

***Algebra and Trigonometry 12th edition Sullivan
Solutions Manual***

SKU: 9780138251338-SOLUTIONS

INSTRUCTOR'S SOLUTIONS MANUAL

TIM BRITT

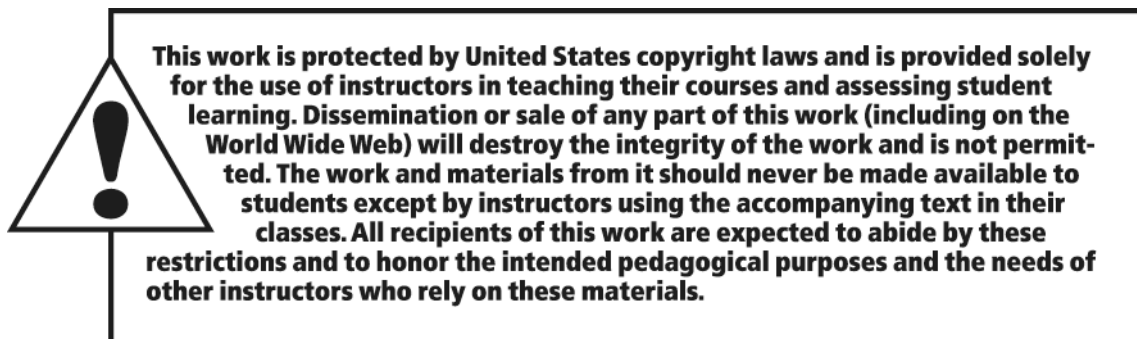
Jackson State Community College

ALGEBRA AND TRIGONOMETRY TWELFTH EDITION

Michael Sullivan

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Table of Contents

Chapter R Review

R.1 Real Numbers.....	1
R.2 Algebra Essentials.....	5
R.3 Geometry Essentials.....	11
R.4 Polynomials.....	16
R.5 Factoring Polynomials	23
R.6 Synthetic Division.....	28
R.7 Rational Expressions.....	30
R.8 n th Roots; Rational Exponents.....	40

Chapter 1 Equations and Inequalities

1.1 Linear Equations	50
1.2 Quadratic Equations.....	68
1.3 Complex Numbers; Quadratic Equations in the Complex Number System.....	86
1.4 Radical Equations; Equations Quadratic in Form; Factorable Equations.....	92
1.5 Solving Inequalities	116
1.6 Equations and Inequalities Involving Absolute Value	127
1.7 Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications	137
Chapter Review	145
Chapter Test.....	153
Chapter Projects	155

Chapter 2 Graphs

2.1 The Distance and Midpoint Formulas	156
2.2 Graphs of Equations in Two Variables; Intercepts; Symmetry	169
2.3 Lines	183
2.4 Circles	201
2.5 Variation	215
Chapter Review	221
Chapter Test.....	227
Cumulative Review	229
Chapter Project	231

Chapter 3 Functions and Their Graphs

3.1 Functions.....	232
3.2 The Graph of a Function.....	250
3.3 Properties of Functions	259
3.4 Library of Functions; Piecewise-defined Functions	276
3.5 Graphing Techniques: Transformations	288
3.6 Mathematical Models: Building Functions	306
Chapter Review	314
Chapter Test.....	321
Cumulative Review	324
Chapter Projects.....	328

Chapter 4 Linear and Quadratic Functions

4.1 Properties of Linear Functions and Linear Models	330
4.2 Building Linear Functions from Data	341
4.3 Quadratic Functions and Their Properties	347
4.4 Build Quadratic Models from Verbal Descriptions and from Data	371
4.5 Inequalities Involving Quadratic Functions.....	378
Chapter Review	398
Chapter Test.....	406
Cumulative Review	408
Chapter Projects.....	411

Chapter 5 Polynomial and Rational Functions

5.1 Polynomial Functions	414
5.2 Graphing Polynomial Functions; Models	424
5.3 Properties of Rational Functions	440
5.4 The Graph of a Rational Function	450
5.5 Polynomial and Rational Inequalities	506
5.6 The Real Zeros of a Polynomial Function	527
5.7 Complex Zeros; Fundamental Theorem of Algebra	558
Chapter Review	567
Chapter Test.....	582
Cumulative Review	586
Chapter Projects.....	591

Chapter 6 Exponential and Logarithmic Functions

6.1 Composite Functions	593
6.2 One-to-One Functions; Inverse Functions	611
6.3 Exponential Functions	634
6.4 Logarithmic Functions	655
6.5 Properties of Logarithms	677
6.6 Logarithmic and Exponential Equations.....	686
6.7 Financial Models	707
6.8 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models	715
6.9 Building Exponential, Logarithmic, and Logistic Models from Data	725
Chapter Review	729
Chapter Test.....	742
Cumulative Review	746
Chapter Projects.....	749

Chapter 7 Trigonometric Functions

7.1 Angles, Arc Length, and Circular Motion	752
7.2 Right Triangle Trigonometry.....	761
7.3 Computing the Values of Trigonometric Functions of Acute Angles	777
7.4 Trigonometric Functions of Any Angle	790
7.5 Unit Circle Approach: Properties of the Trigonometric Functions	806
7.6 Graphs of the Sine and Cosine Functions	815
7.7 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions	837
7.8 Phase Shift; Sinusoidal Curve Fitting.....	847
Chapter Review	860
Chapter Test.....	869
Cumulative Review	873
Chapter Projects	877

Chapter 8 Analytic Trigonometry

8.1 The Inverse Sine, Cosine, and Tangent Functions.....	881
8.2 The Inverse Trigonometric Functions (Continued)	895
8.3 Trigonometric Equations	907
8.4 Trigonometric Identities	928
8.5 Sum and Difference Formulas	941
8.6 Double-angle and Half-angle Formulas.....	966
8.7 Product-to-Sum and Sum-to-Product Formulas	994
Chapter Review	1007
Chapter Test.....	1022
Cumulative Review	1027
Chapter Projects	1033

Chapter 9 Applications of Trigonometric Functions

9.1 Applications Involving Right Triangles	1037
9.2 The Law of Sines	1045
9.3 The Law of Cosines	1060
9.4 Area of a Triangle	1072
9.5 Simple Harmonic Motion; Damped Motion; Combining Waves.....	1082
Chapter Review	1092
Chapter Test.....	1098
Cumulative Review	1101
Chapter Projects	1107

Chapter 10 Polar Coordinates; Vectors

10.1 Polar Coordinates.....	1111
10.2 Polar Equations and Graphs.....	1120
10.3 The Complex Plane; De Moivre's Theorem.....	1149
10.4 Vectors.....	1163
10.5 The Dot Product.....	1176
Chapter Review	1182
Chapter Test.....	1190
Cumulative Review	1194
Chapter Projects	1196

Chapter 11 Analytic Geometry

11.2 The Parabola	1199
11.3 The Ellipse	1215
11.4 The Hyperbola	1232
11.5 Rotation of Axes; General Form of a Conic	1252
11.6 Polar Equations of Conics	1265
11.7 Plane Curves and Parametric Equations	1274
Chapter Review	1289
Chapter Test.....	1298
Cumulative Review	1303
Chapter Projects.....	1305

Chapter 12 Systems of Equations and Inequalities

12.1 Systems of Linear Equations: Substitution and Elimination	1309
12.2 Systems of Linear Equations: Matrices	1332
12.3 Systems of Linear Equations: Determinants.....	1356
12.4 Matrix Algebra.....	1370
12.5 Partial Fraction Decomposition	1389
12.6 Systems of Nonlinear Equations.....	1407
12.7 Systems of Inequalities	1435
12.8 Linear Programming	1450
Chapter Review	1464
Chapter Test.....	1479
Cumulative Review	1487
Chapter Projects.....	1491

Chapter 13 Sequences; Induction; the Binomial Theorem

13.1 Sequences	1493
13.2 Arithmetic Sequences	1503
13.3 Geometric Sequences; Geometric Series	1512
13.4 Mathematical Induction	1524
13.5 The Binomial Theorem.....	1533
Chapter Review	1540
Chapter Test.....	1544
Cumulative Review	1547
Chapter Projects.....	1550

Chapter 14 Counting and Probability

14.1 Counting	1553
14.2 Permutations and Combinations	1556
14.3 Probability	1561
Chapter Review	1568
Chapter Test	1570
Cumulative Review	1571
Chapter Projects	1574

Appendix Graphing Utilities

Section 1 The Viewing Rectangle	1577
Section 2 Using a Graphing Utility to Graph Equations	1578
Section 3 Using a Graphing Utility to Locate Intercepts and Check for Symmetry	1583
Section 5 Square Screens	1585

Chapter R

Review

Section R.1

1. rational
2. $4 + 5 \cdot 6 - 3 = 4 + 30 - 3 = 31$
3. Distributive
4. c
5. a
6. b
7. True
8. False; The Zero-Product Property states that if a product equals 0, then at least one of the factors must equal 0.
9. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.
10. True
11. $A \cup B = \{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
12. $A \cup C = \{1, 3, 4, 5, 9\} \cup \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 5, 6, 9\}$
13. $A \cap B = \{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\} = \{4\}$
14. $A \cap C = \{1, 3, 4, 5, 9\} \cap \{1, 3, 4, 6\} = \{1, 3, 4\}$
15. $(A \cup B) \cap C$
 $= (\{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}) \cap \{1, 3, 4, 6\}$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 6\}$
16. $(A \cap B) \cup C$
 $= (\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}) \cup \{1, 3, 4, 6\}$
 $= \{4\} \cup \{1, 3, 4, 6\}$
 $= \{1, 3, 4, 6\}$
17. $\overline{A} = \{0, 2, 6, 7, 8\}$
18. $\overline{C} = \{0, 2, 5, 7, 8, 9\}$
19. $\overline{A \cap B} = \overline{\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}}$
 $= \overline{\{4\}} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
20. $\overline{B \cup C} = \overline{\{2, 4, 6, 7, 8\} \cup \{1, 3, 4, 6\}}$
 $= \overline{\{1, 2, 3, 4, 6, 7, 8\}} = \{0, 5, 9\}$
21. $\overline{A} \cup \overline{B} = \{0, 2, 6, 7, 8\} \cup \{0, 1, 3, 5, 9\}$
 $= \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
22. $\overline{B} \cap \overline{C} = \{0, 1, 3, 5, 9\} \cap \{0, 2, 5, 7, 8, 9\}$
 $= \{0, 5, 9\}$
23. a. $\{2, 5\}$
b. $\{-6, 2, 5\}$
c. $\left\{-6, \frac{1}{2}, -1.333..., 2, 5\right\}$
d. $\{\pi\}$
e. $\left\{-6, \frac{1}{2}, -1.333..., \pi, 2, 5\right\}$
24. a. $\{1\}$
b. $\{0, 1\}$
c. $\left\{-\frac{5}{3}, 2.060606... = 2.\overline{06}, 1.25, 0, 1\right\}$
d. $\{\sqrt{5}\}$

Chapter R: Review

- e. $\left\{-\frac{5}{3}, 2.060606\dots = 2.\overline{06}, 1.25, 0, 1, \sqrt{5}\right\}$
25. a. $\{1\}$
 b. $\{0, 1\}$
 c. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
 d. None
 e. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
26. a. None
 b. $\{-1\}$
 c. $\{-1.3, -1.2, -1.1, -1\}$
 d. None
 e. $\{-1.3, -1.2, -1.1, -1\}$
27. a. None
 b. None
 c. None
 d. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$
 e. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$
28. a. None
 b. None
 c. $\left\{\frac{1}{2}+10.3\right\}$
 d. $\left\{-\sqrt{2}, \pi+\sqrt{2}\right\}$
 e. $\left\{-\sqrt{2}, \pi+\sqrt{2}, \frac{1}{2}+10.3\right\}$
29. a. 18.953 b. 18.952
30. a. 25.861 b. 25.861
31. a. 28.653 b. 28.653
32. a. 99.052 b. 99.052
33. a. 0.063 b. 0.062
34. a. 0.054 b. 0.053
35. a. 9.999 b. 9.998
36. a. 1.001 b. 1.000
37. a. 0.429 b. 0.428
38. a. 0.556 b. 0.555
39. a. 34.733 b. 34.733
40. a. 16.200 b. 16.200
41. $3+2=5$
42. $5\cdot 2=10$
43. $x+2=3\cdot 4$
44. $3+y=2+2$
45. $3y=1+2$
46. $2x=4\cdot 6$
47. $x-2=6$
48. $2-y=6$
49. $\frac{x}{2}=6$
50. $\frac{2}{x}=6$
51. $9-4+2=5+2=7$
52. $6-4+3=2+3=5$
53. $-6+4\cdot 3=-6+12=6$
54. $8-4\cdot 2=8-8=0$
55. $18-5\cdot 2=18-10=8$
56. $100-10\cdot 2=100-20=80$
57. $4+\frac{1}{3}=\frac{12+1}{3}=\frac{13}{3}$
58. $2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$

$$\begin{aligned} 59. \quad 6 - [3 \cdot 5 + 2 \cdot (3 - 2)] &= 6 - [15 + 2 \cdot (1)] \\ &= 6 - 17 \\ &= -11 \end{aligned}$$

$$\begin{aligned} 60. \quad 2 \cdot [8 - 3 \cdot (4 + 2)] - 3 &= 2 \cdot [8 - 3 \cdot (6)] - 3 \\ &= 2 \cdot [8 - 18] - 3 \\ &= 2 \cdot [-10] - 3 \\ &= -20 - 3 \\ &= -23 \end{aligned}$$

$$\begin{aligned} 61. \quad 4 \cdot (9 + 5) - 6 \cdot 7 + 3 &= 4 \cdot (14) - 42 + 3 \\ &= 56 - 42 + 3 \\ &= 14 + 3 \\ &= 17 \end{aligned}$$

$$\begin{aligned} 62. \quad 1 - (4 \cdot 3 - 2 + 2) &= 1 - (12 - 2 + 2) \\ &= 1 - 12 \\ &= -11 \end{aligned}$$

$$\begin{aligned} 63. \quad 10 - [6 - 2 \cdot 2 + (8 - 3)] \cdot 2 &= 10 - [6 - 4 + 5] \cdot 2 \\ &= 10 - [2 + 5] \cdot 2 \\ &= 10 - [7] \cdot 2 \\ &= 10 - 14 \\ &= -4 \end{aligned}$$

$$\begin{aligned} 64. \quad 2 - 5 \cdot 4 - [6 \cdot (3 - 4)] &= 2 - 20 - [6 \cdot (-1)] \\ &= -18 - [-6] \\ &= -18 + 6 \\ &= -12 \end{aligned}$$

$$65. \quad (5 - 3) \frac{1}{2} = (2) \frac{1}{2} = 1$$

$$66. \quad (5 + 4) \frac{1}{3} = (9) \frac{1}{3} = 3$$

$$67. \quad \frac{4 + 8}{5 - 3} = \frac{12}{2} = 6$$

$$68. \quad \frac{2 - 4}{5 - 3} = \frac{-2}{2} = -1$$

$$69. \quad \frac{3}{5} \cdot \frac{10}{21} = \frac{3 \cdot 2 \cdot 5}{5 \cdot 3 \cdot 7} = \frac{\cancel{3} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{3} \cdot 7} = \frac{2}{7}$$

$$70. \quad \frac{5}{9} \cdot \frac{3}{10} = \frac{5 \cdot 3}{3 \cdot 3 \cdot 5 \cdot 2} = \frac{\cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot 3 \cdot \cancel{5} \cdot 2} = \frac{1}{6}$$

$$71. \quad \frac{6}{25} \cdot \frac{10}{27} = \frac{2 \cdot 3 \cdot 5 \cdot 2}{5 \cdot 5 \cdot 3 \cdot 9} = \frac{2 \cdot \cancel{3} \cdot \cancel{5} \cdot 2}{\cancel{5} \cdot 5 \cdot \cancel{3} \cdot 9} = \frac{4}{45}$$

$$72. \quad \frac{21}{25} \cdot \frac{100}{3} = \frac{3 \cdot 7 \cdot 4 \cdot 25}{25 \cdot 3} = \frac{\cancel{3} \cdot 7 \cdot 4 \cdot \cancel{25}}{\cancel{25} \cdot \cancel{3}} = 28$$

$$73. \quad \frac{3}{4} + \frac{2}{5} = \frac{15 + 8}{20} = \frac{23}{20}$$

$$74. \quad \frac{4}{3} + \frac{1}{2} = \frac{8 + 3}{6} = \frac{11}{6}$$

$$75. \quad \frac{7}{8} + \frac{4}{7} = \frac{49 + 32}{56} = \frac{81}{56}$$

$$76. \quad \frac{8}{9} + \frac{15}{2} = \frac{16 + 135}{18} = \frac{151}{18}$$

$$77. \quad \frac{5}{18} + \frac{1}{12} = \frac{10 + 3}{36} = \frac{13}{36}$$

$$78. \quad \frac{2}{15} + \frac{8}{9} = \frac{6 + 40}{45} = \frac{46}{45}$$

$$79. \quad \frac{5}{24} - \frac{8}{15} = \frac{25 - 64}{120} = -\frac{39}{120} = -\frac{13}{40}$$

$$80. \quad \frac{3}{14} - \frac{2}{21} = \frac{9 - 4}{42} = \frac{5}{42}$$

$$81. \quad \frac{3}{20} - \frac{2}{15} = \frac{9 - 8}{60} = \frac{1}{60}$$

$$82. \quad \frac{6}{35} - \frac{3}{14} = \frac{12 - 15}{70} = -\frac{3}{70}$$

$$83. \quad \left(\frac{5}{18} \right) \cdot \left(\frac{11}{27} \right) = \frac{5}{18} \cdot \frac{27}{11} = \frac{5 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 11} = \frac{5 \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot 11} = \frac{15}{22}$$

$$84. \quad \left(\frac{5}{21} \right) \cdot \left(\frac{2}{35} \right) = \frac{5}{21} \cdot \frac{35}{2} = \frac{5 \cdot 7 \cdot 5}{7 \cdot 3 \cdot 2} = \frac{5 \cdot \cancel{7} \cdot 5}{\cancel{7} \cdot 3 \cdot 2} = \frac{25}{6}$$

Chapter R: Review

$$85. \frac{1}{3} \cdot \frac{4}{7} + \frac{17}{21} = \frac{4}{21} + \frac{17}{21} = \frac{21}{21} = 1$$

$$86. \frac{2}{3} + \frac{4}{5} \cdot \frac{1}{6} = \frac{2}{3} + \frac{2 \cdot 2}{5 \cdot 3 \cdot 2} = \frac{2}{3} + \frac{2 \cdot \cancel{2}}{5 \cdot 3 \cdot \cancel{2}} = \frac{2}{3} + \frac{2}{15}$$

$$= \frac{2}{3} \cdot \frac{5}{5} + \frac{2}{15} = \frac{10}{15} + \frac{2}{15} = \frac{10+2}{15} = \frac{12}{15}$$

$$= \frac{4 \cdot 3}{5 \cdot 3} = \frac{4 \cdot \cancel{3}}{5 \cdot \cancel{3}} = \frac{4}{5}$$

$$87. 2 \cdot \frac{3}{4} + \frac{3}{8} = \frac{2}{1} \cdot \frac{3}{4} + \frac{3}{8} = \frac{6}{4} + \frac{3}{8} = \frac{6}{4} \cdot \frac{2}{2} + \frac{3}{8}$$

$$= \frac{12}{8} + \frac{3}{8} = \frac{12+3}{8} = \frac{15}{8}$$

$$88. 3 \cdot \frac{5}{6} - \frac{1}{2} = \frac{3}{1} \cdot \frac{5}{6} - \frac{1}{2} = \frac{3 \cdot 5}{3 \cdot 2} - \frac{1}{2} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 2} - \frac{1}{2}$$

$$= \frac{5}{2} - \frac{1}{2} = \frac{5-1}{2} = \frac{4}{2} = 2$$

$$89. 6(x+4) = 6x+24$$

$$90. 4(2x-1) = 8x-4$$

$$91. x(x-4) = x^2-4x$$

$$92. 4x(x+3) = 4x^2+12x$$

$$93. 2\left(\frac{3}{4}x - \frac{1}{2}\right) = 2 \cdot \frac{3}{4}x - 2 \cdot \frac{1}{2} = \frac{2 \cdot 3x}{2 \cdot 2} - \frac{2}{2}$$

$$= \frac{\cancel{2} \cdot 3x}{\cancel{2} \cdot 2} - \frac{2}{2} = \frac{3}{2}x - 1$$

$$94. 3\left(\frac{2}{3}x + \frac{1}{6}\right) = 3 \cdot \frac{2}{3}x + 3 \cdot \frac{1}{6} = \frac{3 \cdot 2x}{3} + \frac{3}{3 \cdot 2}$$

$$= \frac{\cancel{3} \cdot 2x}{\cancel{3}} + \frac{\cancel{3}}{\cancel{3} \cdot 2} = 2x + \frac{1}{2}$$

$$95. (x+2)(x+4) = x^2+4x+2x+8$$

$$= x^2+6x+8$$

$$96. (x+5)(x+1) = x^2+x+5x+5$$

$$= x^2+6x+5$$

$$97. (x+9)(2x-7) = 2x^2-7x+18x-63$$

$$= 2x^2+11x-63$$

$$98. (3x-1)(x+5) = 3x^2+15x-x-5$$

$$= 3x^2+14x-5$$

$$99. (x-8)(x-2) = x^2-2x-8x+16$$

$$= x^2-10x+16$$

$$100. (x-4)(x-2) = x^2-2x-4x+8$$

$$= x^2-6x+8$$

$$101. 3x(x-5k) = 3x^2-60x$$

$$3x^2-15xk = 3x^2-60x$$

$$-15xk = -60x$$

$$k = 4$$

$$102. (x-k)(x+3k) = x^2+4x-12$$

$$x^2+3kx-kx-3k^2 = x^2+4x-12$$

$$x^2+x(3k-k)-3k^2 = x^2+4x-12$$

$$x^2+x(3k-k)-3k^2 = x^2+4x-12$$

$$x^2+x(2k)-3k^2 = x^2+4x-12$$

$$2k = 4$$

$$k = 2$$

$$103. 2x+3x = 2 \cdot x + 3 \cdot x$$

$$= (2+3) \cdot x$$

$$= (5) \cdot x$$

$$= 5x$$

$$104. 2+3 \cdot 4 = 2+12 = 14$$

since multiplication comes before addition in the order of operations for real numbers.

$$(2+3) \cdot 4 = 5 \cdot 4 = 20$$

since operations inside parentheses come before multiplication in the order of operations for real numbers.

$$105. 2(3 \cdot 4) = 2(12) = 24$$

$$(2 \cdot 3) \cdot (2 \cdot 4) = (6)(8) = 48$$

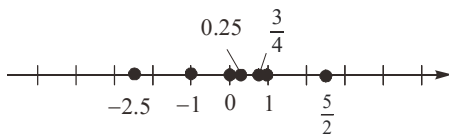
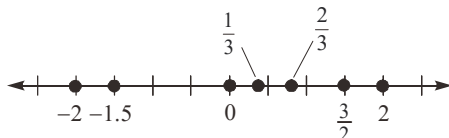
$$106. \frac{4+3}{2+5} = \frac{7}{7} = 1, \text{ but}$$

$$\frac{4}{2} + \frac{3}{5} = \frac{4 \cdot 5 + 3 \cdot 2}{10} = \frac{20+6}{10} = \frac{26}{10} = \frac{13}{5} = 2.6$$

$$107. \text{ Subtraction is not commutative; for example: } 2-3 = -1 \neq 1 = 3-2.$$

108. Subtraction is not associative; for example: $(5 - 2) - 1 = 2 \neq 4 = 5 - (2 - 1)$.
109. Division is not commutative; for example:
 $\frac{2}{3} \neq \frac{3}{2}$.
110. Division is not associative; for example: $(12 \div 2) \div 2 = 6 \div 2 = 3$, but
 $12 \div (2 \div 2) = 12 \div 1 = 12$.
111. The Symmetric Property implies that if $2 = x$, then $x = 2$.
112. From the *principle of substitution*, if $x = 5$, then
 $(x)(x) = (5)(5)$
 $\Rightarrow x^2 = 25$
 $\Rightarrow x^2 + x = 25 + 5$
 $\Rightarrow x^2 + x = 30$
113. There are no real numbers that are both rational and irrational, since an irrational number, by definition, is a number that cannot be expressed as the ratio of two integers; that is, not a rational number.
 Every real number is either a rational number or an irrational number, since the decimal form of a real number either involves an infinitely repeating pattern of digits or an infinite, non-repeating string of digits.
114. The sum of an irrational number and a rational number must be irrational. Otherwise, the irrational number would then be the difference of two rational numbers, and therefore would have to be rational.
115. Answers will vary.
116. Since 1 day = 24 hours, we compute
 $\frac{12997}{24} = 541.541\bar{6}$.
 Now we only need to consider the decimal part of the answer in terms of a 24 hour day. That is,
 $(0.541\bar{6})(24) \approx 13$ hours. So it must be 13 hours later than 12 noon, which makes the time 1 a.m. CST.
117. Answers will vary.

Section R.2

- variable
- origin
- strict
- base; exponent (or power)
- 1.2345678×10^3
- d
- a
- b
- True
- False; the absolute value of a real number is nonnegative. $|0| = 0$ which is not a positive number.
- False; a number in scientific notation is expressed as the product of a number, x , $1 \leq x < 10$ or $-10 < x \leq -1$, and a power of 10.
- True
- 
- 
- $\frac{1}{2} > 0$
- $5 < 6$
- $-1 > -2$
- $-3 < -\frac{5}{2}$
- $\pi > 3.14$
- $\sqrt{2} > 1.41$

Chapter R: Review

21. $\frac{1}{2} = 0.5$

22. $\frac{1}{3} > 0.33$

23. $\frac{2}{3} < 0.67$

24. $\frac{1}{4} = 0.25$

25. $x > 0$

26. $z < 0$

27. $x < 2$

28. $y > -5$

29. $x \leq 1$

30. $x \geq 2$

31. Graph on the number line: $x \geq -2$



32. Graph on the number line: $x < 4$



33. Graph on the number line: $x > -1$



34. Graph on the number line: $x \leq 7$



35. $d(C, D) = d(0, 1) = |1 - 0| = |1| = 1$

36. $d(C, A) = d(0, -3) = |-3 - 0| = |-3| = 3$

37. $d(D, E) = d(1, 3) = |3 - 1| = |2| = 2$

38. $d(C, E) = d(0, 3) = |3 - 0| = |3| = 3$

39. $d(A, E) = d(-3, 3) = |3 - (-3)| = |6| = 6$

40. $d(D, B) = d(1, -1) = |-1 - 1| = |-2| = 2$

41. $x + 2y = -2 + 2 \cdot 3 = -2 + 6 = 4$

42. $3x + y = 3(-2) + 3 = -6 + 3 = -3$

43. $5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

44. $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = -2$

45. $\frac{2x}{x-y} = \frac{2(-2)}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$

46. $\frac{x+y}{x-y} = \frac{-2+3}{-2-3} = \frac{1}{-5} = -\frac{1}{5}$

47. $\frac{3x+2y}{2+y} = \frac{3(-2)+2(3)}{2+3} = \frac{-6+6}{5} = \frac{0}{5} = 0$

48. $\frac{2x-3}{y} = \frac{2(-2)-3}{3} = \frac{-4-3}{3} = -\frac{7}{3}$

49. $|x+y| = |3+(-2)| = |1| = 1$

50. $|x-y| = |3-(-2)| = |5| = 5$

51. $|x|+|y| = |3|+|-2| = 3+2 = 5$

52. $|x|-|y| = |3|-|-2| = 3-2 = 1$

53. $\frac{|x|}{x} = \frac{|3|}{3} = \frac{3}{3} = 1$

54. $\frac{|y|}{y} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

55. $|4x-5y| = |4(3)-5(-2)|$
 $= |12+10|$
 $= |22|$
 $= 22$

56. $|3x+2y| = |3(3)+2(-2)| = |9-4| = |5| = 5$

57. $||4x|-|5y|| = ||4(3)|-|5(-2)||$
 $= ||12|-|10||$
 $= |12-10|$
 $= |2|$
 $= 2$

$$\begin{aligned}
 58. \quad 3|x| + 2|y| &= 3|3| + 2|-2| \\
 &= 3 \cdot 3 + 2 \cdot 2 \\
 &= 9 + 4 \\
 &= 13
 \end{aligned}$$

$$59. \quad \frac{x^2 - 1}{x}$$

Part (c) must be excluded. The value $x = 0$ must be excluded from the domain because it causes division by 0.

$$60. \quad \frac{x^2 + 1}{x}$$

Part (c) must be excluded. The value $x = 0$ must be excluded from the domain because it causes division by 0.

$$61. \quad \frac{x}{x^2 - 9} = \frac{x}{(x-3)(x+3)}$$

Part (a), $x = 3$, must be excluded because it causes the denominator to be 0.

$$62. \quad \frac{x}{x^2 + 9}$$

None of the given values are excluded. The domain is all real numbers.

$$63. \quad \frac{x^2}{x^2 + 1}$$

None of the given values are excluded. The domain is all real numbers.

$$64. \quad \frac{x^3}{x^2 - 1} = \frac{x^3}{(x-1)(x+1)}$$

Parts (b) and (d) must be excluded. The values $x = 1$, and $x = -1$ must be excluded from the domain because they cause division by 0.

$$65. \quad \frac{x^2 + 5x - 10}{x^3 - x} = \frac{x^2 + 5x - 10}{x(x-1)(x+1)}$$

Parts (b), (c), and (d) must be excluded. The values $x = 0$, $x = 1$, and $x = -1$ must be excluded from the domain because they cause division by 0.

$$66. \quad \frac{-9x^2 - x + 1}{x^3 + x} = \frac{-9x^2 - x + 1}{x(x^2 + 1)}$$

Part (c) must be excluded. The value $x = 0$ must

be excluded from the domain because it causes division by 0.

$$67. \quad \frac{4}{x-5}$$

$x = 5$ must be excluded because it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq 5\}$$

$$68. \quad \frac{-6}{x+4}$$

$x = -4$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq -4\}$$

$$69. \quad \frac{x}{x+4}$$

$x = -4$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq -4\}$$

$$70. \quad \frac{x-2}{x-6}$$

$x = 6$ must be excluded since it makes the denominator equal 0.

$$\text{Domain} = \{x | x \neq 6\}$$

$$71. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(32 - 32) = \frac{5}{9}(0) = 0^\circ\text{C}$$

$$72. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(212 - 32) = \frac{5}{9}(180) = 100^\circ\text{C}$$

$$73. \quad C = \frac{5}{9}(F - 32) = \frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25^\circ\text{C}$$

$$\begin{aligned}
 74. \quad C &= \frac{5}{9}(F - 32) = \frac{5}{9}(-4 - 32) \\
 &= \frac{5}{9}(-36) \\
 &= -20^\circ\text{C}
 \end{aligned}$$

$$75. \quad (-9)^2 = (-9)(-9) = 81$$

$$76. \quad -4^2 = -(4)^2 = -16$$

$$77. \quad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Chapter R: Review

$$78. -4^{-2} = -\frac{1}{4^2} = -\frac{1}{16}$$

$$79. 3^{-6} \cdot 3^4 = 3^{-6+4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$80. 4^{-2} \cdot 4^3 = 4^{-2+3} = 4^1 = 4$$

$$81. (4^{-3})^{-1} = 4^{(-3)(-1)} = 4^3 = 64$$

$$82. (2^{-1})^{-3} = 2^{(-1)(-3)} = 2^3 = 8$$

$$83. \sqrt{100} = \sqrt{10^2} = 10$$

$$84. \sqrt{36} = \sqrt{6^2} = 6$$

$$85. \sqrt{(-4)^2} = |-4| = 4$$

$$86. \sqrt{(-3)^2} = |-3| = 3$$

$$87. (9x^4)^2 = 9^2 (x^4)^2 = 81x^8$$

$$88. (-4x^2)^{-1} = \frac{1}{-4x^2} = -\frac{1}{4x^2}$$

$$89. (x^2 y^{-1})^2 = (x^2)^2 \cdot (y^{-1})^2 = x^4 y^{-2} = \frac{x^4}{y^2}$$

$$90. (x^{-1} y)^3 = (x^{-1})^3 \cdot y^3 = x^{-3} y^3 = \frac{y^3}{x^3}$$

$$91. \frac{x^2 y^5}{x^3 y^4} = x^{2-3} y^{5-4} = x^{-1} y^1 = \frac{y}{x}$$

$$92. \frac{x^{-2} y}{x y^2} = x^{-2-1} y^{1-2} = x^{-3} y^{-1} = \frac{1}{x^3 y}$$

$$\begin{aligned} 93. \frac{(-4)^2 y^5 (xz)^3}{(-3)^3 x y^7 z^2} &= \frac{16 y^5 x^3 z^3}{-27 x y^7 z^2} \\ &= -\frac{16}{27} x^{3-1} y^{5-7} z^{3-2} \\ &= -\frac{16}{27} x^2 y^{-2} z^1 \\ &= -\frac{16 x^2 z}{27 y^2} \end{aligned}$$

$$\begin{aligned} 94. \frac{4x^{-2} (yz)^{-1}}{2^3 x^4 y} &= \frac{4x^{-2} y^{-1} z^{-1}}{8x^4 y} \\ &= \frac{4}{8} x^{-2-4} y^{-1-1} z^{-1} \\ &= \frac{1}{2} x^{-6} y^{-2} z^{-1} \\ &= \frac{1}{2x^6 y^2 z} \end{aligned}$$

$$95. \left(\frac{2x^{-3}}{3y^{-1}} \right)^{-2} = \left(\frac{3y}{2x^3} \right)^{-2} = \left(\frac{3x^3}{2y} \right)^2 = \frac{3^2 x^6}{2^2 y^2} = \frac{9x^6}{4y^2}$$

$$\begin{aligned} 96. \left(\frac{5x^{-2}}{6y^{-2}} \right)^{-3} &= \left(\frac{5y^2}{6x^2} \right)^{-3} = \left(\frac{6x^2}{5y^2} \right)^3 \\ &= \frac{6^3 (x^2)^3}{5^3 (y^2)^3} = \frac{216x^6}{125y^6} \end{aligned}$$

$$97. 2xy^{-1} = \frac{2x}{y} = \frac{2(2)}{(-1)} = -4$$

$$98. -3x^{-1}y = \frac{-3y}{x} = \frac{-3(-1)}{(2)} = \frac{3}{2}$$

$$99. x^2 + y^2 = (2)^2 + (-1)^2 = 4 + 1 = 5$$

$$100. x^2 y^2 = (2)^2 (-1)^2 = 4 \cdot 1 = 4$$

$$101. (xy)^2 = (2 \cdot (-1))^2 = (-2)^2 = 4$$

$$102. (x+y)^2 = (2+(-1))^2 = (1)^2 = 1$$

$$103. \sqrt{x^2} = |x| = |2| = 2$$

$$104. (\sqrt{x})^2 = x = 2$$

$$105. \sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$106. \sqrt{x^2} + \sqrt{y^2} = |x| + |y| = |2| + |-1| = 2 + 1 = 3$$

$$107. x^y = 2^{-1} = \frac{1}{2}$$

108. $y^x = (-1)^2 = 1$

109. If $x = 2$,

$$2x^3 - 3x^2 + 5x - 4 = 2 \cdot 2^3 - 3 \cdot 2^2 + 5 \cdot 2 - 4$$

$$= 16 - 12 + 10 - 4$$

$$= 10$$

If $x = 1$,

$$2x^3 - 3x^2 + 5x - 4 = 2 \cdot 1^3 - 3 \cdot 1^2 + 5 \cdot 1 - 4$$

$$= 2 - 3 + 5 - 4$$

$$= 0$$

110. If $x = 1$,

$$4x^3 + 3x^2 - x + 2 = 4 \cdot 1^3 + 3 \cdot 1^2 - 1 + 2$$

$$= 4 + 3 - 1 + 2$$

$$= 8$$

If $x = 2$,

$$4x^3 + 3x^2 - x + 2 = 4 \cdot 2^3 + 3 \cdot 2^2 - 2 + 2$$

$$= 32 + 12 - 2 + 2$$

$$= 44$$

111. $\left(\frac{666}{222}\right)^4 = \left(\frac{666}{222}\right)^4 = 3^4 = 81$

112. $(0.1)^3 (20)^3 = \left(\frac{1}{10}\right)^3 \cdot (2 \cdot 10)^3$

$$= \frac{1}{10^3} \cdot 2^3 \cdot 10^3$$

$$= 2^3 = 8$$

113. $(8.2)^6 \approx 304,006.671$

114. $(3.7)^5 \approx 693.440$

115. $(6.1)^{-3} \approx 0.004$

116. $(2.2)^{-5} \approx 0.019$

117. $(-2.8)^6 \approx 481.890$

118. $-(2.8)^6 \approx -481.890$

119. $(-8.11)^{-4} \approx 0.000$

120. $-(8.11)^{-4} \approx -0.000$

121. $454.2 = 4.542 \times 10^2$

122. $32.14 = 3.214 \times 10^1$

123. $0.013 = 1.3 \times 10^{-2}$

124. $0.00421 = 4.21 \times 10^{-3}$

125. $32,155 = 3.2155 \times 10^4$

126. $21,210 = 2.121 \times 10^4$

127. $0.000423 = 4.23 \times 10^{-4}$

128. $0.0514 = 5.14 \times 10^{-2}$

129. $6.15 \times 10^4 = 61,500$

130. $9.7 \times 10^3 = 9700$

131. $1.214 \times 10^{-3} = 0.001214$

132. $9.88 \times 10^{-4} = 0.000988$

133. $1.1 \times 10^8 = 110,000,000$

134. $4.112 \times 10^2 = 411.2$

135. $8.1 \times 10^{-2} = 0.081$

136. $6.453 \times 10^{-1} = 0.6453$

137. $A = lw$

138. $P = 2(l + w)$

139. $C = \pi d$

140. $A = \frac{1}{2}bh$

141. $A = \frac{\sqrt{3}}{4}x^2$

142. $P = 3x$

143. $V = \frac{4}{3}\pi r^3$

Chapter R: Review

144. $S = 4\pi r^2$

145. $V = x^3$

146. $S = 6x^2$

147. a. If $x = 1000$,
 $C = 4000 + 2x$
 $= 4000 + 2(1000)$
 $= 4000 + 2000$
 $= \$6000$
 The cost of producing 1000 watches is \$6000.

b. If $x = 2000$,
 $C = 4000 + 2x$
 $= 4000 + 2(2000)$
 $= 4000 + 4000$
 $= \$8000$
 The cost of producing 2000 watches is \$8000.

148. $210 + 80 - 120 + 25 - 60 - 32 - 5 = \$98$
 His balance at the end of the month was \$98.

149. We want the difference between x and 4 to be at least 6 units. Since we don't care whether the value for x is larger or smaller than 4, we take the absolute value of the difference. We want the inequality to be non-strict since we are dealing with an 'at least' situation. Thus, we have
 $|x - 4| \geq 6$

150. We want the difference between x and 2 to be more than 5 units. Since we don't care whether the value for x is larger or smaller than 2, we take the absolute value of the difference. We want the inequality to be strict since we are dealing with a 'more than' situation. Thus, we have
 $|x - 2| > 5$

151. a. $|x - 110| = |108 - 110| = |-2| = 2 \leq 5$
 108 volts is acceptable.

b. $|x - 110| = |104 - 110| = |-6| = 6 > 5$
 104 volts is not acceptable.

152. a. $|x - 220| = |214 - 220| = |-6| = 6 \leq 8$
 214 volts is acceptable.

b. $|x - 220| = |209 - 220| = |-11| = 11 > 8$
 209 volts is not acceptable.

153. a. $|x - 3| = |2.999 - 3|$
 $= |-0.001|$
 $= 0.001 \leq 0.01$
 A radius of 2.999 centimeters is acceptable.

b. $|x - 3| = |2.89 - 3|$
 $= |-0.11|$
 $= 0.11 \not\leq 0.01$
 A radius of 2.89 centimeters is not acceptable.

154. a. $|x - 98.6| = |97 - 98.6|$
 $= |-1.6|$
 $= 1.6 \geq 1.5$
 97°F is unhealthy.

b. $|x - 98.6| = |100 - 98.6|$
 $= |1.4|$
 $= 1.4 < 1.5$
 100°F is not unhealthy.

155. The distance from Earth to the Moon is about
 $4 \times 10^8 = 400,000,000$ meters.

156. The height of Mt. Everest is about
 $8848 = 8.848 \times 10^3$ meters.

157. The wavelength of visible light is about
 $5 \times 10^{-7} = 0.0000005$ meters.

158. The diameter of an atom is about
 $1 \times 10^{-10} = 0.0000000001$ meters.

159. The diameter is about $0.0403 = 4.03 \times 10^{-2}$ inches.

160. The tiniest motor is less than $0.00004 = 4 \times 10^{-5}$ millimeters tall.

161. $186,000 \cdot 60 \cdot 60 \cdot 24 \cdot 365$
 $= (1.86 \times 10^5) (6 \times 10^1)^2 (2.4 \times 10^1) (3.65 \times 10^2)$
 $= 586.5696 \times 10^{10} = 5.865696 \times 10^{12}$
 There are about 5.9×10^{12} miles in one light-year.

162. $\frac{93,000,000}{186,000} = \frac{9.3 \times 10^7}{1.86 \times 10^5} = 5 \times 10^2$
 $= 500$ seconds $\approx$ 8 min. 20 sec.
 It takes about 8 minutes 20 seconds for a beam of light to reach Earth from the Sun.

163. $\frac{1}{3} = 0.333333 \dots > 0.333$
 $\frac{1}{3}$ is larger by approximately 0.0003333 ...

164. $\frac{2}{3} = 0.666666 \dots > 0.666$
 $\frac{2}{3}$ is larger by approximately 0.000666 ...

165. $(5.24 \times 10^6) (6.5 \times 10^{13}) = (5.24 \times 6.5) (10^6 \times 10^{13})$
 $34.06 \times 10^{19} = 3.406 \times 10^{20}$

166. $\frac{1.62 \times 10^{-4}}{4.5 \times 10^{-10}} = \frac{1.62}{4.5} \times \frac{10^{-4}}{10^{-10}} = 0.36 \times 10^6$
 $= 3.6 \times 10^5$

167. No. For any positive number a , the value $\frac{a}{2}$ is smaller and therefore closer to 0.

168. We are given that $1 < x^2 < 10$. This implies that $1 < x < \sqrt{10}$. Since $x < \sqrt{10} \approx 3.162$ and $x > \pi \approx 3.142$, the number could be 3.15 or 3.16 (which are between 1 and 10 as required). The number could also be 3.14 since numbers such as 3.146 which lie between π and $\sqrt{10}$ would equal 3.14 when truncated to two decimal places.

169. Answers will vary.

170. Answers will vary.
 $5 < 8$ is a true statement because 5 is further to the left than 8 on a real number line.

2. $A = \frac{1}{2}bh$

3. $C = 2\pi r$

4. similar

5. c

6. b

7. True.

8. True. $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

9. False; the surface area of a sphere of radius r is given by $V = 4\pi r^2$.

10. True. The lengths of the corresponding sides are equal.

11. True. Two corresponding angles are equal.

12. False. The sides are not proportional.

13. $a = 5, b = 12,$
 $c^2 = a^2 + b^2$
 $= 5^2 + 12^2$
 $= 25 + 144$
 $= 169 \Rightarrow c = 13$

14. $a = 6, b = 8,$
 $c^2 = a^2 + b^2$
 $= 6^2 + 8^2$
 $= 36 + 64$
 $= 100 \Rightarrow c = 10$

15. $a = 10, b = 24,$
 $c^2 = a^2 + b^2$
 $= 10^2 + 24^2$
 $= 100 + 576$
 $= 676 \Rightarrow c = 26$

16. $a = 4, b = 3,$
 $c^2 = a^2 + b^2$
 $= 4^2 + 3^2$
 $= 16 + 9$
 $= 25 \Rightarrow c = 5$

Section R.3

- right; hypotenuse

Chapter R: Review

17. $a = 7$, $b = 24$,

$$c^2 = a^2 + b^2$$

$$= 7^2 + 24^2$$

$$= 49 + 576$$

$$= 625 \Rightarrow c = 25$$

18. $a = 14$, $b = 48$,

$$c^2 = a^2 + b^2$$

$$= 14^2 + 48^2$$

$$= 196 + 2304$$

$$= 2500 \Rightarrow c = 50$$

19. $5^2 = 3^2 + 4^2$

$$25 = 9 + 16$$

$$25 = 25$$

The given triangle is a right triangle. The hypotenuse is 5.

20. $10^2 = 6^2 + 8^2$

$$100 = 36 + 64$$

$$100 = 100$$

The given triangle is a right triangle. The hypotenuse is 10.

21. $6^2 = 4^2 + 5^2$

$$36 = 16 + 25$$

$$36 = 41 \text{ false}$$

The given triangle is not a right triangle.

22. $3^2 = 2^2 + 2^2$

$$9 = 4 + 4$$

$$9 = 8 \text{ false}$$

The given triangle is not a right triangle.

23. $25^2 = 7^2 + 24^2$

$$625 = 49 + 576$$

$$625 = 625$$

The given triangle is a right triangle. The hypotenuse is 25.

24. $26^2 = 10^2 + 24^2$

$$676 = 100 + 576$$

$$676 = 676$$

The given triangle is a right triangle. The hypotenuse is 26.

25. $6^2 = 3^2 + 4^2$

$$36 = 9 + 16$$

$$36 = 25 \text{ false}$$

The given triangle is not a right triangle.

26. $7^2 = 5^2 + 4^2$

$$49 = 25 + 16$$

$$49 = 41 \text{ false}$$

The given triangle is not a right triangle.

27. $A = l \cdot w = 6 \cdot 7 = 42 \text{ in}^2$

28. $A = l \cdot w = 9 \cdot 4 = 36 \text{ cm}^2$

29. $A = \frac{1}{2}b \cdot h = \frac{1}{2}(14)(4) = 28 \text{ in}^2$

30. $A = \frac{1}{2}b \cdot h = \frac{1}{2}(4)(9) = 18 \text{ cm}^2$

31. $A = \pi r^2 = \pi(5)^2 = 25\pi \text{ m}^2$

$$C = 2\pi r = 2\pi(5) = 10\pi \text{ m}$$

32. $A = \pi r^2 = \pi(2)^2 = 4\pi \text{ ft}^2$

$$C = 2\pi r = 2\pi(2) = 4\pi \text{ ft}$$

33. $V = lwh = 6 \cdot 8 \cdot 5 = 240 \text{ ft}^3$

$$S = 2lw + 2lh + 2wh$$

$$= 2(6)(8) + 2(6)(5) + 2(8)(5)$$

$$= 96 + 60 + 80$$

$$= 236 \text{ ft}^2$$

34. $V = lwh = 9 \cdot 4 \cdot 8 = 288 \text{ in}^3$

$$S = 2lw + 2lh + 2wh$$

$$= 2(9)(4) + 2(9)(8) + 2(4)(8)$$

$$= 72 + 144 + 64$$

$$= 280 \text{ in}^2$$

35. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{500}{3}\pi \text{ cm}^3$

$$S = 4\pi r^2 = 4\pi \cdot 5^2 = 100\pi \text{ cm}^2$$

36. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 3^3 = 36\pi \text{ ft}^3$

$$S = 4\pi r^2 = 4\pi \cdot 3^2 = 36\pi \text{ ft}^2$$

37. $V = \pi r^2 h = \pi(9)^2(8) = 648\pi \text{ in}^3$

$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi(9)^2 + 2\pi(9)(8)$$

$$= 162\pi + 144\pi$$

$$= 306\pi \text{ in}^2$$

38. $V = \pi r^2 h = \pi(8)^2(9) = 576\pi \text{ in}^3$

$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi(8)^2 + 2\pi(8)(9)$$

$$= 128\pi + 144\pi$$

$$= 272\pi \text{ in}^2$$

39. The diameter of the circle is 2, so its radius is 1.

$$A = \pi r^2 = \pi(1)^2 = \pi \text{ square units}$$

40. The diameter of the circle is 2, so its radius is 1.

$$A = 2^2 - \pi(1)^2 = 4 - \pi \text{ square units}$$

41. The diameter of the circle is the length of the diagonal of the square.

$$d^2 = 2^2 + 2^2$$

$$= 4 + 4$$

$$= 8$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area of the circle is:

$$A = \pi r^2 = \pi(\sqrt{2})^2 = 2\pi \text{ square units}$$

42. The diameter of the circle is the length of the diagonal of the square.

$$d^2 = 2^2 + 2^2$$

$$= 4 + 4$$

$$= 8$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area is:

$$A = \pi(\sqrt{2})^2 - 2^2 = 2\pi - 4 \text{ square units}$$

43. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{4} = \frac{x}{2}$$

$$\frac{8 \cdot 2}{4} = x$$

$$4 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have

$$A = 90^\circ, B = 60^\circ, \text{ and } C = 30^\circ.$$

44. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{6}{12} = \frac{x}{16}$$

$$\frac{6 \cdot 16}{12} = x$$

$$8 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have

$$A = 30^\circ, B = 75^\circ, \text{ and } C = 75^\circ.$$

45. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{30}{20} = \frac{x}{45}$$

$$\frac{30 \cdot 45}{20} = x$$

$$\frac{135}{2} = x \text{ or } x = 67.5$$

In addition, corresponding angles must have the same angle measure. Therefore, we have

$$A = 60^\circ, B = 95^\circ, \text{ and } C = 25^\circ.$$

46. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{10} = \frac{x}{50}$$

$$\frac{8 \cdot 50}{10} = x$$

$$40 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have

$$A = 50^\circ, B = 125^\circ, \text{ and } C = 5^\circ.$$

Chapter R: Review

47. The total distance traveled is 4 times the circumference of the wheel.
 Total Distance = $4C = 4(\pi d) = 4\pi \cdot 16$
 $= 64\pi \approx 201.1$ inches ≈ 16.8 feet

48. The distance traveled in one revolution is the circumference of the disk 4π .
 The number of revolutions =
 $\frac{\text{dist. traveled}}{\text{circumference}} = \frac{20}{4\pi} = \frac{5}{\pi} \approx 1.6$ revolutions

49. Area of the border = area of EFGH – area of ABCD
 $= 10^2 - 6^2 = 100 - 36 = 64$ ft²

50. FG = 4 feet; BG = 4 feet and BC = 10 feet, so CG = 6 feet. The area of the triangle CGF is:
 $A = \frac{1}{2} \cdot (4)(6) = 12$ ft²

51. Area of the window = area of the rectangle + area of the semicircle.

$$A = (6)(4) + \frac{1}{2} \cdot \pi \cdot 2^2 = 24 + 2\pi \approx 30.28$$
 ft²

Perimeter of the window = 2 heights + width + one-half the circumference.

$$P = 2(6) + 4 + \frac{1}{2} \cdot \pi(4) = 12 + 4 + 2\pi$$

 $= 16 + 2\pi \approx 22.28$ feet

52. Area of the deck = area of the pool and deck – area of the pool.

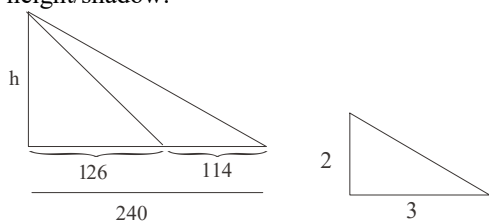
$$A = \pi(13)^2 - \pi(10)^2 = 169\pi - 100\pi$$

 $= 69\pi$ ft² ≈ 216.77 ft²

The amount of fence is the circumference of the circle with radius 13 feet.

$$C = 2\pi(13) = 26\pi$$
 ft ≈ 81.68 ft

53. We can form similar triangles using the Great Pyramid's height/shadow and Thales' height/shadow:



This allows us to write

$$\frac{h}{240} = \frac{2}{3}$$

$$h = \frac{2 \cdot 240}{3} = 160$$

The height of the Great Pyramid is 160 paces.

