 20)}{0.17 + 0.81 + 1.32 + 19 + 0.45 + 0.68} = 320 \frac{\text{Btu}}{\text{hr}}$$

1.9

with the help of a psychometric chart, using a dry bulb Temp. of 68°F and $\phi = 40\%$, we identify condensation temperature to be 42°F . Thus condensation will occur between surfaces 4 and 5.



1.10

$$1000 \begin{bmatrix} \frac{1}{1000} & 0 & & & \\ \cancel{1.47} & \cancel{-1.47} & & & \\ -1.47 & 1.47 + 0.053 & -0.053 & & \\ & -0.053 & 0.053 + 2.22 & -2.22 & \\ & & -2.22 & 2.22 + 1.47 & -1.47 \\ & & & \cancel{-1.47} & \cancel{1.47} \\ & & & 0 & \frac{1}{1000} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 15 \\ 16.81 \\ 66.99 \\ 68.19 \\ 70 \end{Bmatrix} ^\circ\text{F} \quad \Longleftarrow$$

$$q = UA\Delta T$$

as an example:

$$q = (1.47)(1000)(70 - 68.19) = 2660 \frac{\text{Btu}}{\text{hr}} \quad \Longleftarrow$$

or

$$q = (1.47)(1000)(16.81 - 15) = 2660 \frac{\text{Btu}}{\text{hr}}$$

1.11

$$22.5 \begin{bmatrix} \frac{1}{22.5} & 0 & & \\ \cancel{5.88} & \cancel{-5.88} & & \\ -5.88 & 5.88 + 2.56 & -2.56 & \\ & -2.56 & 2.56 + 1.47 & -1.47 \\ & & \cancel{-1.47} & \cancel{1.47} \\ & & 0 & \frac{1}{22.5} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 26.85 \\ 42.59 \\ 70 \end{Bmatrix} ^\circ\text{F}$$

$$q = U_1 A \Delta T = (5.88)(22.5)(26.85 - 20) = 910 \frac{\text{Btu}}{\text{hr}}$$

or as another example:



$$q = U_2 A \Delta T = (2.56)(22.5)(42.59 - 26.85) = 910 \frac{\text{Btu}}{\text{hr}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{1}{(0.17 + 0.39 + 0.68)} (22.5)(70 - 20)$$

$$q = 910 \frac{\text{Btu}}{\text{hr}}$$



1.12

$$(A_1)_c = (A_5)_c = \left(\frac{12 + 10.5}{2} \right) (3) - (3) \frac{\pi}{4} (0.5)^2 = 33.161 \text{ in}^2$$

$$(A_2)_c = (A_4)_c = \frac{(10.5 + 9)}{2} (3) - 3 \frac{\pi}{4} (0.5)^2 = 28.661 \text{ in}^2$$

$$(A_3)_c = (9)(3) - 3 \frac{\pi}{4} (0.5)^2 = 26.411 \text{ in}^2$$

$$(K_1)_c = (K_5)_c = \frac{(33.161)(3.27 \times 10^6)}{6} = 18,072,745 \frac{\text{lb}}{\text{in}}$$

$$(K_2)_c = (K_4)_c = \frac{(28.661)(3.27 \times 10^6)}{6} = 15,620,245 \frac{\text{lb}}{\text{in}}$$

$$(K_3)_c = \frac{(26.411)(3.27 \times 10^6)}{4} = 21,590,992 \frac{\text{lb}}{\text{in}}$$

$$(K_1)_s = (K_2)_s = (K_4)_s = (K_5)_s = \frac{(3) \left(\frac{\pi}{4} \right) (0.5)^2 (29 \times 10^6)}{6} = 2,847,068 \frac{\text{lb}}{\text{in}}$$

$$(K_3)_s = \frac{(3) \left(\frac{\pi}{4} \right) (0.5)^2 (29 \times 10^6)}{4} = 4,270,602 \frac{\text{lb}}{\text{in}}$$

The Combined stiffnesses:

$$K_1 = K_5 = 18,072,745 + 2,847,068 = 20,919,813$$

$$K_2 = K_4 = 15,620,245 + 2,847,068 = 18,467,313$$

$$K_3 = 21,590,992 + 4,270,602 = 25,861,594$$

~~20,919,813~~ 1	~~-20,919,813~~ 0				
~~-20,919,813~~ 0	20,919,813 +18,467,313	-18,467,313			
	-18,467,313	18,467,313 +25,861,594	-25,861,594		
		-25,861,594	25,861,594 +18,467,313	-18,467,313	
			-18,467,313	18,467,313 +20,919,813	-20,919,813
				-20,919,813	20,919,813

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1000 \end{Bmatrix}$$

$$u_1 = 0 \quad u_2 = -4.78016 \times 10^{-5} \text{ in} \quad u_3 = -1.01951 \times 10^{-4} \text{ in}$$

$$\underline{\underline{u_4 = -1.40618 \times 10^{-4}}} \quad \underline{\underline{u_5 = -1.94768 \times 10^{-4} \text{ in}}} \quad \underline{\underline{u_6 = -2.42570 \times 10^{-4} \text{ in}}}$$