54. Let x = the approximate distance from San Juan to Hamilton and y = the approximate distance from Hamilton to Fort Lauderdale. Using similar triangles, we get

$$\frac{1046}{58} = \frac{x}{53.5} \qquad \frac{1046}{58} = \frac{y}{57}$$

$$\frac{1046 \cdot 53.5}{58} = x \qquad \frac{1046 \cdot 57}{58} = y$$

$$964.8 \approx x \qquad 1028.0 \approx y$$

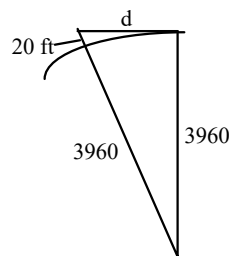
The approximate distance between San Juan and Hamilton is 965 miles and the approximate distance between Hamilton and Fort Lauderdale is 1028 miles.

55. Convert 20 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$20 \text{ feet} = 20 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.003788 \text{ miles}$$

$$d^2 = (3960 + 0.003788)^2 - 3960^2 = 30 \text{ sq. miles}$$

$$d \approx 5.477 \text{ miles}$$

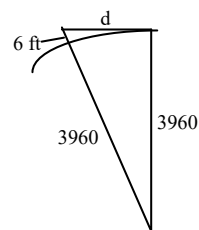


56. Convert 6 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$6 \text{ feet} = 6 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.001136 \text{ miles}$$

$$d^2 = (3960 + 0.001136)^2 - 3960^2 = 9 \text{ sq. miles}$$

$$d \approx 3 \text{ miles}$$



57. Convert 100 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$100 \text{ feet} = 100 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.018939 \text{ miles}$$

$$d^2 = (3960 + 0.018939)^2 - 3960^2 \approx 150 \text{ sq. miles}$$

$$d \approx 12.2 \text{ miles}$$

Convert 150 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$150 \text{ feet} = 150 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.028409 \text{ miles}$$

$$d^2 = (3960 + 0.028409)^2 - 3960^2 \approx 225 \text{ sq. miles}$$

$$d \approx 15.0 \text{ miles}$$

58. Given $m > 0$, $n > 0$ and $m > n$,

if $a = m^2 - n^2$, $b = 2mn$ and $c = m^2 + n^2$, then

$$\begin{aligned} a^2 + b^2 &= (m^2 - n^2)^2 + (2mn)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \end{aligned}$$

$$\text{and } c^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$$

$\therefore a^2 + b^2 = c^2 \rightarrow a, b$ and c represent the sides of a right triangle.

59. $V = \pi r^2 h$

$$= \pi(10)^2(4.5)$$

$$= 450\pi \text{ ft}^3$$

So, $1 \text{ ft}^3 \approx 7.48052 \text{ gal}$ so

$$(450\pi \text{ ft}^3)(7.48052 \text{ gal/ft}^3) \approx 10,575 \text{ gal}$$

60. $10000(5.61458) = 56145.8 \text{ ft}^3$

$$V = \pi r^2 h$$

$$56145.8 = \pi(25)^2 h$$

$$h = \frac{56145.8}{625\pi} = 28.6 \text{ ft}$$

61. $A = \pi r^2$

$$A_2 = \pi(2r)^2$$

$$= \pi 4r^2$$

$$= 4\pi r^2 = 4A$$

If you double the radius, the area is four times the original area.

62. $V = \frac{4}{3}\pi r^3$

$$V_2 = \frac{4}{3}\pi(2r)^3$$

$$= \frac{4}{3}\pi \cdot 8r^3$$

$$= 8 \cdot \frac{4}{3}\pi r^3 = 8V$$

If you double the radius the volume is 8 times the original volume.

63. Let l = length of the rectangle and w = width of the rectangle.

Notice that

$$\begin{aligned} (l+w)^2 - (l-w)^2 &= [(l+w) + (l-w)][(l+w) - (l-w)] \\ &= (2l)(2w) = 4lw = 4A \end{aligned}$$

$$\text{So } A = \frac{1}{4}[(l+w)^2 - (l-w)^2]$$

Since $(l-w)^2 \geq 0$, the largest area will occur when $l-w=0$ or $l=w$; that is, when the rectangle is a square. But

$$1000 = 2l + 2w = 2(l+w)$$

$$500 = l + w = 2l$$

$$250 = l = w$$

The largest possible area is $250^2 = 62500 \text{ sq ft}$.

A circular pool with circumference = 1000 feet

yields the equation: $2\pi r = 1000 \Rightarrow r = \frac{500}{\pi}$

The area enclosed by the circular pool is:

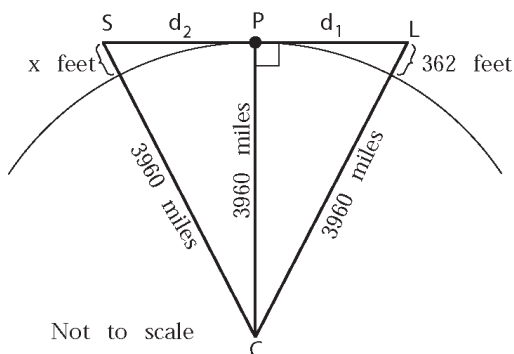
$$A = \pi r^2 = \pi \left(\frac{500}{\pi} \right)^2 = \frac{500^2}{\pi} \approx 79577.47 \text{ ft}^2$$

Thus, a circular pool will enclose the most area.

64. Consider the diagram showing the lighthouse at point L, relative to the center of Earth, using the radius of Earth as 3960 miles. Let P refer to the furthest point on the horizon from which the light is visible. Note also that

$$362 \text{ feet} = \frac{362}{5280} \text{ miles.}$$

Chapter R: Review



Apply the Pythagorean Theorem to $\triangle CPL$:

$$(3960)^2 + (d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2$$

$$(d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2 - (3960)^2$$

$$d_1 = \sqrt{\left(3960 + \frac{362}{5280}\right)^2 - (3960)^2} \approx 23.30 \text{ mi.}$$

Therefore, the light from the lighthouse can be seen at point P on the horizon, where point P is approximately 23.30 miles away from the lighthouse. Brochure information is slightly overstated.

Verify the ship information:

Let S refer to the ship's location, and let x equal the height, in feet, of the ship.

We need $d_1 + d_2 \geq 40$.

Since $d_1 \approx 23.30$ miles we need

$$d_2 \geq 40 - 23.30 = 16.70 \text{ miles.}$$

Apply the Pythagorean Theorem to $\triangle CPS$:

$$(3960)^2 + (16.7)^2 = (3960 + x)^2$$

$$\sqrt{(3960)^2 + (16.7)^2} = 3960 + x$$

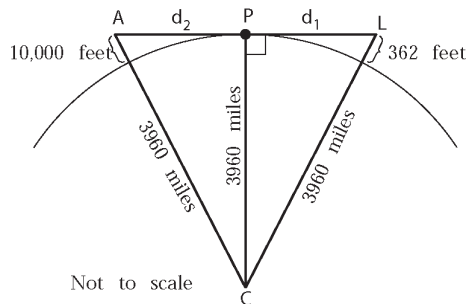
$$\sqrt{(3960)^2 + (16.7)^2} - 3960 = x$$

$$x \approx 0.035 \text{ miles}$$

$$x \approx 185.93 \text{ feet.}$$

The ship would have to be at least 186 feet tall to see the lighthouse from 40 miles away.

Verify the airplane information:



Let A refer to the airplane's location. The distance from the plane to point P is d_2 .

We want to show that $d_1 + d_2 \geq 120$.

Assume the altitude of the airplane is

$$10,000 \text{ feet} = \frac{10000}{5280} \text{ miles.}$$

Apply the Pythagorean Theorem to $\triangle CPA$:

$$(3960)^2 + (d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2$$

$$(d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2$$

$$d_2 = \sqrt{\left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2} \approx 122.49 \text{ miles.}$$

$$\text{re, } d_1 + d_2 \approx 23.30 + 122.49 = 145.79 \geq 120.$$

The brochure information is slightly understated.

Note that a plane at an altitude of 6233 feet could see the lighthouse from 120 miles away.

Section R.4

- 4; 3
- $x^4 - 16$
- $x^3 - 8$
- a
- c
- False; monomials cannot have negative degrees.
- True
- False; the dividend = (quotient)(divisor) + remainder

9. $2x^3$ Monomial; Variable: x ; Coefficient: 2; Degree: 3
10. $-4x^2$ Monomial; Variable: x ; Coefficient: -4 ; Degree: 2
11. $\frac{8}{x} = 8x^{-1}$ Not a monomial; when written in the form ax^k , the variable has a negative exponent.
12. $-2x^{-3}$ Not a monomial; when written in the form ax^k , the variable has a negative exponent.
13. $-2xy^2$ Monomial; Variables: x, y ; Coefficient: -2 ; Degree: 3
14. $5x^2y^3$ Monomial; Variables: x, y ; Coefficient: 5; Degree: 5
15. $\frac{8x}{y} = 8xy^{-1}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.
16. $-\frac{2x^2}{y^3} = -2x^2 y^{-3}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.
17. $x^2 + y^2$ Not a monomial; the expression contains more than one term. This expression is a binomial.
18. $3x^2 + 4$ Not a monomial; the expression contains more than one term. This expression is a binomial.
19. $3x^2 - 5$ Polynomial; Degree: 2
20. $1 - 4x$ Polynomial; Degree: 1
21. 5 Polynomial; Degree: 0
22. $-\pi$ Polynomial; Degree: 0
23. $3x^2 - \frac{5}{x}$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.
24. $\frac{3}{x} + 2$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.
25. $2y^3 - \sqrt{2}$ Polynomial; Degree: 3
26. $10z^2 + z$ Polynomial; Degree: 2
27. $\frac{x^2 + 5}{x^3 - 1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.
28. $\frac{3x^3 + 2x - 1}{x^2 + x + 1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.
29. $(x^2 + 6x + 8) + (3x^2 - 4x + 7)$
 $= (x^2 + 3x^2) + (6x - 4x) + (8 + 7)$
 $= 4x^2 + 2x + 15$
30. $(x^3 + 3x^2 + 2) + (x^2 - 4x + 4)$
 $= x^3 + (3x^2 + x^2) + (-4x) + (2 + 4)$
 $= x^3 + 4x^2 - 4x + 6$
31. $(x^3 - 2x^2 + 5x + 10) - (2x^2 - 4x + 3)$
 $= x^3 - 2x^2 + 5x + 10 - 2x^2 + 4x - 3$
 $= x^3 + (-2x^2 - 2x^2) + (5x + 4x) + (10 - 3)$
 $= x^3 - 4x^2 + 9x + 7$
32. $(x^2 - 3x - 4) - (x^3 - 3x^2 + x + 5)$
 $= x^2 - 3x - 4 - x^3 + 3x^2 - x - 5$
 $= -x^3 + (x^2 + 3x^2) + (-3x - x) + (-4 - 5)$
 $= -x^3 + 4x^2 - 4x - 9$
33. $(6x^5 + x^3 + x) + (5x^4 - x^3 + 3x^2)$
 $= 6x^5 + 5x^4 + 3x^2 + x$
34. $(10x^5 - 8x^2) + (3x^3 - 2x^2 + 6)$
 $= 10x^5 + 3x^3 - 10x^2 + 6$
35. $(x^2 - 6x + 4) + 3(2x^2 + x - 5)$
 $= x^2 - 6x + 4 + 6x^2 + 3x - 15$
 $= 7x^2 - 3x - 11$

Chapter R: Review

$$\begin{aligned} 36. \quad & -2(x^2 + x + 1) + (-5x^2 - x + 2) \\ & = -2x^2 - 2x - 2 - 5x^2 - x + 2 \\ & = -7x^2 - 3x \end{aligned}$$

$$\begin{aligned} 37. \quad & 6(x^3 + x^2 - 3) - 4(2x^3 - 3x^2) \\ & = 6x^3 + 6x^2 - 18 - 8x^3 + 12x^2 \\ & = -2x^3 + 18x^2 - 18 \end{aligned}$$

$$\begin{aligned} 38. \quad & 8(4x^3 - 3x^2 - 1) - 6(4x^3 + 8x - 2) \\ & = 32x^3 - 24x^2 - 8 - 24x^3 - 48x + 12 \\ & = 8x^3 - 24x^2 - 48x + 4 \end{aligned}$$

$$\begin{aligned} 39. \quad & (x^2 - x + 2) + (2x^2 - 3x + 5) - (x^2 + 1) \\ & = x^2 - x + 2 + 2x^2 - 3x + 5 - x^2 - 1 \\ & = 2x^2 - 4x + 6 \end{aligned}$$

$$\begin{aligned} 40. \quad & (x^2 + 1) - (4x^2 + 5) + (x^2 + x - 2) \\ & = x^2 + 1 - 4x^2 - 5 + x^2 + x - 2 \\ & = -2x^2 + x - 6 \end{aligned}$$

$$\begin{aligned} 41. \quad & 7(y^2 - 5y + 3) - 4(3 - y^2) \\ & = 7y^2 - 35y + 21 - 12 + 4y^2 \\ & = 11y^2 - 35y + 9 \end{aligned}$$

$$\begin{aligned} 42. \quad & 8(1 - y^3) + 4(1 + y + y^2 + y^3) \\ & = 8 - 8y^3 + 4 + 4y + 4y^2 + 4y^3 \\ & = -4y^3 + 4y^2 + 4y + 12 \end{aligned}$$

$$43. \quad x^2(x^2 + 2x - 5) = x^4 + 2x^3 - 5x^2$$

$$44. \quad 4x^2(x^3 - x + 2) = 4x^5 - 4x^3 + 8x^2$$

$$45. \quad -2x^2(4x^3 + 5) = -8x^5 - 10x^2$$

$$46. \quad 5x^3(3x - 4) = 15x^4 - 20x^3$$

$$\begin{aligned} 47. \quad & (x + 1)(x^2 + 2x - 4) \\ & = x(x^2 + 2x - 4) + 1(x^2 + 2x - 4) \\ & = x^3 + 2x^2 - 4x + x^2 + 2x - 4 \\ & = x^3 + 3x^2 - 2x - 4 \end{aligned}$$

$$\begin{aligned} 48. \quad & (2x - 3)(x^2 + x + 1) \\ & = 2x(x^2 + x + 1) - 3(x^2 + x + 1) \\ & = 2x^3 + 2x^2 + 2x - 3x^2 - 3x - 3 \\ & = 2x^3 - x^2 - x - 3 \end{aligned}$$

$$\begin{aligned} 49. \quad & (x + 2)(x + 4) = x^2 + 4x + 2x + 8 \\ & = x^2 + 6x + 8 \end{aligned}$$

$$\begin{aligned} 50. \quad & (x + 3)(x + 5) = x^2 + 5x + 3x + 15 \\ & = x^2 + 8x + 15 \end{aligned}$$

$$\begin{aligned} 51. \quad & (2x + 7)(x + 5) = 2x^2 + 7x + 10x + 35 \\ & = 2x^2 + 17x + 35 \end{aligned}$$

$$\begin{aligned} 52. \quad & (3x + 1)(2x + 1) = 6x^2 + 3x + 2x + 1 \\ & = 6x^2 + 5x + 1 \end{aligned}$$

$$\begin{aligned} 53. \quad & (x - 4)(x + 2) = x^2 + 2x - 4x - 8 \\ & = x^2 - 2x - 8 \end{aligned}$$

$$\begin{aligned} 54. \quad & (x + 4)(x - 2) = x^2 - 2x + 4x - 8 \\ & = x^2 + 2x - 8 \end{aligned}$$

$$\begin{aligned} 55. \quad & (x - 6)(x - 3) = x^2 - 6x - 3x + 18 \\ & = x^2 - 9x + 18 \end{aligned}$$

$$\begin{aligned} 56. \quad & (x - 5)(x - 1) = x^2 - x - 5x + 5 \\ & = x^2 - 6x + 5 \end{aligned}$$

$$\begin{aligned} 57. \quad & (2x + 3)(x - 2) = 2x^2 - 4x + 3x - 6 \\ & = 2x^2 - x - 6 \end{aligned}$$

$$\begin{aligned} 58. \quad & (2x - 4)(3x + 1) = 6x^2 + 2x - 12x - 4 \\ & = 6x^2 - 10x - 4 \end{aligned}$$

$$\begin{aligned} 59. \quad & (-3x + 4)(x - 2) = -3x^2 + 4x + 6x - 8 \\ & = -3x^2 + 10x - 8 \end{aligned}$$

$$\begin{aligned} 60. \quad & (-3x - 1)(x + 1) = -3x^2 - 3x - x - 1 \\ & = -3x^2 - 4x - 1 \end{aligned}$$

$$\begin{aligned} 61. \quad & (-x - 5)(-2x - 7) = 2x^2 + 10x + 7x + 35 \\ & = 2x^2 + 17x + 35 \end{aligned}$$

$$\begin{aligned} 62. \quad (-2x-3)(3-x) &= -6x + 2x^2 - 9 + 3x \\ &= 2x^2 - 3x - 9 \end{aligned}$$

$$\begin{aligned} 63. \quad (x-2y)(x+y) &= x^2 + xy - 2xy - 2y^2 \\ &= x^2 - xy - 2y^2 \end{aligned}$$

$$\begin{aligned} 64. \quad (2x+3y)(x-y) &= 2x^2 - 2xy + 3xy - 3y^2 \\ &= 2x^2 + xy - 3y^2 \end{aligned}$$

$$\begin{aligned} 65. \quad (-2x-3y)(3x+2y) &= -6x^2 - 4xy - 9xy - 6y^2 \\ &= -6x^2 - 13xy - 6y^2 \end{aligned}$$

$$\begin{aligned} 66. \quad (x-3y)(-2x+y) &= -2x^2 + xy + 6xy - 3y^2 \\ &= -2x^2 + 7xy - 3y^2 \end{aligned}$$

$$67. \quad (x-7)(x+7) = x^2 - 7^2 = x^2 - 49$$

$$68. \quad (x-1)(x+1) = x^2 - 1^2 = x^2 - 1$$

$$69. \quad (2x+3)(2x-3) = (2x)^2 - 3^2 = 4x^2 - 9$$

$$70. \quad (3x+2)(3x-2) = (3x)^2 - 2^2 = 9x^2 - 4$$

$$71. \quad (x+4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2 = x^2 + 8x + 16$$

$$72. \quad (x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$$

$$73. \quad (x-4)^2 = x^2 - 2 \cdot x \cdot 4 + 4^2 = x^2 - 8x + 16$$

$$74. \quad (x-5)^2 = x^2 - 2 \cdot x \cdot 5 + 5^2 = x^2 - 10x + 25$$

$$75. \quad (3x+4)(3x-4) = (3x)^2 - 4^2 = 9x^2 - 16$$

$$76. \quad (5x-3)(5x+3) = (5x)^2 - 3^2 = 25x^2 - 9$$

$$\begin{aligned} 77. \quad (2x-3)^2 &= (2x)^2 - 2(2x)(3) + 3^2 \\ &= 4x^2 - 12x + 9 \end{aligned}$$

$$\begin{aligned} 78. \quad (3x-4)^2 &= (3x)^2 - 2(3x)(4) + 4^2 \\ &= 9x^2 - 24x + 16 \end{aligned}$$

$$79. \quad (x+y)(x-y) = (x)^2 - (y)^2 = x^2 - y^2$$

$$80. \quad (x+3y)(x-3y) = (x)^2 - (3y)^2 = x^2 - 9y^2$$

$$81. \quad (3x+y)(3x-y) = (3x)^2 - (y)^2 = 9x^2 - y^2$$

$$82. \quad (3x+4y)(3x-4y) = (3x)^2 - (4y)^2 = 9x^2 - 16y^2$$

$$83. \quad (x+y)^2 = x^2 + 2xy + y^2$$

$$84. \quad (x-y)^2 = x^2 - 2xy + y^2$$

$$\begin{aligned} 85. \quad (x-2y)^2 &= x^2 + 2(x \cdot (-2y)) + (-2y)^2 \\ &= x^2 - 4xy + 4y^2 \end{aligned}$$

$$\begin{aligned} 86. \quad (2x+3y)^2 &= (2x)^2 + 2(2x \cdot 3y) + (3y)^2 \\ &= 4x^2 + 12xy + 9y^2 \end{aligned}$$

$$\begin{aligned} 87. \quad (x-2)^3 &= x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3 \\ &= x^3 - 6x^2 + 12x - 8 \end{aligned}$$

$$\begin{aligned} 88. \quad (x+1)^3 &= x^3 + 3 \cdot x^2 \cdot 1 + 3 \cdot x \cdot 1^2 + 1^3 \\ &= x^3 + 3x^2 + 3x + 1 \end{aligned}$$

$$\begin{aligned} 89. \quad (2x+1)^3 &= (2x)^3 + 3(2x)^2(1) + 3(2x) \cdot 1^2 + 1^3 \\ &= 8x^3 + 12x^2 + 6x + 1 \end{aligned}$$

$$\begin{aligned} 90. \quad (3x-2)^3 &= (3x)^3 - 3(3x)^2(2) + 3(3x) \cdot 2^2 - 2^3 \\ &= 27x^3 - 54x^2 + 36x - 8 \end{aligned}$$

$$\begin{array}{r} 4x^2 - 11x + 23 \\ 91. \quad x+2 \overline{) 4x^3 - 3x^2 + x + 1} \\ \underline{4x^3 + 8x^2} \\ -11x^2 + x \\ \underline{-11x^2 - 22x} \\ 23x + 1 \\ \underline{23x + 46} \\ -45 \end{array}$$

Check:

$$\begin{aligned} (x+2)(4x^2 - 11x + 23) &+ (-45) \\ &= 4x^3 - 11x^2 + 23x + 8x^2 - 22x + 46 - 45 \\ &= 4x^3 - 3x^2 + x + 1 \end{aligned}$$

The quotient is $4x^2 - 11x + 23$; the remainder is -45 .

Chapter R: Review

$$\begin{array}{r}
 3x^2 - 7x + 15 \\
 92. \quad x + 2 \overline{) 3x^3 - x^2 + x - 2} \\
 \underline{3x^3 + 6x^2} \\
 -7x^2 + x \\
 \underline{-7x^2 - 14x} \\
 15x - 2 \\
 \underline{15x + 30} \\
 -32
 \end{array}$$

Check:

$$\begin{aligned}
 (x+2)(3x^2-7x+15) + (-32) \\
 = 3x^3 - 7x^2 + 15x + 6x^2 - 14x + 30 - 32 \\
 = 3x^3 - x^2 + x - 2
 \end{aligned}$$

The quotient is $3x^2 - 7x + 15$; the remainder is -32 .

$$\begin{array}{r}
 4x - 3 \\
 93. \quad x^2 \overline{) 4x^3 - 3x^2 + x + 1} \\
 \underline{4x^3} \\
 -3x^2 + x + 1 \\
 \underline{-3x^2} \\
 x + 1
 \end{array}$$

Check:

$$(x^2)(4x-3) + (x+1) = 4x^3 - 3x^2 + x + 1$$

The quotient is $4x-3$; the remainder is $x+1$.

$$\begin{array}{r}
 3x - 1 \\
 94. \quad x^2 \overline{) 3x^3 - x^2 + x - 2} \\
 \underline{3x^3} \\
 -x^2 + x - 2 \\
 \underline{-x^2} \\
 x - 2
 \end{array}$$

Check:

$$(x^2)(3x-1) + (x-2) = 3x^3 - x^2 + x - 2$$

The quotient is $3x-1$; the remainder is $x-2$.

$$\begin{array}{r}
 5x^2 - 13 \\
 95. \quad x^2 + 2 \overline{) 5x^4 + 0x^3 - 3x^2 + x + 1} \\
 \underline{5x^4} + 10x^2 \\
 -13x^2 + x + 1 \\
 \underline{-13x^2} - 26 \\
 x + 27
 \end{array}$$

Check:

$$\begin{aligned}
 (x^2+2)(5x^2-13) + (x+27) \\
 = 5x^4 + 10x^2 - 13x^2 - 26 + x + 27 \\
 = 5x^4 - 3x^2 + x + 1
 \end{aligned}$$

The quotient is $5x^2 - 13$; the remainder is $x + 27$.

$$\begin{array}{r}
 5x^2 - 11 \\
 96. \quad x^2 + 2 \overline{) 5x^4 + 0x^3 - x^2 + x - 2} \\
 \underline{5x^4} + 10x^2 \\
 -11x^2 + x - 2 \\
 \underline{-11x^2} - 22 \\
 x + 20
 \end{array}$$

Check:

$$\begin{aligned}
 (x^2+2)(5x^2-11) + (x+20) \\
 = 5x^4 + 10x^2 - 11x^2 - 22 + x + 20 \\
 = 5x^4 - x^2 + x - 2
 \end{aligned}$$

The quotient is $5x^2 - 11$; the remainder is $x + 20$.

$$\begin{array}{r}
 2x^2 \\
 97. \quad 2x^3 - 1 \overline{) 4x^5 + 0x^4 + 0x^3 - 3x^2 + x + 1} \\
 \underline{4x^5} - 2x^2 \\
 -x^2 + x + 1
 \end{array}$$

Check:

$$\begin{aligned}
 (2x^3-1)(2x^2) + (-x^2+x+1) \\
 = 4x^5 - 2x^2 - x^2 + x + 1 = 4x^5 - 3x^2 + x + 1
 \end{aligned}$$

The quotient is $2x^2$; the remainder is $-x^2 + x + 1$.

$$98. \quad 3x^3 - 1 \overline{) 3x^5 + 0x^4 + 0x^3 - x^2 + x - 2}$$

$$\underline{3x^5} -x^2 $$

$$ x - 2$$

Check:

$$(3x^3 - 1)(x^2) + (x - 2) = 3x^5 - x^2 + x - 2$$

The quotient is x^2 ; the remainder is $x - 2$.

$$99. \quad 2x^2 + x + 1 \overline{) 2x^4 - 3x^3 + 0x^2 + x + 1}$$

$$\underline{2x^4 + x^3 + x^2} $$

$$ -4x^3 - x^2 + x $$

$$\underline{-4x^3 - 2x^2 - 2x} $$

$$ x^2 + 3x + 1$$

$$\underline{x^2 + \frac{1}{2}x + \frac{1}{2}}$$

$$ \frac{5}{2}x + \frac{1}{2}$$

Check:

$$(2x^2 + x + 1)(x^2 - 2x + \frac{1}{2}) + \frac{5}{2}x + \frac{1}{2}$$

$$= 2x^4 - 4x^3 + x^2 + x^3 - 2x^2 + \frac{1}{2}x$$

$$+ x^2 - 2x + \frac{1}{2} + \frac{5}{2}x + \frac{1}{2}$$

$$= 2x^4 - 3x^3 + x + 1$$

The quotient is $x^2 - 2x + \frac{1}{2}$; the remainder is

$$\frac{5}{2}x + \frac{1}{2}.$$

$$100. \quad 3x^2 + x + 1 \overline{) 3x^4 - x^3 + 0x^2 + x - 2}$$

$$\underline{3x^4 + x^3 + x^2} $$

$$ -2x^3 - x^2 + x $$

$$\underline{-2x^3 - \frac{2}{3}x^2 - \frac{2}{3}x} $$

$$ -\frac{1}{3}x^2 + \frac{5}{3}x - 2$$

$$\underline{-\frac{1}{3}x^2 - \frac{1}{9}x - \frac{1}{9}}$$

$$ \phantom{-\frac{1}{3}x^2 +} \frac{16}{9}x - \frac{17}{9}$$

Check:

$$(3x^2 + x + 1)(x^2 - \frac{2}{3}x - \frac{1}{9}) + (\frac{16}{9}x - \frac{17}{9})$$

$$= 3x^4 + x^3 + x^2 - 2x^3 - \frac{2}{3}x^2 - \frac{2}{3}x$$

$$- \frac{1}{3}x^2 - \frac{1}{9}x - \frac{1}{9} + \frac{16}{9}x - \frac{17}{9}$$

$$= 3x^4 - x^3 + x - 2$$

The quotient is $x^2 - \frac{2}{3}x - \frac{1}{9}$; the remainder is

$$\frac{16}{9}x - \frac{17}{9}.$$

$$101. \quad x - 1 \overline{) -4x^2 - 3x - 3}$$

$$\underline{-4x^3 + x^2 + 0x - 4}$$

$$ -4x^3 + 4x^2$$

$$ -3x^2$$

$$\underline{-3x^2 + 3x}$$

$$ -3x - 4$$

$$\underline{-3x + 3}$$

$$ -7$$

Check:

$$(x - 1)(-4x^2 - 3x - 3) + (-7)$$

$$= -4x^3 - 3x^2 - 3x + 4x^2 + 3x + 3 - 7$$

$$= -4x^3 + x^2 - 4$$

The quotient is $-4x^2 - 3x - 3$; the remainder is -7 .

$$102. \quad x - 1 \overline{) -3x^3 - 3x^2 - 3x - 5}$$

$$\underline{-3x^4 + 3x^3}$$

$$ -3x^3$$

$$\underline{-3x^3 + 3x^2}$$

$$ -3x^2 - 2x$$

$$\underline{-3x^2 + 3x}$$

$$ -5x - 1$$

$$\underline{-5x + 5}$$

$$ -6$$

Chapter R: Review

Check:

$$\begin{aligned} & (x-1)(-3x^3-3x^2-3x-5)+(-6) \\ &= -3x^4-3x^3-3x^2-5x+3x^3+3x^2 \\ & \quad +3x+5-6 \\ &= -3x^4-2x-1 \end{aligned}$$

The quotient is $-3x^3-3x^2-3x-5$; the remainder is -6 .

$$\begin{array}{r} 103. \quad x^2+x+1 \overline{) x^4+0x^3-x^2+0x+1} \\ \underline{x^4+0x^3+x^2+0x+1} \\ -x^3-2x^2 \\ \underline{-x^3-x^2-x} \\ -x^2+x+1 \\ \underline{-x^2-x-1} \\ 2x+2 \end{array}$$

Check:

$$\begin{aligned} & (x^2+x+1)(x^2-x-1)+2x+2 \\ &= x^4+x^3+x^2-x^3-x^2-x-x^2-x \\ & \quad -1+2x+2 \\ &= x^4-x^2+1 \end{aligned}$$

The quotient is x^2-x-1 ; the remainder is $2x+2$.

$$\begin{array}{r} 104. \quad x^2-x+1 \overline{) x^4+0x^3-x^2+0x+1} \\ \underline{x^4-0x^3+x^2+0x+1} \\ x^3-2x^2 \\ \underline{x^3-x^2+x} \\ -x^2-x+1 \\ \underline{-x^2+x-1} \\ -2x+2 \end{array}$$

Check:

$$\begin{aligned} & (x^2-x+1)(x^2+x-1)+(-2x+2) \\ &= x^4+x^3-x^2-x^3-x^2+x+x^2+x \\ & \quad -1-2x+2 \\ &= x^4-x^2+1 \end{aligned}$$

The quotient is x^2+x-1 ; the remainder is $-2x+2$.

$$\begin{array}{r} 105. \quad x-a \overline{) x^3+0x^2+0x-a^3} \\ \underline{x^3-ax^2} \\ ax^2 \\ \underline{ax^2-a^2x} \\ a^2x-a^3 \\ \underline{a^2x-a^3} \\ 0 \end{array}$$

Check:

$$\begin{aligned} & (x-a)(x^2+ax+a^2)+0 \\ &= x^3+ax^2+a^2x-ax^2-a^2x-a^3 \\ &= x^3-a^3 \end{aligned}$$

The quotient is x^2+ax+a^2 ; the remainder is 0 .

$$\begin{array}{r} 106. \quad x-a \overline{) x^5+0x^4+0x^3+0x^2+0x-a^5} \\ \underline{x^5-ax^4} \\ ax^4 \\ \underline{ax^4-a^2x^3} \\ a^2x^3 \\ \underline{a^2x^3-a^3x^2} \\ a^3x^2 \\ \underline{a^3x^2-a^4x} \\ a^4x-a^5 \\ \underline{a^4x-a^5} \\ 0 \end{array}$$

Check:

$$\begin{aligned}(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4)+0 \\&= x^5+ax^4+a^2x^3+a^3x^2+a^4x-ax^4 \\&\quad -a^2x^3-a^3x^2-a^4x-a^5 \\&= x^5-a^5\end{aligned}$$

The quotient is $x^4+ax^3+a^2x^2+a^3x+a^4$; the remainder is 0.

107. $(3x-2k)(4x+3k)=12x^2+kx-96$
 $12x^2-8kx+9kx-6k^2=12x^2+kx-96$
 $kx-6k^2=kx-96$
 $-6k^2=-96$
 $k^2=16$
 $k=\pm 4$
108. The products $(x+y)(x-y)$ and $(z+w)(z-w)$ will each result in a binomial that is the difference of squares. The product of those resulting binomials will have 4 terms.
109. When we multiply polynomials $p_1(x)$ and $p_2(x)$, each term of $p_1(x)$ will be multiplied by each term of $p_2(x)$. So when the highest-powered term of $p_1(x)$ multiplies by the highest powered term of $p_2(x)$, the exponents on the variables in those terms will add according to the basic rules of exponents. Therefore, the highest powered term of the product polynomial will have degree equal to the sum of the degrees of $p_1(x)$ and $p_2(x)$.
110. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x) \neq$ the degree of $p_2(x)$, each term of $p_1(x)$ will be added to each term of $p_2(x)$. Since only the terms with equal degrees will combine via addition, the degree of the sum polynomial will be the degree of the highest powered term overall, that is, the degree of the polynomial that had the higher degree.
111. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)=$ the degree

of $p_2(x)$, the new polynomial will have degree $\leq$ the degree of $p_1(x)$ and $p_2(x)$.

112. Answers will vary.

113. Answers will vary.

Section R.5

1. $3x(x-2)(x+2)$
2. prime
3. c
4. b
5. d
6. c
7. True; x^2+4 is prime over the set of real numbers.
8. False; $3x^3-2x^2-6x+4=(3x-2)(x^2-2)$
9. $3x+6=3(x+2)$
10. $7x-14=7(x-2)$
11. $ax^2+a=a(x^2+1)$
12. $ax-a=a(x-1)$
13. $x^3+x^2+x=x(x^2+x+1)$
14. $x^3-x^2+x=x(x^2-x+1)$
15. $2x^2-2x=2x(x-1)$
16. $3x^2-3x=3x(x-1)$
17. $3x^2y-6xy^2+12xy=3xy(x-2y+4)$
18. $60x^2y-48xy^2+72x^3y=12xy(5x-4y+6x^2)$
19. $x^2-1=x^2-1^2=(x-1)(x+1)$

Chapter R: Review

20. $x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$

21. $4x^2 - 1 = (2x)^2 - 1^2 = (2x-1)(2x+1)$

22. $9x^2 - 1 = (3x)^2 - 1^2 = (3x-1)(3x+1)$

23. $x^2 - 16 = x^2 - 4^2 = (x-4)(x+4)$

24. $x^2 - 25 = x^2 - 5^2 = (x-5)(x+5)$

25. $25x^2 - 4 = (5x-2)(5x+2)$

26. $36x^2 - 9 = 9(4x^2 - 1) = 9(2x-1)(2x+1)$

27. $x^2 + 2x + 1 = (x+1)^2$

28. $x^2 - 4x + 4 = (x-2)^2$

29. $x^2 + 4x + 4 = (x+2)^2$

30. $x^2 - 2x + 1 = (x-1)^2$

31. $x^2 - 10x + 25 = (x-5)^2$

32. $x^2 + 10x + 25 = (x+5)^2$

33. $4x^2 + 4x + 1 = (2x+1)^2$

34. $9x^2 + 6x + 1 = (3x+1)^2$

35. $16x^2 + 8x + 1 = (4x+1)^2$

36. $25x^2 + 10x + 1 = (5x+1)^2$

37. $x^3 - 27 = x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

38. $x^3 + 125 = x^3 + 5^3 = (x+5)(x^2 - 5x + 25)$

39. $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$

40. $27 - 8x^3 = 3^3 - (2x)^3$
 $= (3-2x)(9+6x+4x^2)$
 $= -(2x-3)(4x^2+6x+9)$

41. $8x^3 + 27 = (2x)^3 + 3^3$
 $= (2x+3)(4x^2 - 6x + 9)$

42. $64 - 27x^3 = 4^3 - (3x)^3$
 $= (4-3x)(16+12x+9x^2)$
 $= -(3x-4)(9x^2+12x+16)$

43. $x^2 + 5x + 6 = (x+2)(x+3)$

44. $x^2 + 6x + 8 = (x+2)(x+4)$

45. $x^2 + 7x + 6 = (x+6)(x+1)$

46. $x^2 + 9x + 8 = (x+8)(x+1)$

47. $x^2 + 7x + 10 = (x+2)(x+5)$

48. $x^2 + 11x + 10 = (x+10)(x+1)$

49. $x^2 - 10x + 16 = (x-2)(x-8)$

50. $x^2 - 17x + 16 = (x-16)(x-1)$

51. $x^2 - 7x - 8 = (x+1)(x-8)$

52. $x^2 - 2x - 8 = (x+2)(x-4)$

53. $x^2 + 7x - 8 = (x+8)(x-1)$

54. $x^2 + 2x - 8 = (x+4)(x-2)$

55. $2x^2 + 4x + 3x + 6 = 2x(x+2) + 3(x+2)$
 $= (x+2)(2x+3)$

56. $3x^2 - 3x + 2x - 2 = 3x(x-1) + 2(x-1)$
 $= (x-1)(3x+2)$

57. $5x^2 - 15x + x - 3 = 5x(x-3) + 1(x-3)$
 $= (x-3)(5x+1)$

58. $3x^2 + 6x - x - 2 = 3x(x+2) - 1(x+2)$
 $= (x+2)(3x-1)$

Section R.5: Factoring Polynomials

$$59. \quad 6x^2 + 21x + 8x + 28 = 3x(2x + 7) + 4(2x + 7) \\ = (2x + 7)(3x + 4)$$

$$60. \quad 9x^2 - 6x + 3x - 2 = 3x(3x - 2) + 1(3x - 2) \\ = (3x - 2)(3x + 1)$$

$$61. \quad 3x^2 + 4x + 1 = (3x + 1)(x + 1)$$

$$62. \quad 2x^2 + 3x + 1 = (2x + 1)(x + 1)$$

$$63. \quad 2z^2 + 9z + 7 = (2z + 7)(z + 1)$$

$$64. \quad 6z^2 + 5z + 1 = (3z + 1)(2z + 1)$$

$$65. \quad 5x^2 + 6x - 8 = (5x - 4)(x + 2)$$

$$66. \quad 3x^2 + 10x + 8 = (3x + 4)(x + 2)$$

$$67. \quad 5x^2 - 6x - 8 = (5x + 4)(x - 2)$$

$$68. \quad 3x^2 - 10x + 8 = (3x - 4)(x - 2)$$

$$69. \quad 5x^2 + 22x + 8 = (5x + 2)(x + 4)$$

$$70. \quad 3x^2 - 14x + 8 = (3x - 2)(x - 4)$$

$$71. \quad 5x^2 + 18x - 8 = (5x - 2)(x + 4)$$

$$72. \quad 3x^2 - 10x - 8 = (3x + 2)(x - 4)$$

$$73. \quad \text{Since } b \text{ is } 10 \text{ then we need half of } 10 \text{ squared to} \\ \text{be the last term in our trinomial. Thus}$$

$$\frac{1}{2}(10) = 5; (5)^2 = 25 \\ x^2 + 10x + 25 = (x + 5)^2$$

$$74. \quad \text{Since } b \text{ is } 14 \text{ then we need half of } 14 \text{ squared to} \\ \text{be the last term in our trinomial. Thus}$$

$$\frac{1}{2}(14) = 7; (7)^2 = 49 \\ p^2 + 14p + 49 = (p + 7)^2$$

$$75. \quad \text{Since } b \text{ is } -6 \text{ then we need half of } -6 \text{ squared to} \\ \text{be the last term in our trinomial. Thus}$$

$$\frac{1}{2}(-6) = -3; (-3)^2 = 9 \\ y^2 - 6y + 9 = (y - 3)^2$$

$$76. \quad \text{Since } b \text{ is } -4 \text{ then we need half of } -4 \text{ squared to} \\ \text{be the last term in our trinomial. Thus}$$

$$\frac{1}{2}(-4) = -2; (-2)^2 = 4 \\ x^2 - 4x + 4 = (x - 2)^2$$

$$77. \quad \text{Since } b \text{ is } -\frac{1}{2} \text{ then we need half of } -\frac{1}{2} \text{ squared} \\ \text{to be the last term in our trinomial. Thus}$$

$$\frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}; \left(-\frac{1}{4}\right)^2 = \frac{1}{16} \\ x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2$$

$$78. \quad \text{Since } b \text{ is } \frac{1}{3} \text{ then we need half of } \frac{1}{3} \text{ squared to} \\ \text{be the last term in our trinomial. Thus}$$

$$\frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{6}; \left(\frac{1}{6}\right)^2 = \frac{1}{36} \\ x^2 + \frac{1}{3}x + \frac{1}{36} = \left(x + \frac{1}{6}\right)^2$$

$$79. \quad x^2 - 36 = (x - 6)(x + 6)$$

$$80. \quad x^2 - 9 = (x - 3)(x + 3)$$

$$81. \quad 2 - 8x^2 = 2(1 - 4x^2) = 2(1 - 2x)(1 + 2x)$$

$$82. \quad 3 - 27x^2 = 3(1 - 9x^2) = 3(1 - 3x)(1 + 3x)$$

$$83. \quad x^2 + 11x + 10 = (x + 1)(x + 10)$$

$$84. \quad x^2 + 5x + 4 = (x + 4)(x + 1)$$

$$85. \quad x^2 - 10x + 21 = (x - 7)(x - 3)$$

$$86. \quad x^2 - 6x + 8 = (x - 2)(x - 4)$$

$$87. \quad 4x^2 - 8x + 32 = 4(x^2 - 2x + 8)$$

$$88. \quad 3x^2 - 12x + 15 = 3(x^2 - 4x + 5)$$

$$89. \quad x^2 + 4x + 16 \text{ is prime over the reals because} \\ \text{there are no factors of } 16 \text{ whose sum is } 4.$$

$$90. \quad x^2 + 12x + 36 = (x + 6)^2$$

$$91. \quad 15 + 2x - x^2 = -(x^2 - 2x - 15) = -(x - 5)(x + 3)$$

Chapter R: Review

92. $14 + 6x - x^2 = -(x^2 - 6x - 14)$ is prime over the integers because there are no factors of -14 whose sum is -6 .
93. $3x^2 - 12x - 36 = 3(x^2 - 4x - 12)$
 $= 3(x - 6)(x + 2)$
94. $x^3 + 8x^2 - 20x = x(x^2 + 8x - 20)$
 $= x(x + 10)(x - 2)$
95. $y^4 + 11y^3 + 30y^2 = y^2(y^2 + 11y + 30)$
 $= y^2(y + 5)(y + 6)$
96. $3y^3 - 18y^2 - 48y = 3y(y^2 - 6y - 16)$
 $= 3y(y + 2)(y - 8)$
97. $4x^2 + 12x + 9 = (2x + 3)^2$
98. $9x^2 - 12x + 4 = (3x - 2)^2$
99. $6x^2 + 8x + 2 = 2(3x^2 + 4x + 1)$
 $= 2(3x + 1)(x + 1)$
100. $8x^2 + 6x - 2 = 2(4x^2 + 3x - 1)$
 $= 2(4x - 1)(x + 1)$
101. $x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9)$
 $= (x - 3)(x + 3)(x^2 + 9)$
102. $x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1)$
 $= (x - 1)(x + 1)(x^2 + 1)$
103. $x^6 - 2x^3 + 1 = (x^3 - 1)^2$
 $= [(x - 1)(x^2 + x + 1)]^2$
 $= (x - 1)^2(x^2 + x + 1)^2$
104. $x^6 + 2x^3 + 1 = (x^3 + 1)^2$
 $= [(x + 1)(x^2 - x + 1)]^2$
 $= (x + 1)^2(x^2 - x + 1)^2$
105. $x^7 - x^5 = x^5(x^2 - 1) = x^5(x - 1)(x + 1)$
106. $x^8 - x^5 = x^5(x^3 - 1) = x^5(x - 1)(x^2 + x + 1)$
107. $16x^2 + 24x + 9 = (4x + 3)^2$
108. $9x^2 - 24x + 16 = (3x - 4)^2$
109. $5 + 16x - 16x^2 = -(16x^2 - 16x - 5)$
 $= -(4x - 5)(4x + 1)$
110. $5 + 11x - 16x^2 = -(16x^2 - 11x - 5)$
 $= -(16x + 5)(x - 1)$
111. $4y^2 - 16y + 15 = (2y - 5)(2y - 3)$
112. $9y^2 + 9y - 4 = (3y + 4)(3y - 1)$
113. $1 - 8x^2 - 9x^4 = -(9x^4 + 8x^2 - 1)$
 $= -(9x^2 - 1)(x^2 + 1)$
 $= -(3x - 1)(3x + 1)(x^2 + 1)$
114. $4 - 14x^2 - 8x^4 = -2(4x^4 + 7x^2 - 2)$
 $= -2(4x^2 - 1)(x^2 + 2)$
 $= -2(2x - 1)(2x + 1)(x^2 + 2)$
115. $x(x + 3) - 6(x + 3) = (x + 3)(x - 6)$
116. $5(3x - 7) + x(3x - 7) = (3x - 7)(x + 5)$
117. $(x + 2)^2 - 5(x + 2) = (x + 2)[(x + 2) - 5]$
 $= (x + 2)(x - 3)$
118. $(x - 1)^2 - 2(x - 1) = (x - 1)[(x - 1) - 2]$
 $= (x - 1)(x - 3)$
119. $(3x - 2)^3 - 27$
 $= (3x - 2)^3 - 3^3$
 $= [(3x - 2) - 3][(3x - 2)^2 + 3(3x - 2) + 9]$
 $= (3x - 5)(9x^2 - 12x + 4 + 9x - 6 + 9)$
 $= (3x - 5)(9x^2 - 3x + 7)$

$$\begin{aligned}
 120. \quad & (5x+1)^3 - 1 \\
 &= (5x+1)^3 - 1^3 \\
 &= [(5x+1)-1][(5x+1)^2 + (1)(5x+1)+1] \\
 &= 5x(25x^2 + 10x + 1 + 5x + 1 + 1) \\
 &= 5x(25x^2 + 15x + 3)
 \end{aligned}$$

$$\begin{aligned}
 121. \quad & 3(x^2 + 10x + 25) - 4(x+5) \\
 &= 3(x+5)^2 - 4(x+5) \\
 &= (x+5)[3(x+5) - 4] \\
 &= (x+5)(3x+15-4) \\
 &= (x+5)(3x+11)
 \end{aligned}$$

$$\begin{aligned}
 122. \quad & 7(x^2 - 6x + 9) + 5(x-3) \\
 &= 7(x-3)^2 + 5(x-3) \\
 &= (x-3)[7(x-3) + 5] \\
 &= (x-3)(7x-21+5) \\
 &= (x-3)(7x-16)
 \end{aligned}$$

$$\begin{aligned}
 123. \quad & x^3 + 2x^2 - x - 2 = x^2(x+2) - 1(x+2) \\
 &= (x+2)(x^2 - 1) \\
 &= (x+2)(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 124. \quad & x^3 - 3x^2 - x + 3 = x^2(x-3) - 1(x-3) \\
 &= (x-3)(x^2 - 1) \\
 &= (x-3)(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 125. \quad & x^4 - x^3 + x - 1 = x^3(x-1) + 1(x-1) \\
 &= (x-1)(x^3 + 1) \\
 &= (x-1)(x+1)(x^2 - x + 1)
 \end{aligned}$$

$$\begin{aligned}
 126. \quad & x^4 + x^3 + x + 1 = x^3(x+1) + 1(x+1) \\
 &= (x+1)(x^3 + 1) \\
 &= (x+1)(x+1)(x^2 - x + 1) \\
 &= (x+1)^2(x^2 - x + 1)
 \end{aligned}$$

$$\begin{aligned}
 127. \quad & 2(3x+4)^2 + (2x+3) \cdot 2(3x+4) \cdot 3 \\
 &= 2(3x+4)((3x+4) + (2x+3) \cdot 3) \\
 &= 2(3x+4)(3x+4+6x+9) \\
 &= 2(3x+4)(9x+13)
 \end{aligned}$$

$$\begin{aligned}
 128. \quad & 5(2x+1)^2 + (5x-6) \cdot 2(2x+1) \cdot 2 \\
 &= (2x+1)(5(2x+1) + (5x-6) \cdot 4) \\
 &= (2x+1)(10x+5+20x-24) \\
 &= (2x+1)(30x-19)
 \end{aligned}$$

$$\begin{aligned}
 129. \quad & 2x(2x+5) + x^2 \cdot 2 = 2x((2x+5) + x) \\
 &= 2x(2x+5+x) \\
 &= 2x(3x+5)
 \end{aligned}$$

$$\begin{aligned}
 130. \quad & 3x^2(8x-3) + x^3 \cdot 8 = x^2(3(8x-3) + 8x) \\
 &= x^2(24x-9+8x) \\
 &= x^2(32x-9)
 \end{aligned}$$

$$\begin{aligned}
 131. \quad & 2(x+3)(x-2)^3 + (x+3)^2 \cdot 3(x-2)^2 \\
 &= (x+3)(x-2)^2(2(x-2) + (x+3) \cdot 3) \\
 &= (x+3)(x-2)^2(2x-4+3x+9) \\
 &= (x+3)(x-2)^2(5x+5) \\
 &= 5(x+3)(x-2)^2(x+1)
 \end{aligned}$$

$$\begin{aligned}
 132. \quad & 4(x+5)^3(x-1)^2 + (x+5)^4 \cdot 2(x-1) \\
 &= 2(x+5)^3(x-1)(2(x-1) + (x+5)) \\
 &= 2(x+5)^3(x-1)(2x-2+x+5) \\
 &= 2(x+5)^3(x-1)(3x+3) \\
 &= 2 \cdot 3(x+5)^3(x-1)(x+1) \\
 &= 6(x+5)^3(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 133. \quad & (4x-3)^2 + x \cdot 2(4x-3) \cdot 4 \\
 &= (4x-3)((4x-3) + 8x) \\
 &= (4x-3)(4x-3+8x) \\
 &= (4x-3)(12x-3) \\
 &= 3(4x-3)(4x-1)
 \end{aligned}$$

Chapter R: Review

134. $3x^2(3x+4)^2 + x^3 \cdot 2(3x+4) \cdot 3$
 $= 3x^2(3x+4)((3x+4)+2x)$
 $= 3x^2(3x+4)(3x+4+2x)$
 $= 3x^2(3x+4)(5x+4)$
135. $2(3x-5) \cdot 3(2x+1)^3 + (3x-5)^2 \cdot 3(2x+1)^2 \cdot 2$
 $= 6(3x-5)(2x+1)^2((2x+1)+(3x-5))$
 $= 6(3x-5)(2x+1)^2(2x+1+3x-5)$
 $= 6(3x-5)(2x+1)^2(5x-4)$
136. $3(4x+5)^2 \cdot 4(5x+1)^2 + (4x+5)^3 \cdot 2(5x+1) \cdot 5$
 $= 2(4x+5)^2(5x+1)(6(5x+1)+5(4x+5))$
 $= 2(4x+5)^2(5x+1)(30x+6+20x+25)$
 $= 2(4x+5)^2(5x+1)(50x+31)$
137. $x^{-4} + x^{-2} = x^{-4}(1+x^{-2})$
138. $2x^{-5} - 6x^{-4} + 8x^{-3} = 2x^{-5}(1-3x+4x^2)$
139. $x^{-2}(x+1) + x^{-1}(x-1) = x^{-2}[(x+1)+x(x-1)]$
 $= x^{-2}(x+1+x^2-x)$
 $= x^{-2}(x^2+1)$
140.
 $x(x+3)^{-1} + 4x^2(x+3)^{-2} = x(x+3)^{-2}[(x+3)+4x]$
 $= x(x+3)^{-2}(5x+3)$
141. The possible factorizations are
 $(x \pm 1)(x \pm 4) = x^2 \pm 5x + 4$ or
 $(x \pm 2)(x \pm 2) = x^2 \pm 4x + 4$, none of which
equals $x^2 + 4$.
142. The possible factorizations are
 $(x \pm 1)^2 = x^2 \pm 2x + 1$, neither of which equals
 $x^2 + x + 1$.
143. Answers will vary.
144. Answers will vary.

Section R.6

1. quotient; divisor; remainder

$$2. \begin{array}{r} -3 \overline{) 2 \ 0 \ -5 \ 1} \end{array}$$

3. d

4. a

5. True

6. True

$$7. \begin{array}{r} 2 \overline{) 1 \ -7 \ 5 \ 10} \\ \underline{2 \ -10 \ -10} \\ 1 \ -5 \ -5 \ 0 \end{array}$$

Quotient: $x^2 - 5x - 5$

Remainder: 0

$$8. \begin{array}{r} -1 \overline{) 1 \ 2 \ -3 \ 1} \\ \underline{-1 \ -1 \ 4} \\ 1 \ 1 \ -4 \ 5 \end{array}$$

Quotient: $x^2 + x - 4$

Remainder: 5

$$9. \begin{array}{r} 3 \overline{) 3 \ 2 \ -1 \ 3} \\ \underline{9 \ 33 \ 96} \\ 3 \ 11 \ 32 \ 99 \end{array}$$

Quotient: $3x^2 + 11x + 32$

Remainder: 99

$$10. \begin{array}{r} -2 \overline{) -4 \ 2 \ -1 \ 1} \\ \underline{8 \ -20 \ 42} \\ -4 \ 10 \ -21 \ 43 \end{array}$$

Quotient: $-4x^2 + 10x - 21$

Remainder: 43

$$11. \begin{array}{r} -3 \overline{) 1 \ 0 \ -4 \ 0 \ 1 \ 0} \\ \underline{-3 \ 9 \ -15 \ 45 \ -138} \\ 1 \ -3 \ 5 \ -15 \ 46 \ -138 \end{array}$$

Quotient: $x^4 - 3x^3 + 5x^2 - 15x + 46$

Remainder: -138

Section R.6: Synthetic Division

$$\begin{array}{r} 12. \quad 2 \overline{) 1 \ 0 \ 1 \ 0 \ 2} \\ \underline{2 \ 4 \ 10 \ 20} \\ 1 \ 2 \ 5 \ 10 \ 22 \end{array}$$

Quotient: $x^3 + 2x^2 + 5x + 10$
 Remainder: 22

$$\begin{array}{r} 13. \quad 1 \overline{) 4 \ 0 \ -3 \ 0 \ 1 \ 0 \ 5} \\ \underline{4 \ 4 \ 1 \ 1 \ 2 \ 2} \\ 4 \ 4 \ 1 \ 1 \ 2 \ 2 \ 7 \end{array}$$

Quotient: $4x^5 + 4x^4 + x^3 + x^2 + 2x + 2$
 Remainder: 7

$$\begin{array}{r} 14. \quad -1 \overline{) 1 \ 0 \ 5 \ 0 \ 0 \ -10} \\ \underline{-1 \ 1 \ -6 \ 6 \ -6} \\ 1 \ -1 \ 6 \ -6 \ 6 \ -16 \end{array}$$

Quotient: $x^4 - x^3 + 6x^2 - 6x + 6$
 Remainder: -16

$$\begin{array}{r} 15. \quad -1.1 \overline{) 0.1 \ 0 \ 0.2 \ 0} \\ \underline{-0.11 \ 0.121 \ -0.3531} \\ 0.1 \ -0.11 \ 0.321 \ -0.3531 \end{array}$$

Quotient: $0.1x^2 - 0.11x + 0.321$
 Remainder: -0.3531

$$\begin{array}{r} 16. \quad -2.1 \overline{) 0.1 \ 0 \ -0.2} \\ \underline{-0.21 \ 0.441} \\ 0.1 \ -0.21 \ 0.241 \end{array}$$

Quotient: $0.1x - 0.21$
 Remainder: 0.241

$$\begin{array}{r} 17. \quad 2 \overline{) 1 \ 0 \ 0 \ 0 \ 0 \ -32} \\ \underline{2 \ 4 \ 8 \ 16 \ 32} \\ 1 \ 2 \ 4 \ 8 \ 16 \ 0 \end{array}$$

Quotient: $x^4 + 2x^3 + 4x^2 + 8x + 16$
 Remainder: 0

$$\begin{array}{r} 18. \quad -1 \overline{) 1 \ 0 \ 0 \ 0 \ 0 \ 1} \\ \underline{-1 \ 1 \ -1 \ 1 \ -1} \\ 1 \ -1 \ 1 \ -1 \ 1 \ 0 \end{array}$$

Quotient: $x^4 - x^3 + x^2 - x + 1$
 Remainder: 0

$$\begin{array}{r} 19. \quad 2 \overline{) 4 \ -3 \ -8 \ 4} \\ \underline{8 \ 10 \ 4} \\ 4 \ 5 \ 2 \ 8 \end{array}$$

Remainder = $8 \neq 0$. Therefore, $x - 2$ is not a factor of $4x^3 - 3x^2 - 8x + 4$.

$$\begin{array}{r} 20. \quad -3 \overline{) -4 \ 5 \ 0 \ 8} \\ \underline{12 \ -51 \ 153} \\ -4 \ 17 \ -51 \ 161 \end{array}$$

Remainder = $161 \neq 0$. Therefore, $x + 3$ is not a factor of $-4x^3 + 5x^2 + 8$.

$$\begin{array}{r} 21. \quad 3 \overline{) 2 \ -6 \ 0 \ -7 \ 21} \\ \underline{6 \ 0 \ 0 \ -21} \\ 2 \ 0 \ 0 \ -7 \ 0 \end{array}$$

Remainder = 0. Therefore, $x - 3$ is a factor of $2x^4 - 6x^3 - 7x + 21$.

$$\begin{array}{r} 22. \quad 2 \overline{) 4 \ 0 \ -15 \ 0 \ -4} \\ \underline{8 \ 16 \ 2 \ 4} \\ 4 \ 8 \ 1 \ 2 \ 0 \end{array}$$

Remainder = 0. Therefore, $x - 2$ is a factor of $4x^4 - 15x^2 - 4$.

$$\begin{array}{r} 23. \quad -2 \overline{) 5 \ 0 \ 0 \ 43 \ 0 \ 0 \ 24} \\ \underline{-10 \ 20 \ -40 \ -6 \ 12 \ -24} \\ 3 \ -10 \ 20 \ 3 \ -6 \ 12 \ 0 \end{array}$$

Remainder = 0. Therefore, $x + 3$ is a factor of $5x^6 + 43x^3 + 24$.

$$\begin{array}{r} 24. \quad -3 \overline{) 2 \ 0 \ -18 \ 0 \ 1 \ 0 \ -9} \\ \underline{-6 \ 18 \ 0 \ 0 \ -3 \ 9} \\ 2 \ -6 \ 0 \ 0 \ 1 \ -3 \ 0 \end{array}$$

Remainder = 0. Therefore, $x + 2$ is a factor of $2x^6 - 18x^4 + x^2 - 9$.

Chapter R: Review

$$\begin{array}{r} 25. \quad -4 \overline{) 1 \quad 0 \quad -16 \quad -1 \quad 0 \quad 19} \\ \underline{-4 \quad 16 \quad 0 \quad 4 \quad -16} \\ 1 \quad -4 \quad 0 \quad -1 \quad 4 \quad 3 \end{array}$$

Remainder = $1 \neq 0$. Therefore, $x+4$ is not a factor of $x^5 - 16x^3 - x^2 + 19$.

$$\begin{array}{r} 26. \quad -4 \overline{) 1 \quad 0 \quad -16 \quad 0 \quad 1 \quad 0 \quad -16} \\ \underline{-4 \quad 16 \quad 0 \quad 0 \quad -4 \quad 16} \\ 1 \quad -4 \quad 0 \quad 0 \quad 1 \quad -4 \quad 0 \end{array}$$

Remainder = 0. Therefore, $x+4$ is a factor of $x^6 - 16x^4 + x^2 - 16$.

$$\begin{array}{r} 27. \quad \frac{1}{3} \overline{) 3 \quad -1 \quad 0 \quad 6 \quad -2} \\ \underline{1 \quad 0 \quad 0 \quad 2} \\ 3 \quad 0 \quad 0 \quad 6 \quad 0 \end{array}$$

Remainder = 0; therefore $x - \frac{1}{3}$ is a factor of $3x^4 - x^3 + 6x - 2$.

$$\begin{array}{r} 28. \quad -\frac{1}{3} \overline{) 3 \quad 1 \quad 0 \quad -3 \quad 1} \\ \underline{-1 \quad 0 \quad 0 \quad 1} \\ 3 \quad 0 \quad 0 \quad -3 \quad 2 \end{array}$$

Remainder = $2 \neq 0$; therefore $x + \frac{1}{3}$ is not a factor of $3x^4 + x^3 - 3x + 1$.

$$\begin{array}{r} 29. \quad -2 \overline{) 1 \quad -2 \quad 3 \quad 5} \\ \underline{-2 \quad 8 \quad -22} \\ 1 \quad -4 \quad 11 \quad -17 \end{array}$$

$$\frac{x^3 - 2x^2 + 3x + 5}{x + 2} = x^2 - 4x + 11 + \frac{-17}{x + 2}$$

$$a + b + c + d = 1 - 4 + 11 - 17 = -9$$

$$\begin{array}{r} 30. \quad h \overline{) 1 \quad 3-h \quad -2h \quad -2h^2 \quad h^3} \\ \underline{h \quad 3h \quad h^2 \quad -h^3} \\ 1 \quad 3 \quad h \quad -h^2 \quad 0 \end{array}$$

$x^3 + 3x^2 + hx - h^2$ is the quotient and 0 is the remainder.

$$\begin{array}{r} 31. \quad -y \overline{) 1 \quad 3y \quad -3y^2 \quad -y^3 \quad 4y^4} \\ \underline{-y \quad -2y^2 \quad 5y^3 \quad 4y^3} \\ 1 \quad 2y \quad -5y^2 \quad 4y^3 \quad 0 \end{array}$$

Yes, $x+y$ is a factor of

$$x^4 + 3x^3y - 3x^2y^2 - xy^3 + 4y^4.$$

32. Answers will vary.

Section R.7

1. lowest terms

2. Least Common Multiple

3. d

4. a

$$\begin{aligned} 5. \text{ True; } \frac{\frac{1}{3} + \frac{1}{x}}{\frac{1}{5} - \frac{1}{x}} &= \frac{\frac{x+3}{3x}}{\frac{x-5}{5x}} \\ &= \frac{x+3}{3x} \cdot \frac{5x}{x-5} = \frac{5(x+3)}{3(x-5)} \end{aligned}$$

6. False;

$$2x^3 + 6x^2 = 2x^2(x+3)$$

$$6x^4 + 4x^3 = 2x^3(3x+2)$$

$$LCM = 2x^3(x+3)(3x+2)$$

$$7. \quad \frac{3x+9}{x^2-9} = \frac{3(x+3)}{(x-3)(x+3)} = \frac{3}{x-3}$$

$$8. \quad \frac{4x^2+8x}{12x+24} = \frac{4x(x+2)}{12(x+2)} = \frac{x}{3}$$

$$9. \quad \frac{x^2-2x}{3x-6} = \frac{x(x-2)}{3(x-2)} = \frac{x}{3}$$

$$10. \quad \frac{15x^2+24x}{3x^2} = \frac{3x(5x+8)}{3x^2} = \frac{5x+8}{x}$$

$$11. \quad \frac{24x^2}{12x^2-6x} = \frac{24x^2}{6x(2x-1)} = \frac{4x}{2x-1}$$

$$12. \quad \frac{x^2+4x+4}{x^2-4} = \frac{(x+2)(x+2)}{(x-2)(x+2)} = \frac{x+2}{x-2}$$

$$\begin{aligned}
 13. \quad \frac{y^2 - 49}{3y^2 - 18y - 21} &= \frac{(y+7)(y-7)}{3(y^2 - 6y - 7)} \\
 &= \frac{(y+7)(y-7)}{3(y-7)(y+1)} \\
 &= \frac{y+7}{3(y+1)}
 \end{aligned}$$

$$14. \quad \frac{3y^2 - y - 2}{3y^2 + 5y + 2} = \frac{(3y+2)(y-1)}{(3y+2)(y+1)} = \frac{y-1}{y+1}$$

$$15. \quad \frac{x^2 + 4x - 12}{x^2 - 4x + 4} = \frac{(x+6)(x-2)}{(x-2)(x-2)} = \frac{x+6}{x-2}$$

$$16. \quad \frac{x - x^2}{x^2 + x - 2} = \frac{-x(x-1)}{(x+2)(x-1)} = \frac{-x}{x+2} = -\frac{x}{x+2}$$

$$\begin{aligned}
 17. \quad \frac{x^2 + x - 20}{4 - x} &= \frac{(x+5)(x-4)}{2 - x} \\
 &= \frac{(x+5)(x-4)}{-1(x-4)} \\
 &= -(x+5)
 \end{aligned}$$

$$18. \quad \frac{2x^2 + 5x - 3}{1 - 2x} = \frac{(2x-1)(x+3)}{-1(2x-1)} = -(x+3) = -x-3$$

$$\begin{aligned}
 19. \quad \frac{3x+6}{5x^2} \cdot \frac{x}{x^2-4} &= \frac{3(x+2)}{5x^2} \cdot \frac{x}{(x-2)(x+2)} \\
 &= \frac{3}{5x(x-2)}
 \end{aligned}$$

$$20. \quad \frac{3}{2x} \cdot \frac{x^2}{6x+10} = \frac{3}{2} \cdot \frac{x}{2(3x+5)} = \frac{3x}{4(3x+5)}$$

$$\begin{aligned}
 21. \quad \frac{4x^2}{x^2-16} \cdot \frac{x^3-64}{2x} &= \frac{4x^2}{(x-4)(x+4)} \cdot \frac{(x-4)(x^2+4x+16)}{2x} \\
 &= \frac{2x \cdot 2x(x-4)(x^2+4x+16)}{2x(x-4)(x+4)} \\
 &= \frac{2x(x^2+4x+16)}{x+4}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{12}{x^2+x} \cdot \frac{x^3+1}{4x-2} &= \frac{12}{x(x+1)} \cdot \frac{(x+1)(x^2-x+1)}{2(2x-1)} \\
 &= \frac{2 \cdot 6(x+1)(x^2-x+1)}{2x(x+1)(2x-1)} \\
 &= \frac{6(x^2-x+1)}{x(2x-1)}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \frac{4x-8}{-3x} \cdot \frac{12}{12-6x} &= \frac{4(x-2)}{-3x} \cdot \frac{12}{6(2-x)} \\
 &= \frac{4(x-2)}{-3x} \cdot \frac{2}{(-1)(x-2)} \\
 &= \frac{8}{3x}
 \end{aligned}$$

$$24. \quad \frac{6x-27}{5x} \cdot \frac{2}{4x-18} = \frac{3(2x-9)}{5x} \cdot \frac{2}{2(2x-9)} = \frac{3}{5x}$$

$$\begin{aligned}
 25. \quad \frac{x^2-4x-12}{x^2+2x-48} \cdot \frac{x^2+4x-32}{x^2+10x+16} &= \frac{(x-6)(x+2)}{(x+8)(x-6)} \cdot \frac{(x+8)(x-4)}{(x+8)(x+2)} \\
 &= \frac{x-4}{x+8}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \frac{x^2+x-6}{x^2+4x-5} \cdot \frac{x^2-25}{x^2+2x-15} &= \frac{(x-2)(x+3)}{(x+5)(x-1)} \cdot \frac{(x+5)(x-5)}{(x+5)(x-3)} \\
 &= \frac{(x-2)(x+3)(x-5)}{(x+5)(x-1)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \frac{6x}{\frac{x^2-4}{3x-9}} &= \frac{6x}{x^2-4} \cdot \frac{2x+4}{3x-9} \\
 &= \frac{6x}{(x-2)(x+2)} \cdot \frac{2(x+2)}{3(x-3)} \\
 &= \frac{4x}{(x-2)(x-3)}
 \end{aligned}$$

Chapter R: Review

$$\begin{aligned} 28. \frac{\frac{12x}{5x+20}}{\frac{4x^2}{x^2-16}} &= \frac{12x}{5x+20} \cdot \frac{x^2-16}{4x^2} \\ &= \frac{12x}{5(x+4)} \cdot \frac{(x+4)(x-4)}{4x^2} \\ &= \frac{3(x-4)}{5x} \end{aligned}$$

$$\begin{aligned} 29. \frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}} &= \frac{8x}{x^2-1} \cdot \frac{x+1}{10x} \\ &= \frac{8x}{(x-1)(x+1)} \cdot \frac{x+1}{10x} \\ &= \frac{4}{5(x-1)} \end{aligned}$$

$$\begin{aligned} 30. \frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}} &= \frac{x-2}{4x} \cdot \frac{12x}{x^2-4x+4} \\ &= \frac{x-2}{4x} \cdot \frac{12x}{(x-2)(x-2)} \\ &= \frac{3}{x-2} \end{aligned}$$

$$\begin{aligned} 31. \frac{\frac{4-x}{4+x}}{\frac{4x}{x^2-16}} &= \frac{4-x}{4+x} \cdot \frac{x^2-16}{4x} \\ &= \frac{4-x}{4+x} \cdot \frac{(x+4)(x-4)}{4x} \\ &= \frac{(4-x)(x-4)}{4x} \\ &= -\frac{(x-4)^2}{4x} \end{aligned}$$

$$\begin{aligned} 32. \frac{\frac{3+x}{3-x}}{\frac{x^2-9}{9x^3}} &= \frac{3+x}{3-x} \cdot \frac{9x^3}{x^2-9} \\ &= \frac{3+x}{3-x} \cdot \frac{9x^3}{(x+3)(x-3)} \\ &= \frac{9x^3}{(3-x)(x-3)} \\ &= \frac{9x^3}{-(x-3)^2} \\ &= -\frac{9x^3}{(x-3)^2} \end{aligned}$$

$$\begin{aligned} 33. \frac{\frac{x^2+7x+12}{x^2-7x+12}}{\frac{x^2+x-12}{x^2-x-12}} &= \frac{x^2+7x+12}{x^2-7x+12} \cdot \frac{x^2-x-12}{x^2+x-12} \\ &= \frac{(x+3)(x+4)}{(x-3)(x-4)} \cdot \frac{(x-4)(x+3)}{(x+4)(x-3)} \\ &= \frac{(x+3)^2}{(x-3)^2} \end{aligned}$$

$$\begin{aligned} 34. \frac{\frac{x^2+7x+6}{x^2+x-6}}{\frac{x^2+5x-6}{x^2+5x+6}} &= \frac{x^2+7x+6}{x^2+x-6} \cdot \frac{x^2+5x+6}{x^2+5x-6} \\ &= \frac{(x+6)(x+1)}{(x+3)(x-2)} \cdot \frac{(x+2)(x+3)}{(x+6)(x-1)} \\ &= \frac{(x+1)(x+2)}{(x-2)(x-1)} \end{aligned}$$

$$\begin{aligned} 35. \frac{\frac{5x^2-7x-6}{2x^2+3x-5}}{\frac{15x^2+14x+3}{2x^2+13x+20}} &= \frac{5x^2-7x-6}{2x^2+3x-5} \cdot \frac{2x^2+13x+20}{15x^2+14x+3} \\ &= \frac{(5x+3)(x-2)}{(x-1)(2x+5)} \cdot \frac{(2x+5)(x+4)}{(5x+3)(3x+1)} \\ &= \frac{(x-2)(x+4)}{(x-1)(3x+1)} \end{aligned}$$

$$\begin{aligned}
 36. \quad \frac{\frac{9x^2+3x-2}{12x^2+5x-2}}{\frac{9x^2-6x+1}{8x^2-10x-3}} &= \frac{9x^2+3x-2}{12x^2+5x-2} \cdot \frac{8x^2-10x-3}{9x^2-6x+1} \\
 &= \frac{(3x+2)(3x-1)}{(3x+2)(4x-1)} \cdot \frac{(4x+1)(2x-3)}{(3x-1)(3x-1)} \\
 &= \frac{(4x+1)(2x-3)}{(4x-1)(3x-1)}
 \end{aligned}$$

$$37. \quad \frac{x}{2} + \frac{5}{2} = \frac{x+5}{2}$$

$$38. \quad \frac{3}{x} - \frac{6}{x} = \frac{3-6}{x} = \frac{-3}{x} = -\frac{3}{x}$$

$$39. \quad \frac{x^2}{2x-3} - \frac{4}{2x-3} = \frac{x^2-4}{2x-3} = \frac{(x+2)(x-2)}{2x-3}$$

$$40. \quad \frac{3x^2}{2x-1} - \frac{9}{2x-1} = \frac{3x^2-9}{2x-1} = \frac{3(x^2-3)}{2x-1}$$

$$41. \quad \frac{x+5}{x-4} + \frac{3x-2}{x-4} = \frac{x+5+3x-2}{x-4} = \frac{4x+3}{x-4}$$

$$42. \quad \frac{2x-5}{3x+2} + \frac{x+4}{3x+2} = \frac{2x-5+x+4}{3x+2} = \frac{3x-1}{3x+2}$$

$$\begin{aligned}
 43. \quad \frac{3x+5}{2x-1} - \frac{2x-4}{2x-1} &= \frac{(3x+5)-(2x-4)}{2x-1} \\
 &= \frac{3x+5-2x+4}{2x-1} \\
 &= \frac{x+9}{2x-1}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{5x-4}{3x+4} - \frac{x+1}{3x+4} &= \frac{(5x-4)-(x+1)}{3x+4} \\
 &= \frac{5x-4-x-1}{3x+4} \\
 &= \frac{4x-5}{3x+4}
 \end{aligned}$$

$$45. \quad \frac{4}{x-2} + \frac{x}{2-x} = \frac{4}{x-2} - \frac{x}{x-2} = \frac{4-x}{x-2}$$

$$46. \quad \frac{6}{x-1} - \frac{x}{1-x} = \frac{6}{x-1} + \frac{x}{x-1} = \frac{x+6}{x-1}$$

$$\begin{aligned}
 47. \quad \frac{7}{x-3} - \frac{3}{x+1} &= \frac{7(x+1)}{(x-3)(x+1)} - \frac{3(x-3)}{(x+1)(x-3)} \\
 &= \frac{7x+7-3x+9}{(x+1)(x-3)} \\
 &= \frac{4x+16}{(x+1)(x-3)} \\
 &= \frac{4(x+4)}{(x+1)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{2}{x+5} - \frac{5}{x-5} &= \frac{2(x-5)}{(x+5)(x-5)} - \frac{5(x+5)}{(x+5)(x-5)} \\
 &= \frac{2x-10-5x-25}{(x+5)(x-5)} \\
 &= \frac{-3x-35}{(x+5)(x-5)} \\
 &= -\frac{3x+35}{(x+5)(x-5)}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{x}{x+1} + \frac{2x-3}{x-1} &= \frac{x(x-1)}{(x+1)(x-1)} + \frac{(2x-3)(x+1)}{(x-1)(x+1)} \\
 &= \frac{x^2-x+2x^2-x-3}{(x-1)(x+1)} \\
 &= \frac{3x^2-2x-3}{(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad \frac{3x}{x-4} + \frac{2x}{x+3} &= \frac{3x(x+3)}{(x-4)(x+3)} + \frac{2x(x-4)}{(x-4)(x+3)} \\
 &= \frac{3x^2+9x+2x^2-8x}{(x-4)(x+3)} \\
 &= \frac{5x^2+x}{(x-4)(x+3)} \\
 &= \frac{x(5x+1)}{(x-4)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{x-3}{x+2} - \frac{x+4}{x-2} &= \frac{(x-3)(x-2)}{(x+2)(x-2)} - \frac{(x+4)(x+2)}{(x-2)(x+2)} \\
 &= \frac{x^2-5x+6-(x^2+6x+8)}{(x+2)(x-2)} \\
 &= \frac{x^2-5x+6-x^2-6x-8}{(x+2)(x-2)} \\
 &= \frac{-11x-2}{(x+2)(x-2)} \quad \text{or} \quad \frac{-(11x+2)}{(x+2)(x-2)}
 \end{aligned}$$

Chapter R: Review

$$\begin{aligned}
 52. \quad \frac{2x-3}{x-1} - \frac{2x+1}{x+1} &= \frac{(2x-3)(x+1)}{(x-1)(x+1)} - \frac{(2x+1)(x-1)}{(x+1)(x-1)} \\
 &= \frac{2x^2 - x - 3 - (2x^2 - x - 1)}{(x+1)(x-1)} \\
 &= \frac{2x^2 - x - 3 - 2x^2 + x + 1}{(x+1)(x-1)} \\
 &= \frac{-2}{(x+1)(x-1)} \\
 &= -\frac{2}{(x+1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{x}{x^2-4} + \frac{1}{x} &= \frac{x^2 + x^2 - 4}{x(x^2 - 4)} \\
 &= \frac{2x^2 - 4}{x(x^2 - 4)} \\
 &= \frac{2(x^2 - 2)}{x(x-2)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad \frac{x-1}{x^3} + \frac{x}{x^2+1} &= \frac{(x-1)(x^2+1) + x^4}{x^3(x^2+1)} \\
 &= \frac{x^3 - x^2 + x - 1 + x^4}{x^3(x^2+1)} \\
 &= \frac{x^4 + x^3 - x^2 + x - 1}{x^3(x^2+1)}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad x^2 - 4 &= (x+2)(x-2) \\
 x^2 - x - 2 &= (x+1)(x-2) \\
 \text{Therefore, LCM} &= (x+2)(x-2)(x+1).
 \end{aligned}$$

$$\begin{aligned}
 56. \quad x^2 - x - 12 &= (x+3)(x-4) \\
 x^2 - 8x + 16 &= (x-4)(x-4) \\
 \text{Therefore, LCM} &= (x+3)(x-4)^2.
 \end{aligned}$$

$$\begin{aligned}
 57. \quad x^3 - x &= x(x^2 - 1) = x(x+1)(x-1) \\
 x^2 - x &= x(x-1) \\
 \text{Therefore, LCM} &= x(x+1)(x-1).
 \end{aligned}$$

$$\begin{aligned}
 58. \quad 3x^2 - 27 &= 3(x^2 - 9) = 3(x+3)(x-3) \\
 2x^2 - x - 15 &= (2x+5)(x-3) \\
 \text{Therefore, LCM} &= 3(2x+5)(x-3)(x+3).
 \end{aligned}$$

$$\begin{aligned}
 59. \quad 4x^3 - 4x^2 + x &= x(4x^2 - 4x + 1) \\
 &= x(2x-1)(2x-1) \\
 2x^3 - x^2 &= x^2(2x-1) \\
 x^3 & \\
 \text{Therefore, LCM} &= x^3(2x-1)^2.
 \end{aligned}$$

$$\begin{aligned}
 60. \quad x-3 & \\
 x^2 + 3x &= x(x+3) \\
 x^3 - 9x &= x(x^2 - 9) = x(x+3)(x-3) \\
 \text{Therefore, LCM} &= x(x+3)(x-3).
 \end{aligned}$$

$$\begin{aligned}
 61. \quad x^3 - x &= x(x^2 - 1) = x(x+1)(x-1) \\
 x^3 - 2x^2 + x &= x(x^2 - 2x + 1) = x(x-1)^2 \\
 x^3 - 1 &= (x-1)(x^2 + x + 1) \\
 \text{Therefore, LCM} &= x(x+1)(x-1)^2(x^2 + x + 1).
 \end{aligned}$$

$$\begin{aligned}
 62. \quad x^2 + 4x + 4 &= (x+2)^2 \\
 x^3 + 2x^2 &= x^2(x+2) \\
 (x+2)^3 & \\
 \text{Therefore, LCM} &= x^2(x+2)^3.
 \end{aligned}$$

$$\begin{aligned}
 63. \quad \frac{x}{x^2-7x+6} - \frac{x}{x^2-2x-24} & \\
 &= \frac{x}{(x-6)(x-1)} - \frac{x}{(x-6)(x+4)} \\
 &= \frac{x(x+4)}{(x-6)(x-1)(x+4)} - \frac{x(x-1)}{(x-6)(x+4)(x-1)} \\
 &= \frac{x^2 + 4x - x^2 + x}{(x-6)(x+4)(x-1)} = \frac{5x}{(x-6)(x+4)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 64. \quad \frac{x}{x-3} - \frac{x+1}{x^2+5x-24} & \\
 &= \frac{x}{(x-3)} - \frac{x+1}{(x-3)(x+8)} \\
 &= \frac{x(x+8)}{(x-3)(x+8)} - \frac{x+1}{(x-3)(x+8)} \\
 &= \frac{x^2 + 8x - x - 1}{(x-3)(x+8)} = \frac{x^2 + 7x - 1}{(x-3)(x+8)}
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & \frac{4x}{x^2-4} - \frac{2}{x^2+x-6} \\
 &= \frac{4x}{(x-2)(x+2)} - \frac{2}{(x+3)(x-2)} \\
 &= \frac{4x(x+3)}{(x-2)(x+2)(x+3)} - \frac{2(x+2)}{(x+3)(x-2)(x+2)} \\
 &= \frac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)} \\
 &= \frac{4x^2+10x-4}{(x-2)(x+2)(x+3)} \\
 &= \frac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 66. \quad & \frac{3x}{x-1} - \frac{x-4}{x^2-2x+1} = \frac{3x}{(x-1)} - \frac{x-4}{(x-1)^2} \\
 &= \frac{3x(x-1)}{(x-1)(x-1)} - \frac{x-4}{(x-1)^2} \\
 &= \frac{3x^2-3x-x+4}{(x-1)^2} \\
 &= \frac{3x^2-4x+4}{(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 67. \quad & \frac{3}{(x-1)^2(x+1)} + \frac{2}{(x-1)(x+1)^2} \\
 &= \frac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2} \\
 &= \frac{3x+3+2x-2}{(x-1)^2(x+1)^2} \\
 &= \frac{5x+1}{(x-1)^2(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 68. \quad & \frac{2}{(x+2)^2(x-1)} - \frac{6}{(x+2)(x-1)^2} \\
 &= \frac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2} \\
 &= \frac{2x-2-6x-12}{(x+2)^2(x-1)^2} \\
 &= \frac{-4x-14}{(x+2)^2(x-1)^2} \\
 &= \frac{-2(2x+7)}{(x+2)^2(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 69. \quad & \frac{x+4}{x^2-x-2} - \frac{2x+3}{x^2+2x-8} \\
 &= \frac{x+4}{(x-2)(x+1)} - \frac{2x+3}{(x+4)(x-2)} \\
 &= \frac{(x+4)(x+4)}{(x-2)(x+1)(x+4)} - \frac{(2x+3)(x+1)}{(x+4)(x-2)(x+1)} \\
 &= \frac{x^2+8x+16-(2x^2+5x+3)}{(x-2)(x+1)(x+4)} \\
 &= \frac{-x^2+3x+13}{(x-2)(x+1)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 70. \quad & \frac{2x-3}{x^2+8x+7} - \frac{x-2}{(x+1)^2} \\
 &= \frac{2x-3}{(x+1)(x+7)} - \frac{x-2}{(x+1)^2} \\
 &= \frac{(2x-3)(x+1)}{(x+1)(x+7)(x+1)} - \frac{(x-2)(x+7)}{(x+1)^2(x+7)} \\
 &= \frac{2x^2-x-3-(x^2+5x-14)}{(x+1)^2(x+7)} \\
 &= \frac{x^2-6x+11}{(x+1)^2(x+7)}
 \end{aligned}$$

$$\begin{aligned}
 71. \quad & \frac{1}{x} - \frac{2}{x^2+x} + \frac{3}{x^3-x^2} \\
 &= \frac{1}{x} - \frac{2}{x(x+1)} + \frac{3}{x^2(x-1)} \\
 &= \frac{x(x+1)(x-1)-2x(x-1)+3(x+1)}{x^2(x+1)(x-1)} \\
 &= \frac{x(x^2-1)-2x^2+2x+3x+3}{x^2(x+1)(x-1)} \\
 &= \frac{x^3-x-2x^2+5x+3}{x^2(x+1)(x-1)} \\
 &= \frac{x^3-2x^2+4x+3}{x^2(x+1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 72. \quad & \frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x^3-x^2} \\
 &= \frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x^2(x-1)} \\
 &= \frac{x^3+2x(x-1)^2-(x+1)(x-1)}{x^2(x-1)^2} \\
 &= \frac{x^3+2x(x^2-2x+1)-(x^2-1)}{x^2(x-1)^2} \\
 &= \frac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2} \\
 &= \frac{3x^3-5x^2+2x+1}{x^2(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 73. \quad & \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right) = \frac{1}{h} \left(\frac{1 \cdot x}{(x+h)x} - \frac{1(x+h)}{x(x+h)} \right) \\
 &= \frac{1}{h} \left(\frac{x-x-h}{x(x+h)} \right) \\
 &= \frac{-h}{hx(x+h)} \\
 &= \frac{-1}{x(x+h)}
 \end{aligned}$$

$$\begin{aligned}
 74. \quad & \frac{1}{h} \left(\frac{1}{(x+h)^2} - \frac{1}{x^2} \right) \\
 &= \frac{1}{h} \left(\frac{1 \cdot x^2}{(x+h)^2 x^2} - \frac{1(x+h)^2}{x^2(x+h)^2} \right) \\
 &= \frac{1}{h} \left(\frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2} \right) \\
 &= \frac{-2xh - h^2}{hx^2(x+h)^2} \\
 &= \frac{h(-2x-h)}{hx^2(x+h)^2} \\
 &= \frac{-2x-h}{x^2(x+h)^2} \\
 &= -\frac{2x+h}{x^2(x+h)^2}
 \end{aligned}$$

$$75. \quad \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\left(\frac{x}{x} + \frac{1}{x}\right)}{\left(\frac{x}{x} - \frac{1}{x}\right)} = \frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)} = \frac{x+1}{x} \cdot \frac{x}{x-1} = \frac{x+1}{x-1}$$

$$\begin{aligned}
 76. \quad & \frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}} = \frac{\left(\frac{4x^2}{x^2} + \frac{1}{x^2}\right)}{\left(\frac{3x^2}{x^2} - \frac{1}{x^2}\right)} = \frac{\left(\frac{4x^2+1}{x^2}\right)}{\left(\frac{3x^2-1}{x^2}\right)} \\
 &= \frac{4x^2+1}{x^2} \cdot \frac{x^2}{3x^2-1} \\
 &= \frac{4x^2+1}{3x^2-1}
 \end{aligned}$$

$$\begin{aligned}
 77. \quad & \frac{2-\frac{x+1}{x}}{3+\frac{x-1}{x+1}} = \frac{\frac{2x}{x} - \frac{x+1}{x}}{\frac{3(x+1)}{x+1} + \frac{x-1}{x+1}} = \frac{\frac{2x-x-1}{x}}{\frac{3x+3+x-1}{x+1}} \\
 &= \frac{\frac{x-1}{x}}{\frac{4x+2}{x+1}} = \frac{x-1}{x} \cdot \frac{x+1}{2(2x+1)} \\
 &= \frac{(x-1)(x+1)}{2x(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & \frac{1-\frac{x}{x+1}}{2-\frac{x-1}{x}} = \frac{\left(\frac{x+1}{x+1} - \frac{x}{x+1}\right)}{\left(\frac{2x}{x} - \frac{x-1}{x}\right)} = \frac{\left(\frac{1}{x+1}\right)}{\left(\frac{x+1}{x}\right)} \\
 &= \frac{1}{x+1} \cdot \frac{x}{x+1} \\
 &= \frac{x}{(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 79. \quad & \frac{\frac{x+4}{x-2} - \frac{x-3}{x+1}}{x+1} \\
 &= \frac{\left(\frac{(x+4)(x+1)}{(x-2)(x+1)} - \frac{(x-3)(x-2)}{(x+1)(x-2)}\right)}{x+1} \\
 &= \frac{\left(\frac{x^2+5x+4-(x^2-5x+6)}{(x-2)(x+1)}\right)}{x+1} \\
 &= \frac{10x-2}{(x-2)(x+1)} \cdot \frac{1}{x+1} \\
 &= \frac{2(5x-1)}{(x-2)(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 80. \quad & \frac{\frac{x-2}{x+1} - \frac{x}{x-2}}{x+3} \\
 &= \frac{\left(\frac{(x-2)(x-2)}{(x+1)(x-2)} - \frac{x(x+1)}{(x-2)(x+1)} \right)}{x+3} \\
 &= \frac{\left(\frac{x^2 - 4x + 4 - (x^2 + x)}{(x-2)(x+1)} \right)}{x+3} \\
 &= \frac{-5x + 4}{(x-2)(x+1)} \cdot \frac{1}{x+3} \\
 &= \frac{-5x + 4}{(x-2)(x+1)(x+3)} \\
 &= \frac{-(5x-4)}{(x-2)(x+1)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 81. \quad & \frac{\frac{x-2}{x+2} + \frac{x-1}{x+1}}{\frac{x}{x+1} - \frac{2x-3}{x}} \\
 &= \frac{\left(\frac{(x-2)(x+1)}{(x+2)(x+1)} + \frac{(x-1)(x+2)}{(x+1)(x+2)} \right)}{\left(\frac{x^2}{(x+1)(x)} - \frac{(2x-3)(x+1)}{x(x+1)} \right)} \\
 &= \frac{\left(\frac{x^2 - x - 2 + x^2 + x - 2}{(x+2)(x+1)} \right)}{\left(\frac{x^2 - (2x^2 - x - 3)}{x(x+1)} \right)} \\
 &= \frac{\left(\frac{2x^2 - 4}{(x+2)(x+1)} \right)}{\left(\frac{-x^2 + x + 3}{x(x+1)} \right)} \\
 &= \frac{2(x^2 - 2)}{(x+2)(x+1)} \cdot \frac{x(x+1)}{-(x^2 - x - 3)} \\
 &= \frac{2x(x^2 - 2)}{-(x+2)(x^2 - x - 3)} \\
 &= \frac{-2x(x^2 - 2)}{(x+2)(x^2 - x - 3)}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & \frac{\frac{2x+5}{x} - \frac{x}{x-3}}{\frac{x^2}{x-3} - \frac{(x+1)^2}{x+3}} \\
 &= \frac{\left(\frac{(2x+5)(x-3)}{x(x-3)} - \frac{x(x)}{x(x-3)} \right)}{\left(\frac{x^2(x+3)}{(x-3)(x+3)} - \frac{(x-3)(x+1)^2}{(x-3)(x+3)} \right)} \\
 &= \frac{\left(\frac{2x^2 - x - 15 - x^2}{x(x-3)} \right)}{\left(\frac{x^3 + 3x^2 - (x^3 - x^2 - 5x - 3)}{(x-3)(x+3)} \right)} \\
 &= \frac{\left(\frac{x^2 - x - 15}{x(x-3)} \right)}{\left(\frac{4x^2 + 5x + 3}{(x-3)(x+3)} \right)} \\
 &= \frac{x^2 - x - 15}{x(x-3)} \cdot \frac{(x-3)(x+3)}{4x^2 + 5x + 3} \\
 &= \frac{(x^2 - x - 15)(x+3)}{x(4x^2 + 5x + 3)}
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{1}{\frac{x-1}{x}} \\
 &= 1 - \frac{x}{x-1} \\
 &= \frac{x-1-x}{x-1} \\
 &= \frac{-1}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 84. \quad & 1 - \frac{1}{1 - \frac{1}{1-x}} = 1 - \frac{1}{\frac{1-x-1}{1-x}} = 1 - \frac{1}{\frac{-x}{1-x}} \\
 &= 1 - \frac{1-x}{-x} = 1 + \frac{1-x}{x} \\
 &= \frac{x+1-x}{x} \\
 &= \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 85. \quad \frac{2(x-1)^{-1} + 3}{3(x-1)^{-1} + 2} &= \frac{\frac{2}{x-1} + 3}{\frac{3}{x-1} + 2} = \frac{\frac{2}{x-1} + \frac{3(x-1)}{x-1}}{\frac{3}{x-1} + \frac{2(x-1)}{x-1}} \\
 &= \frac{\frac{2+3(x-1)}{x-1}}{\frac{3+2(x-1)}{x-1}} \\
 &= \frac{2+3(x-1)}{3+2(x-1)} \\
 &= \frac{2+3(x-1)}{3+2(x-1)} = \frac{2+3x-3}{3+2x-2} \\
 &= \frac{3x-1}{2x+1}
 \end{aligned}$$

$$\begin{aligned}
 86. \quad \frac{4(x+2)^{-1} - 3}{3(x+2)^{-1} - 1} &= \frac{\frac{4}{x+2} - 3}{\frac{3}{x+2} - 1} = \frac{\frac{4}{x+2} - \frac{3(x+2)}{x+2}}{\frac{3}{x+2} - \frac{1(x+2)}{x+2}} \\
 &= \frac{\frac{4-3(x+2)}{x+2}}{\frac{3-(x+2)}{x+2}} \\
 &= \frac{4-3(x+2)}{3-(x+2)} \cdot \frac{x+2}{x+2} \\
 &= \frac{4-3(x+2)}{3-(x+2)} = \frac{4-3x-6}{3-x-2} \\
 &= \frac{-3x-2}{-x+1} = \frac{3x+2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 87. \quad \frac{(2x+3) \cdot 3 - (3x-5) \cdot 2}{(3x-5)^2} &= \frac{6x+9-6x+10}{(3x-5)^2} \\
 &= \frac{19}{(3x-5)^2}
 \end{aligned}$$

$$\begin{aligned}
 88. \quad \frac{(4x+1) \cdot 5 - (5x-2) \cdot 4}{(5x-2)^2} &= \frac{20x+5-20x+8}{(5x-2)^2} \\
 &= \frac{13}{(5x-2)^2}
 \end{aligned}$$

$$\begin{aligned}
 89. \quad \frac{x \cdot 2x - (x^2+1) \cdot 1}{(x^2+1)^2} &= \frac{2x^2 - x^2 - 1}{(x^2+1)^2} \\
 &= \frac{x^2 - 1}{(x^2+1)^2} \\
 &= \frac{(x-1)(x+1)}{(x^2+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 90. \quad \frac{x \cdot 2x - (x^2-4) \cdot 1}{(x^2-4)^2} &= \frac{2x^2 - x^2 + 4}{(x^2-4)^2} = \frac{x^2 + 4}{(x^2-4)^2} \\
 &= \frac{x^2 + 4}{(x+2)^2 (x-2)^2}
 \end{aligned}$$

$$\begin{aligned}
 91. \quad \frac{(3x+1) \cdot 2x - x^2 \cdot 3}{(3x+1)^2} &= \frac{6x^2 + 2x - 3x^2}{(3x+1)^2} \\
 &= \frac{3x^2 + 2x}{(3x+1)^2} \\
 &= \frac{x(3x+2)}{(3x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 92. \quad \frac{(2x-5) \cdot 3x^2 - x^3 \cdot 2}{(2x-5)^2} &= \frac{6x^3 - 15x^2 - 2x^3}{(2x-5)^2} \\
 &= \frac{4x^3 - 15x^2}{(2x-5)^2} \\
 &= \frac{x^2(4x-15)}{(2x-5)^2}
 \end{aligned}$$

$$\begin{aligned}
 93. \quad \frac{(x^2+1) \cdot 3 - (3x+4) \cdot 2x}{(x^2+1)^2} &= \frac{3x^2 + 3 - 6x^2 - 8x}{(x^2+1)^2} \\
 &= \frac{-3x^2 - 8x + 3}{(x^2+1)^2} \\
 &= \frac{-(3x^2 + 8x - 3)}{(x^2+1)^2} \\
 &= -\frac{(3x-1)(x+3)}{(x^2+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 94. \quad \frac{(x^2+9) \cdot 2 - (2x-5) \cdot 2x}{(x^2+9)^2} &= \frac{2x^2+18-4x^2+10x}{(x^2+9)^2} \\
 &= \frac{-2x^2+10x+18}{(x^2+9)^2} \\
 &= \frac{-2(x^2-5x-9)}{(x^2+9)^2}
 \end{aligned}$$

$$\begin{aligned}
 95. \quad \frac{1}{f} &= (n-1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\
 \frac{1}{f} &= (n-1) \left(\frac{R_2 + R_1}{R_1 \cdot R_2} \right) \\
 \frac{R_1 \cdot R_2}{f} &= (n-1)(R_2 + R_1) \\
 \frac{f}{R_1 \cdot R_2} &= \frac{1}{(n-1)(R_2 + R_1)} \\
 f &= \frac{R_1 \cdot R_2}{(n-1)(R_2 + R_1)} \\
 f &= \frac{0.1(0.2)}{(1.5-1)(0.2+0.1)} \\
 &= \frac{0.02}{0.5(0.3)} = \frac{0.02}{0.15} = \frac{2}{15} \text{ meters}
 \end{aligned}$$

$$\begin{aligned}
 96. \quad \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3} \\
 R &= \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \\
 &= \frac{5 \cdot 4 \cdot 10}{4 \cdot 10 + 5 \cdot 10 + 5 \cdot 4} \\
 &= \frac{200}{110} = \frac{20}{11} \text{ ohms}
 \end{aligned}$$

$$\begin{aligned}
 97. \quad \frac{x^2+3xk-3x-9k}{x^2+2x-15} &= \frac{x(x+3k)-3(x+3k)}{(x-3)(x+5)} \\
 \frac{(x-3)(x+3k)}{(x-3)(x+5)} &= \frac{(x+3k)}{(x+5)} \\
 &= \frac{x+12}{x+5} \\
 x+3k &= x+12 \\
 3k &= 12 \\
 k &= 4
 \end{aligned}$$

$$\begin{aligned}
 98. \quad \frac{A}{x+2} + \frac{7}{x-3} &= \frac{12x-1}{x^2-x+6} \\
 A(x-3) + 7(x+2) &= 12x-1 \\
 Ax-3A+7x+14 &= 12x-1 \\
 x(A+7)-3A+14 &= 12x-1 \\
 &\text{so} \\
 A+7 &= 12 \\
 A &= 5
 \end{aligned}$$

$$\begin{aligned}
 99. \quad 1 + \frac{1}{x} &= \frac{x+1}{x} \Rightarrow a=1, b=1, c=0 \\
 1 + \frac{1}{1+\frac{1}{x}} &= 1 + \frac{1}{\left(\frac{x+1}{x}\right)} = 1 + \frac{x}{x+1} \\
 &= \frac{x+1+x}{x+1} = \frac{2x+1}{x+1} \\
 \Rightarrow a &= 2, b=1, c=1
 \end{aligned}$$

$$\begin{aligned}
 1 + \frac{1}{1+\frac{1}{1+\frac{1}{x}}} &= 1 + \frac{1}{\left(\frac{2x+1}{x+1}\right)} = 1 + \frac{x+1}{2x+1} \\
 &= \frac{2x+1+x+1}{2x+1} = \frac{3x+2}{2x+1} \\
 \Rightarrow a &= 3, b=2, c=1
 \end{aligned}$$

$$\begin{aligned}
 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}} &= 1 + \frac{1}{\left(\frac{3x+2}{2x+1}\right)} = 1 + \frac{2x+1}{3x+2} \\
 &= \frac{3x+2+2x+1}{3x+2} = \frac{5x+3}{3x+2} \\
 \Rightarrow a &= 5, b=3, c=2
 \end{aligned}$$

If we continue this process, the values of a , b and c produce the following sequences:

$a: 1, 2, 3, 5, 8, 13, 21, \dots$

$b: 1, 1, 2, 3, 5, 8, 13, 21, \dots$

$c: 0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from the sum of the previous 2 values in the list.