1.12 Cont.

$$\sigma_{\text{Concrete}}^{(1)} = E_c \frac{(u_2 - u_1)}{\ell} = \frac{(3.27 \times 10^6)(-4.78016 \times 10^{-5})}{6} = \underline{\underline{26 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma_{\text{Steel}}^{(1)} = E_s \frac{(u_2 - u_1)}{\ell} = \underline{\underline{231 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

as a check:

$$\sigma_{\text{Concrete}}^{(1)}(A_1)_c + \sigma_s^{(1)}(A_1)_s = (26)(33.161) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = \underline{\underline{1000 \text{ lb}}}$$

$$\sigma_c^{(2)} = E_c \left(\frac{u_3 - u_2}{\ell} \right) = \frac{(3.27 \times 10^6)(-1.01951 \times 10^{-4} + 4.78016 \times 10^{-5})}{6} = \underline{\underline{30 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma_s^{(2)} = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

$$\text{Check: } \sigma_c^{(2)}(A_2)_c + \sigma_s^{(2)}(A_2)_s = (30)(28.661) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$$

$$\sigma_c^{(3)} = E_c \frac{(u_4 - u_3)}{\ell} = \frac{(3.27 \times 10^6)(-1.40618 \times 10^{-4} + 1.01951 \times 10^{-4})}{4} = \underline{\underline{32 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma_s^{(3)} = E_s \frac{(u_4 - u_3)}{\ell} = \frac{(29 \times 10^6)(-1.40618 \times 10^{-4} + 1.01951 \times 10^{-4})}{4} = \underline{\underline{280 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\text{Check: } \sigma_c^{(3)}(A_3)_c + \sigma_s^{(3)}(A_3)_s = (32)(26.411) + (280)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$$

$$\sigma_c^{(4)} = E_c \frac{(u_5 - u_4)}{\ell} = \frac{(3.27 \times 10^6)(-1.94768 \times 10^{-4} + 1.40618 \times 10^{-4})}{6} = \underline{\underline{30 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma_s^{(4)} = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

$$\sigma_c^{(5)} = E_c \frac{(u_6 - u_5)}{\ell} = \frac{(3.27 \times 10^6)(-2.42570 + 1.94768) \times 10^{-4}}{6} = \underline{\underline{26 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma_s^{(5)} = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

note $\sigma_c^{(1)} = \sigma_c^{(5)}$ and $\sigma_c^{(2)} = \sigma_c^{(4)}$ as expected.

1.13

$$\Lambda^{(e)} = \frac{A_{\text{avg}} E}{2\ell} (u_{i+1} - u_i)^2$$

$$\Lambda_{\text{total}} = \Lambda^{(1)} + \Lambda^{(2)} + \Lambda^{(3)} + \Lambda^{(4)} + \Lambda^{(5)}$$

$$\begin{aligned} \Lambda^{(1)} = \Lambda^{(5)} &= \frac{(A_1)_c E_c}{2\ell} (u_{i+1} - u_i)^2 + \frac{(A_1)_s E_s}{2\ell} (u_{i+1} - u_i)^2 \\ &= \frac{(A_1)_c E_c + (A_1)_s E_s}{2\ell} (u_2 - u_1)^2 \end{aligned}$$

$$\Lambda^{(1)} = \left[\frac{(33.161)(3.27 \times 10^6) + 3 \left(\frac{\pi}{4} \right) (0.5)^2 (29 \times 10^6)}{(2)(6)} \right] (-4.78016 \times 10^{-5})^2 = 0.024 \text{ lb} \cdot \text{in}$$

or

$$\Lambda^{(5)} = \left[\frac{(33.161)(3.27 \times 10^6) + 3 \left(\frac{\pi}{4} \right) (0.5)^2 (29 \times 10^6)}{(2)(6)} \right] = [(-2.4257 + 1.94768) \times 10^{-4}]^2 = 0.024 \checkmark$$

$$\begin{aligned} \Lambda^{(2)} &= \frac{(A_2)_c E_c + (A_2)_s E_s}{2\ell} (u_3 - u_2) \\ &= \left[\frac{(28.661)(3.27 \times 10^6) + 3 \frac{\pi}{4} (0.5)^2 (29 \times 10^6)}{2(6)} \right] [(-1.01951 \times 10^{-4} + 4.78016 \times 10^{-5})]^2 = 0.027 \end{aligned}$$

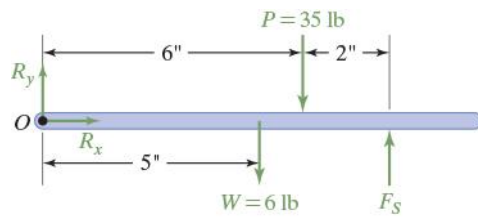
or

$$\Lambda^{(4)} = \left[\frac{(28.661)(3.27 \times 10^6) + 3 \frac{\pi}{4} (0.5)^2 (29 \times 10^6)}{2(6)} \right] = [(-1.94768 + 1.40618) \times 10^{-4}]^2 = 0.027 \checkmark$$

$$\begin{aligned} \Lambda^{(3)} &= \frac{(A_3)_c E_c + (A_3)_s E_s}{2\ell} (u_4 - u_3) \\ \Lambda^{(3)} &= \left[\frac{(26.411)(3.27 \times 10^6) + 3 \frac{\pi}{4} (0.5)^2 (29 \times 10^6)}{2(4)} \right] [(-1.40618 + 1.01951 \times 10^{-4})]^2 \\ &= 0.019 \text{ lb} \cdot \text{in} \end{aligned}$$

$$\Lambda_{\text{total}} = (2)(0.024) + 0.019 + 2(0.027) = \underline{\underline{0.121 \text{ lb} \cdot \text{in}}} \quad \leftarrow$$