Chapter R: Review

100. Answers will vary.

101. Answers will vary.

Section R.8

1. 9; -9

2. 4; $|-4| = 4$

3. index

4. cube root

5. b

6. d

7. c

8. c

9. true

10. False; $\sqrt[4]{(-3)^4} = |-3| = 3$

11. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

12. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

13. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

14. $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$

15. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

16. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

17. $\sqrt{700} = \sqrt{100 \cdot 7} = 10\sqrt{7}$

18. $\sqrt{45x^3} = \sqrt{9 \cdot 5 \cdot x^2 \cdot x} = 3x\sqrt{5x}$

19. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = 2\sqrt[3]{4}$

20. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

21. $\sqrt[3]{-8x^4} = \sqrt[3]{-8x^3 \cdot x} = -2x\sqrt[3]{x}$

22. $\sqrt[3]{192x^5} = \sqrt[3]{64 \cdot 3x^3 \cdot x^2} = 4x\sqrt[3]{3x^2}$

23. $\sqrt[4]{243} = \sqrt[4]{81 \cdot 3} = 3\sqrt[4]{3}$

24. $\sqrt[4]{48x^5} = \sqrt[4]{16x^4 \cdot 3x} = 2x\sqrt[4]{3x}$

25. $\sqrt[4]{x^{12}y^8} = \sqrt[4]{(x^3)^4(y^2)^4} = x^3y^2$

26. $\sqrt[5]{x^{10}y^5} = \sqrt[5]{(x^2)^5y^5} = x^2y$

27. $\sqrt[4]{\frac{x^9y^7}{xy^3}} = \sqrt[4]{x^8y^4} = x^2y$

28. $\sqrt[3]{\frac{3xy^2}{81x^4y^2}} = \sqrt[3]{\frac{1}{27x^3}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27x^3}} = \frac{1}{3x}$

29. $\sqrt{64x} = 8\sqrt{x}$

30. $\sqrt{9x^5} = 3\sqrt{x^4 \cdot x} = 3x^2\sqrt{x}$

31. $\sqrt[4]{162x^9y^{12}} = \sqrt[4]{2(3)^4x(x^2)^4(y^3)^4}$
 $= 3x^2y^3\sqrt[4]{2x}$

32. $\sqrt[3]{-40x^{14}y^{10}} = \sqrt[3]{5(-2)^3x^2(x^4)^3y(y^3)^3}$
 $= -2x^4y^3\sqrt[3]{5x^2y}$

33. $\sqrt{15x^2}\sqrt{5x} = \sqrt{75x^2 \cdot x} = \sqrt{25 \cdot 3 \cdot x^2 \cdot x} = 5x\sqrt{3x}$

34. $\sqrt{5x}\sqrt{20x^3} = \sqrt{100x^4} = 10x^2$

35. $(\sqrt{5}\sqrt[3]{9})^2 = (\sqrt{5})^2(\sqrt[3]{9})^2$
 $= 5 \cdot \sqrt[3]{9^2} = 5\sqrt[3]{81} = 5 \cdot 3\sqrt[3]{3} = 15\sqrt[3]{3}$

36. $(\sqrt[3]{3}\sqrt{10})^4 = (\sqrt[3]{3})^4(\sqrt{10})^4$
 $= \sqrt[3]{3^4} \cdot 10^2 = 3\sqrt[3]{3} \cdot 100 = 300\sqrt[3]{3}$

37. $(3\sqrt{6})(2\sqrt{2}) = 6\sqrt{12} = 6\sqrt{4 \cdot 3} = 12\sqrt{3}$

$$38. (5\sqrt{8})(-3\sqrt{3}) = -15\sqrt{24} = -30\sqrt{6}$$

$$39. 3\sqrt{2} + 4\sqrt{2} = (3+4)\sqrt{2} = 7\sqrt{2}$$

$$40. 6\sqrt{5} - 4\sqrt{5} = (6-4)\sqrt{5} = 2\sqrt{5}$$

$$\begin{aligned} 41. -\sqrt{48} + 5\sqrt{12} &= -\sqrt{16 \cdot 3} + 5\sqrt{4 \cdot 3} \\ &= -4\sqrt{3} + 5 \cdot 2\sqrt{3} \\ &= (-4+10)\sqrt{3} \\ &= 6\sqrt{3} \end{aligned}$$

$$\begin{aligned} 42. 2\sqrt{12} - 3\sqrt{27} &= 2\sqrt{4 \cdot 3} - 3\sqrt{9 \cdot 3} \\ &= 4\sqrt{3} - 9\sqrt{3} \\ &= (4-9)\sqrt{3} \\ &= -5\sqrt{3} \end{aligned}$$

$$\begin{aligned} 43. (\sqrt{3}+3)(\sqrt{3}-1) &= (\sqrt{3})^2 + 3\sqrt{3} - \sqrt{3} - 3 \\ &= 3 + 2\sqrt{3} - 3 \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 44. (\sqrt{5}-2)(\sqrt{5}+3) &= (\sqrt{5})^2 - 2\sqrt{5} + 3\sqrt{5} - 6 \\ &= 5 + \sqrt{5} - 6 \\ &= \sqrt{5} - 1 \end{aligned}$$

$$\begin{aligned} 45. 5\sqrt[3]{2} - 2\sqrt[3]{54} &= 5\sqrt[3]{2} - 2 \cdot 3\sqrt[3]{2} \\ &= 5\sqrt[3]{2} - 6\sqrt[3]{2} \\ &= (5-6)\sqrt[3]{2} \\ &= -\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} 46. 9\sqrt[3]{24} - \sqrt[3]{81} &= 9 \cdot 2\sqrt[3]{3} - 3\sqrt[3]{3} \\ &= 18\sqrt[3]{3} - 3\sqrt[3]{3} \\ &= (18-3)\sqrt[3]{3} \\ &= 15\sqrt[3]{3} \end{aligned}$$

$$\begin{aligned} 47. (\sqrt{x}-1)^2 &= (\sqrt{x})^2 - 2\sqrt{x} + 1 \\ &= x - 2\sqrt{x} + 1 \end{aligned}$$

$$\begin{aligned} 48. (\sqrt{x} + \sqrt{5})^2 &= (\sqrt{x})^2 + 2(\sqrt{x})(\sqrt{5}) + (\sqrt{5})^2 \\ &= x + 2\sqrt{5x} + 5 \end{aligned}$$

$$\begin{aligned} 49. \sqrt[3]{16x^4} - \sqrt[3]{2x} &= \sqrt[3]{8x^3 \cdot 2x} - \sqrt[3]{2x} \\ &= 2x\sqrt[3]{2x} - \sqrt[3]{2x} \\ &= (2x-1)\sqrt[3]{2x} \end{aligned}$$

$$\begin{aligned} 50. \sqrt[4]{32x} + \sqrt[4]{2x^5} &= \sqrt[4]{16 \cdot 2x} + \sqrt[4]{x^4 \cdot 2x} \\ &= 2\sqrt[4]{2x} + x\sqrt[4]{2x} \\ &= (2+x)\sqrt[4]{2x} \text{ or } (x+2)\sqrt[4]{2x} \end{aligned}$$

$$\begin{aligned} 51. \sqrt{8x^3} - 3\sqrt{50x} &= \sqrt{4x^2 \cdot 2x} - 3\sqrt{25 \cdot 2x} \\ &= 2x\sqrt{2x} - 15\sqrt{2x} \\ &= (2x-15)\sqrt{2x} \end{aligned}$$

$$\begin{aligned} 52. 3x\sqrt{9y} + 4\sqrt{25y} &= 9x\sqrt{y} + 20\sqrt{y} \\ &= (9x+20)\sqrt{y} \end{aligned}$$

$$\begin{aligned} 53. \sqrt[3]{16x^4y} - 3x\sqrt[3]{2xy} + 5\sqrt[3]{-2xy^4} \\ &= \sqrt[3]{8x^3 \cdot 2xy} - 3x\sqrt[3]{2xy} + 5\sqrt[3]{-y^3 \cdot 2xy} \\ &= 2x\sqrt[3]{2xy} - 3x\sqrt[3]{2xy} - 5y\sqrt[3]{2xy} \\ &= (2x-3x-5y)\sqrt[3]{2xy} \\ &= (-x-5y)\sqrt[3]{2xy} \text{ or } -(x+5y)\sqrt[3]{2xy} \end{aligned}$$

$$\begin{aligned} 54. 8xy - \sqrt{25x^2y^2} + \sqrt[3]{8x^3y^3} &= 8xy - 5xy + 2xy \\ &= (8-5+2)xy \\ &= 5xy \end{aligned}$$

$$55. \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$56. \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$57. \frac{-\sqrt{3}}{\sqrt{5}} = \frac{-\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{15}}{5}$$

$$58. \frac{-\sqrt{3}}{\sqrt{8}} = \frac{-\sqrt{3}}{2\sqrt{2}} = \frac{-\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{6}}{2 \cdot 2} = \frac{-\sqrt{6}}{4}$$

Chapter R: Review

$$\begin{aligned}
 59. \quad \frac{\sqrt{3}}{5-\sqrt{2}} &= \frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} \\
 &= \frac{\sqrt{3}(5+\sqrt{2})}{25-2} \\
 &= \frac{\sqrt{3}(5+\sqrt{2})}{23} \quad \text{or} \quad \frac{5\sqrt{3}+\sqrt{6}}{23}
 \end{aligned}$$

$$\begin{aligned}
 60. \quad \frac{\sqrt{2}}{\sqrt{7}+2} &= \frac{\sqrt{2}}{\sqrt{7}+2} \cdot \frac{\sqrt{7}-2}{\sqrt{7}-2} \\
 &= \frac{\sqrt{2}(\sqrt{7}-2)}{7-4} \\
 &= \frac{\sqrt{2}(\sqrt{7}-2)}{3} \quad \text{or} \quad \frac{\sqrt{14}-2\sqrt{2}}{3}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad \frac{2-\sqrt{5}}{2+3\sqrt{5}} &= \frac{2-\sqrt{5}}{2+3\sqrt{5}} \cdot \frac{2-3\sqrt{5}}{2-3\sqrt{5}} \\
 &= \frac{4-2\sqrt{5}-6\sqrt{5}+15}{4-45} \\
 &= \frac{19-8\sqrt{5}}{-41} = \frac{8\sqrt{5}-19}{41}
 \end{aligned}$$

$$\begin{aligned}
 62. \quad \frac{\sqrt{3}-1}{2\sqrt{3}+3} &= \frac{\sqrt{3}-1}{2\sqrt{3}+3} \cdot \frac{2\sqrt{3}-3}{2\sqrt{3}-3} \\
 &= \frac{6-2\sqrt{3}-3\sqrt{3}+3}{12-9} = \frac{9-5\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 63. \quad \frac{5}{\sqrt{2}-1} &= \frac{5}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} \\
 &= \frac{5\sqrt{2}+5}{2-1} = 5\sqrt{2}+5
 \end{aligned}$$

$$\begin{aligned}
 64. \quad \frac{-3}{\sqrt{5}+4} &= \frac{-3}{\sqrt{5}+4} \cdot \frac{\sqrt{5}-4}{\sqrt{5}-4} \\
 &= \frac{-3\sqrt{5}+12}{5-16} = \frac{-3\sqrt{5}+12}{-11} \\
 &= \frac{3\sqrt{5}-12}{11}
 \end{aligned}$$

$$65. \quad \frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{5\sqrt[3]{4}}{2}$$

$$66. \quad \frac{-2}{\sqrt[3]{9}} = \frac{-2}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{-2\sqrt[3]{3}}{3}$$

$$\begin{aligned}
 67. \quad \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} &= \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \cdot \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}} \\
 &= \frac{(x+h)-2\sqrt{x(x+h)}+x}{(x+h)-x} \\
 &= \frac{x+h-2\sqrt{x^2+xh}+x}{x+h-x} \\
 &= \frac{2x+h-2\sqrt{x^2+xh}}{h}
 \end{aligned}$$

$$\begin{aligned}
 68. \quad \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}} &= \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}} \cdot \frac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}+\sqrt{x-h}} \\
 &= \frac{(x+h)+2\sqrt{(x-h)(x+h)}+(x-h)}{(x+h)-(x-h)} \\
 &= \frac{x+h+2\sqrt{x^2-h^2}+x-h}{x+h-x+h} \\
 &= \frac{2x+2\sqrt{x^2-h^2}}{2h} \\
 &= \frac{x+\sqrt{x^2-h^2}}{h}
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{\sqrt{11}+1}{2} &= \frac{\sqrt{11}+1}{2} \cdot \frac{\sqrt{11}-1}{\sqrt{11}-1} \\
 &= \frac{11-1}{2(\sqrt{11}-1)} = \frac{10}{2(\sqrt{11}-1)} \\
 &= \frac{5}{\sqrt{11}-1}
 \end{aligned}$$

$$\begin{aligned}
 70. \quad \frac{5-\sqrt{43}}{3} &= \frac{5-\sqrt{43}}{3} \cdot \frac{5+\sqrt{43}}{5+\sqrt{43}} = \frac{25-43}{3(5+\sqrt{43})} \\
 &= \frac{-18}{3(5+\sqrt{43})} = \frac{-6}{5+\sqrt{43}} = -\frac{6}{5+\sqrt{43}}
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \frac{\sqrt{6}-\sqrt{15}}{\sqrt{15}} &= \frac{\sqrt{6}-\sqrt{15}}{\sqrt{15}} \cdot \frac{\sqrt{6}+\sqrt{15}}{\sqrt{6}+\sqrt{15}} \\
 &= \frac{6-15}{\sqrt{90}-15} = \frac{-9}{3\sqrt{10}-15} \\
 &= \frac{-9}{3(\sqrt{10}-5)} = -\frac{3}{\sqrt{10}-5}
 \end{aligned}$$

$$72. \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{5-3}{5-\sqrt{15}} = \frac{2}{5-\sqrt{15}}$$

$$73. \frac{\sqrt{x}-\sqrt{c}}{x-c} = \frac{\sqrt{x}-\sqrt{c}}{x-c} \cdot \frac{\sqrt{x}+\sqrt{c}}{\sqrt{x}+\sqrt{c}}$$

$$= \frac{x-c}{(x-c)(\sqrt{x}+\sqrt{c})} = \frac{1}{\sqrt{x}+\sqrt{c}}$$

$$74. \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \frac{x-4}{(x-4)(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$$

$$75. \frac{\sqrt{x-7}-1}{x-8} = \frac{\sqrt{x-7}-1}{x-8} \cdot \frac{\sqrt{x-7}+1}{\sqrt{x-7}+1}$$

$$= \frac{x-7-1}{(x-8)(\sqrt{x-7}+1)}$$

$$= \frac{x-8}{(x-8)(\sqrt{x-7}+1)}$$

$$= \frac{1}{\sqrt{x-7}+1}$$

$$76. \frac{4-\sqrt{x-9}}{x-25} = \frac{4-\sqrt{x-9}}{x-25} \cdot \frac{4+\sqrt{x-9}}{4+\sqrt{x-9}}$$

$$= \frac{16-(x-9)}{(x-25)(4+\sqrt{x-9})}$$

$$= \frac{25-x}{(x-25)(4+\sqrt{x-9})}$$

$$= -\frac{x-25}{(x-25)(4+\sqrt{x-9})}$$

$$= -\frac{1}{4+\sqrt{x-9}}$$

$$77. 8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$$

$$78. 4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$$

$$79. (-64)^{1/3} = \sqrt[3]{-64} = -4$$

$$80. 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$81. 100^{3/2} = (\sqrt{100})^3 = 10^3 = 1000$$

$$82. 25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$$

$$83. 4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$84. 16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$$

$$85. \left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3 = \frac{3^3}{2^3(\sqrt{2})^3}$$

$$= \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}} = \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{27\sqrt{2}}{32}$$

$$86. \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$87. \left(\frac{8}{9}\right)^{-3/2} = \left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3$$

$$= \frac{3^3}{2^3(\sqrt{2})^3} = \frac{27}{8 \cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}}$$

$$= \frac{27}{16\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{27\sqrt{2}}{32}$$

$$88. \left(\frac{8}{27}\right)^{-2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$89. (-1000)^{-1/3} = \left(-\frac{1}{1000}\right)^{1/3} = \sqrt[3]{-\frac{1}{1000}} = -\frac{1}{10}$$

$$90. -25^{-1/2} = -\left(\frac{1}{25}\right)^{1/2} = -\sqrt{\frac{1}{25}} = -\frac{1}{5}$$

Chapter R: Review

$$91. \left(-\frac{64}{125}\right)^{-2/3} = \left(-\frac{125}{64}\right)^{2/3} = \left(\sqrt[3]{-\frac{125}{64}}\right)^2 \\ = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$$

$$92. -81^{-3/4} = -\left(\frac{1}{81}\right)^{3/4} = -\left(\sqrt[4]{\frac{1}{81}}\right)^3 \\ = -\left(\frac{1}{3}\right)^3 = -\frac{1}{27}$$

$$93. x^{3/4} x^{1/3} x^{-1/2} = x^{3/4+1/3-1/2} = x^{7/12}$$

$$94. x^{2/3} x^{1/2} x^{-1/4} = x^{2/3+1/2-1/4} = x^{11/12}$$

$$95. (x^3 y^6)^{1/3} = (x^3)^{1/3} (y^6)^{1/3} = xy^2$$

$$96. (x^4 y^8)^{3/4} = (x^4)^{3/4} (y^8)^{3/4} = x^3 y^6$$

$$97. \frac{(x^2 y)^{1/3} (xy^2)^{2/3}}{x^{2/3} y^{2/3}} = \frac{(x^2)^{1/3} (y)^{1/3} (x)^{2/3} (y^2)^{2/3}}{x^{2/3} y^{2/3}} \\ = \frac{x^{2/3} y^{1/3} x^{2/3} y^{4/3}}{x^{2/3} y^{2/3}} \\ = x^{2/3+2/3-2/3} y^{1/3+4/3-2/3} \\ = x^{2/3} y^1 = x^{2/3} y$$

$$98. \frac{(xy)^{1/4} (x^2 y^2)^{1/2}}{(x^2 y)^{3/4}} = \frac{x^{1/4} y^{1/4} (x^2)^{1/2} (y^2)^{1/2}}{(x^2)^{3/4} y^{3/4}} \\ = \frac{x^{1/4} y^{1/4} xy}{x^{3/2} y^{3/4}} \\ = x^{1/4+1-3/2} y^{1/4+1-3/4} \\ = x^{-1/4} y^{1/2} = \frac{y^{1/2}}{x^{1/4}}$$

$$99. \frac{(16x^2 y^{-1/3})^{3/4}}{(xy^2)^{1/4}} = \frac{16^{3/4} (x^2)^{3/4} (y^{-1/3})^{3/4}}{x^{1/4} (y^2)^{1/4}} \\ = \frac{(\sqrt[4]{16})^3 x^{3/2} y^{-1/4}}{x^{1/4} y^{1/2}} \\ = 2^3 x^{3/2-1/4} y^{-1/4-1/2} \\ = 8x^{5/4} y^{-3/4} \\ = \frac{8x^{5/4}}{y^{3/4}}$$

$$100. \frac{(4x^{-1} y^{1/3})^{3/2}}{(xy)^{3/2}} = \frac{4^{3/2} (x^{-1})^{3/2} (y^{1/3})^{3/2}}{x^{3/2} y^{3/2}} \\ = \frac{(\sqrt{4})^3 x^{-3/2} y^{1/2}}{x^{3/2} y^{3/2}} \\ = 2^3 x^{-3/2-3/2} y^{1/2-3/2} \\ = 8x^{-3} y^{-1} \\ = \frac{8}{x^3 y}$$

$$101. \frac{x}{(1+x)^{1/2}} + 2(1+x)^{1/2} = \frac{x+2(1+x)^{1/2} (1+x)^{1/2}}{(1+x)^{1/2}} \\ = \frac{x+2(1+x)}{(1+x)^{1/2}} \\ = \frac{x+2+2x}{(1+x)^{1/2}} \\ = \frac{3x+2}{(1+x)^{1/2}}$$

$$102. \frac{1+x}{2x^{1/2}} + x^{1/2} = \frac{1+x+x^{1/2} \cdot 2x^{1/2}}{2x^{1/2}} \\ = \frac{1+x+2x}{2x^{1/2}} = \frac{3x+1}{2x^{1/2}}$$

$$\begin{aligned}
 103. \quad & 2x(x^2+1)^{1/2} + x^2 \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x \\
 &= 2x(x^2+1)^{1/2} + \frac{x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x(x^2+1)^{1/2} \cdot (x^2+1)^{1/2} + x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x(x^2+1)^{1/2+1/2} + x^3}{(x^2+1)^{1/2}} = \frac{2x(x^2+1)^1 + x^3}{(x^2+1)^{1/2}} \\
 &= \frac{2x^3 + 2x + x^3}{(x^2+1)^{1/2}} = \frac{3x^3 + 2x}{(x^2+1)^{1/2}} \\
 &= \frac{x(3x^2 + 2)}{(x^2+1)^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 104. \quad & (x+1)^{1/3} + x \cdot \frac{1}{3}(x+1)^{-2/3}, x \neq -1 \\
 &= (x+1)^{1/3} + \frac{x}{3(x+1)^{2/3}} \\
 &= \frac{3(x+1)^{2/3}(x+1)^{1/3} + x}{3(x+1)^{2/3}} \\
 &= \frac{3(x+1)^{2/3+1/3} + x}{3(x+1)^{2/3}} = \frac{3(x+1)^1 + x}{3(x+1)^{2/3}} \\
 &= \frac{3x+3+x}{3(x+1)^{2/3}} = \frac{4x+3}{3(x+1)^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 105. \quad & \sqrt{4x+3} \cdot \frac{1}{2\sqrt{x-5}} + \sqrt{x-5} \cdot \frac{1}{5\sqrt{4x+3}}, x > 5 \\
 &= \frac{\sqrt{4x+3}}{2\sqrt{x-5}} + \frac{\sqrt{x-5}}{5\sqrt{4x+3}} \\
 &= \frac{\sqrt{4x+3} \cdot 5 \cdot \sqrt{4x+3} + \sqrt{x-5} \cdot 2 \cdot \sqrt{x-5}}{10\sqrt{x-5}\sqrt{4x+3}} \\
 &= \frac{5(4x+3) + 2(x-5)}{10\sqrt{(x-5)(4x+3)}} \\
 &= \frac{20x+15+2x-10}{10\sqrt{(x-5)(4x+3)}} \\
 &= \frac{22x+5}{10\sqrt{(x-5)(4x+3)}}
 \end{aligned}$$

$$\begin{aligned}
 106. \quad & \frac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}} + \frac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}}, x \neq 2, x \neq -\frac{1}{8} \\
 &= \frac{8\sqrt[3]{8x+1} \cdot \sqrt[3]{(8x+1)^2} + \sqrt[3]{x-2} \cdot \sqrt[3]{(x-2)^2}}{24\sqrt[3]{(x-2)^2} \cdot \sqrt[3]{(8x+1)^2}} \\
 &= \frac{8\sqrt[3]{(8x+1)^3} + \sqrt[3]{(x-2)^3}}{24\sqrt[3]{(x-2)^2} \cdot \sqrt[3]{(8x+1)^2}} \\
 &= \frac{8(8x+1) + x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}} \\
 &= \frac{64x+8+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}} \\
 &= \frac{65x+6}{24\sqrt[3]{(x-2)^2(8x+1)^2}}
 \end{aligned}$$

$$\begin{aligned}
 107. \quad & \frac{\left(\sqrt{1+x} - x \cdot \frac{1}{2\sqrt{1+x}}\right)}{1+x} = \frac{\left(\sqrt{1+x} - \frac{x}{2\sqrt{1+x}}\right)}{1+x} \\
 &= \frac{\left(\frac{2\sqrt{1+x}\sqrt{1+x} - x}{2\sqrt{1+x}}\right)}{1+x} \\
 &= \frac{2(1+x) - x}{2(1+x)^{1/2}} \cdot \frac{1}{1+x} \\
 &= \frac{2+x}{2(1+x)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 108. \quad & \frac{\left(\sqrt{x^2+1} - x \cdot \frac{2x}{2\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} - \frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1} \\
 &= \frac{\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1} = \frac{1}{\sqrt{x^2+1}} \cdot \frac{1}{x^2+1} \\
 &= \frac{1}{(x^2+1)^{3/2}}
 \end{aligned}$$

Chapter R: Review

$$\begin{aligned}
 109. \quad & \frac{(x+4)^{1/2} - 2x(x+4)^{-1/2}}{x+4} \\
 &= \frac{\left((x+4)^{1/2} - \frac{2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{\left((x+4)^{1/2} \cdot \frac{(x+4)^{1/2}}{(x+4)^{1/2}} - \frac{2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{\left(\frac{x+4-2x}{(x+4)^{1/2}} \right)}{x+4} \\
 &= \frac{-x+4}{(x+4)^{1/2}} \cdot \frac{1}{x+4} \\
 &= \frac{-x+4}{(x+4)^{3/2}} \\
 &= \frac{4-x}{(x+4)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 110. \quad & \frac{(9-x^2)^{1/2} + x^2(9-x^2)^{-1/2}}{9-x^2}, -3 < x < 3 \\
 &= \frac{\left((9-x^2)^{1/2} + \frac{x^2}{(9-x^2)^{1/2}} \right)}{9-x^2} \\
 &= \frac{\left(\frac{(9-x^2)^{1/2} \cdot (9-x^2)^{1/2} + x^2}{(9-x^2)^{1/2}} \right)}{9-x^2} \\
 &= \frac{(9-x^2)^{1/2} \cdot (9-x^2)^{1/2} + x^2}{(9-x^2)^{1/2}} \cdot \frac{1}{9-x^2} \\
 &= \frac{9-x^2+x^2}{(9-x^2)^{1/2}} \cdot \frac{1}{9-x^2} \\
 &= \frac{9}{(9-x^2)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 111. \quad & \frac{\frac{x^2}{(x^2-1)^{1/2}} - (x^2-1)^{1/2}}{x^2}, x < -1 \text{ or } x > 1 \\
 &= \frac{\left(\frac{x^2 - (x^2-1)^{1/2} \cdot (x^2-1)^{1/2}}{(x^2-1)^{1/2}} \right)}{x^2} \\
 &= \frac{x^2 - (x^2-1)^{1/2} \cdot (x^2-1)^{1/2}}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{x^2 - (x^2-1)}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{x^2 - x^2 + 1}{(x^2-1)^{1/2}} \cdot \frac{1}{x^2} \\
 &= \frac{1}{x^2(x^2-1)^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 112. \quad & \frac{(x^2+4)^{1/2} - x^2(x^2+4)^{-1/2}}{x^2+4} \\
 &= \frac{\left((x^2+4)^{1/2} - \frac{x^2}{(x^2+4)^{1/2}} \right)}{x^2+4} \\
 &= \frac{\left(\frac{(x^2+4)^{1/2} \cdot (x^2+4)^{1/2} - x^2}{(x^2+4)^{1/2}} \right)}{x^2+4} \\
 &= \frac{(x^2+4)^{1/2} \cdot (x^2+4)^{1/2} - x^2}{(x^2+4)^{1/2}} \cdot \frac{1}{x^2+4} \\
 &= \frac{x^2+4-x^2}{(x^2+4)^{1/2}} \cdot \frac{1}{x^2+4} = \frac{4}{(x^2+4)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 113. \quad & \frac{\frac{1+x^2}{2\sqrt{x}} - 2x\sqrt{x}}{(1+x^2)^2}, x > 0 \\
 & = \frac{\left(\frac{1+x^2 - (2\sqrt{x})(2x\sqrt{x})}{2\sqrt{x}} \right)}{(1+x^2)^2} \\
 & = \frac{1+x^2 - (2\sqrt{x})(2x\sqrt{x})}{2\sqrt{x}} \cdot \frac{1}{(1+x^2)^2} \\
 & = \frac{1+x^2 - 4x^2}{2\sqrt{x}} \cdot \frac{1}{(1+x^2)^2} = \frac{1-3x^2}{2\sqrt{x}(1+x^2)^2}
 \end{aligned}$$

$$\begin{aligned}
 114. \quad & \frac{2x(1-x^2)^{1/3} + \frac{2}{3}x^3(1-x^2)^{-2/3}}{(1-x^2)^{2/3}}, x \neq -1, x \neq 1 \\
 & = \frac{\left(2x(1-x^2)^{1/3} + \frac{2x^3}{3(1-x^2)^{2/3}} \right)}{(1-x^2)^{2/3}} \\
 & = \frac{\left(\frac{2x(1-x^2)^{1/3} \cdot 3(1-x^2)^{2/3} + 2x^3}{3(1-x^2)^{2/3}} \right)}{(1-x^2)^{2/3}} \\
 & = \frac{6x(1-x^2)^{1/3+2/3} + 2x^3}{3(1-x^2)^{2/3}} \cdot \frac{1}{(1-x^2)^{2/3}} \\
 & = \frac{6x(1-x^2) + 2x^3}{3(1-x^2)^{2/3+2/3}} = \frac{6x - 6x^3 + 2x^3}{3(1-x^2)^{4/3}} \\
 & = \frac{6x - 4x^3}{3(1-x^2)^{4/3}} = \frac{2x(3-2x^2)}{3(1-x^2)^{4/3}}
 \end{aligned}$$

$$\begin{aligned}
 115. \quad & (x+1)^{3/2} + x \cdot \frac{3}{2}(x+1)^{1/2} \\
 & = (x+1)^{1/2} \left(x+1 + \frac{3}{2}x \right) \\
 & = (x+1)^{1/2} \left(\frac{5}{2}x + 1 \right) \\
 & = \frac{1}{2}(x+1)^{1/2} (5x+2)
 \end{aligned}$$

$$\begin{aligned}
 116. \quad & (x^2+4)^{4/3} + x \cdot \frac{4}{3}(x^2+4)^{1/3} \cdot 2x \\
 & = (x^2+4)^{1/3} \left(x^2+4 + \frac{8}{3}x^2 \right) \\
 & = (x^2+4)^{1/3} \left(\frac{11}{3}x^2 + 4 \right) \\
 & = \frac{1}{3}(x^2+4)^{1/3} (11x^2+12)
 \end{aligned}$$

$$\begin{aligned}
 117. \quad & 6x^{1/2}(x^2+x) - 8x^{3/2} - 8x^{1/2} \\
 & = 2x^{1/2}(3(x^2+x) - 4x - 4) \\
 & = 2x^{1/2}(3x^2 - x - 4) \\
 & = 2x^{1/2}(3x-4)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 118. \quad & 6x^{1/2}(2x+3) + x^{3/2} \cdot 8 \\
 & = 2x^{1/2}(3(2x+3) + 4x) \\
 & = 2x^{1/2}(10x+9)
 \end{aligned}$$

$$\begin{aligned}
 119. \quad & 3(x^2+4)^{4/3} + x \cdot 4(x^2+4)^{1/3} \cdot 2x \\
 & = (x^2+4)^{1/3} [3(x^2+4) + 8x^2] \\
 & = (x^2+4)^{1/3} [3x^2 + 12 + 8x^2] \\
 & = (x^2+4)^{1/3} (11x^2 + 12)
 \end{aligned}$$

$$\begin{aligned}
 120. \quad & 2x(3x+4)^{4/3} + x^2 \cdot 4(3x+4)^{1/3} \\
 & = 2x(3x+4)^{1/3} [(3x+4) + 2x] \\
 & = 2x(3x+4)^{1/3} (5x+4)
 \end{aligned}$$

$$\begin{aligned}
 121. \quad & 4(3x+5)^{1/3}(2x+3)^{3/2} + 3(3x+5)^{4/3}(2x+3)^{1/2} \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} [4(2x+3) + 3(3x+5)] \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} (8x+12+9x+15) \\
 & = (3x+5)^{1/3}(2x+3)^{1/2} (17x+27)
 \end{aligned}$$

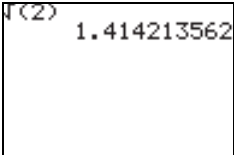
where $x \geq -\frac{3}{2}$.

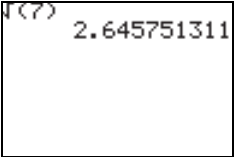
Chapter R: Review

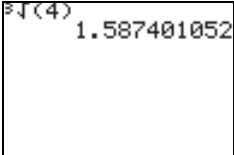
$$\begin{aligned}
 122. \quad & 6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2} \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}[(4x-3) + (6x+1)] \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2) \\
 &= 6(6x+1)^{1/3}(4x-3)^{1/2}(2)(5x-1) \\
 &= 12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1) \\
 &\text{where } x \geq \frac{3}{4}.
 \end{aligned}$$

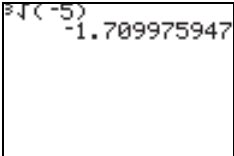
$$\begin{aligned}
 123. \quad & 3x^{-1/2} + \frac{3}{2}x^{1/2}, x > 0 \\
 &= \frac{3}{x^{1/2}} + \frac{3}{2}x^{1/2} \\
 &= \frac{3 \cdot 2 + 3x^{1/2} \cdot x^{1/2}}{2x^{1/2}} = \frac{6 + 3x}{2x^{1/2}} = \frac{3(x+2)}{2x^{1/2}}
 \end{aligned}$$

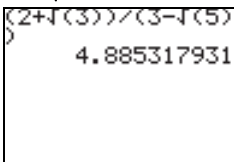
$$\begin{aligned}
 124. \quad & 8x^{1/3} - 4x^{-2/3}, x \neq 0 \\
 &= 8x^{1/3} - \frac{4}{x^{2/3}} \\
 &= \frac{8x^{1/3} \cdot x^{2/3} - 4}{x^{2/3}} = \frac{8x - 4}{x^{2/3}} = \frac{4(2x-1)}{x^{2/3}}
 \end{aligned}$$

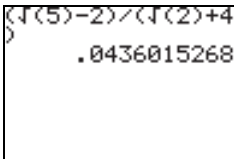
$$125. \quad \sqrt{2} \approx 1.41$$


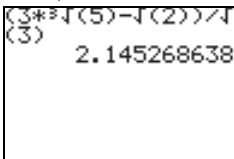
$$126. \quad \sqrt{7} \approx 2.65$$


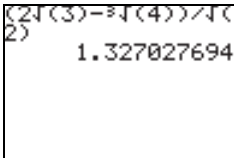
$$127. \quad \sqrt[3]{4} \approx 1.59$$


$$128. \quad \sqrt[3]{-5} \approx -1.71$$


$$129. \quad \frac{2+\sqrt{3}}{3-\sqrt{5}} \approx 4.89$$


$$130. \quad \frac{\sqrt{5}-2}{\sqrt{2}+4} \approx 0.04$$


$$131. \quad \frac{3\sqrt[3]{5}-\sqrt{2}}{\sqrt{3}} \approx 2.15$$


$$132. \quad \frac{2\sqrt{3}-\sqrt[3]{4}}{\sqrt{2}} \approx 1.33$$


$$133. \quad \text{a. } V = 40(12)^2 \sqrt{\frac{96}{12}} - 0.608 \approx 15,660.4 \text{ gallons}$$

$$\text{b. } V = 40(1)^2 \sqrt{\frac{96}{1}} - 0.608 \approx 390.7 \text{ gallons}$$

$$134. \quad \text{a. } v = \sqrt{64 \cdot 4 + 0^2} = \sqrt{256} = 16 \text{ feet per second}$$

$$\text{b. } v = \sqrt{64 \cdot 16 + 0^2} = \sqrt{1024} = 32 \text{ feet per second}$$

$$\text{c. } v = \sqrt{64 \cdot 2 + 4^2} = \sqrt{144} = 12 \text{ feet per second}$$

$$135. \quad T = 2\pi \sqrt{\frac{64}{32}} = 2\pi \sqrt{2} \approx 8.89 \text{ seconds}$$

$$\begin{aligned}
 136. \quad T &= 2\pi\sqrt{\frac{16}{32}} = 2\pi\sqrt{\frac{1}{2}} = \frac{2\pi}{\sqrt{2}} \\
 &= \pi\sqrt{2} \approx 4.44 \text{ seconds}
 \end{aligned}$$

$$\begin{array}{r}
 137. \quad \sqrt{3} \overline{) 1 \quad -4 \quad -3 \quad 13} \\
 \underline{ \sqrt{3} \quad 3-4\sqrt{3} \quad -12} \\
 1 \quad \sqrt{3}-4 \quad -4\sqrt{3} \quad 1
 \end{array}$$

The quotient is $x^2 + (\sqrt{3} - 4)x - 4\sqrt{3}$

The remainder is 1

$$\begin{array}{r}
 138. \quad 1+\sqrt{2} \overline{) 1 \quad -9 \quad 13 \quad 7} \\
 \underline{ 1+\sqrt{2} \quad -6-7\sqrt{2} \quad -7} \\
 1 \quad \sqrt{3}-8 \quad -7-7\sqrt{2} \quad 0
 \end{array}$$

Yes, $1+\sqrt{2}$ is a factor of $x^3 - 9x^2 + 13x + 7$.

139. Answers may vary. One possibility follows: If

$$a = -5, \text{ then } \sqrt{a^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq a.$$

Since we use the principal square root, which is always non-negative,

$$\sqrt{a^2} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

which is the definition of $|a|$, so $\sqrt{a^2} = |a|$.

Chapter 1

Equations and Inequalities

Section 1.1

1. Distributive

2. Zero-Product

3. $\{x|x \neq 4\}$

4. False. Multiplying both sides of an equation by zero will not result in an equivalent equation.

5. identity

6. linear; first-degree

7. False. The solution is $\frac{8}{3}$.

8. True

9. b

10. d

11. $7x = 21$

$$\frac{7x}{7} = \frac{21}{7}$$

$$x = 3$$

The solution set is $\{3\}$.

12. $6x = -24$

$$\frac{6x}{6} = \frac{-24}{6}$$

$$x = -4$$

The solution set is $\{-4\}$.

13. $3x + 15 = 0$

$$3x + 15 - 15 = 0 - 15$$

$$3x = -15$$

$$\frac{3x}{3} = \frac{-15}{3}$$

$$x = -5$$

The solution set is $\{-5\}$.

14. $6x + 18 = 0$

$$6x + 18 - 18 = 0 - 18$$

$$6x = -18$$

$$\frac{6x}{6} = \frac{-18}{6}$$

$$x = -3$$

The solution set is $\{-3\}$.

15. $2x - 3 = 0$

$$2x - 3 + 3 = 0 + 3$$

$$2x = 3$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

16. $3x + 4 = 0$

$$3x + 4 - 4 = 0 - 4$$

$$3x = -4$$

$$\frac{3x}{3} = \frac{-4}{3}$$

$$x = -\frac{4}{3}$$

The solution set is $\left\{-\frac{4}{3}\right\}$.

17. $\frac{1}{4}x = \frac{7}{20}$

$$4\left(\frac{1}{4}x\right) = 4\left(\frac{7}{20}\right)$$

$$x = \frac{28}{20} = \frac{7}{5}$$

The solution set is $\left\{\frac{7}{5}\right\}$.

$$\begin{aligned}
 18. \quad \frac{2}{3}x &= \frac{9}{2} \\
 6\left(\frac{2}{3}x\right) &= 6\left(\frac{9}{2}\right) \\
 4x &= 27 \\
 \frac{4x}{4} &= \frac{27}{4} \\
 x &= \frac{27}{4}
 \end{aligned}$$

The solution set is $\left\{\frac{27}{4}\right\}$.

$$\begin{aligned}
 19. \quad 3x + 4 &= x \\
 3x + 4 - 4 &= x - 4 \\
 3x &= x - 4 \\
 3x - x &= x - 4 - x \\
 2x &= -4 \\
 \frac{2x}{2} &= \frac{-4}{2} \\
 x &= -2
 \end{aligned}$$

The solution set is $\{-2\}$.

$$\begin{aligned}
 20. \quad 2x + 9 &= 5x \\
 2x + 9 - 9 &= 5x - 9 \\
 2x &= 5x - 9 \\
 2x - 5x &= 5x - 9 - 5x \\
 -3x &= -9 \\
 \frac{-3x}{-3} &= \frac{-9}{-3} \\
 x &= 3
 \end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}
 21. \quad 2t - 6 &= 3 - t \\
 2t - 6 + 6 &= 3 - t + 6 \\
 2t &= 9 - t \\
 2t + t &= 9 - t + t \\
 3t &= 9 \\
 \frac{3t}{3} &= \frac{9}{3} \\
 t &= 3
 \end{aligned}$$

The solution set is $\{3\}$.

$$\begin{aligned}
 22. \quad 5y + 6 &= -18 - y \\
 5y + 6 - 6 &= -18 - y - 6 \\
 5y &= -y - 24 \\
 5y + y &= -y - 24 + y \\
 6y &= -24 \\
 \frac{6y}{6} &= \frac{-24}{6} \\
 y &= -4
 \end{aligned}$$

The solution set is $\{-4\}$.

$$\begin{aligned}
 23. \quad 6 - x &= 2x + 9 \\
 6 - x - 6 &= 2x + 9 - 6 \\
 -x &= 2x + 3 \\
 -x - 2x &= 2x + 3 - 2x \\
 -3x &= 3 \\
 \frac{-3x}{-3} &= \frac{3}{-3} \\
 x &= -1
 \end{aligned}$$

The solution set is $\{-1\}$.

$$\begin{aligned}
 24. \quad 3 - 2x &= 2 - x \\
 3 - 2x - 3 &= 2 - x - 3 \\
 -2x &= -x - 1 \\
 -2x + x &= -x - 1 + x \\
 -x &= -1 \\
 \frac{-x}{-1} &= \frac{-1}{-1} \\
 x &= 1
 \end{aligned}$$

The solution set is $\{1\}$.

$$\begin{aligned}
 25. \quad 3 + 2n &= 4n + 7 \\
 3 + 2n - 3 &= 4n + 7 - 3 \\
 2n &= 4n + 4 \\
 2n - 4n &= 4n + 4 - 4n \\
 -2n &= 4 \\
 \frac{-2n}{-2} &= \frac{4}{-2} \\
 n &= -2
 \end{aligned}$$

The solution set is $\{-2\}$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}26. \quad & 6 - 2m = 3m + 1 \\ & 6 - 2m - 6 = 3m + 1 - 6 \\ & -2m = 3m - 5 \\ & -2m - 3m = 3m - 5 - 3m \\ & -5m = -5 \\ & \frac{-5m}{-5} = \frac{-5}{-5} \\ & m = 1\end{aligned}$$

The solution set is $\{1\}$.

$$\begin{aligned}27. \quad & 3(5 + 3x) = 8(x - 1) \\ & 15 + 9x = 8x - 8 \\ & 9x - 8x + 15 = 8x - 8x - 8 \\ & x + 15 - 15 = -8 - 15 \\ & x = -23\end{aligned}$$

The solution set is $\{-23\}$.

$$\begin{aligned}28. \quad & 3(2 - x) = 2x - 1 \\ & 6 - 3x = 2x - 1 \\ & 6 - 3x - 6 = 2x - 1 - 6 \\ & -3x = 2x - 7 \\ & -3x - 2x = 2x - 7 - 2x \\ & -5x = -7 \\ & \frac{-5x}{-5} = \frac{-7}{-5} \\ & x = \frac{7}{5}\end{aligned}$$

The solution set is $\left\{\frac{7}{5}\right\}$.

$$\begin{aligned}29. \quad & 8x - (3x + 2) = 3x - 10 \\ & 8x - 3x - 2 = 3x - 10 \\ & 5x - 2 = 3x - 10 \\ & 5x - 2 + 2 = 3x - 10 + 2 \\ & 5x = 3x - 8 \\ & 5x - 3x = 3x - 8 - 3x \\ & 2x = -8 \\ & \frac{2x}{2} = \frac{-8}{2} \\ & x = -4\end{aligned}$$

The solution set is $\{-4\}$.

$$\begin{aligned}30. \quad & 7 - (2x - 1) = 10 \\ & 7 - 2x + 1 = 10 \\ & 8 - 2x = 10 \\ & 8 - 2x - 8 = 10 - 8 \\ & -2x = 2 \\ & \frac{-2x}{-2} = \frac{2}{-2} \\ & x = -1\end{aligned}$$

The solution set is $\{-1\}$.

$$\begin{aligned}31. \quad & \frac{3}{2}x + 2 = \frac{1}{2} - \frac{1}{2}x \\ & 2\left(\frac{3}{2}x + 2\right) = 2\left(\frac{1}{2} - \frac{1}{2}x\right) \\ & 3x + 4 = 1 - x \\ & 3x + 4 - 4 = 1 - x - 4 \\ & 3x = -3 - x \\ & 3x + x = -3 - x + x \\ & 4x = -3 \\ & \frac{4x}{4} = \frac{-3}{4} \\ & x = -\frac{3}{4}\end{aligned}$$

The solution set is $\left\{-\frac{3}{4}\right\}$.

$$\begin{aligned}32. \quad & \frac{1}{3}x = 2 - \frac{2}{3}x \\ & 3\left(\frac{1}{3}x\right) = 3\left(2 - \frac{2}{3}x\right) \\ & x = 6 - 2x \\ & x + 2x = 6 - 2x + 2x \\ & 3x = 6 \\ & \frac{3x}{3} = \frac{6}{3} \\ & x = 2\end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 33. \quad & \frac{1}{2}x - 5 = \frac{3}{4}x \\
 & 4\left(\frac{1}{2}x - 5\right) = 4\left(\frac{3}{4}x\right) \\
 & 2x - 20 = 3x \\
 & 2x - 20 - 2x = 3x - 2x \\
 & -20 = x \\
 & x = -20 \\
 & \text{The solution set is } \{-20\}.
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & 1 - \frac{1}{2}x = 6 \\
 & 2\left(1 - \frac{1}{2}x\right) = 2(6) \\
 & 2 - x = 12 \\
 & 2 - x - 2 = 12 - 2 \\
 & -x = 10 \\
 & \frac{-x}{-1} = \frac{10}{-1} \\
 & x = -10 \\
 & \text{The solution set is } \{-10\}.
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \frac{2}{3}p = \frac{1}{2}p + \frac{1}{3} \\
 & 6\left(\frac{2}{3}p\right) = 6\left(\frac{1}{2}p + \frac{1}{3}\right) \\
 & 4p = 3p + 2 \\
 & 4p - 3p = 3p + 2 - 3p \\
 & p = 2 \\
 & \text{The solution set is } \{2\}.
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & \frac{1}{2} - \frac{1}{3}p = \frac{4}{3} \\
 & 6\left(\frac{1}{2} - \frac{1}{3}p\right) = 6\left(\frac{4}{3}\right) \\
 & 3 - 2p = 8 \\
 & 3 - 2p - 3 = 8 - 3 \\
 & -2p = 5 \\
 & \frac{-2p}{-2} = \frac{5}{-2} \\
 & p = -\frac{5}{2} \\
 & \text{The solution set is } \left\{-\frac{5}{2}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & 0.2m = 0.9 + 0.5m \\
 & 0.2m - 0.5m = 0.9 + 0.5m - 0.5m \\
 & -0.3m = 0.9 \\
 & \frac{-0.3m}{-0.3} = \frac{0.9}{-0.3} \\
 & m = -3 \\
 & \text{The solution set is } \{-3\}.
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & 0.9t = 1 + t \\
 & 0.9t - t = 1 + t - t \\
 & -0.1t = 1 \\
 & \frac{-0.1t}{-0.1} = \frac{1}{-0.1} \\
 & t = -10 \\
 & \text{The solution set is } \{-10\}.
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & \frac{x+1}{3} + \frac{x+2}{7} = 2 \\
 & 21\left(\frac{x+1}{3} + \frac{x+2}{7}\right) = 21(2) \\
 & 7(x+1) + (3)(x+2) = 42 \\
 & 7x + 7 + 3x + 6 = 42 \\
 & 10x + 13 = 42 \\
 & 10x + 13 - 13 = 42 - 13 \\
 & 10x = 29 \\
 & \frac{10x}{10} = \frac{29}{10} \\
 & x = \frac{29}{10} \\
 & \text{The solution set is } \left\{\frac{29}{10}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & \frac{2x+1}{3} + 16 = 3x \\
 & 3\left(\frac{2x+1}{3} + 16\right) = 3(3x) \\
 & 2x + 1 + 48 = 9x \\
 & 2x + 49 = 9x \\
 & 2x + 49 - 2x = 9x - 2x \\
 & 49 = 7x \\
 & \frac{49}{7} = \frac{7x}{7} \\
 & x = 7 \\
 & \text{The solution set is } \{7\}.
 \end{aligned}$$