1.14

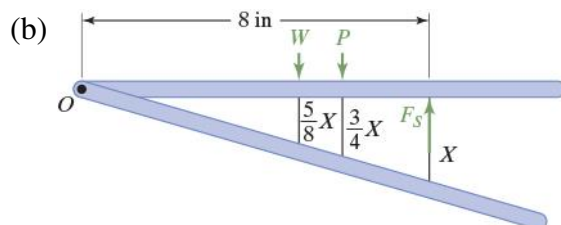


(a) $\sum M_0 = 0$

$$(-6 \text{ lb})(5 \text{ in}) + (35 \text{ lb})(6 \text{ in}) + F_s(8 \text{ in}) = 0$$

$$F_s = 30 \text{ lb}$$

$$F_s = KX \quad 30 \text{ lb} = 60 X \quad \underline{\underline{X = 0.5 \text{ in}}} \quad \leftarrow$$



$$\frac{\partial \pi}{\partial u_i} = \frac{\partial}{\partial u_i} \sum_{i=1}^n \Lambda^{(e)} - \frac{\partial}{\partial u_i} \sum_{i=1}^m F_i u_i = 0$$

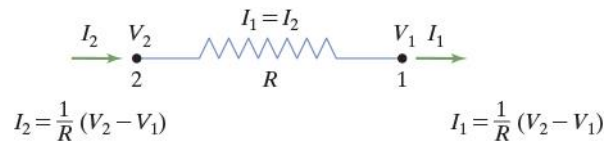
$$\frac{\partial}{\partial X} \left(\frac{1}{2} KX^2 \right) - \frac{\partial}{\partial X} \left(35 \left(\frac{3}{4} X \right) \right) - (6) \left(\frac{5}{8} X \right) = 0$$

$$KX - (35) \left(\frac{3}{4} \right) - (6) \left(\frac{5}{8} \right) = 0$$

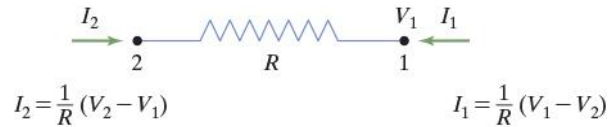
$$X = \frac{(35) \left(\frac{3}{4} \right) + (6) \left(\frac{5}{8} \right)}{60}$$

$$\underline{\underline{X = 0.5 \text{ in}}} \quad \leftarrow$$

1.15



(or)



Because of the fact that charge is conserved in a circuit (Kirchhoff's current law), at any time, the algebraic sum of the currents entering any node must be zero. Thus we can write

$$I_1 = \frac{1}{R} (V_1 - V_2)$$

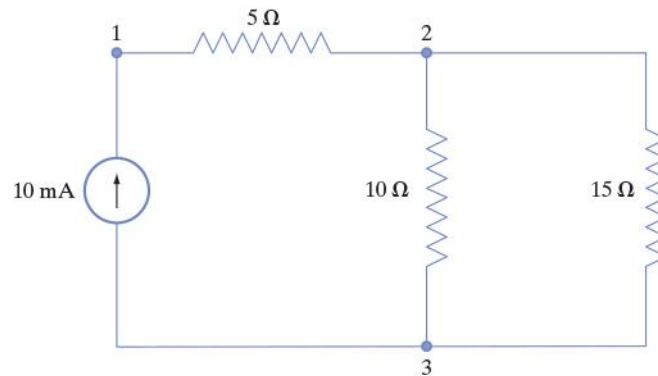
$$I_2 = \frac{1}{R} (V_2 - V_1)$$

Note $I_1 + I_2 = 0$

In matrix form:

$$\frac{1}{R} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

1.16



$$[K]^{(1)} = \frac{1}{5} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = \frac{1}{10} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(3)} = \frac{1}{15} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \text{ Apply } V_3 = 0 \text{ as a boundary condition}$$

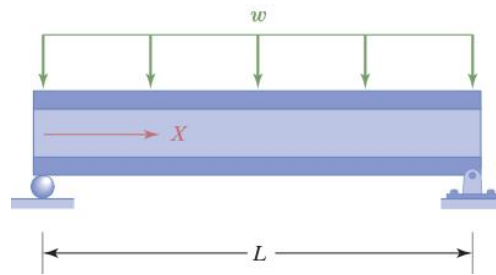
$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5} + \frac{1}{10} + \frac{1}{15} & -\frac{1}{10} - \frac{1}{15} \\ 0 & -\frac{1}{10} - \frac{1}{15} & \frac{1}{10} + \frac{1}{15} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 0 \\ 0 \end{Bmatrix}$$

$$V_1 = 0.11 \quad V_2 = 0.06$$

$$\underline{\underline{V_1 - V_2 = 0.05 \text{ Volts}}} \quad \leftarrow$$

$$\underline{\underline{V_2 - V_3 = 0.06 \text{ Volts}}} \quad \leftarrow$$

1.17



$$\frac{d^2Y}{dX^2} = \frac{M(X)}{EI} = \frac{wX(L-X)}{2EI}$$

$$\frac{dY}{dX} = \frac{1}{2EI} \left(\frac{wX^2L}{2} - \frac{wX^3}{3} \right) + C_1$$

$$Y = \frac{1}{2EI} \left(\frac{wX^3L}{6} - \frac{wX^4}{12} \right) + C_1X + C_2$$

Applying Boundary Conditions:

$$Y = 0 \quad @ \quad X = 0 \quad C_2 = 0$$

$$Y = 0 \quad @ \quad X = L \quad C_1 = -\frac{wL^3}{24EI}$$

$$\underline{\underline{Y_{\text{exact}} = \frac{-wX}{24EI} (X^3 - 2LX^2 + L^3)}} \quad \leftarrow$$