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 41. \quad & \frac{5}{8}(p+3) - 2 = \frac{1}{4}(2p-3) + \frac{11}{16} \\
 & 10(p+3) - 32 = 4(2p-3) + 11 \\
 & 10p + 30 - 32 = 8p - 12 + 11 \\
 & 10p - 2 = 8p - 1 \\
 & 10p - 8p - 2 = 8p - 8p - 1 \\
 & 2p - 2 + 2 = -1 + 2 \\
 & 2p = 1 \\
 & \frac{2}{2}p = \frac{1}{2} \\
 & p = \frac{1}{2}
 \end{aligned}$$

The solution set is $\left\{\frac{1}{2}\right\}$.

$$\begin{aligned}
 42. \quad & \frac{1}{3}(w+1) - 3 = \frac{2}{5}(w-4) - \frac{2}{15} \\
 & 5(w+1) - 45 = 6(w-4) - 2 \\
 & 5w + 5 - 45 = 6w - 24 - 2 \\
 & 5w - 40 = 6w - 26 \\
 & 5w - 6w - 40 = 6w - 6w - 26 \\
 & -w - 40 + 40 = -26 + 40 \\
 & -w = 14 \\
 & \frac{-1}{-1}w = 14 \\
 & w = -14
 \end{aligned}$$

The solution set is $\{-14\}$.

$$\begin{aligned}
 43. \quad & \frac{2}{y} + \frac{4}{y} = 3 \\
 & y\left(\frac{2}{y} + \frac{4}{y}\right) = y(3) \\
 & 2 + 4 = 3y \\
 & 6 = 3y \\
 & \frac{6}{3} = \frac{3y}{3} \\
 & 2 = y
 \end{aligned}$$

Since $y = 2$ does not cause a denominator to equal zero, the solution set is $\{2\}$.

$$\begin{aligned}
 44. \quad & \frac{4}{y} - 5 = \frac{5}{2y} \\
 & 2y\left(\frac{4}{y} - 5\right) = 2y\left(\frac{5}{2y}\right) \\
 & 8 - 10y = 5 \\
 & 8 - 10y - 8 = 5 - 8 \\
 & -10y = -3 \\
 & \frac{-10y}{-10} = \frac{-3}{-10} \\
 & y = \frac{3}{10}
 \end{aligned}$$

Since $y = \frac{3}{10}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{3}{10}\right\}$.

$$\begin{aligned}
 45. \quad & \frac{1}{2} + \frac{2}{x} = \frac{3}{4} \\
 & 4x\left(\frac{1}{2} + \frac{2}{x}\right) = 4x\left(\frac{3}{4}\right) \\
 & 2x + 8 = 3x \\
 & 2x + 8 - 2x = 3x - 2x \\
 & 8 = x
 \end{aligned}$$

Since $x = 8$ does not cause any denominator to equal zero, the solution set is $\{8\}$.

$$\begin{aligned}
 46. \quad & \frac{3}{x} - \frac{1}{3} = \frac{1}{6} \\
 & 6x\left(\frac{3}{x} - \frac{1}{3}\right) = 6x\left(\frac{1}{6}\right) \\
 & 18 - 2x = x \\
 & 18 - 2x + 2x = x + 2x \\
 & 18 = 3x \\
 & \frac{18}{3} = \frac{3x}{3} \\
 & 6 = x
 \end{aligned}$$

Since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

$$\begin{aligned}
 47. \quad (x+7)(x-1) &= (x+1)^2 \\
 x^2 + 6x - 7 &= x^2 + 2x + 1 \\
 x^2 + 6x - 7 - x^2 &= x^2 + 2x + 1 - x^2 \\
 6x - 7 &= 2x + 1 \\
 6x - 7 + 7 &= 2x + 1 + 7 \\
 6x &= 2x + 8 \\
 6x - 2x &= 2x + 8 - 2x \\
 4x &= 8 \\
 \frac{4x}{4} &= \frac{8}{4} \\
 x &= 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 48. \quad (x+2)(x-3) &= (x+3)^2 \\
 x^2 - x - 6 &= x^2 + 6x + 9 \\
 x^2 - x - 6 - x^2 &= x^2 + 6x + 9 - x^2 \\
 -x - 6 &= 6x + 9 \\
 -x - 6 + 6 &= 6x + 9 + 6 \\
 -x &= 6x + 15 \\
 -x - 6x &= 6x + 15 - 6x \\
 -7x &= 15 \\
 \frac{-7x}{-7} &= \frac{15}{-7} \\
 x &= -\frac{15}{7}
 \end{aligned}$$

The solution set is $\left\{-\frac{15}{7}\right\}$.

$$\begin{aligned}
 49. \quad x(2x-3) &= (2x+1)(x-4) \\
 2x^2 - 3x &= 2x^2 - 7x - 4 \\
 2x^2 - 3x - 2x^2 &= 2x^2 - 7x - 4 - 2x^2 \\
 -3x &= -7x - 4 \\
 -3x + 7x &= -7x - 4 + 7x \\
 4x &= -4 \\
 \frac{4x}{4} &= \frac{-4}{4} \\
 x &= -1
 \end{aligned}$$

The solution set is $\{-1\}$.

$$\begin{aligned}
 50. \quad x(1+2x) &= (2x-1)(x-2) \\
 x + 2x^2 &= 2x^2 - 5x + 2 \\
 x + 2x^2 - 2x^2 &= 2x^2 - 5x + 2 - 2x^2 \\
 x &= -5x + 2 \\
 x + 5x &= -5x + 2 + 5x \\
 6x &= 2 \\
 \frac{6x}{6} &= \frac{2}{6} \Rightarrow x = \frac{1}{3}
 \end{aligned}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

$$\begin{aligned}
 51. \quad p(p^2+3) &= 12 + p^3 \\
 p^3 + 3p &= 12 + p^3 \\
 p^3 - p^3 + 3p &= 12 + p^3 - p^3 \\
 3p &= 12 \\
 \frac{3p}{3} &= \frac{12}{3} \Rightarrow p = 4
 \end{aligned}$$

The solution set is $\{4\}$.

$$\begin{aligned}
 52. \quad w(4-w^2) &= 8 - w^3 \\
 4w - w^3 &= 8 - w^3 \\
 4w - w^3 + w^3 &= 8 - w^3 + w^3 \\
 4w &= 8 \\
 \frac{4w}{4} &= \frac{8}{4} \\
 w &= 2
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 53. \quad \frac{x}{x-2} + 3 &= \frac{2}{x-2} \\
 \left(\frac{x}{x-2} + 3\right)(x-2) &= \left(\frac{2}{x-2}\right)(x-2) \\
 x + 3(x-2) &= 2 \\
 x + 3x - 6 &= 2 \\
 4x - 6 &= 2 \\
 4x - 6 + 6 &= 2 + 6 \\
 4x &= 8 \\
 \frac{4x}{4} &= \frac{8}{4} \\
 x &= 2
 \end{aligned}$$

Since $x = 2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

Chapter 1: Equations and Inequalities

54.
$$\frac{2x}{x+3} = \frac{-6}{x+3} - 2$$
$$\left(\frac{2x}{x+3}\right)(x+3) = \left(\frac{-6}{x+3} - 2\right)(x+3)$$
$$2x = -6 - (2)(x+3)$$
$$2x = -6 - 2x - 6$$
$$2x = -12 - 2x$$
$$2x + 2x = -12 - 2x + 2x$$
$$4x = -12$$
$$\frac{4x}{4} = \frac{-12}{4}$$
$$x = -3$$

Since $x = -3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

55.
$$\frac{2x}{x^2 - 4} = \frac{4}{x^2 - 4} - \frac{3}{x+2}$$
$$\frac{2x}{(x+2)(x-2)} = \frac{4}{(x+2)(x-2)} - \frac{3}{x+2}$$
$$\left(\frac{2x}{(x+2)(x-2)}\right)(x+2)(x-2) = \left(\frac{4}{(x+2)(x-2)} - \frac{3}{x+2}\right)(x+2)(x-2)$$
$$2x = 4 - 3(x-2)$$
$$2x = 4 - 3x + 6$$
$$2x = 10 - 3x$$
$$2x + 3x = 10 - 3x + 3x$$
$$5x = 10$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$x = 2$$

Since $x = 2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

$$\begin{aligned}
 56. \quad & \frac{x}{x^2-9} + \frac{4}{x+3} = \frac{3}{x^2-9} \\
 & \frac{x}{(x+3)(x-3)} + \frac{4}{x+3} = \frac{3}{(x+3)(x-3)} \\
 & \left(\frac{x}{(x+3)(x-3)} + \frac{4}{x+3} \right) (x+3)(x-3) = \left(\frac{3}{(x+3)(x-3)} \right) (x+3)(x-3) \\
 & \quad x + 4(x-3) = 3 \\
 & \quad x + 4x - 12 = 3 \\
 & \quad 5x - 12 = 3 \\
 & \quad 5x - 12 + 12 = 3 + 12 \\
 & \quad 5x = 15 \\
 & \quad \frac{5x}{5} = \frac{15}{5} \\
 & \quad x = 3
 \end{aligned}$$

Since $x = 3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

$$\begin{aligned}
 57. \quad & \frac{x}{x+2} = \frac{3}{2} \\
 & 2(x+2) \left(\frac{x}{x+2} \right) = 2(x+2) \left(\frac{3}{2} \right) \\
 & \quad 2x = 3(x+2) \\
 & \quad 2x = 3x + 6 \\
 & \quad 2x - 3x = 3x + 6 - 3x \\
 & \quad -x = 6 \\
 & \quad \frac{-x}{-1} = \frac{6}{-1} \\
 & \quad x = -6
 \end{aligned}$$

Since $x = -6$ does not cause any denominator to equal zero, the solution set is $\{-6\}$.

$$\begin{aligned}
 58. \quad & \frac{3x}{x-1} = 2 \\
 & \left(\frac{3x}{x-1} \right) (x-1) = 2(x-1) \\
 & \quad 3x = 2x - 2 \\
 & \quad 3x - 2x = 2x - 2 - 2x \\
 & \quad x = -2
 \end{aligned}$$

Since $x = -2$ does not cause any denominator to equal zero, the solution set is $\{-2\}$.

$$\begin{aligned}
 59. \quad & \frac{7}{3x+10} = \frac{2}{x-3} \\
 & \left(\frac{7}{3x+10} \right) (3x+10)(x-3) = \left(\frac{2}{x-3} \right) (3x+10)(x-3) \\
 & \quad 7(x-3) = 2(3x+10) \\
 & \quad 7x - 21 = 6x + 20 \\
 & \quad 7x - 21 - 6x = 6x + 20 - 6x \\
 & \quad -21 + x = 20 \\
 & \quad -21 + x + 21 - 25 = 20 + 21 \\
 & \quad x = 41
 \end{aligned}$$

Since $x = 41$ does not cause any denominator to equal zero, the solution is $\{41\}$.

$$\begin{aligned}
 60. \quad & \frac{-4}{x+4} = \frac{-3}{x+6} \\
 & \left(\frac{-4}{x+4} \right) (x+6)(x+4) = \left(\frac{-3}{x+6} \right) (x+6)(x+4) \\
 & \quad -4(x+6) = -3(x+4) \\
 & \quad -4x - 24 = -3x - 12 \\
 & \quad -4x - 24 + 4x = -3x - 12 + 4x \\
 & \quad -24 = -12 + x \\
 & \quad -24 + 12 = -12 + x + 12 \\
 & \quad -12 = x
 \end{aligned}$$

Since $x = -12$ does not cause any denominator to equal zero, the solution set is $\{-12\}$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 61. \quad & \frac{6t+7}{4t-1} = \frac{3t+8}{2t-4} \\
 & \left(\frac{6t+7}{4t-1}\right)(4t-1)(2t-4) = \left(\frac{3t+8}{2t-4}\right)(4t-1)(2t-4) \\
 & (6t+7)(2t-4) = (3t+8)(4t-1) \\
 & 12t^2 - 24t + 14t - 28 = 12t^2 - 3t + 32t - 8 \\
 & 12t^2 - 10t - 28 = 12t^2 + 29t - 8 \\
 & 12t^2 - 10t - 28 - 12t^2 = 12t^2 + 29t - 8 - 12t^2 \\
 & -10t - 28 = 29t - 8 \\
 & -10t - 28 - 29t = 29t - 8 - 29t \\
 & -28 - 39t = -8 \\
 & -28 - 39t + 28 = -8 + 28 \\
 & -39t = 20 \\
 & \frac{-39t}{-39} = \frac{20}{-39} \\
 & t = -\frac{20}{39}
 \end{aligned}$$

Since $t = -\frac{20}{39}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{20}{39}\right\}$.

$$\begin{aligned}
 62. \quad & \frac{8w+5}{10w-7} = \frac{4w-3}{5w+7} \\
 & \left(\frac{8w+5}{10w-7}\right)(10w-7)(5w+7) = \left(\frac{4w-3}{5w+7}\right)(10w-7)(5w+7) \\
 & (8w+5)(5w+7) = (4w-3)(10w-7) \\
 & 40w^2 + 56w + 25w + 35 = 40w^2 - 28w - 30w + 21 \\
 & 40w^2 + 81w + 35 = 40w^2 - 58w + 21 \\
 & 40w^2 + 81w + 35 - 40w^2 = 40w^2 - 58w + 21 - 40w^2 \\
 & 81w + 35 = -58w + 21 \\
 & 81w + 35 + 58w = -58w + 21 + 58w \\
 & 139w + 35 = 21 \\
 & 139w + 35 - 35 = 21 - 35 \\
 & 139w = -14 \\
 & \frac{139w}{139} = \frac{-14}{139} \\
 & w = -\frac{14}{139}
 \end{aligned}$$

Since $w = -\frac{14}{139}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{14}{139}\right\}$.

$$\begin{aligned}
 63. \quad & \frac{4}{x-2} = \frac{-3}{x+5} + \frac{7}{(x+5)(x-2)} \\
 & \left(\frac{4}{x-2} \right) (x+5)(x-2) = \left(\frac{-3}{x+5} + \frac{7}{(x+5)(x-2)} \right) (x+5)(x-2) \\
 & 4(x+5) = -3(x-2) + 7 \\
 & 4x + 20 = -3x + 6 + 7 \\
 & 4x + 20 = -3x + 13 \\
 & 4x + 20 + 3x = -3x + 13 + 3x \\
 & 7x + 20 = 13 \\
 & 7x + 20 - 20 = 13 - 20 \\
 & 7x = -7 \\
 & \frac{7x}{7} = \frac{-7}{7} \\
 & x = -1
 \end{aligned}$$

Since $x = -1$ does not cause any denominator to equal zero, the solution set is $\{-1\}$.

$$\begin{aligned}
 64. \quad & \frac{-4}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)} \\
 & \left(\frac{-4}{2x+3} + \frac{1}{x-1} \right) (2x+3)(x-1) = \left(\frac{1}{(2x+3)(x-1)} \right) (2x+3)(x-1) \\
 & -4(x-1) + 1(2x+3) = 1 \\
 & -4x + 4 + 2x + 3 = 1 \\
 & -2x + 7 = 1 \\
 & -2x + 7 - 7 = 1 - 7 \\
 & -2x = -6 \\
 & \frac{-2x}{-2} = \frac{-6}{-2} \\
 & x = 3
 \end{aligned}$$

Since $x = 3$ does not cause any denominator to equal zero, the solution set is $\{3\}$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 65. \quad & \frac{2}{y+3} + \frac{3}{y-4} = \frac{5}{y+6} \\
 & \left(\frac{2}{y+3} + \frac{3}{y-4} \right) (y+3)(y-4)(y+6) = \left(\frac{5}{y+6} \right) (y+3)(y-4)(y+6) \\
 & 2(y-4)(y+6) + 3(y+3)(y+6) = 5(y+3)(y-4) \\
 & 2(y^2 + 6y - 4y - 24) + 3(y^2 + 6y + 3y + 18) = 5(y^2 - 4y + 3y - 12) \\
 & 2(y^2 + 2y - 24) + 3(y^2 + 9y + 18) = 5(y^2 - y - 12) \\
 & 2y^2 + 4y - 48 + 3y^2 + 27y + 54 = 5y^2 - 5y - 60 \\
 & 5y^2 + 31y + 6 = 5y^2 - 5y - 60 \\
 & 5y^2 + 31y + 6 - 5y^2 = 5y^2 - 5y - 60 - 5y^2 \\
 & 31y + 6 = -5y - 60 \\
 & 31y + 6 + 5y = -5y - 60 + 5y \\
 & 36y + 6 = -60 \\
 & 36y + 6 - 6 = -60 - 6 \\
 & 36y = -66 \\
 & \frac{36y}{36} = \frac{-66}{36} \\
 & y = -\frac{11}{6}
 \end{aligned}$$

Since $y = -\frac{11}{6}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{11}{6}\right\}$.

$$\begin{aligned}
 66. \quad & \frac{5}{5z-11} + \frac{4}{2z-3} = \frac{-3}{5-z} \\
 & \left(\frac{5}{5z-11} + \frac{4}{2z-3} \right) (5z-11)(2z-3)(5-z) = \left(\frac{-3}{5-z} \right) (5z-11)(2z-3)(5-z) \\
 & 5(2z-3)(5-z) + 4(5z-11)(5-z) = -3(5z-11)(2z-3) \\
 & 5(10z - 2z^2 - 15 + 3z) + 4(25z - 5z^2 - 55 + 11z) = -3(10z^2 - 15z - 22z + 33) \\
 & 5(-2z^2 + 13z - 15) + 4(-5z^2 + 36z - 55) = -3(10z^2 - 37z + 33) \\
 & -10z^2 + 65z - 75 - 20z^2 + 144z - 220 = -30z^2 + 111z - 99 \\
 & -30z^2 + 209z - 295 = -30z^2 + 111z - 99 \\
 & -30z^2 + 209z - 295 + 30z^2 = -30z^2 + 111z - 99 + 30z^2 \\
 & 209z - 295 = 111z - 99 \\
 & 209z - 295 - 209z = 111z - 99 - 209z \\
 & -295 = -98z - 99 \\
 & -295 + 99 = -98z - 99 + 99 \\
 & -196 = -98z \\
 & \frac{-196}{-98} = \frac{-118z}{-98} \\
 & 2 = z
 \end{aligned}$$

Since $z = 2$ does not cause any denominator to equal zero, the solution set is $\{2\}$.

$$\begin{aligned}
67. \quad & \frac{x}{x^2-9} - \frac{x-4}{x^2+3x} = \frac{10}{x^2-3x} \\
& \frac{x}{(x+3)(x-3)} - \frac{x-4}{x(x+3)} = \frac{10}{x(x-3)} \\
& \left(\frac{x}{(x+3)(x-3)} - \frac{x-4}{x(x+3)} \right) x(x+3)(x-3) = \left(\frac{10}{x(x-3)} \right) x(x+3)(x-3) \\
& (x)(x) - (x-4)(x-3) = 10(x+3) \\
& x^2 - (x^2 - 3x - 4x + 12) = 10x + 30 \\
& x^2 - (x^2 - 7x + 12) = 10x + 30 \\
& x^2 - x^2 + 7x - 12 = 10x + 30 \\
& 7x - 12 = 10x + 30 \\
& 7x - 12 + 12 = 10x + 30 + 12 \\
& 7x = 10x + 42 \\
& 7x - 10x = 10x - 10x + 42 \\
& -3x = 42 \\
& \frac{-3x}{-3} = \frac{42}{-3} \\
& x = -14
\end{aligned}$$

Since $x = -6$ does not cause any denominator to equal zero, the solution set is $\{-14\}$.

$$\begin{aligned}
68. \quad & \frac{x+1}{x^2+2x} - \frac{x+4}{x^2+x} = \frac{-3}{x^2+3x+2} \\
& \frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} = \frac{-3}{(x+2)(x+1)} \\
& \left(\frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} \right) x(x+2)(x+1) = \left(\frac{-3}{(x+2)(x+1)} \right) x(x+2)(x+1) \\
& (x+1)(x+1) - (x+4)(x+2) = -3x \\
& (x^2 + x + x + 1) - (x^2 + 2x + 4x + 8) = -3x \\
& x^2 + 2x + 1 - (x^2 + 6x + 8) = -3x \\
& x^2 + 2x + 1 - x^2 - 6x - 8 = -3x \\
& 2x + 1 - 6x - 8 = -3x \\
& -4x - 7 = -3x \\
& -4x - 7 + 4x = -3x + 4x \\
& -7 = x
\end{aligned}$$

Since $x = -7$ does not cause any denominator to equal zero, the solution set is $\{-7\}$.

Chapter 1: Equations and Inequalities

69.
$$3.2x + \frac{21.3}{65.871} = 19.23$$
$$3.2x + \frac{21.3}{65.871} - \frac{21.3}{65.871} = 19.23 - \frac{21.3}{65.871}$$
$$3.2x = 19.23 - \frac{21.3}{65.871}$$
$$\left(\frac{1}{3.2}\right)(3.2x) = \left(19.23 - \frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)$$
$$x = \left(19.23 - \frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right) \approx 5.91$$

The solution set is approximately $\{5.91\}$.

70.
$$6.2x - \frac{19.1}{83.72} = 0.195$$
$$6.2x - \frac{19.1}{83.72} + \frac{19.1}{83.72} = 0.195 + \frac{19.1}{83.72}$$
$$6.2x = 0.195 + \frac{19.1}{83.72}$$
$$\left(\frac{1}{6.2}\right)(6.2x) = \left(0.195 + \frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)$$
$$x = \left(0.195 + \frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right) \approx 0.07$$

The solution set is approximately $\{0.07\}$.

71.
$$14.72 - 21.58x = \frac{18}{2.11}x + 2.4$$
$$14.72 - 21.58x - \frac{18}{2.11}x = \frac{18}{2.11}x + 2.4 - \frac{18}{2.11}x$$
$$14.72 - 21.58x - \frac{18}{2.11}x = 2.4$$
$$14.72 - 21.58x - \frac{18}{2.11}x - 14.72 = 2.4 - 14.72$$
$$-21.58x - \frac{18}{2.11}x = -12.32$$
$$\left(-21.58 - \frac{18}{2.11}\right)x = -12.32$$
$$\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right)\left(-21.58 - \frac{18}{2.11}\right)x = -12.32\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right)$$
$$x = -12.32\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right) \approx 0.41$$

The solution set is approximately $\{0.41\}$.

$$\begin{aligned}
 72. \quad & 18.63x - \frac{21.2}{2.6} = \frac{14}{2.32}x - 20 \\
 & 18.63x - \frac{21.2}{2.6} - \frac{14}{2.32}x = \frac{14}{2.32}x - 20 - \frac{14}{2.32}x \\
 & 18.63x - \frac{21.2}{2.6} - \frac{14}{2.32}x = -20 \\
 & 18.63x - \frac{21.2}{2.6} - \frac{14}{2.32}x + \frac{21.2}{2.6} = -20 + \frac{21.2}{2.6} \\
 & 18.63x - \frac{14}{2.32}x = -20 + \frac{21.2}{2.6} \\
 & \left(18.63 - \frac{14}{2.32}\right)x = -20 + \frac{21.2}{2.6} \\
 & \left(\frac{1}{18.63 - \frac{14}{2.32}}\right)\left(18.63 - \frac{14}{2.32}\right)x = \left(-20 + \frac{21.2}{2.6}\right)\left(\frac{1}{18.63 - \frac{14}{2.32}}\right) \\
 & x = \left(-20 + \frac{21.2}{2.6}\right)\left(\frac{1}{18.63 - \frac{14}{2.32}}\right) \approx -0.94
 \end{aligned}$$

The solution set is approximately $\{-0.94\}$.

$$73. \quad ax - b = c, \quad a \neq 0$$

$$ax - b + b = c + b$$

$$ax = b + c$$

$$\frac{ax}{a} = \frac{b+c}{a}$$

$$x = \frac{b+c}{a}$$

$$74. \quad 1 - ax = b, \quad a \neq 0$$

$$1 - ax - 1 = b - 1$$

$$-ax = b - 1$$

$$\frac{-ax}{-a} = \frac{b-1}{-a}$$

$$x = \frac{b-1}{-a} = \frac{1-b}{a}$$

$$75. \quad \frac{x}{a} + \frac{x}{b} = c, \quad a \neq 0, b \neq 0, a \neq -b$$

$$ab\left(\frac{x}{a} + \frac{x}{b}\right) = ab \cdot c$$

$$bx + ax = abc$$

$$(a+b)x = abc$$

$$\frac{(a+b)x}{a+b} = \frac{abc}{a+b}$$

$$x = \frac{abc}{a+b}$$

$$76. \quad \frac{a}{x} + \frac{b}{x} = c, \quad c \neq 0$$

$$x\left(\frac{a}{x} + \frac{b}{x}\right) = x \cdot c$$

$$a + b = cx$$

$$\frac{a+b}{c} = \frac{cx}{c}$$

$$x = \frac{a+b}{c}$$

Chapter 1: Equations and Inequalities

77. $x + 2a = 16 + ax - 6a$, if $x = 4$

$$4 + 2a = 16 + a(4) - 6a$$

$$4 + 2a = 16 + 4a - 6a$$

$$4 + 2a = 16 - 2a$$

$$4a = 12$$

$$\frac{4a}{4} = \frac{12}{4}$$

$$a = 3$$

78. $x + 2b = x - 4 + 2bx$, for $x = 2$

$$2 + 2b = 2 - 4 + 2b(2)$$

$$2 + 2b = 2 - 4 + 4b$$

$$2 + 2b = -2 + 4b$$

$$4 = 2b$$

$$\frac{4}{2} = b$$

$$b = 2$$

79. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$RR_1R_2\left(\frac{1}{R}\right) = RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 = R(R_2 + R_1)$$

$$\frac{R_1R_2}{R_2 + R_1} = \frac{R(R_2 + R_1)}{R_2 + R_1}$$

$$\frac{R_1R_2}{R_2 + R_1} = R$$

80. $A = P(1 + rt)$

$$A = P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

81. $F = \frac{mv^2}{R}$

$$RF = R\left(\frac{mv^2}{R}\right)$$

$$RF = mv^2$$

$$\frac{RF}{F} = \frac{mv^2}{F}$$

$$R = \frac{mv^2}{F}$$

82. $PV = nRT$

$$\frac{PV}{nR} = \frac{nRT}{nR}$$

$$\frac{PV}{nR} = T$$

83. $S = \frac{a}{1-r}$

$$S(1-r) = \left(\frac{a}{1-r}\right)(1-r)$$

$$S - Sr = a$$

$$S - Sr - S = a - S$$

$$-Sr = a - S$$

$$\frac{-Sr}{-S} = \frac{a - S}{-S}$$

$$r = \frac{S - a}{S}$$

84. $v = -gt + v_0$

$$v - v_0 = -gt$$

$$\frac{v - v_0}{-g} = \frac{-gt}{-g}$$

$$t = \frac{v - v_0}{-g} = \frac{v_0 - v}{g}$$

85.

Amount in bonds	Amount in CDs	Total
x	$x - 3000$	20,000

$$x + (x - 3000) = 20,000$$

$$2x - 3000 = 20,000$$

$$2x = 23,000$$

$$x = 11,500$$

\$11,500 will be invested in bonds and \$8500 will be invested in CD's.

Section 1.1: Linear Equations

86.

Sean's Amount	George's Amount	Total
x	$x - 3000$	10,000

$$x + (x - 3000) = 10,000$$

$$2x - 3000 = 10,000$$

$$2x = 13,000$$

$$x = 6500$$

Sean will receive \$6500 and Jorge will receive \$3500.

87.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	8	$8(1.5x)$

$$40x + 8(1.5x) = 910$$

$$40x + 12x = 910$$

$$52x = 910$$

$$x = \frac{910}{52} = 17.50$$

Sandra's regular hourly wage is \$17.50.

88.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	6	$6(1.5x)$
Sunday wage	$2x$	4	$4(2x)$

$$40x + 6(1.5x) + 4(2x) = 1083$$

$$40x + 9x + 8x = 1083$$

$$57x = 1083$$

$$x = \frac{1083}{57} = 19$$

Leigh's regular hourly wage is \$19.00.

89. Let x represent the score on the final exam.

$$\frac{80 + 83 + 71 + 61 + 95 + x + x}{7} = 80$$

$$\frac{390 + 2x}{7} = 80$$

$$390 + 2x = 560$$

$$2x = 170$$

$$x = 85$$

Camila needs a score of 85 on the final exam.

90. Let x represent the score on the final exam.

Note: since the final exam counts for two-thirds of the overall grade, the average of the four test scores count for one-third of the overall grade.

For a B, the average score must be 80.

$$\frac{1}{3} \left(\frac{86 + 80 + 84 + 90}{4} \right) + \frac{2}{3}x = 80$$

$$\frac{1}{3} \left(\frac{340}{4} \right) + \frac{2}{3}x = 80$$

$$\frac{85}{3} + \frac{2}{3}x = 80$$

$$3 \left(\frac{85}{3} + \frac{2}{3}x \right) = 3(80)$$

$$85 + 2x = 240$$

$$2x = 155$$

$$x = 77.5$$

Ali needs a score of 78 to earn a B.

For an A, the average score must be 90.

$$\frac{1}{3} \left(\frac{86 + 80 + 84 + 90}{4} \right) + \frac{2}{3}x = 90$$

$$\frac{1}{3} \left(\frac{340}{4} \right) + \frac{2}{3}x = 90$$

$$\frac{85}{3} + \frac{2}{3}x = 90$$

$$3 \left(\frac{85}{3} + \frac{2}{3}x \right) = 3(90)$$

$$85 + 2x = 270$$

$$2x = 185$$

$$x = 92.5$$

Ali needs a score of 93 to earn an A.

Chapter 1: Equations and Inequalities

91. Let x represent the original price of the phone.
Then $0.12x$ represents the reduction in the price of the phone.

The new price of the phone is \$572.
original price – reduction = new price

$$x - 0.12x = 572$$

$$0.88x = 572$$

$$x = 650$$

The original price of the phone was \$650.
The amount of the reduction (i.e., the savings) is $0.12(\$650) = \$78$.

92. Let x represent the original price of the car.
Then $0.15x$ represents the reduction in the price of the car.

The new price of the car is \$8000.
list price – reduction = new price

$$x - 0.15x = 18000$$

$$0.85x = 18000$$

$$x \approx 21176.47$$

The list price of the car was \$21,176.47.
The amount of the reduction (i.e., the savings) is $0.15(\$21176.47) \approx \$3176.47$.

93. Let x represent the price the theater pays for the candy.
Then $2.75x$ represents the markup on the candy.
The selling price of the candy is \$4.50.
supplier price + markup = selling price

$$x + 2.75x = 4.50$$

$$3.75x = 4.50$$

$$x = 1.20$$

The theater paid \$1.20 for the candy.

94. Let x represent selling price for the new car.
The dealer's cost is $0.85(\$24,000) = \$20,400$.
The markup is \$300.
selling price = dealer's cost + markup
 $x = 20,400 + 300 = \$20,700$
At \$300 over the dealer's cost, the price of the care is \$20,700.

95.

	Tickets sold	Price per ticket	Money earned
Adults	x	7.50	$7.50x$
Children	$5200 - x$	4.50	$4.50(5200 - x)$

$$7.50x + 4.50(5200 - x) = 29,961$$

$$7.50x + 23,400 - 4.50x = 29,961$$

$$3.00x + 23,400 = 29,961$$

$$3.00x = 6561$$

$$x = 2187$$

There were 2187 adult patrons.

96. Let p represent the original price for the boots.
Then, $0.30p$ represents the discounted amount.
original price – discount = clearance price

$$p - 0.30p = 399$$

$$0.70p = 399$$

$$p = 570$$

The boots originally cost \$570.

97. Let w represent the width of the rectangle.
Then $w + 8$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.
 $2(w + 8) + 2w = 60$
 $2w + 16 + 2w = 60$
 $4w + 16 = 60$
 $4w = 44$

$$w = 11$$

Now, $11 + 8 = 19$.

The width of the rectangle is 11 feet and the length is 19 feet.

98. Let w represent the width of the rectangle.
Then $2w$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.
 $2(2w) + 2w = 42$
 $4w + 2w = 42$
 $6w = 42$

$$w = 7$$

Now, $2(7) = 14$.

The width of the rectangle is 7 meters and the length is 14 meters.

99. We will let B be the calories from breakfast, L the calories from lunch and D the calories from dinner. So we have the following equations:

$$B = L + 125$$

$$D = 2L - 300$$

$$2025 = B + L + D$$

Now we substitute the first two into the last one and solve for L.

$$2025 = (L + 125) + L + (2L - 300)$$

$$2025 = 4L - 175$$

$$2200 = 4L$$

$$L = 550$$

Now we substitute L into the first two equations to get B and D.

$$B = 550 + 125 = 675$$

$$D = 2(550) - 300 = 800$$

So Herschel took in 675 calories from breakfast, 550 calories from lunch and 800 calories from dinner.

- 100.** We will let B be the calories from breakfast, L the calories from lunch, D the calories from dinner and S the calories from snacks. So we have the following equations:

$$L = 0.5B \quad D = B + 200$$

$$S = B - 120 \quad E = 700$$

$$1480 = B + L + D - E$$

Now we substitute the first four into the last one and solve for B.

$$1480 = B + 0.5B + (B + 200) + (B - 200) - 700$$

$$1480 = 3.5B - 620$$

$$2100 = 3.5B$$

$$B = 600$$

Now we substitute B to get S.

$$S = B - 120 = 600 - 120 = 480$$

So Tyshira took in 480 calories from snacks.

101.

Judy's Amount	Tom's Amount	Total
x	$\frac{2}{3}x$	18

$$x + \frac{2}{3}x = 18$$

$$\frac{5}{3}x = 18$$

$$x = \frac{3}{5}(18)$$

$$x = 10.80$$

Judy pays \$10.80 and Tom pays \$7.20.

- 102.** An isosceles triangle has three equal sides. Therefore: $4x + 10 = 2x + 40 = 3x + 18$. Solve each set separately:

$$4x + 10 = 2x + 40$$

$$2x = 30$$

$$x = 15$$

$$4x + 10 = 3x + 18$$

$$x = 8$$

$$2x + 40 = 3x + 18$$

$$-x = -22$$

$$x = 22$$

Since 22 is the largest of the numbers then the largest perimeter is:

$$4(22) + 10 + 2(22) + 40 + 3(22) + 18 = 266$$

$$\mathbf{103.} \quad \frac{3}{4}x - \frac{1}{5}\left(\frac{1}{2} - 3x\right) + 1 = \frac{1}{4}\left(\frac{1}{20}x + 6\right) - \frac{4}{5}$$

$$\frac{3}{4}x - \frac{1}{10} + \frac{3}{5}x + 1 = \frac{1}{80}x + \frac{3}{2} - \frac{4}{5}$$

Multiply both sides by the LCD 80 to clear fractions.

$$60x - 8 + 48x + 80 = x + 120 - 64$$

$$108x + 72 = x + 58$$

$$107x = -16$$

$$x = -\frac{16}{107}$$

- 104.** If a hexagon is inscribed in a circle then the sides of the hexagon are equal to the radius of the circle. Let the $P = 6r$ be the perimeter of the hexagon. Let r be the radius of the circle.

$$6r = r + 10$$

$$5r = 10$$

$$r = 2$$

Thus $r = 2$ inches is the radius of the circle where the perimeter of the hexagon is 10 inches more than the radius.

- 105.** To move from step (6) to step (7), we divided both sides of the equation by the expression $x - 2$. From step (1), however, we know $x = 2$, so this means we divided both sides of the equation by zero.

106–107. Answers will vary.

Chapter 1: Equations and Inequalities

Section 1.2

1. $x^2 - 5x - 6 = (x - 6)(x + 1)$

2. $2x^2 - x - 3 = (2x - 3)(x + 1)$

3. $\left\{-\frac{5}{3}, 3\right\}$

4. True

5. $\frac{1}{2} \cdot 5 = \frac{5}{2}; \left(\frac{5}{2}\right)^2 = \frac{25}{4}; x^2 + 5x + \frac{25}{4}$
 $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

6. discriminant; negative

7. False; a quadratic equation may have no real solutions.

8. False; If $x^2 = p$ then x could also be negative.

9. b

10. d

11. $x^2 - 9x = 0$

$$x(x - 9) = 0$$

$$x = 0 \text{ or } x - 9 = 0$$

$$x = 0 \text{ or } x = 9$$

The solution set is $\{0, 9\}$.

12. $x^2 + 4x = 0$

$$x(x + 4) = 0$$

$$x = 0 \text{ or } x + 4 = 0$$

$$x = 0 \text{ or } x = -4$$

The solution set is $\{-4, 0\}$.

13. $x^2 - 25 = 0$

$$(x + 5)(x - 5) = 0$$

$$x + 5 = 0 \text{ or } x - 5 = 0$$

$$x = -5 \text{ or } x = 5$$

The solution set is $\{-5, 5\}$.

14. $x^2 - 9 = 0$

$$(x + 3)(x - 3) = 0$$

$$x + 3 = 0 \text{ or } x - 3 = 0$$

$$x = -3 \text{ or } x = 3$$

The solution set is $\{-3, 3\}$.

15. $z^2 + z - 6 = 0$

$$(z + 3)(z - 2) = 0$$

$$z + 3 = 0 \text{ or } z - 2 = 0$$

$$z = -3 \text{ or } z = 2$$

The solution set is $\{-3, 2\}$.

16. $v^2 + 7v + 6 = 0$

$$(v + 6)(v + 1) = 0$$

$$v + 6 = 0 \text{ or } v + 1 = 0$$

$$v = -6 \text{ or } v = -1$$

The solution set is $\{-6, -1\}$

17. $2x^2 - 5x - 3 = 0$

$$(2x + 1)(x - 3) = 0$$

$$2x + 1 = 0 \text{ or } x - 3 = 0$$

$$x = -\frac{1}{2} \text{ or } x = 3$$

The solution set is $\left\{-\frac{1}{2}, 3\right\}$

18. $3x^2 + 5x + 2 = 0$

$$(3x + 2)(x + 1) = 0$$

$$3x + 2 = 0 \text{ or } x + 1 = 0$$

$$x = -\frac{2}{3} \text{ or } x = -1$$

The solution set is $\left\{-1, -\frac{2}{3}\right\}$.

19. $-5w^2 + 180 = 0$

$$-5(w^2 - 36) = 0$$

$$-5(w + 6)(w - 6) = 0$$

$$w + 6 = 0 \text{ or } w - 6 = 0$$

$$w = -6 \text{ or } w = 6$$

The solution set is $\{-6, 6\}$.

Section 1.2: Quadratic Equations

20. $2y^2 - 50 = 0$
 $2(y^2 - 25) = 0$
 $2(y+5)(y-5) = 0$
 $y+5 = 0$ or $y-5 = 0$
 $y = -5$ or $y = 5$
The solution set is $\{-5, 5\}$.

21. $x(x+3) - 10 = 0$
 $x^2 + 3x - 10 = 0$
 $(x-2)(x+5) = 0$
 $x-2 = 0$ or $x+5 = 0$
 $x = 2$ or $x = -5$
The solution set is $\{-5, 2\}$.

22. $x(x+4) = 12$
 $x^2 + 4x - 12 = 0$
 $(x+6)(x-2) = 0$
 $x+6 = 0$ or $x-2 = 0$
 $x = -6$ or $x = 2$
The solution set is $\{-6, 2\}$.

23. $4x^2 + 9 = 12x$
 $4x^2 - 12x + 9 = 0$
 $(2x-3)^2 = 0$
 $2x-3 = 0$
 $x = \frac{3}{2}$
The solution set is $\left\{\frac{3}{2}\right\}$.

24. $25x^2 + 16 = 40x$
 $25x^2 - 40x + 16 = 0$
 $(5x-4)^2 = 0$
 $5x-4 = 0$
 $x = \frac{4}{5}$
The solution set is $\left\{\frac{4}{5}\right\}$.

25. $6(p^2 - 1) = 5p$
 $6p^2 - 6 = 5p$
 $6p^2 - 5p - 6 = 0$
 $(3p+2)(2p-3) = 0$
 $3p+2 = 0$ or $2p-3 = 0$
 $p = -\frac{2}{3}$ or $p = \frac{3}{2}$
The solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

26. $2(2u^2 - 4u) + 3 = 0$
 $4u^2 - 8u + 3 = 0$
 $(2u-1)(2u-3) = 0$
 $2u-1 = 0$ or $2u-3 = 0$
 $u = \frac{1}{2}$ or $u = \frac{3}{2}$
The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

27. $6x - 5 = \frac{6}{x}$
 $(6x-5)x = \left(\frac{6}{x}\right)x$
 $6x^2 - 5x = 6$
 $6x^2 - 5x - 6 = 0$
 $(3x+2)(2x-3) = 0$
 $3x+2 = 0$ or $2x-3 = 0$
 $x = -\frac{2}{3}$ or $x = \frac{3}{2}$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

28. $x + \frac{12}{x} = 7$
 $\left(x + \frac{12}{x}\right)x = 7x$
 $x^2 + 12 = 7x$
 $x^2 - 7x + 12 = 0$
 $(x-3)(x-4) = 0$
 $x-3 = 0$ or $x-4 = 0$
 $x = 3$ or $x = 4$

Chapter 1: Equations and Inequalities

Neither of these values causes a denominator to equal zero, so the solution set is $\{3, 4\}$.

$$\begin{aligned}
 29. \quad & \frac{4(x-2)}{x-3} + \frac{3}{x} = \frac{-3}{x(x-3)} \\
 & \left(\frac{4(x-2)}{x-3} + \frac{3}{x} \right) x(x-3) = \left(\frac{-3}{x(x-3)} \right) x(x-3) \\
 & 4x(x-2) + 3(x-3) = -3 \\
 & 4x^2 - 8x + 3x - 9 = -3 \\
 & 4x^2 - 5x - 6 = 0 \\
 & (4x+3)(x-2) = 0 \\
 & 4x+3=0 \quad \text{or} \quad x-2=0 \\
 & x = -\frac{3}{4} \quad \text{or} \quad x = 2
 \end{aligned}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{3}{4}, 2\right\}$.

$$\begin{aligned}
 30. \quad & \frac{5}{x+4} = 4 + \frac{3}{x-2} \\
 & \left(\frac{5}{x+4} \right) (x+4)(x-2) = \left(4 + \frac{3}{x-2} \right) (x+4)(x-2) \\
 & 5(x-2) = 4(x+4)(x-2) + 3(x+4) \\
 & 5x - 10 = 4(x^2 + 2x - 8) + 3x + 12 \\
 & 5x - 10 = 4x^2 + 8x - 32 + 3x + 12 \\
 & 0 = 4x^2 + 6x - 10 \\
 & 0 = 2(2x^2 + 3x - 5) \\
 & 0 = 2(2x+5)(x-1) \\
 & 2x+5=0 \quad \text{or} \quad x-1=0 \\
 & x = -\frac{5}{2} \quad \text{or} \quad x = 1
 \end{aligned}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{5}{2}, 1\right\}$.

$$\begin{aligned}
 31. \quad & x^2 = 25 \\
 & x = \pm\sqrt{25} \\
 & x = \pm 5 \\
 & \text{The solution set is } \{-5, 5\}.
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & x^2 = 36 \\
 & x = \pm\sqrt{36} \\
 & x = \pm 6 \\
 & \text{The solution set is } \{-6, 6\}.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & (x-1)^2 = 4 \\
 & x-1 = \pm\sqrt{4} \\
 & x-1 = \pm 2 \\
 & x-1 = 2 \quad \text{or} \quad x-1 = -2 \\
 & x = 3 \quad \text{or} \quad x = -1 \\
 & \text{The solution set is } \{-1, 3\}.
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & (x+2)^2 = 1 \\
 & x+2 = \pm\sqrt{1} \\
 & x+2 = \pm 1 \\
 & x+2 = 1 \quad \text{or} \quad x+2 = -1 \\
 & x = -1 \quad \text{or} \quad x = -3 \\
 & \text{The solution set is } \{-3, -1\}.
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \left(\frac{1}{3}h + 4 \right)^2 = 16 \\
 & \frac{1}{3}h + 4 = \pm\sqrt{16} \\
 & \frac{1}{3}h + 4 = \pm 4 \\
 & \frac{1}{3}h + 4 = 4 \quad \text{or} \quad \frac{1}{3}h + 4 = -4 \\
 & \frac{1}{3}h = 0 \quad \text{or} \quad \frac{1}{3}h = -8 \\
 & h = 0 \quad \text{or} \quad h = -24 \\
 & \text{The solution set is } \{-24, 0\}.
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & (3z-2)^2 = 4 \\
 & 3z-2 = \pm\sqrt{4} \\
 & 3z-2 = \pm 2 \\
 & 3z-2 = 2 \quad \text{or} \quad 3z-2 = -2 \\
 & 3z = 4 \quad \text{or} \quad 3z = 0 \\
 & z = \frac{4}{3} \quad \text{or} \quad z = 0 \\
 & \text{The solution set is } \left\{0, \frac{4}{3}\right\}.
 \end{aligned}$$

$$37. \quad x^2 + 4x = 21$$

$$x^2 + 4x + 4 = 21 + 4$$

$$(x+2)^2 = 25$$

$$x+2 = \pm\sqrt{25}$$

$$x+2 = \pm 5$$

$$x = -2 \pm 5$$

$$x = 3 \text{ or } x = -7$$

The solution set is $\{-7, 3\}$.

$$38. \quad x^2 - 6x = 13$$

$$x^2 - 6x + 9 = 13 + 9$$

$$(x-3)^2 = 22$$

$$x-3 = \pm\sqrt{22}$$

$$x = 3 \pm \sqrt{22}$$

The solution set is $\{3 - \sqrt{22}, 3 + \sqrt{22}\}$.

$$39. \quad x^2 - \frac{1}{2}x - \frac{3}{16} = 0$$

$$x^2 - \frac{1}{2}x = \frac{3}{16}$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$$x = \frac{1}{4} \pm \frac{1}{2}$$

$$x = \frac{3}{4} \text{ or } x = -\frac{1}{4}$$

The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

$$40. \quad x^2 + \frac{2}{3}x - \frac{1}{3} = 0$$

$$x^2 + \frac{2}{3}x = \frac{1}{3}$$

$$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$$

$$x + \frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$$

$$x = -\frac{1}{3} \pm \frac{2}{3}$$

$$x = \frac{1}{3} \text{ or } x = -1$$

The solution set is $\left\{-1, \frac{1}{3}\right\}$.

$$41. \quad 3x^2 + x - \frac{1}{2} = 0$$

$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$

$$x^2 + \frac{1}{3}x = \frac{1}{6}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$

$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$

$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$$

$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$

$$x = \frac{-1 \pm \sqrt{7}}{6}$$

The solution set is $\left\{\frac{-1 - \sqrt{7}}{6}, \frac{-1 + \sqrt{7}}{6}\right\}$.

$$42. \quad 2x^2 - 3x - 1 = 0$$

$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x = \frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$

Chapter 1: Equations and Inequalities

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm \sqrt{\frac{17}{16}}$$

$$x - \frac{3}{4} = \pm \frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4}\right\}$.

43. $x^2 - 4x + 2 = 0$

$a = 1, \quad b = 2, \quad c = -13$

$$x = \frac{-(-2) \pm \sqrt{(2)^2 - 4(1)(-13)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 52}}{2}$$

$$= \frac{-2 \pm \sqrt{56}}{2} = \frac{-2 \pm 2\sqrt{14}}{2} = -1 \pm \sqrt{14}$$

The solution set is $\{-1 - \sqrt{14}, -1 + \sqrt{14}\}$.

44. $x^2 + 4x + 2 = 0$

$a = 1, \quad b = 4, \quad c = 2$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The solution set is $\{-2 - \sqrt{2}, -2 + \sqrt{2}\}$.

45. $x^2 - 4x - 1 = 0$

$a = 1, \quad b = -4, \quad c = -1$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

The solution set is $\{2 - \sqrt{5}, 2 + \sqrt{5}\}$.

46. $x^2 + 6x + 1 = 0$

$a = 1, \quad b = 6, \quad c = 1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$

The solution set is $\{-3 - 2\sqrt{2}, -3 + 2\sqrt{2}\}$.

47. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

$$x = \frac{5+1}{4} \text{ or } x = \frac{5-1}{4}$$

$$x = \frac{6}{4} \text{ or } x = \frac{4}{4}$$

$$x = \frac{3}{2} \text{ or } x = 1$$

The solution set is $\left\{1, \frac{3}{2}\right\}$.

48. $2x^2 + 5x + 3 = 0$

$a = 2, \quad b = 5, \quad c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}$$

$$x = \frac{-5+1}{4} \text{ or } x = \frac{-5-1}{4}$$

$$x = \frac{-4}{4} \text{ or } x = \frac{-6}{4}$$

$$x = -1 \text{ or } x = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, -1\right\}$.

49. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 - 32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

50. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

51. $9x^2 + 8x = 5$

$9x^2 + 8x - 5 = 0$

$a = 9, \quad b = 8, \quad c = -5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(9)(-5)}}{2(9)}$$

$$= \frac{-8 \pm \sqrt{64 + 180}}{18} = \frac{-8 \pm \sqrt{244}}{18}$$

$$= \frac{-8 \pm 2\sqrt{61}}{18} = \frac{-4 \pm \sqrt{61}}{9}$$

The solution set is $\left\{ \frac{-4 - \sqrt{61}}{9}, \frac{-4 + \sqrt{61}}{9} \right\}$.

52. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4 + 8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{ \frac{-1 - \sqrt{3}}{2}, \frac{-1 + \sqrt{3}}{2} \right\}$.

53. $4x^2 = 9x$

$4x^2 - 9x = 0$

$x(4x - 9) = 0$

$x = 0 \quad \text{or} \quad 4x - 9 = 0$

$x = 0 \quad \text{or} \quad x = \frac{9}{4}$

The solution set is $\left\{ 0, \frac{9}{4} \right\}$.

54. $5x = 4x^2$

$0 = 4x^2 - 5x$

$0 = x(4x - 5)$

$x = 0 \quad \text{or} \quad 4x - 5 = 0$

$x = 0 \quad \text{or} \quad x = \frac{5}{4}$

The solution set is $\left\{ 0, \frac{5}{4} \right\}$.

55. $9t^2 - 6t + 1 = 0$

$a = 9, \quad b = -6, \quad c = 1$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{36 - 36}}{18} = \frac{6 \pm 0}{18} = \frac{1}{3}$$

The solution set is $\left\{ \frac{1}{3} \right\}$.

56. $4u^2 - 6u + 9 = 0$

$a = 4, \quad b = -6, \quad c = 9$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{6 \pm \sqrt{36 - 144}}{8} = \frac{6 \pm \sqrt{-108}}{8}$$

No real solution.

57. $\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2} = 0$

$$4\left(\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2}\right) = 4(0)$$

$3x^2 - x - 2 = 0$

$a = 3, \quad b = -1, \quad c = -2$

Chapter 1: Equations and Inequalities

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1+24}}{6} = \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$

$$x = \frac{1+5}{6} \quad \text{or} \quad x = \frac{1-5}{6}$$

$$x = \frac{6}{6} \quad \text{or} \quad x = \frac{-4}{6}$$

$$x = 1 \quad \text{or} \quad x = -\frac{2}{3}$$

The solution set is $\left\{-\frac{2}{3}, 1\right\}$.

58. $\frac{2}{3}x^2 - x - 3 = 0$

$$3\left(\frac{2}{3}x^2 - x - 3\right) = 3(0)$$

$$2x^2 - 3x - 9 = 0$$

$$a = 2, \quad b = -3, \quad c = -9$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9+72}}{4} = \frac{3 \pm \sqrt{81}}{4} = \frac{3 \pm 9}{4}$$

$$x = \frac{3+9}{4} \quad \text{or} \quad x = \frac{3-9}{4}$$

$$x = \frac{12}{4} \quad \text{or} \quad x = \frac{-6}{4}$$

$$x = 3 \quad \text{or} \quad x = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, 3\right\}$.

59. $\frac{5}{3}x^2 - x = \frac{1}{3}$

$$3\left(\frac{5}{3}x^2 - x\right) = 3\left(\frac{1}{3}\right)$$

$$5x^2 - 3x = 1$$

$$5x^2 - 3x - 1 = 0$$

$$a = 5, \quad b = -3, \quad c = -1$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{9+20}}{10} = \frac{3 \pm \sqrt{29}}{10}$$

The solution set is $\left\{\frac{3-\sqrt{29}}{10}, \frac{3+\sqrt{29}}{10}\right\}$.

60. $\frac{3}{5}x^2 - x = \frac{1}{5}$

$$5\left(\frac{3}{5}x^2 - x\right) = 5\left(\frac{1}{5}\right)$$

$$3x^2 - 5x = 1$$

$$3x^2 - 5x - 1 = 0$$

$$a = 3, \quad b = -5, \quad c = -1$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6}$$

The solution set is $\left\{\frac{5-\sqrt{37}}{6}, \frac{5+\sqrt{37}}{6}\right\}$.

61. $2x(x+2) = 3$

$$2x^2 + 4x - 3 = 0$$

$$a = 2, \quad b = 4, \quad c = -3$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

The solution set is $\left\{\frac{-2-\sqrt{10}}{2}, \frac{-2+\sqrt{10}}{2}\right\}$.

62. $3x(x+2)=1$

$3x^2+6x-1=0$

$a=3, \quad b=6, \quad c=-1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2(3)} = \frac{-6 \pm \sqrt{36+12}}{6}$$

$$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6} = \frac{-3 \pm 2\sqrt{3}}{3}$$

The solution set is $\left\{ \frac{-3-2\sqrt{3}}{3}, \frac{-3+2\sqrt{3}}{3} \right\}$.

63. $4 + \frac{1}{x} - \frac{1}{x^2} = 0$

$x^2 \left(4 + \frac{1}{x} - \frac{1}{x^2} \right) = x^2(0)$

$4x^2 + x - 1 = 0$

$a=4, \quad b=1, \quad c=-1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1+16}}{8} = \frac{-1 \pm \sqrt{17}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$\left\{ \frac{-1-\sqrt{17}}{8}, \frac{-1+\sqrt{17}}{8} \right\}$.

64. $2 + \frac{8}{x} + \frac{3}{x^2} = 0$

$x^2 \left(2 + \frac{8}{x} + \frac{3}{x^2} \right) = x^2(0)$

$2x^2 + 8x + 3 = 0$

$a=2, \quad b=8, \quad c=3$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{64-24}}{4} = \frac{-8 \pm \sqrt{40}}{4}$$

$$= \frac{-8 \pm 2\sqrt{10}}{4} = \frac{-4 \pm \sqrt{10}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$\left\{ \frac{-4-\sqrt{10}}{2}, \frac{-4+\sqrt{10}}{2} \right\}$.

65. $\frac{3x}{x-2} + \frac{1}{x} = 4$

$\left(\frac{3x}{x-2} + \frac{1}{x} \right) x(x-2) = 4x(x-2)$

$3x(x) + (x-2) = 4x^2 - 8x$

$3x^2 + x - 2 = 4x^2 - 8x$

$0 = x^2 - 9x + 2$

$a=1, \quad b=-9, \quad c=2$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{81-8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$\left\{ \frac{9-\sqrt{73}}{2}, \frac{9+\sqrt{73}}{2} \right\}$.

66. $\frac{2x}{x-3} + \frac{1}{x} = 4$

$\left(\frac{2x}{x-3} + \frac{1}{x} \right) x(x-3) = 4x(x-3)$

$2x(x) + (x-3) = 4x^2 - 12x$

$2x^2 + x - 3 = 4x^2 - 12x$

$0 = 2x^2 - 13x + 3$

$a=2, \quad b=-13, \quad c=3$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{13 \pm \sqrt{169-24}}{4} = \frac{13 \pm \sqrt{145}}{4}$$

Neither of these values causes a denominator to equal zero, so the solution set is

$\left\{ \frac{13-\sqrt{145}}{4}, \frac{13+\sqrt{145}}{4} \right\}$.

67. $x^2 - 4.1x + 2.2 = 0$

$a=1, \quad b=-4.1, \quad c=2.2$

$$x = \frac{-(-4.1) \pm \sqrt{(-4.1)^2 - 4(1)(2.2)}}{2(1)}$$

$$= \frac{4.1 \pm \sqrt{16.81-8.8}}{2} = \frac{4.1 \pm \sqrt{8.01}}{2}$$

$x \approx 3.47 \quad \text{or} \quad x \approx 0.63$

The solution set is $\{0.63, 3.47\}$.

Chapter 1: Equations and Inequalities

68. $x^2 + 3.9x + 1.8 = 0$

$a = 1, \quad b = 3.9, \quad c = 1.8$

$$x = \frac{-3.9 \pm \sqrt{(3.9)^2 - 4(1)(1.8)}}{2(1)}$$

$$= \frac{-3.9 \pm \sqrt{15.21 - 7.2}}{2} = \frac{-3.9 \pm \sqrt{8.01}}{2}$$

$x \approx -0.53$ or $x \approx -3.37$

The solution set is $\{-3.37, -0.53\}$.

69. $x^2 + \sqrt{3}x - 3 = 0$

$a = 1, \quad b = \sqrt{3}, \quad c = -3$

$$x = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3+12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$x \approx 1.07$ or $x \approx -2.80$

The solution set is $\{-2.80, 1.07\}$.

70. $x^2 + \sqrt{2}x - 2 = 0$

$a = 1, \quad b = \sqrt{2}, \quad c = -2$

$$x = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

$x \approx 0.87$ or $x \approx -2.29$

The solution set is $\{-2.29, 0.87\}$.

71. $\pi x^2 - x - \pi = 0$

$a = \pi, \quad b = -1, \quad c = -\pi$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)}$$

$$= \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$x \approx 1.17$ or $x \approx -0.85$

The solution set is $\{-0.85, 1.17\}$.

72. $\pi x^2 + \pi x - 2 = 0$

$a = \pi, \quad b = \pi, \quad c = -2$

$$x = \frac{-\pi \pm \sqrt{(\pi)^2 - 4(\pi)(-2)}}{2(\pi)}$$

$$= \frac{-\pi \pm \sqrt{\pi^2 + 8\pi}}{2\pi}$$

$x \approx 0.44$ or $x \approx -1.44$

The solution set is $\{-1.44, 0.44\}$.

73. $2x^2 - 6x + 7 = 0$

$a = 2, \quad b = -6, \quad c = 7$

$$b^2 - 4ac = (-6)^2 - 4(2)(7) = 36 - 56 = -20$$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

74. $x^2 + 4x + 7 = 0$

$a = 1, \quad b = 4, \quad c = 7$

$$b^2 - 4ac = (4)^2 - 4(1)(7) = 16 - 28 = -12$$

Since the $b^2 - 4ac < 0$, the equation has no real solution.

75. $9x^2 - 30x + 25 = 0$

$a = 9, \quad b = -30, \quad c = 25$

$$b^2 - 4ac = (-30)^2 - 4(9)(25) = 900 - 900 = 0$$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

76. $25x^2 - 20x + 4 = 0$

$a = 25, \quad b = -20, \quad c = 4$

$$b^2 - 4ac = (-20)^2 - 4(25)(4) = 400 - 400 = 0$$

Since $b^2 - 4ac = 0$, the equation has one repeated real solution.

77. $3x^2 + 5x - 8 = 0$

$a = 3, \quad b = 5, \quad c = -8$

$$b^2 - 4ac = (5)^2 - 4(3)(-8) = 25 + 96 = 121$$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

78. $2x^2 - 3x - 7 = 0$

$a = 2, \quad b = -3, \quad c = -7$

$$b^2 - 4ac = (-3)^2 - 4(2)(-7) = 9 + 56 = 65$$

Since $b^2 - 4ac > 0$, the equation has two unequal real solutions.

79. $x^2 - 5 = 0$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

80. $x^2 - 6 = 0$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$

The solution set is $\{-\sqrt{6}, \sqrt{6}\}$.

81. $16x^2 - 8x + 1 = 0$

$$(4x-1)(4x-1) = 0$$

$$4x-1 = 0$$

$$x = \frac{1}{4}$$

The solution set is $\left\{\frac{1}{4}\right\}$.

82. $9x^2 - 12x + 4 = 0$

$$(3x-2)(3x-2) = 0$$

$$3x-2 = 0$$

$$x = \frac{2}{3}$$

The solution set is $\left\{\frac{2}{3}\right\}$.

83. $10x^2 - 19x - 15 = 0$

$$(5x+3)(2x-5) = 0$$

$$5x+3 = 0 \quad \text{or} \quad 2x-5 = 0$$

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{5}{2}$$

The solution set is $\left\{-\frac{3}{5}, \frac{5}{2}\right\}$.

84. $6x^2 + 7x - 20 = 0$

$$(3x-4)(2x+5) = 0$$

$$3x-4 = 0 \quad \text{or} \quad 2x+5 = 0$$

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{5}{2}$$

The solution set is $\left\{-\frac{5}{2}, \frac{4}{3}\right\}$.

85. $2 + z = 6z^2$

$$0 = 6z^2 - z - 2$$

$$0 = (3z-2)(2z+1)$$

$$3z-2 = 0 \quad \text{or} \quad 2z+1 = 0$$

$$z = \frac{2}{3} \quad \text{or} \quad z = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$.

86. $2 = y + 6y^2$

$$0 = 6y^2 + y - 2$$

$$0 = (3y+2)(2y-1)$$

$$3y+2 = 0 \quad \text{or} \quad 2y-1 = 0$$

$$y = -\frac{2}{3} \quad \text{or} \quad y = \frac{1}{2}$$

The solution set is $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$.

87. $x^2 + \sqrt{2}x = \frac{1}{2}$

$$x^2 + \sqrt{2}x - \frac{1}{2} = 0$$

$$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = 2(0)$$

$$2x^2 + 2\sqrt{2}x - 1 = 0$$

$$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$$

$$x = \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$

$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-2}{2}, \frac{-\sqrt{2}+2}{2}\right\}$

88. $\frac{1}{2}x^2 = \sqrt{2}x + 1$

$$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$$

$$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = 2(0)$$

$$x^2 - 2\sqrt{2}x - 2 = 0$$

$$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$$

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 x &= \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)} \\
 &= \frac{2\sqrt{2} \pm \sqrt{8+8}}{2} = \frac{2\sqrt{2} \pm \sqrt{16}}{2} \\
 &= \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}
 \end{aligned}$$

The solution set is $\{\sqrt{2}-2, \sqrt{2}+2\}$.

89. $x^2 + x = 4$

$$x^2 + x - 4 = 0$$

$$a = 1, \quad b = 1, \quad c = -4$$

$$\begin{aligned}
 x &= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)} \\
 &= \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}
 \end{aligned}$$

The solution set is $\left\{\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right\}$.

90. $x^2 + x = 1$

$$x^2 + x - 1 = 0$$

$$a = 1, \quad b = 1, \quad c = -1$$

$$\begin{aligned}
 x &= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} \\
 &= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}
 \end{aligned}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

91.

$$\begin{aligned}
 \frac{x}{x-2} + \frac{2}{x+1} &= \frac{7x+1}{x^2-x-2} \\
 \frac{x}{x-2} + \frac{2}{x+1} &= \frac{7x+1}{(x-2)(x+1)}
 \end{aligned}$$

$$\left(\frac{x}{x-2} + \frac{2}{x+1}\right)(x-2)(x+1) = \left(\frac{7x+1}{(x-2)(x+1)}\right)(x-2)(x+1)$$

$$x(x+1) + 2(x-2) = 7x+1$$

$$x^2 + x + 2x - 4 = 7x+1$$

$$x^2 + 3x - 4 = 7x+1$$

$$x^2 - 4x - 5 = 0$$

$$(x+1)(x-5) = 0$$

$$x+1=0 \quad \text{or} \quad x-5=0$$

$$x=-1 \quad \text{or} \quad x=5$$

The value $x = -1$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\{5\}$.

$$\begin{aligned}
 92. \quad & \frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{x^2+x-2} \\
 & \frac{3x}{x+2} + \frac{1}{x-1} = \frac{4-7x}{(x+2)(x-1)} \\
 & \left(\frac{3x}{x+2} + \frac{1}{x-1} \right) (x+2)(x-1) = \left(\frac{4-7x}{(x+2)(x-1)} \right) (x+2)(x-1) \\
 & 3x(x-1) + (x+2) = 4-7x \\
 & 3x^2 - 3x + x + 2 = 4 - 7x \\
 & 3x^2 - 2x + 2 = 4 - 7x \\
 & 3x^2 + 5x - 2 = 0 \\
 & (3x-1)(x+2) = 0 \\
 & 3x-1 = 0 \quad \text{or} \quad x+2 = 0 \\
 & x = \frac{1}{3} \quad \text{or} \quad x = -2
 \end{aligned}$$

The value $x = -2$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\left\{ \frac{1}{3} \right\}$.

93. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$\begin{aligned}
 (2x+3)^2 &= (2x-5)^2 + (x+7)^2 \\
 4x^2 + 12x + 9 &= 4x^2 - 20x + 25 + x^2 + 14x + 49 \\
 12x + 9 &= x^2 - 6x + 74 \\
 0 &= x^2 - 18x + 65 \\
 0 &= (x-5)(x-13) \\
 x-5 &= 0 \quad \text{or} \quad x-13 = 0 \\
 x &= 5 \quad \text{or} \quad x = 13
 \end{aligned}$$

This means there are 2 possible that meet these requirements. Substituting x into the given sides gives:

When $x = 5$: 5m, 12m, 13m
 When $x = 13$: 20m, 21m, 29m
 Thus there are 2 solutions.

94. Since this is a right triangle then we can use the Pythagorean Theorem. So

$$\begin{aligned}
 (4x+5)^2 &= (3x+13)^2 + x^2 \\
 16x^2 + 40x + 25 &= 9x^2 + 78x + 169 + x^2 \\
 6x^2 - 38x - 144 &= 0 \\
 2(3x^2 - 19x - 72) &= 0 \\
 2(3x+8)(x-9) &= 0 \\
 3x+8 &= 0 \quad \text{or} \quad x-9 = 0 \\
 x &= -\frac{8}{3} \quad \text{or} \quad x = 9
 \end{aligned}$$

This means there are 2 possible solutions that meet these requirements. Substituting x into the given sides gives:

When $x = 9$: 41m, 40m, 9m

When $x = -\frac{8}{3}$ at least one side of the triangle

has a negative measurement which is impossible. Thus there is only 1 triangle possible

95. Let w represent the width of window. Then $l = w + 2$ represents the length of the window.

Since the area is 143 square feet, we have:

$$w(w+2) = 143$$

$$w^2 + 2w - 143 = 0$$

$$(w+13)(w-11) = 0$$

$$\cancel{w+13} \quad \text{or} \quad w = 11$$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

96. Let w represent the width of window. Then $l = w + 1$ represents the length of the window. Since the area is 306 square centimeters, we have: $w(w+1) = 306$

$$w^2 + w - 306 = 0$$

$$(w+18)(w-17) = 0$$

$$\cancel{w+18} \quad \text{or} \quad w = 17$$

Discard the negative solution since width cannot

Chapter 1: Equations and Inequalities

be negative. The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

97. Let l represent the length of the rectangle.
Let w represent the width of the rectangle.
The perimeter is 26 meters and the area is 40 square meters.

$$2l + 2w = 26$$

$$l + w = 13 \quad \text{so} \quad w = 13 - l$$

$$lw = 40$$

$$l(13 - l) = 40$$

$$13l - l^2 = 40$$

$$l^2 - 13l + 40 = 0$$

$$(l - 8)(l - 5) = 0$$

$$l = 8 \quad \text{or} \quad l = 5$$

$$w = 5 \quad w = 8$$

The dimensions are 5 meters by 8 meters.

98. Let r represent the radius of the circle.
Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250} = 25\sqrt{2}$ feet. The length of the diagonal is $2r$.

Use the Pythagorean Theorem to solve for r :

$$(2r)^2 = (25\sqrt{2})^2 + (25\sqrt{2})^2$$

$$4r^2 = 1250 + 1250$$

$$4r^2 = 2500$$

$$r^2 = 625$$

$$r = 25$$

The shortest radius setting for the sprinkler is 25 feet.

99. Let x = length of side of original sheet in feet.
Length of box: $x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot

$$V = l \cdot w \cdot h$$

$$4 = (x - 2)(x - 2)(1)$$

$$4 = x^2 - 4x + 4$$

$$0 = x^2 - 4x$$

$$0 = x(x - 4)$$

$$x = 0 \quad \text{or} \quad x = 4$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

100. Let x = width of original sheet in feet.

Length of sheet: $2x$

Length of box: $2x - 2$ feet

Width of box: $x - 2$ feet

Height of box: 1 foot

$$V = l \cdot w \cdot h$$

$$4 = (2x - 2)(x - 2)(1)$$

$$4 = 2x^2 - 6x + 4$$

$$0 = 2x^2 - 6x$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0 \quad \text{or} \quad x = 3$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

101. a. When the ball strikes the ground, the distance from the ground will be 0.

Therefore, we solve

$$96 + 80t - 16t^2 = 0$$

$$-16t^2 + 80t + 96 = 0$$

$$t^2 - 5t - 6 = 0$$

$$(t - 6)(t + 1) = 0$$

$$t = 6 \quad \text{or} \quad t = -1$$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

- b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$96 + 80t - 16t^2 = 96$$

$$-16t^2 + 80t = 0$$

$$t^2 - 5t = 0$$

$$t(t - 5) = 0$$

$$t = 0 \quad \text{or} \quad t = 5$$

The ball is at the top of the building at time $t = 0$ when it is thrown. It will pass the top of the building on the way down after 5 seconds.

102. a. To find when the object will be 15 meters above the ground, we solve

$$-4.9t^2 + 20t = 15$$

$$-4.9t^2 + 20t - 15 = 0$$

$$a = -4.9, b = 20, c = -15$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{106}}{-9.8} = \frac{20 \pm \sqrt{106}}{9.8}$$

$$t \approx 0.99 \quad \text{or} \quad t \approx 3.09$$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

- b. The object will strike the ground when the distance from the ground is 0. Therefore, we solve

$$-4.9t^2 + 20t = 0$$

$$t(-4.9t + 20) = 0$$

$$t = 0 \quad \text{or} \quad -4.9t + 20 = 0$$

$$-4.9t = -20$$

$$t \approx 4.08$$

The object will strike the ground after about 4.08 seconds.

- c. $-4.9t^2 + 20t = 100$

$$-4.9t^2 + 20t - 100 = 0$$

$$a = -4.9, b = 20, c = -100$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

103. Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 90% of the old volume.

$$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$$

$$3x^2 - 57x + 252 = 226.8$$

$$3x^2 - 57x + 25.2 = 0$$

$$x^2 - 19x + 8.4 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \frac{19 \pm \sqrt{327.4}}{2}$$

$$x \approx 0.45 \quad \text{or} \quad x \approx 18.55$$

Since 18.55 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

104. Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 80% of the old volume.

$$(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$$

$$3x^2 - 57x + 252 = 201.6$$

$$3x^2 - 57x + 50.4 = 0$$

$$x^2 - 19x + 16.8 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \frac{19 \pm \sqrt{293.8}}{2}$$

$$x \approx 0.93 \quad \text{or} \quad x \approx 18.07$$

Since 18.07 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

105. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25(\pi(x + 5)^2 - 25\pi)$$

Solving the volume equation:

$$0.25(\pi(x + 5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 108$$

$$\pi x^2 + 10\pi x - 108 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 432\pi}}{6.28}$$

$$x \approx 2.71 \quad \text{or} \quad x \approx -12.71$$

Discard the negative solution. The width of the border is roughly 2.71 feet.

Chapter 1: Equations and Inequalities

- 106.** Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$$A_T = \pi(x + 5)^2.$$

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$$A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi.$$

Since the concrete is 4 inches $= \frac{1}{3}$ foot thick, the

volume of the concrete in the border is

$$\frac{1}{3} A_B = \frac{1}{3} (\pi(x + 5)^2 - 25\pi)$$

Solving the volume equation:

$$\frac{1}{3} (\pi(x + 5)^2 - 25\pi) = 27$$

$$\pi(x^2 + 10x + 25 - 25) = 81$$

$$\pi x^2 + 10\pi x - 81 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 324\pi}}{6.28}$$

$$x \approx 2.13 \text{ or } x \approx -12.13$$

Discard the negative solution. The width of the border is approximately 2.13 feet.

- 107.** Let x represent the width of the border measured in feet.

The total area is $A_T = (6 + 2x)(10 + 2x)$.

The area of the garden is $A_G = 6 \cdot 10 = 60$.

The area of the border is

$$A_B = A_T - A_G = (6 + 2x)(10 + 2x) - 60.$$

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25 A_B = 0.25 ((6 + 2x)(10 + 2x) - 60)$$

Solving the volume equation:

$$0.25 ((6 + 2x)(10 + 2x) - 60) = 27$$

$$60 + 32x + 4x^2 - 60 = 108$$

$$4x^2 + 32x - 108 = 0$$

$$x^2 + 8x - 27 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-27)}}{2(1)} = \frac{-8 \pm \sqrt{172}}{2}$$

$$x \approx 2.56 \text{ or } x \approx -10.56$$

Discard the negative solution. The width of the border is approximately 2.56 feet.

- 108.** Let x = the width and $2x$ = the length of the patio. The height is $\frac{1}{3}$ foot and the concrete available is $8(27) = 216$ cubic feet..

$$V = lwh = x(2x) \cdot \frac{1}{3} = 216$$

$$\frac{2}{3} x^2 = 216$$

$$x^2 = 324 \Rightarrow x = \pm 18$$

The dimensions of the patio are 18 feet by 36 feet.

- 109.** Let x = the length of a 12.9-inch iPad Pro in a 16:9 format.

Then $\frac{9}{16}x$ = the width of the iPad. The diagonal

of the 12.9-inch iPad is 9.7 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{9}{16}x\right)^2 = 12.9^2$$

$$x^2 + \frac{81}{256}x^2 = 166.41$$

$$256\left(x^2 + \frac{81}{256}x^2\right) = 256(166.41)$$

$$256x^2 + 81x^2 = 42600.96$$

$$337x^2 = 42600.96 \Rightarrow x^2 = \frac{42600.96}{337}$$

$$x = \pm \sqrt{\frac{42600.96}{337}} \approx \pm 11.24$$

Since the length cannot be negative, the length of the iPad is $\sqrt{\frac{42600.96}{337}}$ inches and the width is

$$\frac{9}{16} \sqrt{\frac{42600.96}{337}} \approx 6.32 \text{ inches. Thus, the area of the}$$

iPad is $\sqrt{\frac{42600.96}{337}} \cdot \frac{9}{16} \sqrt{\frac{42600.96}{337}} = 71.11$ square inches.

Let y = the length of a 14.4-inch 3:2 format

Microsoft Surface Pro. Then $\frac{2}{3}y$ = the width of

the Surface Pro. The diagonal of a 14.4-inch Surface Pro is 14.4 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{2}{3}y\right)^2 = 14.4^2$$

$$y^2 + \frac{4}{9}y^2 = 207.36$$

$$9\left(y^2 + \frac{4}{9}y^2\right) = 9(207.36)$$

$$9y^2 + 4y^2 = 1866.24$$

$$13y^2 = 1866.24$$

$$y^2 = \frac{1866.24}{13}$$

$$y = \pm \sqrt{\frac{1866.24}{13}} \approx \pm 11.98$$

Since the length cannot be negative, the length of the Surface Pro is 11.98 inches and the width is

$$\frac{2}{3} \sqrt{\frac{1866.24}{13}} \approx 7.99 \text{ inches. Thus, the area of the}$$

14.4-inch 3:2 format Surface Pro is

$$\sqrt{\frac{1866.24}{13}} \cdot \frac{2}{3} \sqrt{\frac{1866.24}{13}}$$

≈ 95.7 square inches.

The Surface Pro format has the larger screen since its area is larger.

- 110.** Let x = the length of a 8.3-inch iPad Mini in a 4:3 format.

Then $\frac{3}{4}x$ = the width of the iPad. The diagonal

of the 8.3-inch iPad is 8.3 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{3}{4}x\right)^2 = 8.3^2$$

$$x^2 + \frac{9}{16}x^2 = 68.89$$

$$16\left(x^2 + \frac{9}{16}x^2\right) = 16(68.89)$$

$$16x^2 + 9x^2 = 1102.24$$

$$25x^2 = 1102.24$$

$$x^2 = 44.0896$$

$$x = \pm \sqrt{44.0896} = \pm 6.64$$

Since the length cannot be negative, the length of the iPad is 6.64 inches and the width is

$$\frac{3}{4}(6.64) = 4.98 \text{ inches. Thus, the area of the}$$

iPad is $(6.64)(4.98) = 33.1$ square inches.

Let y = the length of a 8-inch 16:10 format

Amazon Fire HD 8™. Then $\frac{10}{16}y$ = the width of

the Fire. The diagonal of a 8-inch Fire is 8 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{10}{16}y\right)^2 = 8^2$$

$$y^2 + \frac{100}{256}y^2 = 64$$

$$256\left(y^2 + \frac{100}{256}y^2\right) = 256(64)$$

$$256y^2 + 100y^2 = 16384$$

$$356y^2 = 16384$$

$$y^2 = \frac{16384}{356}$$

$$y = \pm \sqrt{\frac{16384}{356}} \approx \pm 6.78399$$

Since the length cannot be negative, the length of

the Fire is $\sqrt{\frac{16384}{356}} \approx 6.78399$ inches and the

width is $\frac{10}{16} \sqrt{\frac{16384}{356}} \approx 4.240$ inches. Thus, the area

of the Amazon Fire is

$$(6.78399)(4.240) \approx 28.8 \text{ square inches.}$$

The iPad Mini™ 4:3 format has the larger screen since its area is larger.

- 111.** Let h be 1.1. Then

$$1.1 = -0.00025x^2 + 0.04x$$

$$0 = -0.00025x^2 + 0.04x - 1.1$$

$$x = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(-0.00025)(-1.1)}}{2(-0.00025)}$$

$$= 35.3 \text{ ft or } 124.7 \text{ ft}$$

124.7 ft does not make sense in the context of the problem, so the answer is 35.3 ft.

- 112.** Since d is expressed in 1000's we will set $d = 12$ and solve for x using the Quadratic Formula.

$$d = 0.33x^2 + 0.26x + 2.29$$

$$12 = 0.33x^2 + 0.26x + 2.29$$

$$0 = 0.33x^2 + 0.26x - 9.71$$

$$x = \frac{-0.26 \pm \sqrt{(0.26)^2 - 4(0.33)(-9.71)}}{2(0.33)}$$

$$= \frac{-0.26 \pm \sqrt{13.0772}}{0.66}$$

$$x \approx 5.085 \text{ or } x \approx -5.873$$

So the nearest year when the earnings were 12 occurred about 5 years after 2018 or 2023. The negative value -5.873 has no meaning.

Chapter 1: Equations and Inequalities

- 113.** We will set $g = 2.97$ and solve for h using the Quadratic Formula.

$$g = -0.0006x^2 + 0.015x + 3.04$$

$$2.97 = -0.0006x^2 + 0.015x + 3.04$$

$$0 = -0.0006x^2 + 0.015x + 0.07$$

$$x = \frac{-0.015 \pm \sqrt{(0.015)^2 - 4(-0.0006)(0.07)}}{2(-0.0006)}$$

$$= \frac{-0.015 \pm \sqrt{0.000393}}{-0.0012}$$

$$x \approx 29 \text{ or } x \approx -4.02$$

So the estimated numbers of hours worked by a student with a GPA of 2.97 is 29 hours. The value -4.02 has no meaning since it is negative.

- 114.** Let x be the numbers of members in the fraternity and s be the share paid by each member. Then $s = \frac{1470}{x}$. If there are 7

members who cannot contribute then the share goes up by \$5. So we have the following equation:

$$s + 5 = \frac{1470}{x-7} \text{ or } (s+5)(x-7) = 1470$$

Solving these two equations together:

$$(s+5)(x-7) = 1470 \text{ and } s = \frac{1470}{x}$$

$$\left(\frac{1470}{x} + 5\right)(x-7) = 1470$$

$$1470 - \frac{10290}{x} + 5x - 35 = 1470$$

$$5x - \frac{10290}{x} - 35 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$5x^2 - 35x - 10290 = 0$$

$$x^2 - 7x - 2058 = 0$$

$$(x+42)(x-49) = 0$$

$$x = -42 \text{ or } x = 49$$

Since x is the number of members, it must be positive so the number of members is 49.

- 115.** Let a be the age the individual is able to start saving money. Then we need to find where the models are equal. Solving these two equations together:

$$-25a^2 + 2400a - 30700 = 160a + 7840$$

$$25a^2 - 2240a + 38540 = 0$$

$$a = \frac{2240 \pm \sqrt{(2240)^2 - 4(25)(38540)}}{2(25)}$$

$$a = \frac{2240 \pm \sqrt{1163600}}{50}$$

$$a = \frac{2240 \pm 1078.7}{50}$$

$$a = \frac{2240 + 1078.7}{50} \text{ or } a = \frac{2240 - 1078.7}{50}$$

$$a = 66.4 \text{ or } a = 23.2$$

Since x is the age to start saving, it makes sense that the answer is approximate at age 23.

116. $\frac{1}{2}n(n-3) = 65$

$$n(n-3) = 130$$

$$n^2 - 3n - 130 = 0$$

$$(n-13)(n+10) = 0$$

$$n = 13 \text{ or } n = -10$$

Since the number of sides cannot be negative, we discard the negative value. A polygon with 65 diagonals will have 13 sides.

$$\frac{1}{2}n(n-3) = 80$$

$$n(n-3) = 160$$

$$n^2 - 3n - 160 = 0$$

$$a = 1, b = -3, c = -160$$

$$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)} = \frac{3 \pm \sqrt{646}}{2}$$

Neither solution is an integer, so there is no polygon that has 80 diagonals.

117. The roots of a quadratic equation are

$$\begin{aligned}
 x_1 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
 x_1 + x_2 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-b - \sqrt{b^2 - 4ac} - b + \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-2b}{2a} \\
 &= -\frac{b}{a}
 \end{aligned}$$

118. The roots of a quadratic equation are

$$\begin{aligned}
 x_1 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
 x_1 \cdot x_2 &= \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \\
 &= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2} = \frac{b^2 - b^2 + 4ac}{4a^2} \\
 &= \frac{4ac}{4a^2} \\
 &= \frac{c}{a}
 \end{aligned}$$

119. In order to have one repeated solution, we need the discriminant to be 0.

$$\begin{aligned}
 b^2 - 4ac &= 0 \\
 1^2 - 4(k)(k) &= 0 \\
 1 - 4k^2 &= 0 \\
 4k^2 &= 1 \\
 k^2 &= \frac{1}{4} \\
 k &= \pm \sqrt{\frac{1}{4}} \\
 k &= \frac{1}{2} \quad \text{or} \quad k = -\frac{1}{2}
 \end{aligned}$$

120. In order to have one repeated solution, we need the discriminant to be 0.

$$\begin{aligned}
 b^2 - 4ac &= 0 \\
 (-k)^2 - 4(1)(4) &= 0 \\
 k^2 - 16 &= 0 \\
 (k-4)(k+4) &= 0 \\
 k &= 4 \quad \text{or} \quad k = -4
 \end{aligned}$$

121. For
- $ax^2 + bx + c = 0$
- :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^2 - bx + c = 0$:

$$\begin{aligned}
 x &= \frac{b \pm \sqrt{(-b)^2 - 4ac}}{2a} \\
 &= -\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)
 \end{aligned}$$

122. For
- $ax^2 + bx + c = 0$
- :

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $cx^2 + bx + a = 0$:

$$\begin{aligned}
 x_1^* &= \frac{-b - \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b - \sqrt{b^2 - 4ac}}{2c} \\
 &= \frac{-b - \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \\
 &= \frac{b^2 - (b^2 - 4ac)}{2c(-b + \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b + \sqrt{b^2 - 4ac})} \\
 &= \frac{2a}{-b + \sqrt{b^2 - 4ac}} \\
 &= \frac{1}{x_2}
 \end{aligned}$$

Chapter 1: Equations and Inequalities

and

$$\begin{aligned}x_2^* &= \frac{-b + \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b + \sqrt{b^2 - 4ac}}{2c} \\&= \frac{-b + \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}} \\&= \frac{b^2 - (b^2 - 4ac)}{2c(-b - \sqrt{b^2 - 4ac})} = \frac{4ac}{2c(-b - \sqrt{b^2 - 4ac})} \\&= \frac{2a}{-b - \sqrt{b^2 - 4ac}} \\&= \frac{1}{x_1}\end{aligned}$$

- 123.** If x = original width and y = original length, then $xy = 1$ or $x = \frac{1}{y}$. The ratio of side lengths is

$$\frac{x}{y} = \frac{1}{y^2}. \text{ Folding along the longest side results}$$

in sides of length $x = \frac{1}{y}$ and $\frac{y}{2}$ whose ratio is

$$\frac{\frac{y}{2}}{\frac{1}{y}} = \frac{y^2}{2}$$

Equating the ratios gives

$$\frac{1}{y^2} = \frac{y^2}{2}$$

$$y^4 = 2$$

$$y = \sqrt[4]{2} \text{ m}$$

So

$$x = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2} \text{ m.}$$

- 124. a.** $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.
- b.** $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.
- c.** $(x-1)(x-2) = (x-1)^2$ and $x-2 = x-1$ are not equivalent because they do not have the same solution set.

The first equation has the solution set $\{1\}$

while the second equation has no solutions.

- 125.** Answers will vary. Methods may include the quadratic formula, completing the square, graphing, etc.
- 126.** Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.
- 127.** Answers will vary. One possibility:
Two distinct: $x^2 - 3x - 18 = 0$
One repeated: $x^2 - 14x + 49 = 0$
No real: $x^2 + x + 4 = 0$
- 128.** Answers will vary.

Section 1.3

- 1.** Integers: $\{-3, 0\}$

$$\text{Rationals: } \left\{-3, 0, \frac{6}{5}\right\}$$

- 2.** True; the set of real numbers consists of all rational and irrational numbers.

$$\begin{aligned}\text{3. } \frac{3}{2+\sqrt{3}} &= \frac{3}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} \\&= \frac{3(2-\sqrt{3})}{2^2 - (\sqrt{3})^2} \\&= \frac{3(2-\sqrt{3})}{4-3} \\&= 3(2-\sqrt{3})\end{aligned}$$

- 4.** real; imaginary; imaginary unit
- 5.** False; the conjugate of $2+5i$ is $2-5i$.
- 6.** True; the set of real numbers is a subset of the set of complex numbers.
- 7.** False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.

- 8.** b

Section 1.3: Complex Numbers; Quadratic Equations in the Complex Number System

9. a
10. c
11. $(2-3i) + (6+8i) = (2+6) + (-3+8)i = 8+5i$
12. $(4+5i) + (-8+2i) = (4+(-8)) + (5+2)i$
 $= -4+7i$
13. $(-3+2i) - (4-4i) = (-3-4) + (2-(-4))i$
 $= -7+6i$
14. $(3-4i) - (-3-4i) = (3-(-3)) + (-4-(-4))i$
 $= 6+0i = 6$
15. $(2-5i) - (8+6i) = (2-8) + (-5-6)i$
 $= -6-11i$
16. $(-8+4i) - (2-2i) = (-8-2) + (4-(-2))i$
 $= -10+6i$
17. $3(2-6i) = 6-18i$
18. $-4(2+8i) = -8-32i$
19. $-3i(7+6i) = -21i-18i^2 = -21i-18(-1)$
 $= 18-21i$
20. $3i(-3+4i) = -9i+12i^2 = -9i+12(-1) = -12-9i$
21. $(3-4i)(2+i) = 6+3i-8i-4i^2$
 $= 6-5i-4(-1)$
 $= 10-5i$
22. $(5+3i)(2-i) = 10-5i+6i-3i^2$
 $= 10+i-3(-1)$
 $= 13+i$
23. $(-5-i)(-5+i) = 25-5i+5i-i^2$
 $= 25-(-1)$
 $= 26$
24. $(-3+i)(3+i) = -9-3i+3i+i^2$
 $= -9+(-1)$
 $= -10$
25. $\frac{10}{3-4i} = \frac{10}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{30+40i}{9+12i-12i-16i^2}$
 $= \frac{30+40i}{9-16(-1)} = \frac{30+40i}{25}$
 $= \frac{30}{25} + \frac{40}{25}i$
 $= \frac{6}{5} + \frac{8}{5}i$
26. $\frac{13}{5-12i} = \frac{13}{5-12i} \cdot \frac{5+12i}{5+12i}$
 $= \frac{65+156i}{25+60i-60i-144i^2}$
 $= \frac{65+156i}{25-144(-1)} = \frac{65+156i}{169}$
 $= \frac{65}{169} + \frac{156}{169}i$
 $= \frac{5}{13} + \frac{12}{13}i$
27. $\frac{2+i}{i} = \frac{2+i}{i} \cdot \frac{-i}{-i} = \frac{-2i-i^2}{-i^2}$
 $= \frac{-2i-(-1)}{-(-1)} = \frac{1-2i}{1} = 1-2i$
28. $\frac{2-i}{-2i} = \frac{2-i}{-2i} \cdot \frac{i}{i} = \frac{2i-i^2}{-2i^2}$
 $= \frac{2i-(-1)}{-2(-1)} = \frac{1+2i}{2} = \frac{1}{2} + i$
29. $\frac{6-i}{1+i} = \frac{6-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{6-6i-i+i^2}{1-i+i-i^2}$
 $= \frac{6-7i+(-1)}{1-(-1)} = \frac{5-7i}{2} = \frac{5}{2} - \frac{7}{2}i$
30. $\frac{2+3i}{1-i} = \frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i+3i+3i^2}{1+i-i-i^2}$
 $= \frac{2+5i+3(-1)}{1-(-1)} = \frac{-1+5i}{2} = -\frac{1}{2} + \frac{5}{2}i$
31. $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} + 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \frac{3}{4}i^2$
 $= \frac{1}{4} + \frac{\sqrt{3}}{2}i + \frac{3}{4}(-1) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Chapter 1: Equations and Inequalities

$$32. \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)^2 = \frac{3}{4} - 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2}i \right) + \frac{1}{4}i^2$$

$$= \frac{3}{4} - \frac{\sqrt{3}}{2}i + \frac{1}{4}(-1) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$33. (1+i)^2 = 1 + 2i + i^2 = 1 + 2i + (-1) = 2i$$

$$34. (1-i)^2 = 1 - 2i + i^2 = 1 - 2i + (-1) = -2i$$

$$35. i^{23} = i^{22+1} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11}i = -i$$

$$36. i^{14} = (i^2)^7 = (-1)^7 = -1$$

$$37. i^{-20} = \frac{1}{i^{20}} = \frac{1}{i^{20}} = \frac{1}{(i^2)^{10}}$$

$$= \frac{1}{(-1)^{10}} = \frac{1}{1} = 1$$

$$38. i^{-23} = \frac{1}{i^{23}} = \frac{1}{i^{22+1}} = \frac{1}{i^{22} \cdot i} = \frac{1}{(i^2)^{11} \cdot i}$$

$$= \frac{1}{(-1)^{11}i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = i$$

$$39. i^6 - 5 = (i^2)^3 - 5 = (-1)^3 - 5 = -1 - 5 = -6$$

$$40. 4 + i^3 = 4 + i^2 \cdot i = 4 + (-1)i = 4 - i$$

$$41. 6i^3 - 4i^5 = i^3(6 - 4i^2)$$

$$= i^2 \cdot i(6 - 4(-1)) = -1 \cdot i(10) = -10i$$

$$42. 4i^3 - 2i^2 + 1 = 4i^2 \cdot i - 2i^2 + 1$$

$$= 4(-1)i - 2(-1) + 1$$

$$= -4i + 2 + 1$$

$$= 3 - 4i$$

$$43. (1+i)^3 = (1+i)(1+i)(1+i) = (1+2i+i^2)(1+i)$$

$$= (1+2i-1)(1+i) = 2i(1+i)$$

$$= 2i + 2i^2 = 2i + 2(-1)$$

$$= -2 + 2i$$

$$44. (3i)^4 + 1 = 81i^4 + 1 = 81(1) + 1 = 82$$

$$45. i^7(1+i^2) = i^7(1+(-1)) = i^7(0) = 0$$

$$46. 2i^4(1+i^2) = 2(1)(1+(-1)) = 2(0) = 0$$

$$47. i^8 + i^6 - i^4 - i^2 = (i^2)^4 + (i^2)^3 + (i^2)^2 + i^2$$

$$= (-1)^4 + (-1)^3 + (-1)^2 - 1$$

$$= 1 - 1 + 1 - 1$$

$$= 0$$

$$48. i^7 + i^5 + i^3 + i = (i^2)^3 \cdot i + (i^2)^2 \cdot i + i^2 \cdot i + i$$

$$= (-1)^3 \cdot i + (-1)^2 \cdot i + (-1) \cdot i + i$$

$$= -i + i - i + i$$

$$= 0$$

$$49. \sqrt{-4} = 2i$$

$$50. \sqrt{-9} = 3i$$

$$51. \sqrt{-25} = 5i$$

$$52. \sqrt{-64} = 8i$$

$$53. \sqrt{-12} = i\sqrt{4 \cdot 3} = 2\sqrt{3}i$$

$$54. \sqrt{-18} = i\sqrt{9 \cdot 2} = 3\sqrt{2}i$$

$$55. \sqrt{-200} = i\sqrt{100 \cdot 2} = 10\sqrt{2}i$$

$$56. \sqrt{-45} = i\sqrt{9 \cdot 5} = 3\sqrt{5}i$$

$$57. \sqrt{(3+4i)(4i-3)} = \sqrt{12i - 9 + 16i^2 - 12i}$$

$$= \sqrt{-9 + 16(-1)}$$

$$= \sqrt{-25}$$

$$= 5i$$

$$58. \sqrt{(4+3i)(3i-4)} = \sqrt{12i - 16 + 9i^2 - 12i}$$

$$= \sqrt{-16 + 9(-1)}$$

$$= \sqrt{-25}$$

$$= 5i$$

Section 1.3: Complex Numbers; Quadratic Equations in the Complex Number System

59. $x^2 + 4 = 0$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

The solution set is $\{-2i, 2i\}$.

60. $x^2 - 4 = 0$

$$(x+2)(x-2) = 0$$

$$x = -2 \text{ or } x = 2$$

The solution set is $\{-2, 2\}$.

61. $x^2 - 16 = 0$

$$(x+4)(x-4) = 0$$

$$x = -4 \text{ or } x = 4$$

The solution set is $\{-4, 4\}$.

62. $x^2 + 25 = 0$

$$x^2 = -25$$

$$x = \pm\sqrt{-25} = \pm 5i$$

The solution set is $\{-5i, 5i\}$.

63. $x^2 - 6x + 13 = 0$

$$a = 1, b = -6, c = 13,$$

$$b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$$

$$x = \frac{-(-6) \pm \sqrt{-16}}{2(1)} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

The solution set is $\{3 - 2i, 3 + 2i\}$.

64. $x^2 + 4x + 8 = 0$

$$a = 1, b = 4, c = 8$$

$$b^2 - 4ac = 4^2 - 4(1)(8) = 16 - 32 = -16$$

$$x = \frac{-4 \pm \sqrt{-16}}{2(1)} = \frac{-4 \pm 4i}{2} = -2 \pm 2i$$

The solution set is $\{-2 - 2i, -2 + 2i\}$.

65. $x^2 - 6x + 10 = 0$

$$a = 1, b = -6, c = 10$$

$$b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4$$

$$x = \frac{-(-6) \pm \sqrt{-4}}{2(1)} = \frac{6 \pm 2i}{2} = 3 \pm i$$

The solution set is $\{3 - i, 3 + i\}$.

66. $x^2 - 2x + 5 = 0$

$$a = 1, b = -2, c = 5$$

$$b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16$$

$$x = \frac{-(-2) \pm \sqrt{-16}}{2(1)} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

The solution set is $\{1 - 2i, 1 + 2i\}$.

67. $25x^2 - 10x + 2 = 0$

$$a = 25, b = -10, c = 2$$

$$b^2 - 4ac = (-10)^2 - 4(25)(2) = 100 - 200 = -100$$

$$x = \frac{-(-10) \pm \sqrt{-100}}{2(25)} = \frac{10 \pm 10i}{50} = \frac{1 \pm i}{5}$$

The solution set is $\left\{\frac{1}{5} - \frac{1}{5}i, \frac{1}{5} + \frac{1}{5}i\right\}$.

68. $10x^2 + 6x + 1 = 0$

$$a = 10, b = 6, c = 1$$

$$b^2 - 4ac = 6^2 - 4(10)(1) = 36 - 40 = -4$$

$$x = \frac{-6 \pm \sqrt{-4}}{2(10)} = \frac{-6 \pm 2i}{20} = -\frac{3}{10} \pm \frac{1}{10}i$$

The solution set is $\left\{-\frac{3}{10} - \frac{1}{10}i, -\frac{3}{10} + \frac{1}{10}i\right\}$.

69. $5x^2 + 1 = 2x$

$$5x^2 - 2x + 1 = 0$$

$$a = 5, b = -2, c = 1$$

$$b^2 - 4ac = (-2)^2 - 4(5)(1) = 4 - 20 = -16$$

$$x = \frac{-(-2) \pm \sqrt{-16}}{2(5)} = \frac{2 \pm 4i}{10} = \frac{1 \pm 2i}{5}$$

The solution set is $\left\{\frac{1}{5} - \frac{2}{5}i, \frac{1}{5} + \frac{2}{5}i\right\}$.

70. $13x^2 + 1 = 6x$

$$13x^2 - 6x + 1 = 0$$

$$a = 13, b = -6, c = 1$$

$$b^2 - 4ac = (-6)^2 - 4(13)(1) = 36 - 52 = -16$$

$$x = \frac{-(-6) \pm \sqrt{-16}}{2(13)} = \frac{6 \pm 4i}{26} = \frac{3 \pm 2i}{13}$$

The solution set is $\left\{\frac{3}{13} - \frac{2}{13}i, \frac{3}{13} + \frac{2}{13}i\right\}$.

Chapter 1: Equations and Inequalities

71. $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1,$

$b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}.$

72. $x^2 - x + 1 = 0$

$a = 1, b = -1, c = 1$

$b^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3$

$x = \frac{-(-1) \pm \sqrt{-3}}{2(1)} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solution set is $\left\{\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}.$

73. $x^3 - 64 = 0$

$(x - 4)(x^2 + 4x + 16) = 0$

$x - 4 = 0 \Rightarrow x = 4$

or $x^2 + 4x + 16 = 0$

$a = 1, b = 4, c = 16$

$b^2 - 4ac = 4^2 - 4(1)(16) = 16 - 64 = -48$

$x = \frac{-4 \pm \sqrt{-48}}{2(1)} = \frac{-4 \pm 4\sqrt{3}i}{2} = -2 \pm 2\sqrt{3}i$

The solution set is $\{4, -2 - 2\sqrt{3}i, -2 + 2\sqrt{3}i\}.$

74. $x^3 + 27 = 0$

$(x + 3)(x^2 - 3x + 9) = 0$

$x + 3 = 0 \Rightarrow x = -3$

or $x^2 - 3x + 9 = 0$

$a = 1, b = -3, c = 9$

$b^2 - 4ac = (-3)^2 - 4(1)(9) = 9 - 36 = -27$

$x = \frac{-(-3) \pm \sqrt{-27}}{2(1)} = \frac{3 \pm 3\sqrt{3}i}{2} = \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$

The solution set is $\left\{-3, \frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{3}{2} + \frac{3\sqrt{3}}{2}i\right\}.$

75. $x^4 = 16$

$x^4 - 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0$

$(x - 2)(x + 2)(x^2 + 4) = 0$

$x - 2 = 0$ or $x + 2 = 0$ or $x^2 + 4 = 0$

$x = 2$ or $x = -2$ or $x^2 = -4$

$x = 2$ or $x = -2$ or $x = \pm\sqrt{-4} = \pm 2i$

The solution set is $\{-2, 2, -2i, 2i\}.$

76. $x^4 = 1$

$x^4 - 1 = 0$

$(x^2 - 1)(x^2 + 1) = 0$

$(x - 1)(x + 1)(x^2 + 1) = 0$

$x - 1 = 0$ or $x + 1 = 0$ or $x^2 + 1 = 0$

$x = 1$ or $x = -1$ or $x^2 = -1$

$x = 1$ or $x = -1$ or $x = \pm\sqrt{-1} = \pm i$

The solution set is $\{-1, 1, -i, i\}.$

77. $x^4 + 13x^2 + 36 = 0$

$(x^2 + 9)(x^2 + 4) = 0$

$x^2 + 9 = 0$ or $x^2 + 4 = 0$

$x^2 = -9$ or $x^2 = -4$

$x = \pm\sqrt{-9}$ or $x = \pm\sqrt{-4}$

$x = \pm 3i$ or $x = \pm 2i$

The solution set is $\{-3i, 3i, -2i, 2i\}.$

78. $x^4 + 3x^2 - 4 = 0$

$(x^2 - 1)(x^2 + 4) = 0$

$(x - 1)(x + 1)(x^2 + 4) = 0$

$x - 1 = 0$ or $x + 1 = 0$ or $x^2 + 4 = 0$

$x = 1$ or $x = -1$ or $x^2 = -4$

$x = 1$ or $x = -1$ or $x = \pm\sqrt{-4} = \pm 2i$

The solution set is $\{-1, 1, -2i, 2i\}.$

79. $3x^2 - 3x + 4 = 0$

$a = 3, b = -3, c = 4$

$b^2 - 4ac = (-3)^2 - 4(3)(4) = 9 - 48 = -39$

The equation has two complex solutions that are conjugates of each other.