(a) Note that the assumed Solution satisfies the boundary conditions. $Y = C_1 \left[\left(\frac{X}{L} \right)^2 - \left(\frac{X}{L} \right) \right]$

$$\frac{dY}{dX} = C_1 \left[\frac{2X}{L^2} - \frac{1}{L} \right]$$

$$\frac{d^2Y}{dX^2} = C_1 \left(\frac{2}{L^2} \right)$$

$$\frac{2C_1}{L^2} - \frac{wX(L-X)}{2EI} = R$$

We may force the error function to equal zero at $X = \frac{L}{2}$

$$\frac{2C_1}{L^2} - \frac{w \frac{L}{2} \left(L - \frac{L}{2} \right)}{2EI} = 0 \rightarrow C_1 = \frac{wL^4}{16EI}$$

$$\underline{\underline{Y = \frac{wL^4}{16EI} \left[\left(\frac{X}{L} \right)^2 - \left(\frac{X}{L} \right) \right]}} \quad \leftarrow$$

1.17 Cont.

$$(b) \int_0^L R dX = 0$$

$$\int_0^L \left[\frac{2c_1}{L^2} - \frac{wX(L-X)}{2EI} \right] dX = 0$$

$$\frac{2c_1}{L^2} L - \frac{w}{2EI} \left(\frac{L^3}{2} - \frac{L^3}{3} \right) = 0 \rightarrow c_1 = \frac{wL^4}{24EI}$$

$$\underline{\underline{Y = \frac{wL^4}{24EI} \left(\left(\frac{X}{L} \right)^2 - \left(\frac{X}{L} \right) \right)}} \quad \leftarrow$$

$$\text{For } W24 \times 104 \text{ Beam } I = 3100 \text{ in}^4; E = 29 \times 10^6 \frac{\text{lb}}{\text{in}^2}$$

Exact

$$Y_{\max} = \frac{-5wL^4}{384EI} = \frac{-5 \left(5000 \frac{\text{lb}}{\text{ft}} \times \frac{1 \text{ ft}}{12 \text{ in}} \right) \left(20 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} \right)^4}{(384) \left(29 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right) (3100 \text{ in}^4)} = \underline{\underline{-0.20 \text{ in}}}$$

Collocation

$$Y_{\max} = \frac{-wL^4}{64EI} = \frac{- \left(5000 \frac{\text{lb}}{\text{ft}} \times \frac{1 \text{ ft}}{12 \text{ in}} \right) \left(20 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} \right)^4}{(64) \left(29 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right) (3100 \text{ in}^4)} = \underline{\underline{-0.24 \text{ in}}} \quad \leftarrow$$

Subdomain

$$Y_{\max} = \frac{-wL^4}{96EI} = \frac{- \left(5000 \frac{\text{lb}}{\text{ft}} \times \frac{1 \text{ ft}}{12 \text{ in}} \right) \left(20 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} \right)^4}{(96) \left(29 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right) (3100 \text{ in}^4)} = \underline{\underline{-0.16 \text{ in}}}$$

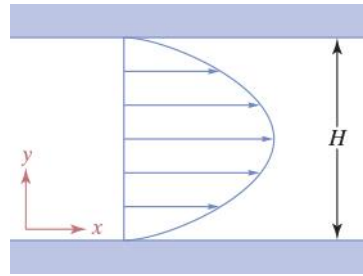
1.18 See Section 1.7

1.19

$$\mu \frac{d^2 u}{dy^2} = \frac{dP}{dX}$$

$$\frac{du}{dy} = \frac{1}{\mu} \frac{dP}{dX} y + c_1$$

$$u(y) = \frac{1}{\mu} \frac{dP}{dX} \frac{y^2}{2} + c_1 y + c_2$$



applying B.Cs $u(0) = 0$ and $u(H) = 0$

$$0 = 0 + 0 + c_2 \quad \text{and} \quad 0 = \frac{1}{\mu} \frac{dP}{dX} \frac{H^2}{2} + c_1 H$$

$$c_2 = 0 \quad \text{and} \quad c_1 = -\frac{H}{2\mu} \frac{dP}{dX}$$

$$\underline{\underline{u(y) = -\frac{1}{2\mu} \frac{dP}{dX} (Hy - y^2)}}$$

note because Pressure drops in the direction of flow, $\frac{dP}{dX} < 0$ in velocity equation.

$$(a) \quad \mu \frac{d^2 u}{dy^2} = \frac{dP}{dX} \quad u_{\text{assumed}} = c_1 \sin\left(\frac{\pi y}{H}\right)$$

note the assumed solution Satisfies the boundary conditions.

$$\frac{du}{dy} = \frac{d}{dy} \left(c_1 \sin\left(\frac{\pi y}{H}\right) \right) = c_1 \frac{\pi}{H} \cos\left(\frac{\pi y}{H}\right)$$

$$\frac{d^2 u}{dy^2} = \frac{d}{dy} \left(c_1 \frac{\pi}{H} \cos\left(\frac{\pi y}{H}\right) \right) = -c_1 \left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)$$

$$\mu \left[-c_1 \left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right) \right] - \frac{dP}{dX} = R$$

We may force the error function to equal zero at $y = \frac{H}{2}$

$$\mu \left[-c_1 \left(\frac{\pi}{H}\right)^2 \right] \sin\left(\frac{\pi}{H} \frac{H}{2}\right) - \frac{dP}{dX} = 0$$

$$c_1 = -\frac{H^2}{\mu \pi^2} \frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{\mu \pi^2} \frac{dP}{dX} \left(\sin\left(\frac{\pi y}{H}\right) \right)}} \quad \leftarrow$$

1.19 Cont.

$$(b) \int_0^H R dy = 0$$

$$\int_0^H \left[\mu \left[-c_1 \left(\frac{\pi}{H} \right)^2 \sin \frac{\pi y}{H} \right] - \frac{dP}{dX} \right] dy = 0$$

$$\mu c_1 \left(\frac{\mu}{H} \right)^2 \left[\frac{1}{\frac{\pi}{H}} \cos \frac{\pi y}{H} \right]_0^H - \frac{dP}{dX} H = 0$$

$$\mu c_1 \frac{\pi}{H} (-2) = H \frac{dP}{dX}$$

$$c_1 = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \left[\sin \left(\frac{\pi y}{H} \right) \right]}} \quad \Longleftarrow$$