Section 1.3: Complex Numbers; Quadratic Equations in the Complex Number System

80. $2x^2 - 4x + 1 = 0$
 $a = 2, b = -4, c = 1$
 $b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$
 The equation has two unequal real number solutions.

81. $2x^2 + 3x = 4$
 $2x^2 + 3x - 4 = 0$
 $a = 2, b = 3, c = -4$
 $b^2 - 4ac = 3^2 - 4(2)(-4) = 9 + 32 = 41$
 The equation has two unequal real solutions.

82. $x^2 + 6 = 2x$
 $x^2 - 2x + 6 = 0$
 $a = 1, b = -2, c = 6$
 $b^2 - 4ac = (-2)^2 - 4(1)(6) = 4 - 24 = -20$
 The equation has two complex solutions that are conjugates of each other.

83. $9x^2 - 12x + 4 = 0$
 $a = 9, b = -12, c = 4$
 $b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$
 The equation has a repeated real solution.

84. $4x^2 + 12x + 9 = 0$
 $a = 4, b = 12, c = 9$
 $b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$
 The equation has a repeated real solution.

85. The other solution is $\overline{2 + 3i} = 2 - 3i$.

86. The other solution is $\overline{4 - i} = 4 + i$.

87. $z + \bar{z} = 3 - 4i + \overline{3 - 4i} = 3 - 4i + 3 + 4i = 6$

88. $w - \bar{w} = 8 + 3i - \overline{(8 + 3i)}$
 $= 8 + 3i - (8 - 3i)$
 $= 8 + 3i - 8 + 3i$
 $= 0 + 6i = 6i$

89. $z \cdot \bar{z} = (3 - 4i)(\overline{3 - 4i})$
 $= (3 - 4i)(3 + 4i)$
 $= 9 + 12i - 12i - 16i^2$
 $= 9 - 16(-1) = 25$

90. $\overline{z - w} = \overline{3 - 4i - (8 + 3i)}$
 $= \overline{3 - 4i - 8 - 3i}$
 $= \overline{-5 - 7i}$
 $= -5 + 7i$

91. $Z = \frac{V}{I} = \frac{18 + i}{3 - 4i} = \frac{18 + i}{3 - 4i} \cdot \frac{3 + 4i}{3 + 4i}$
 $= \frac{54 + 72i + 3i + 4i^2}{9 + 12i - 12i - 16i^2} = \frac{54 + 75i - 4}{9 + 16}$
 $= \frac{50 + 75i}{25} = 2 + 3i$

The impedance is $2 + 3i$ ohms.

92. $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2 + i} + \frac{1}{4 - 3i} = \frac{(4 - 3i) + (2 + i)}{(2 + i)(4 - 3i)}$
 $= \frac{6 - 2i}{8 - 6i + 4i - 3i^2} = \frac{6 - 2i}{8 - 2i + 3} = \frac{6 - 2i}{11 - 2i}$

So, $Z = \frac{11 - 2i}{6 - 2i} = \frac{11 - 2i}{6 - 2i} \cdot \frac{6 + 2i}{6 + 2i}$
 $= \frac{66 + 22i - 12i - 4i^2}{36 + 12i - 12i - 4i^2} = \frac{66 + 10i + 4}{36 + 4}$
 $= \frac{70 + 10i}{40} = \frac{7}{4} + \frac{1}{4}i$

The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

93. $z + \bar{z} = (a + bi) + \overline{(a + bi)}$
 $= a + bi + a - bi$
 $= 2a$

$z - \bar{z} = a + bi - \overline{(a + bi)}$
 $= a + bi - (a - bi)$
 $= a + bi - a + bi$
 $= 2bi$

94. $\overline{\overline{z}} = \overline{a + bi} = \overline{a - bi} = a + bi = z$

95. $\overline{z + w} = \overline{(a + bi) + (c + di)}$
 $= \overline{(a + c) + (b + d)i}$
 $= (a + c) - (b + d)i$
 $= (a - bi) + (c - di)$
 $= \overline{a + bi} + \overline{c + di}$
 $= \bar{z} + \bar{w}$

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 96. \quad \overline{z \cdot w} &= \overline{(a+bi) \cdot (c+di)} \\
 &= \overline{ac + adi + bci + bdi^2} \\
 &= \overline{(ac - bd) + (ad + bc)i} \\
 &= (ac - bd) - (ad + bc)i \\
 \overline{z} \cdot \overline{w} &= \overline{a+bi} \cdot \overline{c+di} \\
 &= (a-bi)(c-di) \\
 &= ac - adi - bci + bdi^2 \\
 &= (ac - bd) - (ad + bc)i
 \end{aligned}$$

$$\begin{aligned}
 97. \quad (a+bi)^2 &= -(a-bi)^2 \\
 a^2 + 2abi + (bi)^2 &= -(a^2 - 2abi + (bi)^2) \\
 a^2 + 2abi + b^2i^2 &= -(a^2 - 2abi + b^2i^2) \\
 a^2 + 2abi - b^2 &= -a^2 + 2abi + b^2 \\
 a^2 - b^2 &= -(a^2 - b^2) \\
 2(a^2 - b^2) &= 0 \\
 a^2 &= b^2 \rightarrow b = \pm a
 \end{aligned}$$

Any complex number of the form $a + ai$ or $a - ai$ will work.

$$\begin{aligned}
 98. \quad \text{Let } u = \sqrt[3]{2} \text{ in } x^3 + 2 = 0 \text{ so that } x^3 + u^3 = 0. \\
 \text{Then, } (x+u)(x^2 - ux + u^2) = 0. \text{ From the first} \\
 \text{factor we find } x = -u = -\sqrt[3]{2}. \text{ From the second} \\
 \text{factor, use the quadratic formula to get} \\
 x = \frac{-(-u) \pm \sqrt{(-u)^2 - 4 \cdot 1 \cdot u^2}}{2 \cdot 1}
 \end{aligned}$$

$$= \frac{u \pm \sqrt{-3u^2}}{2} = \frac{u}{2} \pm \frac{u\sqrt{3}}{2}i = \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i$$

The solution set is:

$$\left\{ -\sqrt[3]{2}, \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i \right\}$$

$$\begin{aligned}
 99. \quad (x+5)(y-5) &= (x+y)^2; \text{ let } u = x+5 \text{ (so} \\
 x &= u-5 \text{ and } v = y-5 \text{ so } y = v+5. \\
 \text{Substituting gives } uv &= (u+v)^2 \text{ or} \\
 u^2 + uv + v^2 &= 0 \text{ which is quadratic in } u. \text{ Using} \\
 \text{the quadratic formula gives} \\
 x = \frac{-v \pm \sqrt{v^2 - 4 \cdot 1 \cdot v^2}}{2 \cdot 1} &= \frac{-v \pm |v|\sqrt{-3}}{2}. \text{ Since } x \\
 \text{is a real number, } u &\text{ must also be a real number.}
 \end{aligned}$$

This is only possible if $v = 0$ which then makes $u = 0$. Therefore, $x = 0 - 5 = -5$ and $y = 0 + 5 = 5$, so $x - y = -5 - 5 = -10$

100 – 102. Answers will vary.

103. Answers will vary. A complex number is the sum or difference of two numbers (real and imaginary parts of the complex number) just as a binomial is the sum or difference of two monomial terms. We multiply two binomials by using the FOIL method, an approach we can also use to multiply two complex numbers.

104. Although the set of real numbers is a subset of the set of complex numbers, not all rules that work in the real number system can be used in the larger complex number system. The rule that allows us to write the product of two square roots as the square root of the product only works in the real number system. That is, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ only when $\sqrt{a}$ and $\sqrt{b}$ are real numbers. In the complex number system we must first convert the radicals to complex form. In this case this means we need to write $\sqrt{-9}$ as $\sqrt{-1 \cdot 9} = \sqrt{9} \cdot \sqrt{-1} = 3i$. Then we can multiply to get $\sqrt{-9} \cdot \sqrt{-9} = 3i \cdot 3i = 9i^2 = 9(-1) = -9$.

Section 1.4

1. True

$$2. \left(\sqrt[3]{x}\right)^3 = x$$

$$3. 6x^3 - 2x^2 = 2x^2(3x - 1)$$

4. False; you can also use the Quadratic Formula or completing the square.

5. quadratic in form

6. True

7. a

8. c

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

9. $\sqrt{2t-1} = 1$

$$(\sqrt{2t-1})^2 = 1^2$$

$$2t-1 = 1$$

$$2t = 2$$

$$t = 1$$

Check: $\sqrt{2(1)-1} = \sqrt{1} = 1$

The solution set is $\{1\}$.

10. $\sqrt{3t+4} = 2$

$$(\sqrt{3t+4})^2 = 2^2$$

$$3t+4 = 4$$

$$3t = 0$$

$$t = 0$$

Check: $\sqrt{3(0)+4} = \sqrt{4} = 2$

The solution set is $\{0\}$.

11. $\sqrt{3t+4} = -6$

Since the principal square root is never negative, the equation has no real solution.

12. $\sqrt{5t+3} = -2$

Since the principal square root is never negative, the equation has no real solution.

13. $\sqrt[3]{1-2x} - 3 = 0$

$$\sqrt[3]{1-2x} = 3$$

$$(\sqrt[3]{1-2x})^3 = 3^3$$

$$1-2x = 27$$

$$-2x = 26$$

$$x = -13$$

Check: $\sqrt[3]{1-2(-13)} - 3 = \sqrt[3]{27} - 3 = 0$

The solution set is $\{-13\}$.

14. $\sqrt[3]{1-2x} - 1 = 0$

$$\sqrt[3]{1-2x} = 1$$

$$(\sqrt[3]{1-2x})^3 = 1^3$$

$$1-2x = 1$$

$$-2x = 0$$

$$x = 0$$

Check: $\sqrt[3]{1-2(0)} - 1 = \sqrt[3]{1} - 1 = 0$

The solution set is $\{0\}$.

15. $\sqrt[5]{x^2+2x} = -1$

$$(\sqrt[5]{x^2+2x})^5 = (-1)^5$$

$$x^2+2x = -1$$

$$x^2+2x+1 = 0$$

$$(x+1)^2 = 0$$

$$x+1 = 0$$

$$x = -1$$

Check: $\sqrt[5]{(-1)^2+2(-1)} = \sqrt[5]{1-2} = \sqrt[5]{-1} = -1$

The solution set is $\{-1\}$.

16. $\sqrt[4]{x^2+16} = \sqrt{5}$

$$(\sqrt[4]{x^2+16})^4 = (\sqrt{5})^4$$

$$x^2+16 = 25$$

$$x^2 = 9$$

$$x = \pm 3$$

Check -3: $\sqrt[4]{(-3)^2+16} = \sqrt[4]{9+16} = \sqrt[4]{25} = \sqrt{5}$

Check 3: $\sqrt[4]{(3)^2+16} = \sqrt[4]{9+16} = \sqrt[4]{25} = \sqrt{5}$

The solution set is $\{-3, 3\}$.

17. $x = 8\sqrt{x}$

$$(x)^2 = (8\sqrt{x})^2$$

$$x^2 = 64x$$

$$x^2 - 64x = 0$$

$$x(x-64) = 0$$

$$x = 0 \text{ or } x = 64$$

Check 0: $0 = 8\sqrt{0}$

$$0 = 0$$

Check 64: $64 = 8\sqrt{64}$

$$64 = 64$$

The solution set is $\{0, 64\}$.

18. $x = 3\sqrt{x}$

$$(x)^2 = (3\sqrt{x})^2$$

$$x^2 = 9x$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0$$

$$x = 0 \text{ or } x = 9$$

Chapter 1: Equations and Inequalities

Check 0: $0 = 3\sqrt{0}$ Check 9: $9 = 3\sqrt{9}$
 $0 = 0$ $9 = 9$

The solution set is $\{0, 9\}$.

19. $\sqrt{15-2x} = x$
 $(\sqrt{15-2x})^2 = x^2$
 $15-2x = x^2$
 $x^2 + 2x - 15 = 0$
 $(x+5)(x-3) = 0$
 $x = -5$ or $x = 3$
 Check -5: $\sqrt{15-2(-5)} = \sqrt{25} = 5 \neq -5$
 Check 3: $\sqrt{15-2(3)} = \sqrt{9} = 3 = 3$
 Disregard $x = -5$ as extraneous.
 The solution set is $\{3\}$.

20. $\sqrt{12-x} = x$
 $(\sqrt{12-x})^2 = x^2$
 $12-x = x^2$
 $x^2 + x - 12 = 0$
 $(x+4)(x-3) = 0$
 $x = -4$ or $x = 3$
 Check -4: $\sqrt{12-(-4)} = \sqrt{16} = 4 \neq -4$
 Check 3: $\sqrt{12-3} = \sqrt{9} = 3 = 3$
 Disregard $x = -4$ as extraneous.
 The solution set is $\{3\}$.

21. $x = 2\sqrt{x-1}$
 $x^2 = (2\sqrt{x-1})^2$
 $x^2 = 4(x-1)$
 $x^2 = 4x - 4$
 $x^2 - 4x + 4 = 0$
 $(x-2)^2 = 0$
 $x = 2$
 Check: $2 = 2\sqrt{2-1}$
 $2 = 2$
 The solution set is $\{2\}$.

22. $x = 2\sqrt{-x-1}$
 $x^2 = (2\sqrt{-x-1})^2$
 $x^2 = 4(-x-1)$
 $x^2 = -4x - 4$
 $x^2 + 4x + 4 = 0$
 $(x+2)^2 = 0$
 $x = -2$

Check: $-2 = 2\sqrt{-(-2)-1}$
 $-2 \neq 2$

The equation has no real solution.

23. $\sqrt{x^2-x-4} = x+2$
 $(\sqrt{x^2-x-4})^2 = (x+2)^2$
 $x^2-x-4 = x^2+4x+4$
 $-8 = 5x$
 $-\frac{8}{5} = x$
 Check: $\sqrt{\left(-\frac{8}{5}\right)^2 - \left(-\frac{8}{5}\right) - 4} = \left(-\frac{8}{5}\right) + 2$
 $\sqrt{\frac{64}{25} + \frac{8}{5} - 4} = \frac{2}{5}$
 $\sqrt{\frac{4}{25}} = \frac{2}{5}$
 $\frac{2}{5} = \frac{2}{5}$

The solution set is $\left\{-\frac{8}{5}\right\}$.

24. $\sqrt{3-x+x^2} = x-2$
 $(\sqrt{3-x+x^2})^2 = (x-2)^2$
 $3-x+x^2 = x^2-4x+4$
 $3x = 1$
 $x = \frac{1}{3}$
 Check: $\sqrt{3-\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2} = \left(\frac{1}{3}\right) - 2$
 $\sqrt{3-\frac{1}{3}+\frac{1}{9}} = -\frac{5}{3}$

Since the principal square root is always a non-negative number; $x = \frac{1}{3}$ does not check.

Therefore this equation has no real solution.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

25. $3 + \sqrt{3x+1} = x$

$$\sqrt{3x+1} = x-3$$

$$(\sqrt{3x+1})^2 = (x-3)^2$$

$$3x+1 = x^2 - 6x + 9$$

$$0 = x^2 - 9x + 8$$

$$0 = (x-1)(x-8)$$

$$x = 1 \text{ or } x = 8$$

Check 1: $3 + \sqrt{3(1)+1} = 3 + \sqrt{4} = 5 \neq 1$

Check 8: $3 + \sqrt{3(8)+1} = 3 + \sqrt{25} = 8 = 8$

Discard $x = 1$ as extraneous.

The solution set is $\{8\}$.

26. $2 + \sqrt{12-2x} = x$

$$\sqrt{12-2x} = x-2$$

$$(\sqrt{12-2x})^2 = (x-2)^2$$

$$12-2x = x^2 - 4x + 4$$

$$0 = x^2 - 2x - 8$$

$$(x+2)(x-4) = 0$$

$$x = -2 \text{ or } x = 4$$

Check -2: $2 + \sqrt{12-2(-2)} = 2 + \sqrt{16} = 6 \neq -2$

Check 4: $2 + \sqrt{12-2(4)} = 2 + \sqrt{4} = 4 = 4$

Discard $x = -2$ as extraneous.

The solution set is $\{4\}$.

27. $\sqrt{3(x+10)} - 4 = x$

$$\sqrt{3(x+10)} = x+4$$

$$(\sqrt{3(x+10)})^2 = (x+4)^2$$

$$3x+30 = x^2 + 8x + 16$$

$$0 = x^2 + 5x - 14$$

$$0 = (x+7)(x-2)$$

$$x = -7 \text{ or } x = 2$$

Check -7: $\sqrt{3(-7+10)} - 4 = \sqrt{9} - 4 = -1 \neq -7$

Check 2: $\sqrt{3(2+10)} - 4 = \sqrt{36} - 4 = 2 = 2$

Discard $x = -7$ as extraneous.

The solution set is $\{2\}$.

28. $\sqrt{1-x} - 3 = x+2$

$$\sqrt{1-x} = x+5$$

$$(\sqrt{1-x})^2 = (x+5)^2$$

$$1-x = x^2 + 10x + 25$$

$$0 = x^2 + 11x + 24$$

$$0 = (x+3)(x+8)$$

$$x = -3 \text{ or } x = -8$$

Check -3: $\sqrt{1-(-3)} - 3 = -3 + 2 \rightarrow -1 = -1$

Check -8: $\sqrt{1-(-8)} - 3 = -8 + 2 \rightarrow 0 = -6$

Discard $x = -8$ as extraneous.

The solution set is $\{-3\}$.

29. $\sqrt{3x-5} - \sqrt{x+7} = 2$

$$\sqrt{3x-5} = 2 + \sqrt{x+7}$$

$$(\sqrt{3x-5})^2 = (2 + \sqrt{x+7})^2$$

$$3x-5 = 4 + 4\sqrt{x+7} + x+7$$

$$2x-16 = 4\sqrt{x+7}$$

$$(2x-16)^2 = (4\sqrt{x+7})^2$$

$$4x^2 - 64x + 256 = 16(x+7)$$

$$4x^2 - 64x + 256 = 16x + 112$$

$$4x^2 - 80x + 144 = 0$$

$$x^2 - 20x + 36 = 0$$

$$(x-2)(x-18) = 0$$

$$x = 2 \text{ or } x = 18$$

Check 2: $\sqrt{3(2)-5} - \sqrt{2+7}$

$$= \sqrt{1} - \sqrt{9} = 1 - 3 = -2 \neq 2$$

Check 18: $\sqrt{3(18)-5} - \sqrt{18+7}$

$$= \sqrt{49} - \sqrt{25} = 7 - 5 = 2 = 2$$

Discard $x = 2$ as extraneous.

The solution set is $\{18\}$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 30. \quad & \sqrt{3x+7} + \sqrt{x+2} = 1 \\
 & \sqrt{3x+7} = 1 - \sqrt{x+2} \\
 & (\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2 \\
 & 3x+7 = 1 - 2\sqrt{x+2} + x+2 \\
 & 2x+4 = -2\sqrt{x+2} \\
 & -x-2 = \sqrt{x+2} \\
 & (-x-2)^2 = (\sqrt{x+2})^2 \\
 & x^2 + 4x + 4 = x+2 \\
 & x^2 + 3x + 2 = 0 \\
 & (x+1)(x+2) = 0 \\
 & x = -1 \text{ or } x = -2
 \end{aligned}$$

$$\begin{aligned}
 \text{Check } -1: & \sqrt{3(-1)+7} + \sqrt{-1+2} \\
 & = \sqrt{4} + \sqrt{1} = 2+1 = 3 \neq 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Check } -2: & \sqrt{3(-2)+7} + \sqrt{-2+2} \\
 & = \sqrt{1} + \sqrt{0} = 1+0 = 1 = 1
 \end{aligned}$$

Discard $x = -1$ as extraneous.

The solution set is $\{-2\}$.

$$\begin{aligned}
 31. \quad & \sqrt{3x+1} - \sqrt{x-1} = 2 \\
 & \sqrt{3x+1} = 2 + \sqrt{x-1} \\
 & (\sqrt{3x+1})^2 = (2 + \sqrt{x-1})^2 \\
 & 3x+1 = 4 + 4\sqrt{x-1} + x-1 \\
 & 2x-2 = 4\sqrt{x-1} \\
 & (2x-2)^2 = (4\sqrt{x-1})^2 \\
 & 4x^2 - 8x + 4 = 16(x-1) \\
 & x^2 - 2x + 1 = 4x - 4 \\
 & x^2 - 6x + 5 = 0 \\
 & (x-1)(x-5) = 0 \\
 & x = 1 \text{ or } x = 5 \\
 \text{Check } 1: & \sqrt{3(1)+1} - \sqrt{1-1} \\
 & = \sqrt{4} - \sqrt{0} = 2 - 0 = 2 = 2 \\
 \text{Check } 5: & \sqrt{3(5)+1} - \sqrt{5-1} \\
 & = \sqrt{16} - \sqrt{4} = 4 - 2 = 2 = 2 \\
 \text{The solution set is } & \{1, 5\}.
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \sqrt{2x+3} - \sqrt{x+1} = 1 \\
 & \sqrt{2x+3} = 1 + \sqrt{x+1} \\
 & (\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2 \\
 & 2x+3 = 1 + 2\sqrt{x+1} + x+1 \\
 & x+1 = 2\sqrt{x+1} \\
 & (x+1)^2 = (2\sqrt{x+1})^2 \\
 & x^2 + 2x + 1 = 4(x+1) \\
 & x^2 + 2x + 1 = 4x + 4 \\
 & x^2 - 2x - 3 = 0 \\
 & (x+1)(x-3) = 0 \\
 & x = -1 \text{ or } x = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{Check } -1: & \sqrt{2(-1)+3} - \sqrt{-1+1} \\
 & = \sqrt{1} - \sqrt{0} = 1 - 0 = 1 = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Check } 3: & \sqrt{2(3)+3} - \sqrt{3+1} \\
 & = \sqrt{9} - \sqrt{4} = 3 - 2 = 1 = 1
 \end{aligned}$$

The solution set is $\{-1, 3\}$.

$$\begin{aligned}
 33. \quad & \sqrt{3-2\sqrt{x}} = \sqrt{x} \\
 & (\sqrt{3-2\sqrt{x}})^2 = (\sqrt{x})^2 \\
 & 3-2\sqrt{x} = x \\
 & -2\sqrt{x} = x-3 \\
 & (-2\sqrt{x})^2 = (x-3)^2 \\
 & 4x = x^2 - 6x + 9 \\
 & 0 = x^2 - 10x + 9 \\
 & 0 = (x-1)(x-9) \\
 & x = 1 \text{ or } x = 9
 \end{aligned}$$

Check 1:	Check 9:
$\sqrt{3-2\sqrt{1}} = \sqrt{1}$	$\sqrt{3-2\sqrt{9}} = \sqrt{9}$
$\sqrt{3-2} = 1$	$\sqrt{3-2 \cdot 3} = 3$
$\sqrt{1} = 1$	$\sqrt{-3} \neq 3$
$1 = 1$	

Discard $x = 9$ as extraneous.

The solution set is $\{1\}$.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

34. $\sqrt{10+3\sqrt{x}} = \sqrt{x}$

$$\left(\sqrt{10+3\sqrt{x}}\right)^2 = (\sqrt{x})^2$$

$$10+3\sqrt{x} = x$$

$$3\sqrt{x} = x-10$$

$$(3\sqrt{x})^2 = (x-10)^2$$

$$9x = x^2 - 20x + 100$$

$$0 = x^2 - 29x + 100$$

$$0 = (x-4)(x-25)$$

$$x = 4 \quad \text{or} \quad x = 25$$

Check 4:

$$\sqrt{10+3\sqrt{4}} = \sqrt{4}$$

$$\sqrt{10+3 \cdot 2} = 2$$

$$\sqrt{16} = 2$$

$$4 \neq 2$$

Check 25:

$$\sqrt{10+3\sqrt{25}} = \sqrt{25}$$

$$\sqrt{10+3 \cdot 5} = 5$$

$$\sqrt{25} = 5$$

$$5 = 5$$

Discard $x = 4$ as extraneous.

The solution set is $\{25\}$.

35. $(3x+1)^{1/2} = 4$

$$\left((3x+1)^{1/2}\right)^2 = (4)^2$$

$$3x+1 = 16$$

$$3x = 15$$

$$x = 5$$

Check: $(3(5)+1)^{1/2} = 16^{1/2} = 4$

The solution set is $\{5\}$.

36. $(3x-5)^{1/2} = 2$

$$\left((3x-5)^{1/2}\right)^2 = (2)^2$$

$$3x-5 = 4$$

$$3x = 9$$

$$x = 3$$

Check: $(3(3)-5)^{1/2} = 4^{1/2} = 2$

The solution set is $\{3\}$.

37. $(5x-2)^{1/3} = 2$

$$\left((5x-2)^{1/3}\right)^3 = (2)^3$$

$$5x-2 = 8$$

$$5x = 10$$

$$x = 2$$

Check: $(5(2)-2)^{1/3} = 8^{1/3} = 2$

The solution set is $\{2\}$.

38. $(2x+1)^{1/3} = -1$

$$\left((2x+1)^{1/3}\right)^3 = (-1)^3$$

$$2x+1 = -1$$

$$2x = -2$$

$$x = -1$$

Check: $(2(-1)+1)^{1/3} = (-1)^{1/3} = -1$

The solution set is $\{-1\}$.

39. $(x^2+9)^{1/2} = 5$

$$\left((x^2+9)^{1/2}\right)^2 = (5)^2$$

$$x^2+9 = 25$$

$$x^2 = 16$$

$$x = \pm\sqrt{16} = \pm 4$$

Check -4 : $\left((-4)^2+9\right)^{1/2} = 25^{1/2} = 5$

Check 4: $\left((4)^2+9\right)^{1/2} = 25^{1/2} = 5$

The solution set is $\{-4, 4\}$.

40. $(x^2-16)^{1/2} = 9$

$$\left((x^2-16)^{1/2}\right)^2 = (9)^2$$

$$x^2-16 = 81$$

$$x^2 = 97$$

$$x = \pm\sqrt{97}$$

Check $-\sqrt{97}$: $\left((-\sqrt{97})^2-16\right)^{1/2} = 81^{1/2} = 9$

Check $\sqrt{97}$: $\left((\sqrt{97})^2-16\right)^{1/2} = 81^{1/2} = 9$

The solution set is $\{-\sqrt{97}, \sqrt{97}\}$.

Chapter 1: Equations and Inequalities

41. $x^{3/2} - 3x^{1/2} = 0$

$$x^{1/2}(x-3) = 0$$

$$x^{1/2} = 0 \text{ or } x-3 = 0$$

$$x = 0 \text{ or } x = 3$$

Check 0: $0^{3/2} - 3 \cdot 0^{1/2} = 0 - 0 = 0$

Check 3: $3^{3/2} - 3 \cdot 3^{1/2} = 3\sqrt{3} - 3\sqrt{3} = 0$

The solution set is $\{0, 3\}$.

42. $x^{3/4} - 9x^{1/4} = 0$

$$x^{1/4}(x^{3/4} - 9) = 0$$

$$x^{1/4} = 0 \text{ or } x^{3/4} = 9$$

$$x = 0 \text{ or } x = 81$$

Check 0: $0^{3/4} - 9 \cdot 0^{1/4} = 0 - 0 = 0$

Check 81: $81^{3/4} - 9 \cdot 81^{1/4} = 27 - 27 = 0$

The solution set is $\{0, 81\}$.

43. $x^4 - 5x^2 + 4 = 0$

$$(x^2 - 4)(x^2 - 1) = 0$$

$$x^2 - 4 = 0 \text{ or } x^2 - 1 = 0$$

$$x = \pm 2 \text{ or } x = \pm 1$$

The solution set is $\{-2, -1, 1, 2\}$.

44. $x^4 - 10x^2 + 25 = 0$

$$(x^2 - 5)(x^2 - 5) = 0$$

$$x^2 - 5 = 0$$

$$x = \pm\sqrt{5}$$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

45. $6x^4 - 5x^2 - 1 = 0$

$$(6x^2 + 1)(x^2 - 1) = 0$$

$$6x^2 + 1 = 0 \text{ or } x^2 - 1 = 0$$

$$6x^2 = -1 \text{ or } x^2 = 1$$

$$\text{Not real or } x = \pm 1$$

The solution set is $\{-1, 1\}$.

46. $2x^4 - 5x^2 - 12 = 0$

$$(2x^2 + 3)(x^2 - 4) = 0$$

$$2x^2 + 3 = 0 \text{ or } x^2 - 4 = 0$$

$$2x^2 = -3 \text{ or } x^2 = 4$$

$$\text{Not real or } x = \pm 2$$

The solution set is $\{-2, 2\}$.

47. $x^6 + 7x^3 - 8 = 0$

$$(x^3 + 8)(x^3 - 1) = 0$$

$$x^3 + 8 = 0 \text{ or } x^3 - 1 = 0$$

$$x^3 = -8 \text{ or } x^3 = 1$$

$$x = -2 \text{ or } x = 1$$

The solution set is $\{-2, 1\}$.

48. $x^6 - 7x^3 - 8 = 0$

$$(x^3 - 8)(x^3 + 1) = 0$$

$$x^3 - 8 = 0 \text{ or } x^3 + 1 = 0$$

$$x^3 = 8 \text{ or } x^3 = -1$$

$$x = 2 \text{ or } x = -1$$

The solution set is $\{-1, 2\}$.

49. $(x+2)^2 + 7(x+2) + 12 = 0$

Let $u = x + 2$, so that $u^2 = (x+2)^2$.

$$u^2 + 7u + 12 = 0$$

$$(u+3)(u+4) = 0$$

$$u+3 = 0 \text{ or } u+4 = 0$$

$$u = -3 \text{ or } u = -4$$

$$x+2 = -3 \text{ or } x+2 = -4$$

$$x = -5 \text{ or } x = -6$$

The solution set is $\{-6, -5\}$.

50. $(2x+5)^2 - (2x+5) - 6 = 0$

Let $u = 2x + 5$ so that $u^2 = (2x+5)^2$.

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u-3 = 0 \text{ or } u+2 = 0$$

$$u = 3 \text{ or } u = -2$$

$$2x+5 = 3 \text{ or } 2x+5 = -2$$

$$x = -1 \text{ or } x = -\frac{7}{2}$$

The solution set is $\{-\frac{7}{2}, -1\}$.

51. $(4x-9)^2 - 10(4x-9) + 25 = 0$

Let $u = 4x - 9$ so that $u^2 = (4x-9)^2$.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

$$u^2 - 10u + 25 = 0$$

$$(u - 5)^2 = 0$$

$$u - 5 = 0$$

$$u = 5$$

$$4x - 9 = 5$$

$$4x = 14$$

$$x = \frac{7}{2}$$

The solution set is $\left\{\frac{7}{2}\right\}$.

$$52. (2 - x)^2 + (2 - x) - 20 = 0$$

Let $u = 2 - x$ so that $u^2 = (2 - x)^2$.

$$u^2 + u - 20 = 0$$

$$(u + 5)(u - 4) = 0$$

$$u + 5 = 0 \quad \text{or} \quad u - 4 = 0$$

$$u = -5 \quad \text{or} \quad u = 4$$

$$2 - x = -5 \quad \text{or} \quad 2 - x = 4$$

$$x = 7 \quad \text{or} \quad x = -2$$

The solution set is $\{-2, 7\}$.

$$53. 2(s + 1)^2 - 5(s + 1) = 3$$

Let $u = s + 1$ so that $u^2 = (s + 1)^2$.

$$2u^2 - 5u = 3$$

$$2u^2 - 5u - 3 = 0$$

$$(2u + 1)(u - 3) = 0$$

$$2u + 1 = 0 \quad \text{or} \quad u - 3 = 0$$

$$u = -\frac{1}{2} \quad \text{or} \quad u = 3$$

$$s + 1 = -\frac{1}{2} \quad \text{or} \quad s + 1 = 3$$

$$s = -\frac{3}{2} \quad \text{or} \quad s = 2$$

The solution set is $\left\{-\frac{3}{2}, 2\right\}$.

$$54. 3(1 - y)^2 + 5(1 - y) + 2 = 0$$

Let $u = 1 - y$ so that $u^2 = (1 - y)^2$.

$$3u^2 + 5u + 2 = 0$$

$$(3u + 2)(u + 1) = 0$$

$$3u + 2 = 0 \quad \text{or} \quad u + 1 = 0$$

$$u = -\frac{2}{3} \quad \text{or} \quad u = -1$$

$$1 - y = -\frac{2}{3} \quad \text{or} \quad 1 - y = -1$$

$$y = \frac{5}{3} \quad \text{or} \quad y = 2$$

The solution set is $\left\{\frac{5}{3}, 2\right\}$.

$$55. x - 4x\sqrt{x} = 0$$

$$x(1 - 4\sqrt{x}) = 0$$

$$x = 0 \quad \text{or} \quad 1 - 4\sqrt{x} = 0$$

$$1 = 4\sqrt{x}$$

$$\frac{1}{4} = \sqrt{x}$$

$$\left(\frac{1}{4}\right)^2 = (\sqrt{x})^2$$

$$\frac{1}{16} = x$$

Check:

$$x = 0: 0 - 4(0)\sqrt{0} = 0$$

$$0 = 0$$

$$x = \frac{1}{16}: \left(\frac{1}{16}\right) - 4\left(\frac{1}{16}\right)\sqrt{\frac{1}{16}} = 0$$

$$\frac{1}{16} - 4\left(\frac{1}{16}\right)\left(\frac{1}{4}\right) = 0$$

$$\frac{1}{16} - \frac{1}{16} = 0$$

$$0 = 0$$

The solution set is $\left\{0, \frac{1}{16}\right\}$.

$$56. x + 8\sqrt{x} = 0$$

$$8\sqrt{x} = -x$$

$$(8\sqrt{x})^2 = (-x)^2$$

$$64x = x^2$$

$$0 = x^2 - 64x$$

$$0 = x(x - 64)$$

$$x = 0 \quad \text{or} \quad x = 64$$

$$\text{Check: } x = 0: 0 + 8\sqrt{0} = 0$$

$$0 = 0$$

$$x = 64: 64 + 8\sqrt{64} = 0$$

$$64 + 64 \neq 0$$

The solution set is $\{0\}$.

Chapter 1: Equations and Inequalities

57. $x + \sqrt{x} = 20$

Let $u = \sqrt{x}$ so that $u^2 = x$.

$$u^2 + u = 20$$

$$u^2 + u - 20 = 0$$

$$(u+5)(u-4) = 0$$

$$u+5=0 \quad \text{or} \quad u-4=0$$

$$u=-5 \quad \text{or} \quad u=4$$

$$\sqrt{x}=-5 \quad \text{or} \quad \sqrt{x}=4$$

$$\text{not possible} \quad \text{or} \quad x=16$$

$$\text{Check: } 16 + \sqrt{16} = 20$$

$$16 + 4 = 20$$

The solution set is $\{16\}$.

58. $x + \sqrt{x} = 6$

Let $u = \sqrt{x}$ so that $u^2 = x$.

$$u^2 + u = 6$$

$$u^2 + u - 6 = 0$$

$$(u+3)(u-2) = 0$$

$$u+3=0 \quad \text{or} \quad u-2=0$$

$$u=-3 \quad \text{or} \quad u=2$$

$$\sqrt{x}=-3 \quad \text{or} \quad \sqrt{x}=2$$

$$\text{not possible} \quad \text{or} \quad x=4$$

$$\text{Check: } 4 + \sqrt{4} = 6$$

$$4 + 2 = 6$$

The solution set is $\{4\}$.

59. $t^{1/2} - 2t^{1/4} + 1 = 0$

Let $u = t^{1/4}$ so that $u^2 = t^{1/2}$.

$$u^2 - 2u + 1 = 0$$

$$(u-1)^2 = 0$$

$$u-1=0$$

$$u=1$$

$$t^{1/4}=1$$

$$t=1$$

$$\text{Check: } 1^{1/2} - 2(1)^{1/4} + 1 = 0$$

$$1 - 2 + 1 = 0$$

$$0 = 0$$

The solution set is $\{1\}$.

60. $z^{1/2} - 4t^{1/4} + 4 = 0$

Let $u = z^{1/4}$ so that $u^2 = z^{1/2}$.

$$u^2 - 4u + 4 = 0$$

$$(u-2)^2 = 0$$

$$u-2=0$$

$$u=2$$

$$z^{1/4}=2$$

$$z=16$$

$$\text{Check: } 16^{1/2} - 4(16)^{1/4} + 4 = 0$$

$$4 - 8 + 4 = 0$$

$$0 = 0$$

The solution set is $\{16\}$.

61. $x^{1/2} - 3x^{1/4} + 2 = 0$

Let $u = x^{1/4}$ so that $u^2 = x^{1/2}$.

$$u^2 - 3u + 2 = 0$$

$$(u-2)(u-1) = 0$$

$$u=2 \quad \text{or} \quad u=1$$

$$x^{1/4}=2 \quad \text{or} \quad x^{1/4}=1$$

$$x=16 \quad \text{or} \quad x=1$$

Check:

$$x=16: 16^{1/2} - 3(16)^{1/4} + 2 = 0$$

$$4 - 6 + 2 = 0$$

$$0 = 0$$

$$x=1: 1^{1/2} - 3(1)^{1/4} + 2 = 0$$

$$1 - 3 + 2 = 0$$

$$0 = 0$$

The solution set is $\{1, 16\}$.

62. $4x^{1/2} - 9x^{1/4} + 4 = 0$

Let $u = x^{1/4}$ so that $u^2 = x^{1/2}$.

$$4u^2 - 9u + 4 = 0$$

$$u = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(4)}}{2(4)} = \frac{9 \pm \sqrt{17}}{8}$$

$$x^{1/4} = \frac{9 \pm \sqrt{17}}{8}$$

$$x = \left(\frac{9 \pm \sqrt{17}}{8} \right)^4$$

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

$$\text{Check } x = \left(\frac{9 + \sqrt{17}}{8} \right)^4 :$$

$$4 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/2} - 9 \left(\left(\frac{9 + \sqrt{17}}{8} \right)^4 \right)^{1/4} + 4 = 0$$

$$4 \left(\frac{9 + \sqrt{17}}{8} \right)^2 - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$$

$$4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 = 0$$

$$64 \left(4 \frac{(9 + \sqrt{17})^2}{64} - 9 \left(\frac{9 + \sqrt{17}}{8} \right) + 4 \right) = (0)(64)$$

$$4(9 + \sqrt{17})^2 - 72(9 + \sqrt{17}) + 256 = 0$$

$$4(81 + 18\sqrt{17} + 17) - 72(9 + \sqrt{17}) + 256 = 0$$

$$324 + 72\sqrt{17} + 68 - 648 - 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

$$\text{Check } x = \left(\frac{9 - \sqrt{17}}{8} \right)^4 :$$

$$4 \left(\left(\frac{9 - \sqrt{17}}{8} \right)^4 \right)^{1/2} - 9 \left(\left(\frac{9 - \sqrt{17}}{8} \right)^4 \right)^{1/4} + 4 = 0$$

$$4 \left(\frac{9 - \sqrt{17}}{8} \right)^2 - 9 \left(\frac{9 - \sqrt{17}}{8} \right) + 4 = 0$$

$$4(81 - 18\sqrt{17} + 17) - 72(9 - \sqrt{17}) + 256 = 0$$

$$324 - 72\sqrt{17} + 68 - 648 + 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

$$\text{The solution set is } \left\{ \left(\frac{9 - \sqrt{17}}{8} \right)^4, \left(\frac{9 + \sqrt{17}}{8} \right)^4 \right\}.$$

$$63. \sqrt[4]{5x^2 - 6} = x$$

$$\left(\sqrt[4]{5x^2 - 6} \right)^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$0 = x^4 - 5x^2 + 6$$

$$\text{Let } u = x^2 \text{ so that } u^2 = x^4.$$

$$0 = u^2 - 5u + 6$$

$$0 = (u - 3)(u - 2)$$

$$u = 3 \quad \text{or} \quad u = 2$$

$$x^2 = 3 \quad \text{or} \quad x^2 = 2$$

$$x = \pm\sqrt{3} \quad \text{or} \quad x = \pm\sqrt{2}$$

Check:

$$x = -\sqrt{3}: \sqrt[4]{5(-\sqrt{3})^2 - 6} = -\sqrt{3}$$

$$\sqrt[4]{15 - 6} = -\sqrt{3}$$

$$\sqrt[4]{9} \neq -\sqrt{3}$$

$$x = \sqrt{3}: \sqrt[4]{5(\sqrt{3})^2 - 6} = \sqrt{3}$$

$$\sqrt[4]{15 - 6} = \sqrt{3}$$

$$\sqrt[4]{9} = \sqrt{3}$$

$$\sqrt{3} = \sqrt{3}$$

$$x = -\sqrt{2}: \sqrt[4]{5(-\sqrt{2})^2 - 6} = -\sqrt{2}$$

$$\sqrt[4]{10 - 6} = -\sqrt{2}$$

$$\sqrt[4]{4} \neq -\sqrt{2}$$

$$x = \sqrt{2}: \sqrt[4]{5(\sqrt{2})^2 - 6} = \sqrt{2}$$

$$\sqrt[4]{10 - 6} = \sqrt{2}$$

$$\sqrt[4]{4} = \sqrt{2}$$

$$\sqrt{2} = \sqrt{2}$$

The solution set is $\{\sqrt{2}, \sqrt{3}\}$.

$$64. \sqrt[4]{4 - 5x^2} = x$$

$$\left(\sqrt[4]{4 - 5x^2} \right)^4 = x^4$$

$$4 - 5x^2 = x^4$$

$$0 = x^4 + 5x^2 - 4$$

$$\text{Let } u = x^2 \text{ so that } u^2 = x^4.$$

$$0 = u^2 + 5u - 4$$

$$u = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

$$x^2 = \frac{-5 \pm \sqrt{41}}{2}$$

$$x = \pm \sqrt{\frac{-5 \pm \sqrt{41}}{2}}$$

Chapter 1: Equations and Inequalities

Since $-5 - \sqrt{41} < 0$, $x = \pm \sqrt{\frac{-5 - \sqrt{41}}{2}}$ is not real.

Since x is a fourth root, $x = -\sqrt{\frac{-5 + \sqrt{41}}{2}}$ is also not real. Therefore, we have only one possible solution to check: $x = \sqrt{\frac{-5 + \sqrt{41}}{2}}$:

Check $x = \sqrt{\frac{-5 + \sqrt{41}}{2}}$:

$$\sqrt[4]{4 - 5 \left(\pm \sqrt{\frac{-5 + \sqrt{41}}{2}} \right)^2} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{4 - 5 \left(\frac{-5 + \sqrt{41}}{2} \right)} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{\frac{8 - 5(-5 + \sqrt{41})}{2}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{\frac{33 - 5\sqrt{41}}{2}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{\frac{66 - 10\sqrt{41}}{4}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{\frac{25 - 10\sqrt{41} + 41}{4}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt[4]{\frac{(-5 + \sqrt{41})^2}{4}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

$$\sqrt{\frac{-5 + \sqrt{41}}{2}} = \sqrt{\frac{-5 + \sqrt{41}}{2}}$$

The solution set is $\left\{ \sqrt{\frac{-5 + \sqrt{41}}{2}} \right\}$.

65. $x^2 + 3x + \sqrt{x^2 + 3x} = 6$

Let $u = \sqrt{x^2 + 3x}$ so that $u^2 = x^2 + 3x$.

$$u^2 + u = 6$$

$$u^2 + u - 6 = 0$$

$$(u + 3)(u - 2) = 0$$

$$u = -3 \quad \text{or} \quad u = 2$$

$$\sqrt{x^2 + 3x} = -3 \quad \text{or} \quad \sqrt{x^2 + 3x} = 2$$

$$\text{Not possible} \quad \text{or} \quad x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = -4 \quad \text{or} \quad x = 1$$

Check $x = -4$:

$$(-4)^2 + 3(-4) + \sqrt{(-4)^2 + 3(-4)} = 6$$

$$16 - 12 + \sqrt{16 - 12} = 6$$

$$16 - 12 + \sqrt{4} = 6$$

$$6 = 6$$

Check $x = 1$:

$$(1)^2 + 3(1) + \sqrt{(1)^2 + 3(1)} = 6$$

$$1 + 3 + \sqrt{1 + 3} = 6$$

$$4 + \sqrt{4} = 6$$

$$6 = 6$$

The solution set is $\{-4, 1\}$.

66. $x^2 - 3x - \sqrt{x^2 - 3x} = 2$

Let $u = \sqrt{x^2 - 3x}$ so that $u^2 = x^2 - 3x$.

$$u^2 - u = 2$$

$$u^2 - u - 2 = 0$$

$$(u + 1)(u - 2) = 0$$

$$u = -1 \quad \text{or}$$

$$u = 2$$

$$\sqrt{x^2 - 3x} = -1 \quad \text{or} \quad \sqrt{x^2 - 3x} = 2$$

$$\text{Not possible} \quad \text{or} \quad x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \quad \text{or} \quad x = -1$$

Check $x = 4$:

$$(4)^2 - 3(4) - \sqrt{(4)^2 - 3(4)} = 16 - 12 - \sqrt{4}$$

$$= 4 - 2 = 2$$

Check $x = -1$:

$$(-1)^2 - 3(-1) - \sqrt{(-1)^2 - 3(-1)} = 1 + 3 - \sqrt{4}$$

$$= 4 - 2 = 2$$

The solution set is $\{-1, 4\}$.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

$$67. \frac{1}{(x+1)^2} = \frac{1}{x+1} + 2$$

$$\text{Let } u = \frac{1}{x+1} \text{ so that } u^2 = \left(\frac{1}{x+1}\right)^2.$$

$$u^2 = u + 2$$

$$u^2 - u - 2 = 0$$

$$(u+1)(u-2) = 0$$

$$u = -1 \quad \text{or} \quad u = 2$$

$$\frac{1}{x+1} = -1 \quad \text{or} \quad \frac{1}{x+1} = 2$$

$$1 = -x - 1 \quad \text{or} \quad 1 = 2x + 2$$

$$x = -2 \quad \text{or} \quad -2x = 1$$

$$x = -\frac{1}{2}$$

Check:

$$x = -2: \frac{1}{(-2+1)^2} = \frac{1}{-2+1} + 2$$

$$1 = -1 + 2$$

$$1 = 1$$

$$x = -\frac{1}{2}: \frac{1}{\left(-\frac{1}{2}+1\right)^2} = \frac{1}{\left(-\frac{1}{2}+1\right)} + 2$$

$$4 = 2 + 2$$

$$4 = 4$$

The solution set is $\{-2, -\frac{1}{2}\}$.

$$68. \frac{1}{(x-1)^2} + \frac{1}{x-1} = 12$$

$$\text{Let } u = \frac{1}{x-1} \text{ so that } u^2 = \left(\frac{1}{x-1}\right)^2.$$

$$u^2 + u = 12$$

$$u^2 + u - 12 = 0$$

$$(u+4)(u-3) = 0$$

$$u = -4 \quad \text{or} \quad u = 3$$

$$\frac{1}{x-1} = -4 \quad \text{or} \quad \frac{1}{x-1} = 3$$

$$1 = -4x + 4 \quad \text{or} \quad 1 = 3x - 3$$

$$4x = 3 \quad \text{or} \quad 4 = 3x$$

$$x = \frac{3}{4} \quad \text{or} \quad x = \frac{4}{3}$$

Check:

$$x = \frac{3}{4}: \frac{1}{\left(\frac{3}{4}-1\right)^2} + \frac{1}{\left(\frac{3}{4}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{16}\right)} + \frac{1}{\left(-\frac{1}{4}\right)} = 12$$

$$16 - 4 = 12$$

$$12 = 12$$

$$x = \frac{4}{3}: \frac{1}{\left(\frac{4}{3}-1\right)^2} + \frac{1}{\left(\frac{4}{3}-1\right)} = 12$$

$$\frac{1}{\left(\frac{1}{9}\right)} + \frac{1}{\left(\frac{1}{3}\right)} = 12$$

$$9 + 3 = 12$$

$$12 = 12$$

The solution set is $\left\{\frac{3}{4}, \frac{4}{3}\right\}$.

$$69. 3x^{-2} - 7x^{-1} - 6 = 0$$

$$\text{Let } u = x^{-1} \text{ so that } u^2 = x^{-2}.$$

$$3u^2 - 7u - 6 = 0$$

$$(3u+2)(u-3) = 0$$

$$u = -\frac{2}{3} \quad \text{or} \quad u = 3$$

$$x^{-1} = -\frac{2}{3} \quad \text{or} \quad x^{-1} = 3$$

$$(x^{-1})^{-1} = \left(-\frac{2}{3}\right)^{-1} \quad \text{or} \quad (x^{-1})^{-1} = (3)^{-1}$$

$$x = -\frac{3}{2} \quad \text{or} \quad x = \frac{1}{3}$$

Check:

$$x = -\frac{3}{2}: 3\left(-\frac{3}{2}\right)^{-2} - 7\left(-\frac{3}{2}\right)^{-1} - 6 = 0$$

$$3\left(\frac{4}{9}\right) - 7\left(-\frac{2}{3}\right) - 6 = 0$$

$$\frac{4}{3} + \frac{14}{3} - 6 = 0$$

$$0 = 0$$

$$x = \frac{1}{3}: 3\left(\frac{1}{3}\right)^{-2} - 7\left(\frac{1}{3}\right)^{-1} - 6 = 0$$

$$3(9) - 7(3) - 6 = 0$$

$$27 - 21 - 6 = 0$$

$$0 = 0$$

The solution set is $\left\{-\frac{3}{2}, \frac{1}{3}\right\}$.