For Comparison, let's use:

$$\mu = 0.02 \frac{\text{N} \cdot \text{S}}{\text{m}^2}; \quad H = 0.01 \text{ mm} = 1 \times 10^{-5} \text{ m}; \quad \frac{dP}{dX} = -1 \times 10^8 \frac{\text{Pa}}{\text{m}}$$

evaluating max vel. @ $y = \frac{H}{2}$

Exact

$$u_{\max} = -\frac{1}{2\mu} \frac{dP}{dX} \left(H \left(\frac{H}{2} \right) - \left(\frac{H}{2} \right)^2 \right) = -\frac{1}{2\mu} \frac{dP}{dX} \frac{H^2}{4} = -\frac{(-1 \times 10^8)(1 \times 10^{-5})^2}{(8)(0.02)} = \underline{\underline{0.06 \frac{\text{m}}{\text{s}}}}$$

Collocation method

$$u_{\max} = -\frac{H^2}{\mu\pi^2} \frac{dP}{dX} \left(\sin \left(\frac{\pi \frac{H}{2}}{H} \right) \right) = -\frac{H^2}{\mu\pi^2} \frac{dP}{dX} = -\frac{(1 \times 10^{-5})^2 (-1 \times 10^8)}{(0.02)\pi^2} = \underline{\underline{0.05 \frac{\text{m}}{\text{s}}}}$$

Subdomain method

$$u_{\max} = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \left(\sin \left(\frac{\pi \frac{H}{2}}{H} \right) \right) = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} = -\frac{(1 \times 10^{-5})^2 (-1 \times 10^8)}{(2)(0.02)\pi} = \underline{\underline{0.08 \frac{\text{m}}{\text{s}}}}$$

1.20

(a) *Galerkin method*

$$\begin{aligned}
 \int_0^H \sin\left(\frac{\pi y}{H}\right) \left[\mu \left(-C_1 \left(\frac{\pi}{H} \right)^2 \sin \frac{\pi y}{H} \right) - \frac{dP}{dX} \right] dy &= 0 \\
 -\mu C_1 \left(\frac{\pi}{H} \right)^2 \int_0^H \sin^2 \left(\frac{\pi y}{H} \right) dy &= \frac{dP}{dX} \int_0^H \sin \left(\frac{\pi y}{H} \right) dy \\
 -\mu C_1 \left(\frac{\pi}{H} \right)^2 \left[\frac{1}{2} y - \frac{1}{4 \left(\frac{\pi}{H} \right)} \sin \left(\frac{2\pi}{H} y \right) \right]_0^H &= \frac{dP}{dX} \left[-\frac{1}{\frac{\pi}{H}} \cos \left(\frac{\pi}{H} y \right) \right]_0^H \\
 -\mu C_1 \left(\frac{\pi}{H} \right)^2 \left(\frac{1}{2} H \right) &= -\frac{H}{\pi} \frac{dP}{dX} (-2) \\
 C_1 = \frac{-4H^2}{\mu\pi^3} \frac{dP}{dX} \quad \text{then, } \underline{\underline{u(y) = -\frac{4H^2}{\mu\pi^3} \frac{dP}{dX} \sin\left(\frac{\pi y}{H}\right)}} &\quad \leftarrow
 \end{aligned}$$

(b) *least-squares method*

$$\int_0^H R \frac{\partial R}{\partial C_1} dy = \int_0^H \overbrace{\sin\left(\frac{\pi y}{H}\right) \left[\mu \left(-C_1 \left(\frac{\pi}{H} \right)^2 \right) \sin \frac{\pi y}{H} - \frac{dP}{dX} \right]}^R \overbrace{\left[-\mu \left(\frac{\pi}{H} \right)^2 \sin \frac{\pi y}{H} \right]}^{\frac{\partial R}{\partial C_1}} dy = 0$$

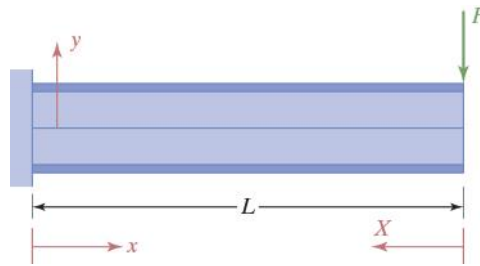
results in:

$$C_1 = -\frac{4H^2}{\mu\pi^3} \frac{dP}{dX} \quad \text{and} \quad \underline{\underline{u(y) = -\frac{4H^2}{\mu\pi^3} \frac{dP}{dX} \sin\left(\frac{\pi y}{H}\right)}} \quad \leftarrow$$

using values from Problem 1.19 for comparison

$$u_{\max} = \frac{-4H^2}{\mu\pi^3} \frac{dP}{dX} = \frac{-4(1 \times 10^{-5})^2 (-1 \times 10^8)}{(0.02)\pi^3} = \underline{\underline{0.06 \frac{\text{m}}{\text{s}}}} \quad \leftarrow$$

1.21



$$\frac{d^2y}{dX^2} = -\frac{PX}{EI}$$

$$\frac{dy}{dX} = -\frac{P}{EI} \frac{X^2}{2} + c_1$$

$$y = -\frac{P}{EI} \frac{X^3}{6} + c_1X + c_2$$

B.Cs ① $y(L) = 0$, ② $\left. \frac{dy}{dX} \right|_{X=L} = 0$

$$\textcircled{2} \quad 0 = -\frac{P}{EI} \frac{L^2}{2} + c_1 \quad c_1 = \frac{P}{EI} \frac{L^2}{2}$$

$$\textcircled{1} \quad 0 = -P \frac{L^3}{6} + \frac{P}{EI} \frac{L^2}{2} L + C_2 \quad C_2 = \frac{P}{EI} \left(-\frac{1}{3} L^3 \right)$$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI} (-X^3 + 3L^2X - 2L^3)}} \quad \leftarrow$$

In terms of x : Substitute for $X = L - x$ and simplify

$$y_{\text{exact}} = \frac{P}{6EI} (-(L-x)^3 + 3L^2(L-x) - 2L^3)$$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI} (x^3 - 3L^2x)}} \quad \leftarrow$$

$$EI \frac{d^2y}{dx^2} + P(L-x) = 0$$

B.Cs $y(0) = 0$ and $\left. \frac{dy}{dx} \right|_{x=0} = 0$

Let us assume: $y_{\text{assumed}} = c_1x^2 + c_2x^3$

note, the assumed solution satisfies the boundary conditions.

1.21 Cont.

$$\frac{dy}{dx} = 2c_1x + 3c_2x^2$$

$$\frac{d^2y}{dx^2} = 2c_1 + 6c_2x$$

then Residual R becomes:

$$R = EI(2c_1 + 6c_2x) + P(L - x)$$

Subdomain method:

$$\int_0^{\frac{L}{2}} R dx = 0 \quad \int_{\frac{L}{2}}^L [EI(2c_1 + 6c_2x) + P(L - x)] dx = 0 \quad (1)$$

$$\int_{\frac{L}{2}}^L R dx = 0 \quad \int_0^{\frac{L}{2}} [EI(2c_1 + 6c_2x) + P(L - x)] dx = 0 \quad (2)$$

Integrating (1):

$$EI \left(2c_1 \left(\frac{L}{2} \right) + 6c_2 \frac{\left(\frac{L}{2} \right)^2}{2} \right) + P \left(L \left(\frac{L}{2} \right) - \frac{\left(\frac{L}{2} \right)^2}{2} \right) = 0$$

Simplifying:

$$2c_1 + \frac{3}{2}c_2L = -\frac{3PL}{4EI} \quad (3)$$

Integrating (2) and Simplifying:

$$c_1 + \frac{9}{4}c_2L = -\frac{PL}{8EI} \quad (4)$$

Solving (3) and (4) Simultaneously:

$$\underline{\underline{c_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{c_2 = \frac{P}{6EI}}} \quad \Leftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

note, because the assumed solution has the same functional form as the exact solution, the coefficients are exact.

1.21 Cont.

Galerkin method:

$$\int_0^L x^2 R dx = 0 \quad \int_0^L x^2 [EI(2c_1 + 6c_2) + P(L - x)] dx = 0 \quad (1)$$

$$\int_0^L x^3 R dx = 0 \quad \int_0^L x^3 [EI(2c_1 + 6c_2) + P(L - x)] dx = 0 \quad (2)$$

Integrating and Simplifying (1) and (2), we get:

$$\begin{cases} \frac{2}{3}c_1 + \frac{3}{2}c_2L = -\frac{PL}{12EI} \end{cases} \quad (3)$$

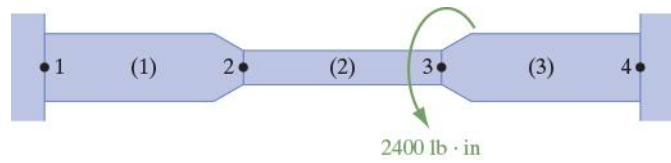
$$\begin{cases} \frac{1}{2}c_1 + \frac{6}{5}c_2L = -\frac{PL}{20EI} \end{cases} \quad (4)$$

Solving (3) and (4) Simultaneously

$$\underline{\underline{c_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{c_2 = \frac{P}{6EI}}} \quad \Longleftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

1.22



$$J_1 = J_3 = \frac{1}{2} \pi r^4$$

$$J_1 = J_3 = \frac{1}{2} \pi \left(\frac{1.5}{2} \right)^4 = 0.497 \text{ in}^4$$

$$J_2 = \frac{1}{2} \pi \left(\frac{1}{2} \right)^4 = 0.0982 \text{ in}^4$$

$$[K]^{(1)} = [K]^{(3)} = \frac{JG}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{(0.497)(9.8 \times 10^6)}{(2)(12)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 202942 & -202942 \\ -202942 & 202942 \end{bmatrix}$$

$$[K]^{(2)} = \frac{(0.0982)(11.2 \times 10^6)}{(1.5)(12)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 61102 & -61102 \\ -61102 & 61102 \end{bmatrix}$$

~~202942~~ 1	~~-202942~~ 0		
~~-202942~~ 0	202942 + 61102	-61102	
	-61102	61102 + 202942	~~-202942~~ 0
		~~-202942~~ 0	~~202942~~ 1

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2400 \\ 0 \end{Bmatrix}$$

$$\theta_1 = 0 \quad \theta_2 = 0.002222 \text{ rad} \quad \theta_3 = 0.009604 \text{ rad} \quad \theta_4 = 0$$

$$R_4 = K_3(\theta_3 - 0) = 202942 \times 0.009604 = 1949 \text{ lb} \cdot \text{in}$$

note they
add up to 2400 lb·in

$$R_1 = K_1(\theta_2 - 0) = 202942 \times 0.002222 = 451 \text{ lb} \cdot \text{in}$$

1.23

$$\begin{bmatrix} 264044 & -61102 \\ -61102 & 264044 \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 1200 \\ 1200 \end{Bmatrix}$$

$$\underline{\underline{\theta_2 = \theta_3 = 0.005913 \text{ rad}}} \quad \longleftarrow \square$$

$$\underline{\underline{R_1 = R_4 = K_1(\theta_2 - 0) = K_3(\theta_3 - 0) = 202942 \times 0.005913 = 1200 \text{ lb} \cdot \text{in}}}$$

1.24

$$A_1 = 0.5 \text{ in}^2$$

$$A_2 = 0.4375 \text{ in}^2$$

$$A_3 = 0.3125 \text{ in}^2$$

$$A_4 = 0.25 \text{ in}^2$$

$$K = \frac{A_{\text{avg}} E}{\ell}$$

$$K_1 = \frac{\left(\frac{0.5 + 0.4375}{2} \right) (10.6 \times 10^6)}{2.5} = 1,987,500 \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{\left(\frac{0.375 + 0.25}{2} \right) (10.6 \times 10^6)}{5} = 662,500 \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{\left(\frac{0.3125 + 0.25}{2} \right) (10.6 \times 10^6)}{2.5} = 1,192,500 \frac{\text{lb}}{\text{in}}$$

$\begin{array}{c} 1 \\ \hline 1987500 \\ \hline -1987500 \\ \hline 0 \end{array}$	$\begin{array}{c} 0 \\ \hline -1987500 \\ \hline 1987500 + 662500 \\ \hline -662500 \end{array}$	$\begin{array}{c} \\ \hline -662500 \\ \hline 662500 + 1192500 \\ \hline -1192500 \end{array}$	$\begin{array}{c} \\ \hline \\ \hline 1192500 \end{array}$
---	--	--	--

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1500 \end{Bmatrix}$$

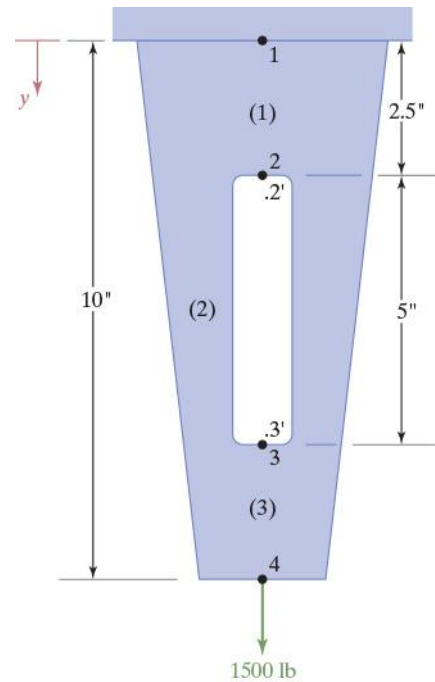
$$u_1 = 0 \quad \underline{\underline{u_2 = 7.5472 \times 10^{-4} \text{ in}}} \quad \underline{\underline{u_3 = 0.003020 \text{ in}}} \quad \underline{\underline{u_4 = 0.004277 \text{ in}}}$$

$$\sigma^{(1)} = (10.6 \times 10^6) \left(\frac{7.5472 \times 10^{-4}}{2.5} \right) = 3200 \frac{\text{lb}}{\text{in}^2}$$

as a check: $\sigma^{(1)} = \frac{1500}{\frac{0.5 + 0.4375}{2}} = 3200 \frac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = (10.6 \times 10^6) \left(\frac{0.00302 - 7.5472 \times 10^{-4}}{5} \right) = 4800 \frac{\text{lb}}{\text{in}^2}$$

check: $\sigma^{(2)} = \frac{1500}{\left(\frac{0.375 + 0.25}{2} \right)} = 4800 \frac{\text{lb}}{\text{in}^2}$



without the hole:

$$A(y) = (4 - 0.2y)(0.125) = 0.5 - 0.025y$$

1.24 Cont.

$$\sigma^{(3)} = (10.6 \times 10^6) \left(\frac{0.004277 - 0.00302}{2.5} \right) = 5300 \frac{\text{lb}}{\text{in}^2}$$

$$\text{check: } \sigma^{(3)} = \frac{1500}{\frac{0.3125 + 0.25}{2}} = 5300 \text{ Psi}$$

1.25

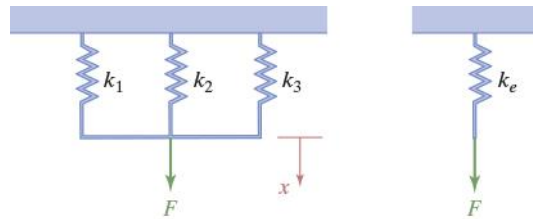
Springs in Parallel

$$F = F_1 + F_2 + F_3$$

$$k_e x = k_1 x + k_2 x + k_3 x$$



$$\underline{\underline{k_e = k_1 + k_2 + k_3}}$$



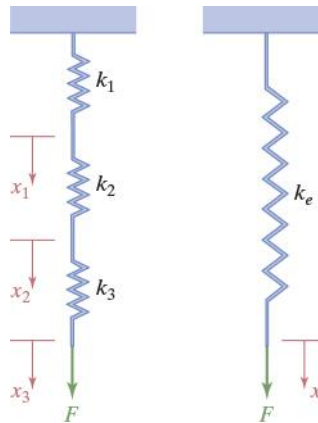
Springs in Series

$$x = x_1 + x_2 + x_3$$

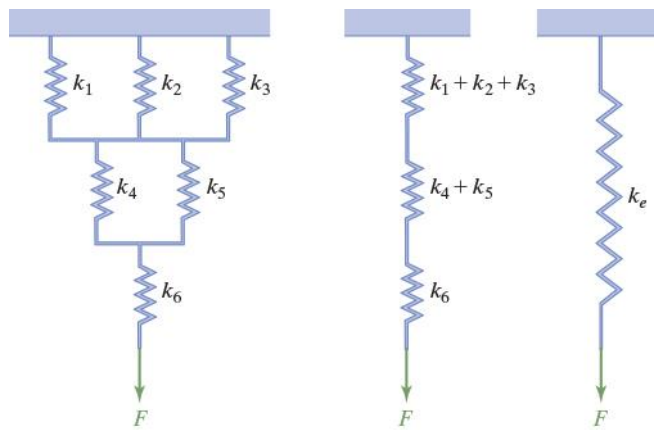
$$\frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$\underline{\underline{k_e = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}}}}}$$



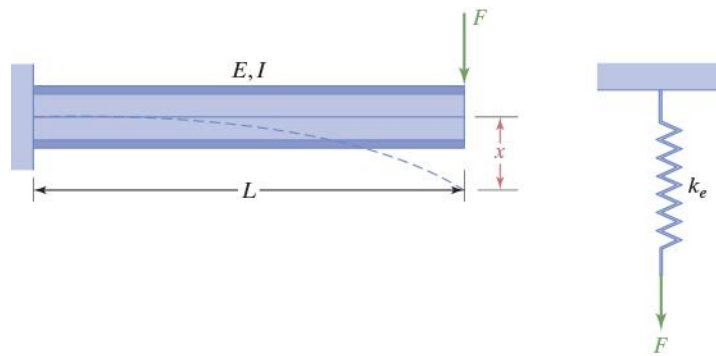
1.26



$$k_e = \frac{1}{\frac{1}{k_1 + k_2 + k_3} + \frac{1}{k_4 + k_5} + \frac{1}{k_6}}$$



1.27



From Table 4.1

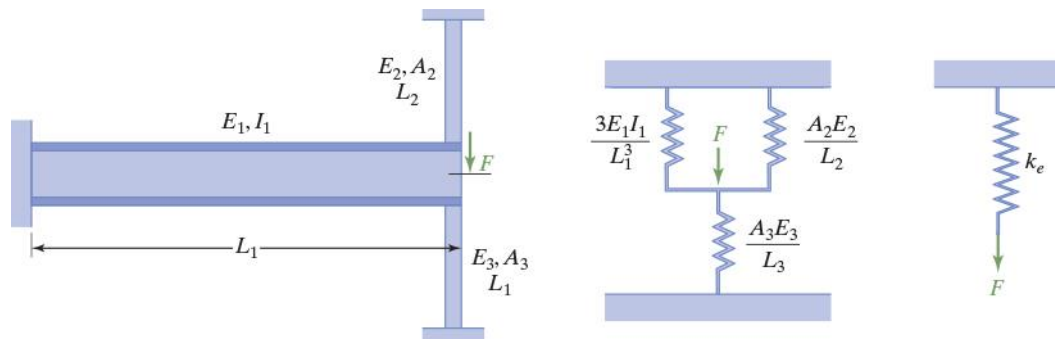
$$X = \frac{FL^3}{3EI}$$

$$F = \frac{3EI}{L^3} x$$

$$\underline{\underline{K_e = \frac{3EI}{L^3}}}$$



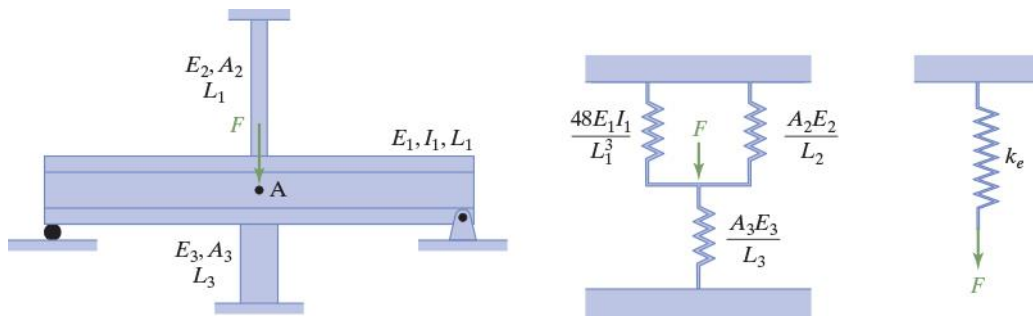
1.28



$$K_e = \frac{3E_1 I_1}{L_1^3} + \frac{A_2 E_2}{L_2} + \frac{A_3 E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in Parallel.

1.29



From Table 4.1, the K_e for the beam is:

$$(K_e)_{\text{beam}} = \frac{48E_1 I_1}{L_1^3}$$

$$K_e = \frac{48E_1 I_1}{L_1^3} + \frac{A_2 E_2}{L_2} + \frac{A_3 E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in parallel.

1.30

$$\sum M_0 = 0$$

$$-FL + (K_2 x_2)(2L) + (K_1 x_1)(L) = 0$$

Note $x_2 = 2x_1$ and simplify

$$(2K_2 x_1)(2L) + K_1 x_1 L = FL$$

$$x_1 = \frac{F}{4K_2 + K_1} \quad \leftarrow$$

$$\pi = \sum_{e=1}^n \Lambda^{(e)} - \sum F_i u_i$$

$$\pi = \frac{1}{2} K_2 x_2^2 + \frac{1}{2} K_1 x_1^2 - F x_1 = \frac{1}{2} K_2 (2x_1)^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\pi = 2K_2 x_1^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\frac{\partial \pi}{\partial x_1} = 4K_2 x_1 + K_1 x_1 - F = 0$$

$$x_1 = \frac{F}{4K_2 + K_1} \quad \leftarrow$$