Chapter 1: Equations and Inequalities

70. $2x^{-2} - 3x^{-1} - 4 = 0$

Let $u = x^{-1}$ so that $u^2 = x^{-2}$.

$$2u^2 - 3u - 4 = 0$$

$$u = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} = \frac{3 \pm \sqrt{41}}{4}$$

$$u = \frac{3 + \sqrt{41}}{4} \quad \text{or} \quad u = \frac{3 - \sqrt{41}}{4}$$

$$x^{-1} = \frac{3 + \sqrt{41}}{4} \quad \text{or} \quad x^{-1} = \frac{3 - \sqrt{41}}{4}$$

$$(x^{-1})^{-1} = \left(\frac{3 + \sqrt{41}}{4}\right)^{-1} \quad \text{or} \quad (x^{-1})^{-1} = \left(\frac{3 - \sqrt{41}}{4}\right)^{-1}$$

$$x = \frac{4}{3 + \sqrt{41}} \left(\frac{3 - \sqrt{41}}{4}\right) \quad \text{or} \quad x = \frac{4}{3 - \sqrt{41}} \left(\frac{3 + \sqrt{41}}{4}\right)$$

$$= \frac{12 - 4\sqrt{41}}{-32} \quad = \frac{12 + 4\sqrt{41}}{-32}$$

$$= \frac{-3 + \sqrt{41}}{8} \quad = \frac{-3 - \sqrt{41}}{8}$$

Check $x = \frac{-3 + \sqrt{41}}{8}$:

$$2\left(\frac{-3 + \sqrt{41}}{8}\right)^{-2} - 3\left(\frac{-3 + \sqrt{41}}{8}\right)^{-1} - 4 = 0$$

$$2\left(\frac{64}{(-3 + \sqrt{41})^2}\right) - 3\left(\frac{8}{-3 + \sqrt{41}}\right) - 4 = 0$$

$$2(64) - 3(8)(-3 + \sqrt{41}) - 4(-3 + \sqrt{41})^2 = 0$$

$$128 + 72 - 24\sqrt{41} - 4(9 - 6\sqrt{41} + 41) = 0$$

$$128 + 72 - 24\sqrt{41} - 36 + 24\sqrt{41} - 164 = 0$$

$$0 = 0$$

Check $x = \frac{-3 - \sqrt{41}}{8}$:

$$2\left(\frac{-3 - \sqrt{41}}{8}\right)^{-2} - 3\left(\frac{-3 - \sqrt{41}}{8}\right)^{-1} - 4 = 0$$

$$2\left(\frac{64}{(-3 - \sqrt{41})^2}\right) - 3\left(\frac{8}{-3 - \sqrt{41}}\right) - 4 = 0$$

$$2(64) - 3(8)(-3 - \sqrt{41}) - 4(-3 - \sqrt{41})^2 = 0$$

$$128 + 72 + 24\sqrt{41} - 4(9 + 6\sqrt{41} + 41) = 0$$

$$128 + 72 + 24\sqrt{41} - 36 - 24\sqrt{41} - 164 = 0$$

$$0 = 0$$

The solution set is $\left\{\frac{-3 - \sqrt{41}}{8}, \frac{-3 + \sqrt{41}}{8}\right\}$.

71. $2x^{2/3} - 5x^{1/3} - 3 = 0$

Let $u = x^{1/3}$ so that $u^2 = x^{2/3}$.

$$2u^2 - 5u - 3 = 0$$

$$(2u + 1)(u - 3) = 0$$

$$u = -\frac{1}{2} \quad \text{or} \quad u = 3$$

$$x^{1/3} = -\frac{1}{2} \quad \text{or} \quad x^{1/3} = 3$$

$$(x^{1/3})^3 = \left(-\frac{1}{2}\right)^3 \quad \text{or} \quad (x^{1/3})^3 = (3)^3$$

$$x = -\frac{1}{8} \quad \text{or} \quad x = 27$$

Check $x = -\frac{1}{8}$: $2\left(-\frac{1}{8}\right)^{2/3} - 5\left(-\frac{1}{8}\right)^{1/3} - 3 = 0$

$$2\left(\frac{1}{4}\right) - 5\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{1}{2} + \frac{5}{2} - 3 = 0$$

$$3 - 3 = 0$$

$$0 = 0$$

Check $x = 27$: $2(27)^{2/3} - 5(27)^{1/3} - 3 = 0$

$$2(9) - 5(3) - 3 = 0$$

$$18 - 15 - 3 = 0$$

$$3 - 3 = 0$$

$$0 = 0$$

The solution set is $\left\{-\frac{1}{8}, 27\right\}$.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

72. $3x^{4/3} + 5x^{2/3} - 2 = 0$

Let $u = x^{2/3}$ so that $u^2 = x^{4/3}$.

$$3u^2 + 5u - 2 = 0$$

$$(3u-1)(u+2) = 0$$

$$u = \frac{1}{3} \quad \text{or} \quad u = -2$$

$$x^{2/3} = \frac{1}{3} \quad \text{or} \quad x^{2/3} = -2$$

$$(x^{2/3})^3 = \left(\frac{1}{3}\right)^3 \quad \text{or} \quad (x^{2/3})^3 = (-2)^3$$

$$x^2 = \frac{1}{27} \quad \text{or} \quad x^2 = -8$$

$$x = \pm\sqrt{\frac{1}{27}} \quad \text{not real}$$

Check: $3\left(\pm\sqrt{\frac{1}{27}}\right)^{4/3} + 5\left(\pm\sqrt{\frac{1}{27}}\right)^{2/3} - 2 = 0$

$$3\left(\frac{1}{27}\right)^{2/3} + 5\left(\pm\frac{1}{27}\right)^{1/3} - 2 = 0$$

$$3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) - 2 = 0$$

$$3\left(\frac{1}{9}\right) + \frac{5}{3} - 2 = 0$$

$$\frac{1}{3} + \frac{5}{3} - 2 = 0$$

$$2 - 2 = 0$$

$$0 = 0$$

Note: $\pm\sqrt{\frac{1}{27}} = \pm\sqrt{\frac{3}{81}} = \pm\frac{\sqrt{3}}{9}$

The solution set is $\left\{-\frac{\sqrt{3}}{9}, \frac{\sqrt{3}}{9}\right\}$.

73. $\left(\frac{v}{v+2}\right)^2 + \frac{3v}{v+2} = 10$

$$\left(\frac{v}{v+2}\right)^2 + 3\left(\frac{v}{v+2}\right) = 10$$

Let $u = \frac{v}{v+2}$ so that $u^2 = \left(\frac{v}{v+2}\right)^2$.

$$u^2 + 3u = 10$$

$$u^2 + 3u - 10 = 0$$

$$(u+5)(u-2) = 0$$

$$u = -5 \quad \text{or} \quad u = 2$$

$$\frac{v}{v+2} = -5 \quad \text{or} \quad \frac{v}{v+2} = 2$$

$$v = -5v - 10 \quad \text{or} \quad v = 2v + 4$$

$$v = -\frac{5}{3} \quad \text{or} \quad v = -4$$

Check $v = -\frac{5}{3}$: $\left(\frac{-\frac{5}{3}}{-\frac{5}{3}+2}\right)^2 + \frac{3\left(-\frac{5}{3}\right)}{-\frac{5}{3}+2} = 10$

$$\frac{\left(\frac{25}{9}\right)}{\left(\frac{1}{9}\right)} + \frac{\left(-\frac{15}{3}\right)}{\left(\frac{1}{3}\right)} = 10$$

$$25 - 15 = 10$$

$$10 = 10$$

Check $v = -4$: $\left(\frac{-4}{-4+2}\right)^2 + \frac{3(-4)}{(-4)+2} = 10$

$$\frac{16}{4} + \frac{-12}{-2} = 10$$

$$4 + 6 = 10$$

$$10 = 10$$

The solution set is $\left\{-4, -\frac{5}{3}\right\}$.

74. $\left(\frac{y}{y-1}\right)^2 = 6\left(\frac{y}{y-1}\right) + 16$

Let $u = \frac{y}{y-1}$ so that $u^2 = \left(\frac{y}{y-1}\right)^2$.

$$u^2 = 6u + 16$$

$$u^2 - 6u - 16 = 0$$

$$(u+2)(u-8) = 0$$

$$u = -2 \quad \text{or} \quad u = 8$$

$$\frac{y}{y-1} = -2 \quad \text{or} \quad \frac{y}{y-1} = 8$$

$$y = -2y + 2 \quad \text{or} \quad y = 8y - 8$$

$$3y = 2 \quad \text{or} \quad -7y = -8$$

$$y = \frac{2}{3} \quad \text{or} \quad y = \frac{8}{7}$$

Chapter 1: Equations and Inequalities

$$\begin{aligned}\text{Check } y = \frac{2}{3} : \left(\frac{\frac{2}{3}}{\frac{2}{3}-1} \right)^2 &= 6 \left(\frac{\frac{2}{3}}{\frac{2}{3}-1} \right) + 16 \\ \frac{\frac{4}{9}}{\frac{1}{9}} &= 6 \cdot \frac{\frac{2}{3}}{\left(-\frac{1}{3}\right)} + 16 \\ 4 &= 6(-2) + 16 \\ 4 &= 4\end{aligned}$$

$$\begin{aligned}\text{Check } y = \frac{8}{7} : \left(\frac{\frac{8}{7}}{\frac{8}{7}-1} \right)^2 &= 6 \left(\frac{\frac{8}{7}}{\frac{8}{7}-1} \right) + 16 \\ \left(\frac{\frac{64}{49}}{\frac{1}{49}} \right) &= 6 \left(\frac{\left(\frac{8}{7}\right)}{\left(\frac{1}{7}\right)} \right) + 16 \\ 64 &= 48 + 16 \\ 64 &= 64\end{aligned}$$

The solution set is $\left\{\frac{2}{3}, \frac{8}{7}\right\}$.

$$\begin{aligned}75. \quad x^3 - 9x &= 0 \\ x(x^2 - 9) &= 0 \\ x(x-3)(x+3) &= 0 \\ x = 0 \text{ or } x-3=0 \text{ or } x+3=0 \\ x &= 3 \quad x = -3\end{aligned}$$

The solution set is $\{-3, 0, 3\}$.

$$\begin{aligned}76. \quad x^4 - x^2 &= 0 \\ x^2(x^2 - 1) &= 0 \\ x^2(x-1)(x+1) &= 0 \\ x^2 = 0 \text{ or } x-1=0 \text{ or } x+1=0 \\ x = 0 \quad x = 1 \quad x = -1 \\ \text{The solution set is } &\{-1, 0, 1\}.\end{aligned}$$

$$\begin{aligned}77. \quad 4x^3 &= 3x^2 \\ 4x^3 - 3x^2 &= 0 \\ x^2(4x-3) &= 0 \\ x^2 = 0 \text{ or } 4x-3 &= 0 \\ x = 0 \quad 4x &= 3 \\ x &= \frac{3}{4}\end{aligned}$$

The solution set is $\left\{0, \frac{3}{4}\right\}$.

$$\begin{aligned}78. \quad x^5 &= 4x^3 \\ x^5 - 4x^3 &= 0 \\ x^3(x^2 - 4) &= 0 \\ x^3(x-2)(x+2) &= 0 \\ x^3 = 0 \text{ or } x-2=0 \text{ or } x+2=0 \\ x = 0 \quad x = 2 \quad x &= -2 \\ \text{The solution set is } &\{-2, 0, 2\}.\end{aligned}$$

$$\begin{aligned}79. \quad x^3 + x^2 - 20x &= 0 \\ x(x^2 + x - 20) &= 0 \\ x(x+5)(x-4) &= 0 \\ x = 0 \text{ or } x+5=0 \text{ or } x-4=0 \\ x &= -5 \quad x = 4 \\ \text{The solution set is } &\{-5, 0, 4\}.\end{aligned}$$

$$\begin{aligned}80. \quad x^3 + 6x^2 - 7x &= 0 \\ x(x^2 + 6x - 7) &= 0 \\ x(x+7)(x-1) &= 0 \\ x = 0 \text{ or } x+7=0 \text{ or } x-1=0 \\ x &= -7 \quad x = 1 \\ \text{The solution set is } &\{-7, 0, 1\}.\end{aligned}$$

$$\begin{aligned}81. \quad x^3 + x^2 - x - 1 &= 0 \\ x^2(x+1) - 1(x+1) &= 0 \\ (x+1)(x^2 - 1) &= 0 \\ (x+1)(x-1)(x+1) &= 0 \\ x+1=0 \text{ or } x-1=0 \\ x &= -1 \quad x = 1 \\ \text{The solution set is } &\{-1, 1\}.\end{aligned}$$

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

$$\begin{aligned}
 82. \quad & x^3 + 4x^2 - x - 4 = 0 \\
 & x^2(x+4) - 1(x+4) = 0 \\
 & (x+4)(x^2 - 1) = 0 \\
 & (x+4)(x-1)(x+1) = 0 \\
 & x+4 = 0 \quad \text{or} \quad x-1 = 0 \quad \text{or} \quad x+1 = 0 \\
 & x = -4 \quad \quad x = 1 \quad \quad x = -1 \\
 & \text{The solution set is } \{-4, -1, 1\}.
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & x^3 - 3x^2 - 16x + 48 = 0 \\
 & x^2(x-3) - 16(x-3) = 0 \\
 & (x-3)(x^2 - 16) = 0 \\
 & (x-3)(x-4)(x+4) = 0 \\
 & x-3 = 0 \quad \text{or} \quad x-4 = 0 \quad \text{or} \quad x+4 = 0 \\
 & x = 3 \quad \quad x = 4 \quad \quad x = -4 \\
 & \text{The solution set is } \{-4, 3, 4\}.
 \end{aligned}$$

$$\begin{aligned}
 84. \quad & x^3 - 3x^2 - x + 3 = 0 \\
 & x^2(x-3) - 1(x-3) = 0 \\
 & (x-3)(x^2 - 1) = 0 \\
 & (x-3)(x-1)(x+1) = 0 \\
 & x-3 = 0 \quad \text{or} \quad x-1 = 0 \quad \text{or} \quad x+1 = 0 \\
 & x = 3 \quad \quad x = 1 \quad \quad x = -1 \\
 & \text{The solution set is } \{-1, 1, 3\}.
 \end{aligned}$$

$$\begin{aligned}
 85. \quad & 2x^{3/4} - 3x^{3/4} = 0 \\
 & x^{3/4}(2 - 3x) = 0 \\
 & x^{3/4} = 0 \quad \text{or} \quad 2 - 3x = 0 \\
 & x = 0 \quad \quad -3x = -2 \\
 & \quad \quad \quad x = \frac{2}{3} \\
 & \text{The solution set is } \left\{0, \frac{2}{3}\right\}.
 \end{aligned}$$

$$\begin{aligned}
 86. \quad & 4x^{-1} - x = 0 \\
 & x^{-1}(4 - x^2) = 0 \\
 & x^{-1} = 0 \quad \quad \text{or} \quad 4 - x^2 = 0 \\
 & \frac{1}{x} = 0 \quad \quad \quad -x^2 = -4 \\
 & \quad \quad \quad \text{no solution} \quad \quad x^2 = 4 \\
 & \quad \quad \quad \quad \quad \quad x = \pm 2 \\
 & \text{The solution set is } \{-2, 2\}.
 \end{aligned}$$

$$\begin{aligned}
 87. \quad & 3(x-3)^{1/3} + 9(x-3)^{1/3} = 0 \\
 & 3(x-3)^{1/3}[1 + 3(x-3)] = 0 \\
 & 3(x-3)^{1/3}(1 + 3x - 9) = 0 \\
 & 3(x-3)^{1/3}(3x - 8) = 0 \\
 & 3(x-3)^{1/3} = 0 \quad \text{or} \quad 3x - 8 = 0 \\
 & (x-3)^{1/3} = 0 \quad \quad 3x = 8 \\
 & x - 3 = 0 \quad \quad \quad x = \frac{8}{3} \\
 & x = 3
 \end{aligned}$$

The solution set is $\left\{\frac{8}{3}, 3\right\}$.

$$\begin{aligned}
 88. \quad & 4(x+2)^{-3} - (x+2)^{-1} = 0 \\
 & (x+2)^{-3}[4 - (x+2)^2] = 0 \\
 & (x+2)^{-3}[4 - (x^2 + 4x + 4)] = 0 \\
 & (x+2)^{-3}[4 - x^2 - 4x - 4] = 0 \\
 & (x+2)^{-3}(-x^2 - 4x) = 0 \\
 & (x+2)^{-3}x(-x-4) = 0 \\
 & (x+2)^{-3} = 0 \quad \text{or} \quad x = 0 \quad \text{or} \quad -x-4 = 0 \\
 & \frac{1}{(x+2)^3} = 0 \quad \quad \quad -x = 4 \\
 & \quad \quad \quad \quad \quad \quad x = -4 \\
 & \text{no solution} \\
 & \text{The solution set is } \{-4, 0\}.
 \end{aligned}$$

$$\begin{aligned}
 89. \quad & x(x^2 - 3x)^{1/3} + 2(x^2 - 3x)^{4/3} = 0 \\
 & (x^2 - 3x)^{1/3}[x + 2(x^2 - 3x)] = 0 \\
 & (x^2 - 3x)^{1/3}(x + 2x^2 - 6x) = 0 \\
 & (x^2 - 3x)^{1/3}(2x^2 - 5x) = 0 \\
 & (x^2 - 3x)^{1/3} = 0 \quad \text{or} \quad 2x^2 - 5x = 0 \\
 & x^2 - 3x = 0 \quad \text{or} \quad 2x^2 - 5x = 0 \\
 & x(x-3) = 0 \quad \text{or} \quad x(2x-5) = 0 \\
 & x = 0 \quad \text{or} \quad x = 3 \quad \text{or} \quad x = 0 \quad \text{or} \quad x = \frac{5}{2} \\
 & \text{The solution set is } \left\{0, \frac{5}{2}, 3\right\}.
 \end{aligned}$$

Chapter 1: Equations and Inequalities

90. $3x(x^2 + 2x)^{1/2} - 2(x^2 + 2x)^{3/2} = 0$

$$(x^2 + 2x)^{1/2} [3x - 2(x^2 + 2x)] = 0$$

$$(x^2 + 2x)^{1/2} (3x - 2x^2 - 4x) = 0$$

$$(x^2 + 2x)^{1/2} (-2x^2 - x) = 0$$

$$(x^2 + 2x)^{1/2} = 0 \quad \text{or} \quad -2x^2 - x = 0$$

$$x^2 + 2x = 0 \quad \text{or} \quad 2x^2 + x = 0$$

$$x(x + 2) = 0 \quad \text{or} \quad x(2x + 1) = 0$$

$$x = 0 \text{ or } x = -2 \quad \text{or} \quad x = 0 \quad \text{or} \quad x = -\frac{1}{2}$$

Check $x = 0$:

$$3 \cdot 0(0^2 + 2 \cdot 0)^{1/2} - 2(0^2 + 2 \cdot 0)^{3/2} = 0$$

$$3 \cdot 0(0)^{1/2} - 2(0)^{3/2} = 0$$

$$0 = 0$$

Check $x = -2$:

$$3(-2)((-2)^2 + 2(-2))^{1/2} - 2((-2)^2 + 2(-2))^{3/2} = 0$$

$$3(-2)(4 - 4)^{1/2} - 2(4 - 4)^{3/2} = 0$$

$$3(-2)(0)^{1/2} - 2(0)^{3/2} = 0$$

$$3(-2)(0) - 2(0) = 0$$

$$0 = 0$$

Check $x = -\frac{1}{2}$:

$$3\left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)\right)^{1/2} - 2\left(\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)\right)^{3/2} = 0$$

$$3\left(-\frac{1}{2}\right)\left(\frac{1}{4} - 1\right)^{1/2} - 2\left(\frac{1}{4} - 1\right)^{3/2} = 0$$

$$3\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{1/2} - 2\left(-\frac{3}{4}\right)^{3/2} = 0$$

Not real

The solution set is $\{-2, 0\}$.

91. $x - 4x^{1/2} + 2 = 0$

Let $u = x^{1/2}$ so that $u^2 = x^2$.

$$u^2 - 4u + 2 = 0$$

$$u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$$u = 2 + \sqrt{2} \quad \text{or} \quad u = 2 - \sqrt{2}$$

$$x^{1/2} = 2 + \sqrt{2} \quad \text{or} \quad x^{1/2} = 2 - \sqrt{2}$$

$$(x^{1/2})^2 = (2 + \sqrt{2})^2 \quad \text{or} \quad (x^{1/2})^2 = (2 - \sqrt{2})^2$$

$$x = (2 + \sqrt{2})^2 \quad \text{or} \quad x = (2 - \sqrt{2})^2$$

Check $x = (2 + \sqrt{2})^2$:

$$(2 + \sqrt{2})^2 - 4(2 + \sqrt{2}) + 2 = 0$$

$$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 = 0$$

$$0 = 0$$

Check $x = (2 - \sqrt{2})^2$:

$$(2 - \sqrt{2})^2 - 4(2 - \sqrt{2}) + 2 = 0$$

$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 = 0$$

$$0 = 0$$

The solution set is

$$\left\{(2 - \sqrt{2})^2, (2 + \sqrt{2})^2\right\} \approx \{0.34, 11.66\}.$$

92. $x^{2/3} + 4x^{1/3} + 2 = 0$

Let $u = x^{1/3}$ so that $u^2 = x^{2/3}$.

$$u^2 + 4u + 2 = 0$$

$$u = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

$$u = -2 + \sqrt{2} \quad \text{or} \quad u = -2 - \sqrt{2}$$

$$x^{1/3} = -2 + \sqrt{2} \quad \text{or} \quad x^{1/3} = -2 - \sqrt{2}$$

$$x = (-2 + \sqrt{2})^3 \quad \text{or} \quad x = (-2 - \sqrt{2})^3$$

Check $x = (-2 + \sqrt{2})^3$:

$$\left((-2 + \sqrt{2})^3\right)^{2/3} + 4\left((-2 + \sqrt{2})^3\right)^{1/3} + 2 = 0$$

$$(-2 + \sqrt{2})^2 + 4(-2 + \sqrt{2}) + 2 = 0$$

$$4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2 = 0$$

$$0 = 0$$

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

Check $x = (-2 - \sqrt{2})^3$:

$$\begin{aligned} \left((-2 - \sqrt{2})^3 \right)^{2/3} + 4 \left((-2 - \sqrt{2})^3 \right)^{1/3} + 2 &= 0 \\ (-2 - \sqrt{2})^2 + 4(-2 - \sqrt{2}) + 2 &= 0 \\ 4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2 &= 0 \\ 0 &= 0 \end{aligned}$$

The solution set is

$$\left\{ (-2 - \sqrt{2})^3, (-2 + \sqrt{2})^3 \right\} \approx \{-39.80, -0.20\}.$$

93. $x^4 + \sqrt{3}x^2 - 3 = 0$

Let $u = x^2$ so that $u^2 = x^4$.

$$u^2 + \sqrt{3}u - 3 = 0$$

$$u = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4(1)(-3)}}{2(1)} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

$$u = \frac{-\sqrt{3} + \sqrt{15}}{2} \quad \text{or} \quad u = \frac{-\sqrt{3} - \sqrt{15}}{2}$$

$$x^2 = \frac{-\sqrt{3} + \sqrt{15}}{2} \quad \text{or} \quad x^2 = \frac{-\sqrt{3} - \sqrt{15}}{2}$$

$$x = \pm \sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \quad \text{or} \quad x = \pm \sqrt{\frac{-\sqrt{3} - \sqrt{15}}{2}}$$

Not real

Check $x = \sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}$:

$$\left(\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \right)^4 + \sqrt{3} \left(\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \right)^2 - 3 = 0$$

$$\left(\frac{-\sqrt{3} + \sqrt{15}}{2} \right)^2 + \sqrt{3} \left(\frac{-\sqrt{3} + \sqrt{15}}{2} \right) - 3 = 0$$

$$\frac{3 - 2\sqrt{3}\sqrt{15} + 15}{4} + \frac{\sqrt{3}(-\sqrt{3}) + \sqrt{3}\sqrt{15}}{2} - 3 = 0$$

$$\frac{18 - 2\sqrt{45}}{4} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$

$$\frac{9 - \sqrt{45}}{2} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$

$$\frac{9 - \sqrt{45} - 3 + \sqrt{45}}{2} - 3 = 0$$

$$3 - 3 = 0$$

$$0 = 0$$

Check $x = -\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}$:

$$\left(-\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \right)^4 + \sqrt{3} \left(-\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \right)^2 - 3 = 0$$

$$\left(\frac{-\sqrt{3} + \sqrt{15}}{2} \right)^2 + \sqrt{3} \left(\frac{-\sqrt{3} + \sqrt{15}}{2} \right) - 3 = 0$$

$$\frac{3 - 2\sqrt{3}\sqrt{15} + 15}{4} + \frac{\sqrt{3}(-\sqrt{3}) + \sqrt{3}\sqrt{15}}{2} - 3 = 0$$

$$\frac{18 - 2\sqrt{45}}{4} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$

$$\frac{9 - \sqrt{45}}{2} + \frac{-3 + \sqrt{45}}{2} - 3 = 0$$

$$\frac{9 - \sqrt{45} - 3 + \sqrt{45}}{2} - 3 = 0$$

$$3 - 3 = 0$$

$$0 = 0$$

The solution set is

$$\left\{ -\sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}}, \sqrt{\frac{-\sqrt{3} + \sqrt{15}}{2}} \right\} \approx \{-1.03, 1.03\}.$$

94. $x^4 + \sqrt{2}x^2 - 2 = 0$

Let $u = x^2$ so that $u^2 = x^4$.

$$u^2 + \sqrt{2}u - 2 = 0$$

$$u = \frac{-\sqrt{2} \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

$$u = \frac{-\sqrt{2} + \sqrt{10}}{2} \quad \text{or} \quad u = \frac{-\sqrt{2} - \sqrt{10}}{2}$$

$$x^2 = \frac{-\sqrt{2} + \sqrt{10}}{2} \quad \text{or} \quad x^2 = \frac{-\sqrt{2} - \sqrt{10}}{2}$$

$$x = \pm \sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}} \quad \text{or} \quad x = \pm \sqrt{\frac{-\sqrt{2} - \sqrt{10}}{2}}$$

Not real

Check $x = \sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}$:

$$\left(\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}} \right)^4 + \sqrt{2} \left(\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}} \right)^2 - 2 = 0$$

$$\left(\frac{-\sqrt{2} + \sqrt{10}}{2} \right)^2 + \sqrt{2} \left(\frac{-\sqrt{2} + \sqrt{10}}{2} \right) - 2 = 0$$

$$\frac{12 - 2\sqrt{20}}{4} + \frac{-2 + \sqrt{20}}{2} - 2 = 0$$

$$\frac{6 - \sqrt{20} - 2 + \sqrt{20}}{2} - 2 = 0$$

$$2 - 2 = 0$$

$$0 = 0$$

Chapter 1: Equations and Inequalities

Check $x = -\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}$:

$$\begin{aligned} \left(-\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}\right)^4 + \sqrt{2}\left(-\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}\right)^2 - 2 &= 0 \\ \left(\frac{-\sqrt{2} + \sqrt{10}}{2}\right)^2 + \sqrt{2}\left(\frac{-\sqrt{2} + \sqrt{10}}{2}\right) - 2 &= 0 \\ \frac{12 - 2\sqrt{20}}{4} + \frac{-2 + \sqrt{20}}{2} - 2 &= 0 \\ \frac{6 - \sqrt{20} - 2 + \sqrt{20}}{2} - 2 &= 0 \\ 2 - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

The solution set is

$$\left\{-\sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}, \sqrt{\frac{-\sqrt{2} + \sqrt{10}}{2}}\right\} \approx \{-0.93, 0.93\}.$$

95. $\pi(1+t)^2 = \pi + 1 + t$

Let $u = 1 + t$ so that $u^2 = (1+t)^2$.

$$\begin{aligned} \pi u^2 &= \pi + u \\ \pi u^2 - u - \pi &= 0 \\ u &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)} = \frac{1 \pm \sqrt{1 + 4\pi^2}}{2\pi} \\ 1 + t &= \frac{1 \pm \sqrt{1 + 4\pi^2}}{2\pi} \\ t &= -1 + \frac{1 \pm \sqrt{1 + 4\pi^2}}{2\pi} \end{aligned}$$

Check $t = -1 + \frac{1 + \sqrt{1 + 4\pi^2}}{2\pi}$:

$$\begin{aligned} \pi \left(\frac{1 + \sqrt{1 + 4\pi^2}}{2\pi}\right)^2 &= \pi + \frac{1 + \sqrt{1 + 4\pi^2}}{2\pi} \\ \pi \left(\frac{1 + 2\sqrt{1 + 4\pi^2} + 1 + 4\pi^2}{4\pi^2}\right) &= \pi + \frac{1 + \sqrt{1 + 4\pi^2}}{2\pi} \\ \frac{2 + 2\sqrt{1 + 4\pi^2} + 4\pi^2}{4\pi} &= \frac{2\pi^2 + 1 + \sqrt{1 + 4\pi^2}}{2\pi} \\ \frac{1 + \sqrt{1 + 4\pi^2} + 2\pi^2}{2\pi} &= \frac{2\pi^2 + 1 + \sqrt{1 + 4\pi^2}}{2\pi} \end{aligned}$$

Check $t = -1 + \frac{1 - \sqrt{1 + 4\pi^2}}{2\pi}$:

$$\begin{aligned} \pi \left(\frac{1 - \sqrt{1 + 4\pi^2}}{2\pi}\right)^2 &= \pi + \frac{1 - \sqrt{1 + 4\pi^2}}{2\pi} \\ \pi \left(\frac{1 - 2\sqrt{1 + 4\pi^2} + 1 + 4\pi^2}{4\pi^2}\right) &= \pi + \frac{1 - \sqrt{1 + 4\pi^2}}{2\pi} \\ \frac{2 - 2\sqrt{1 + 4\pi^2} + 4\pi^2}{4\pi} &= \frac{2\pi^2 + 1 - \sqrt{1 + 4\pi^2}}{2\pi} \\ \frac{1 - \sqrt{1 + 4\pi^2} + 2\pi^2}{2\pi} &= \frac{2\pi^2 + 1 - \sqrt{1 + 4\pi^2}}{2\pi} \end{aligned}$$

The solution set is

$$\begin{aligned} &\left\{-1 + \frac{1 - \sqrt{1 + 4\pi^2}}{2\pi}, -1 + \frac{1 + \sqrt{1 + 4\pi^2}}{2\pi}\right\} \\ &\approx \{-1.85, 0.17\}. \end{aligned}$$

96. $\pi(1+r)^2 = 2 + \pi(1+r)$

Let $u = 1 + r$ so that $u^2 = (1+r)^2$.

$$\begin{aligned} \pi u^2 &= 2 + \pi u \\ \pi u^2 - \pi u - 2 &= 0 \\ u &= \frac{-(-\pi) \pm \sqrt{(-\pi)^2 + 4(\pi)(-2)}}{2(\pi)} \\ &= \frac{\pi \pm \sqrt{\pi^2 - 8\pi}}{2\pi} \\ 1 + r &= \frac{\pi \pm \sqrt{\pi^2 - 8\pi}}{2\pi} \\ r &= -1 + \frac{\pi \pm \sqrt{\pi^2 - 8\pi}}{2\pi} \end{aligned}$$

Check $r = -1 + \frac{\pi + \sqrt{\pi^2 - 8\pi}}{2\pi}$:

$$\begin{aligned} \pi \left(\frac{\pi + \sqrt{\pi^2 - 8\pi}}{2\pi}\right)^2 &= 2 + \pi \left(\frac{\pi + \sqrt{\pi^2 - 8\pi}}{2\pi}\right) \\ \pi \left(\frac{\pi^2 + 2\pi\sqrt{\pi^2 - 8\pi} + \pi^2 - 8\pi}{4\pi^2}\right) &= 2 + \pi \left(\frac{\pi + \sqrt{\pi^2 - 8\pi}}{2\pi}\right) \\ \frac{2\pi^2 + 2\pi\sqrt{\pi^2 - 8\pi} + \pi^2 - 8\pi}{4\pi} &= 2 + \frac{\pi + \sqrt{\pi^2 - 8\pi}}{2} \\ \frac{\pi + \sqrt{\pi^2 - 8\pi} + 4}{2} &= \frac{4 + \pi + \sqrt{\pi^2 - 8\pi}}{2} \end{aligned}$$

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

Check $r = -1 + \frac{\pi - \sqrt{\pi^2 + 8\pi}}{2\pi}$:

$$\begin{aligned} \pi \left(\frac{\pi - \sqrt{\pi^2 + 8\pi}}{2\pi} \right)^2 &= 2 + \pi \left(\frac{\pi - \sqrt{\pi^2 + 8\pi}}{2\pi} \right) \\ \pi \left(\frac{\pi^2 - 2\pi\sqrt{\pi^2 + 8\pi} + \pi^2 + 8\pi}{4\pi^2} \right) &= 2 + \pi \left(\frac{\pi - \sqrt{\pi^2 + 8\pi}}{2\pi} \right) \\ \frac{2\pi^2 - 2\pi\sqrt{\pi^2 + 8\pi} + 8\pi}{4\pi} &= 2 + \frac{\pi - \sqrt{\pi^2 + 8\pi}}{2} \\ \frac{\pi - \sqrt{\pi^2 + 8\pi} + 4}{2} &= \frac{4 + \pi - \sqrt{\pi^2 + 8\pi}}{2} \end{aligned}$$

The solution set is

$$\left\{ -1 + \frac{\pi - \sqrt{\pi^2 + 8\pi}}{2\pi}, -1 + \frac{\pi + \sqrt{\pi^2 + 8\pi}}{2\pi} \right\} \\ \approx \{-1.44, 0.44\}.$$

97. $3x^2 + 7x - 20 = 0$
 $(3x - 5)(x + 4) = 0$
 $3x - 5 = 0$ or $x + 4 = 0$
 $3x = 5$ $x = -4$
 $x = \frac{5}{3}$

The solution set is $\left\{ -4, \frac{5}{3} \right\}$.

98. $2x^2 - 13x + 21 = 0$
 $(2x - 7)(x - 3) = 0$
 $2x - 7 = 0$ or $x - 3 = 0$
 $2x = 7$ $x = 3$
 $x = \frac{7}{2}$

The solution set is $\left\{ \frac{7}{2}, 3 \right\}$.

99. $5a^3 - 45a = -2a^2 + 18$
 $5a^3 + 2a^2 - 45a - 18 = 0$
 $a^2(5a + 2) - 9(5a + 2) = 0$
 $(a^2 - 9)(5a + 2) = 0$
 $(a - 3)(a + 3)(5a + 2) = 0$
 $a - 3 = 0$ or $a + 3 = 0$ or $5a + 2 = 0$
 $a = 3$ $a = -3$ $5a = -2$
 $a = -\frac{2}{5}$

The solution set is $\left\{ -3, -\frac{2}{5}, 3 \right\}$.

100. $3z^3 - 12z = -5z^2 + 20$
 $3z^3 + 5z^2 - 12z - 20 = 0$
 $z^2(3z + 5) - 4(3z + 5) = 0$
 $(z^2 - 4)(3z + 5) = 0$
 $(z - 2)(z + 2)(3z + 5) = 0$
 $z - 2 = 0$ or $z + 2 = 0$ $3z + 5 = 0$
 $z = 2$ $z = -2$ $3z = -5$
 $z = -\frac{5}{3}$

The solution set is $\left\{ -2, -\frac{5}{3}, 2 \right\}$.

101. $4(w - 3) = w + 3$
 $4w - 12 = w + 3$
 $3w = 15$
 $w = 5$
The solution set is $\{5\}$.

102. $6(k + 3) - 2k = 12$
 $6k + 18 - 2k = 12$
 $4k = -6$
 $k = -\frac{3}{2}$
The solution set is $\left\{ -\frac{3}{2} \right\}$.

Chapter 1: Equations and Inequalities

103. $\left(\frac{v}{v+1}\right)^2 + \frac{2v}{v+1} = 8$

Let $u = \frac{v}{v+1}$. Rewrite the equation:

$$u^2 + 2u = 8$$

$$u^2 + 2u - 8 = 0$$

$$(u-2)(u+4) = 0$$

$$u = 2 \text{ or } u = -4$$

Go back in terms of v and solve:

$$\frac{v}{v+1} = 2 \quad \text{or} \quad \frac{v}{v+1} = -4$$

$$v = 2v + 2 \quad v = -4v - 4$$

$$-v = 2 \quad 5v = -4$$

$$v = -2 \quad v = -\frac{4}{5}$$

The solution set is $\left\{-2, -\frac{4}{5}\right\}$.

104. $\left(\frac{y}{y-1}\right)^2 - \frac{6y}{y-1} = 7$

Let $u = \frac{y}{y-1}$. Rewrite the equation:

$$u^2 - 6u = 7$$

$$u^2 - 6u - 7 = 0$$

$$(u-7)(u+1) = 0$$

$$u = 7 \text{ or } u = -1$$

Go back in terms of y and solve:

$$\frac{y}{y-1} = 7 \quad \text{or} \quad \frac{y}{y-1} = -1$$

$$y = 7y - 7 \quad y = -y + 1$$

$$-6y = -7 \quad 2y = 1$$

$$y = \frac{7}{6} \quad y = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}, \frac{7}{6}\right\}$.

105. $\sqrt{2x+5} - x = 1$

$$\sqrt{2x+5} = x+1$$

$$(\sqrt{2x+5})^2 = (x+1)^2$$

$$2x+5 = x^2 + 2x+1$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x = 2 \text{ or } x = -2$$

Check:

$$\sqrt{2(-2)+5} - (-2) = 1 \quad \sqrt{2(2)+5} - (2) = 1$$

$$\sqrt{1} + 2 = 1 \quad \sqrt{9} - 2 = 1$$

$$3 \neq 1 \quad 1 = 1 \quad \text{T}$$

The solution set is $\{2\}$.

106. $\sqrt{3x+1} - 2x = -6$

$$\sqrt{3x+1} = 2x-6$$

$$(\sqrt{3x+1})^2 = (2x-6)^2$$

$$3x+1 = 4x^2 - 24x + 36$$

$$4x^2 - 27x + 35 = 0$$

$$(4x-7)(x-5) = 0$$

$$4x-7 = 0 \quad \text{or} \quad x-5 = 0$$

$$4x = 7 \quad x = 5$$

$$x = \frac{7}{4}$$

Check:

$$\sqrt{3\left(\frac{7}{4}\right)+1} - 2\left(\frac{7}{4}\right) = -6 \quad \sqrt{3(5)+1} - 2(5) = -6$$

$$\frac{5}{2} - \frac{7}{2} = -6 \quad 4 - 10 = -6$$

$$-1 \neq -6$$

$$-6 = -6 \quad \text{T}$$

The solution set is $\{5\}$.

107. $3m^2 + 6m = -1$

$$3m^2 + 6m + 1 = 0$$

$$a = 3, b = 6, c = 1$$

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

$$m = \frac{-6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} = \frac{-6 \pm \sqrt{24}}{6}$$

$$= \frac{-6 \pm 2\sqrt{6}}{6} = \frac{-3 \pm \sqrt{6}}{3}$$

The solution set is $\left\{ \frac{-3 - \sqrt{6}}{3}, \frac{-3 + \sqrt{6}}{3} \right\}$.

108. $4y^2 - 8y = 3$

$$4y^2 - 8y - 3 = 0$$

$$a = 4, b = -8, c = -3$$

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-3)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{112}}{8} = \frac{8 \pm 4\sqrt{7}}{8} = \frac{2 \pm \sqrt{7}}{2}$$

The solution set is $\left\{ \frac{2 - \sqrt{7}}{2}, \frac{2 + \sqrt{7}}{2} \right\}$.

109. $\sqrt[4]{5x^2 - 6} = x$

$$5x^2 - 6 = x^4$$

$$x^4 - 5x^2 + 6 = 0$$

$$(x^2 - 2)(x^2 - 3) = 0$$

$$x^2 - 2 = 0 \quad \text{or} \quad x^2 - 3 = 0$$

$$x^2 = 2 \quad \quad \quad x^2 = 3$$

$$x = \pm\sqrt{2} \quad \quad \quad x = \pm\sqrt{3}$$

Check:

$$\sqrt[4]{5(\sqrt{2})^2 - 6} = \sqrt[4]{5(2) - 6}$$

$$= \sqrt[4]{4} = \sqrt{2}$$

$$\sqrt[4]{5(-\sqrt{2})^2 - 6} = \sqrt[4]{5(2) - 6}$$

$$= \sqrt[4]{4} = \sqrt{2} \neq -\sqrt{2}$$

$$\sqrt[4]{5(\sqrt{3})^2 - 6} = \sqrt[4]{5(3) - 6}$$

$$= \sqrt[4]{9} = \sqrt{3}$$

$$\sqrt[4]{5(-\sqrt{3})^2 - 6} = \sqrt[4]{5(3) - 6}$$

$$= \sqrt[4]{9} = \sqrt{3} \neq -\sqrt{3}$$

The solution set is $\{\sqrt{2}, \sqrt{3}\}$.

110. $\sqrt[4]{4 - 3x^2} = x$

$$4 - 3x^2 = x^4$$

$$x^4 + 3x^2 - 4 = 0$$

$$(x^2 - 1)(x^2 + 4) = 0$$

$$x^2 - 1 = 0 \quad \text{or} \quad x^2 + 4 = 0$$

$$x^2 = 1 \quad \quad \quad x^2 = -4$$

$$x = \pm 1$$

no real
solution

Check:

$$\sqrt[4]{4 - 3(1)^2} = \sqrt[4]{4 - 3} \quad \sqrt[4]{4 - 3(-1)^2} = \sqrt[4]{4 - 3}$$

$$= \sqrt[4]{1} = 1 \quad \quad \quad = \sqrt[4]{1} = 1 \neq -1$$

The solution set is $\{1\}$.

111. $k^2 - k = 12$

$$k^2 - k - 12 = 0$$

$$(k - 4)(k + 3) = 0$$

$$k = 4 \quad \text{or} \quad k = -3$$

$$\frac{x+3}{x-3} = 4 \quad \text{or} \quad \frac{x+3}{x-3} = -3$$

$$x+3 = 4x-12 \quad \text{or} \quad x+3 = -3x+9$$

$$3x = 15 \quad \text{or} \quad 4x = 6$$

$$x = 5 \quad \text{or} \quad x = \frac{6}{4} = \frac{3}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{ \frac{3}{2}, 5 \right\}$.

112. $k^2 - 3k = 28$

$$k^2 - 3k - 28 = 0$$

$$(k + 4)(k - 7) = 0$$

$$k = -4 \quad \text{or} \quad k = 7$$

$$\frac{x+3}{x-4} = -4 \quad \text{or} \quad \frac{x+3}{x-4} = 7$$

$$x+3 = -4x+16 \quad \text{or} \quad x+3 = 7x-28$$

$$5x = 13 \quad \text{or} \quad -6x = -31$$

$$x = \frac{13}{5} = 2.6 \quad \text{or} \quad x = \frac{31}{6} \approx 5.17$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{ \frac{13}{5}, \frac{31}{6} \right\}$.

Chapter 1: Equations and Inequalities

113. Solve the equation $\frac{\sqrt{s}}{4} + \frac{s}{1100} = 4$.

$$\frac{s}{1100} + \frac{\sqrt{s}}{4} - 4 = 0$$

$$(1100)\left(\frac{s}{1100} + \frac{\sqrt{s}}{4} - 4\right) = (0)(1100)$$

$$s + 275\sqrt{s} - 4400 = 0$$

Let $u = \sqrt{s}$, so that $u^2 = s$.

$$u^2 + 275u - 4400 = 0$$

$$u = \frac{-275 \pm \sqrt{275^2 - 4(1)(-4400)}}{2}$$

$$= \frac{-275 \pm \sqrt{93,225}}{2}$$

$$u \approx 15.1638 \text{ or } u \approx -290.1638$$

Since $u = \sqrt{s}$, it must be positive, so

$$s = u^2 \approx (15.1638)^2 \approx 229.94$$

The distance to the water's surface is approximately 229.94 feet.

114. $T = \sqrt[4]{\frac{LH^2}{25}}$

Let $T = 4$ and $H = 10$, and solve for L .

$$4 = \sqrt[4]{\frac{L(10)^2}{25}}$$

$$4 = \sqrt[4]{4L}$$

$$(4)^4 = (\sqrt[4]{4L})^4$$

$$256 = 4L$$

$$64 = L$$

The crushing load is 64 tons.

115. $T = 2\pi\sqrt{\frac{l}{32}}$

Let $T = 16.5$ and solve for l .

$$16.5 = 2\pi\sqrt{\frac{l}{32}}$$

$$\frac{16.5}{2\pi} = \sqrt{\frac{l}{32}}$$

$$\left(\frac{16.5}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{32}}\right)^2$$

$$\left(\frac{16.5}{2\pi}\right)^2 = \frac{l}{32}$$

$$l = 32\left(\frac{16.5}{2\pi}\right)^2 \approx 220.7$$

The length was approximately 220.7 feet.

116. $\sqrt{3x+5} - \sqrt{x-2} = \sqrt{x+3}$

$$(\sqrt{3x+5} - \sqrt{x-2})^2 = (\sqrt{x+3})^2$$

$$3x+5 - 2\sqrt{(3x+5)(x-2)} + (x-2) = x+3$$

$$4x+3 - 2\sqrt{(3x+5)(x-2)} = x+3$$

$$2\sqrt{3x^2 - x - 10} = 3x$$

$$4(3x^2 - x - 10) = 9x^2$$

$$3x^2 - 4x - 40 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-40)}}{6}$$

$$= \frac{4 \pm \sqrt{496}}{6} = \frac{4 \pm 4\sqrt{31}}{6}$$

$$= \frac{4 \pm 4\sqrt{31}}{6}$$

$$= \frac{2 \pm 2\sqrt{31}}{3}$$

Since $x > 2$, the negative solution is extraneous.

The solution set is $\left\{\frac{2 + 2\sqrt{31}}{3}\right\}$.

Section 1.4: Radical Equations; Equations Quadratic in Form; Factorable Equations

117.

$$\begin{aligned}\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18} &= 2 \\ \left(\sqrt[4]{\sqrt[3]{\sqrt{x-7}-10}+18}\right)^4 &= (2)^4 \\ \sqrt[3]{\sqrt{x-7}-10}+18 &= 16 \\ \left(\sqrt[3]{\sqrt{x-7}-10}\right)^3 &= (-2)^3 \\ \sqrt{x-7}-10 &= -8 \\ (\sqrt{x-7})^2 &= (2)^2 \\ x-7 &= 4 \\ x &= 11\end{aligned}$$

The solution set is $\{11\}$

118.

$$\begin{aligned}12x^{2/3} + 3x^{2/3} &= 13x^{2/3} \\ 12x^{14/10} + 3x^{2/10} - 13x^{2/10} &= 0 \\ x^{2/10}(12x - 13x^{1/2} + 3) &= 0 \\ x^{2/10} = 0 &\Rightarrow x = 0 \\ \text{To solve } 12x - 13x^{1/2} + 3 = 0, &\text{ let } u = x^{1/2} \\ \text{Then } 12u^2 - 13u + 3 &= 0 \\ (4u-3)(3u-1) &= 0 \\ u = x^{1/2} = \frac{3}{4} \text{ or } x^{1/2} &= \frac{1}{3} \\ x = \frac{9}{16} \quad x = \frac{1}{9}\end{aligned}$$

The solution set is $\left\{0, \frac{9}{16}, \frac{1}{9}\right\}$

119.

$$\begin{aligned}z^6 + 28z^3 + 27 &= 0 \\ (z^3 + 27)(z^3 + 1) &= 0 \\ (z+3)(z^2-3z+9)(z+1)(z^2-z+1) &= 0 \\ z+3=0 \quad \text{or } z+1=0 \\ z=-3 \quad \text{or } z=-1\end{aligned}$$

$$a=1, b=-3, c=9$$

$$\begin{aligned}x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2} \\ &= \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3\sqrt{3}i}{2}\end{aligned}$$

Also,

$$a=1, b=-1, c=1$$

$$\begin{aligned}x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2} \\ &= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}\end{aligned}$$

The solution set is $\left\{-3, -1, \frac{1 \pm \sqrt{3}i}{2}, \frac{3 \pm 3\sqrt{3}i}{2}\right\}$.

120. Answers will vary. One example: $\sqrt{x+1} = -1$.

121. Answers will vary. One example: $x - \sqrt{x} - 2 = 0$.

122. Answers will vary.

123. Mya did not check her solutions and included the extraneous solution, $x = -1$.

$$\begin{aligned}\sqrt{2x+3} - x &= 0 \\ \sqrt{2x+3} &= x \\ (\sqrt{2x+3})^2 &= x^2 \\ 2x+3 &= x^2 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x=3 \text{ or } x &= -1\end{aligned}$$

Check:

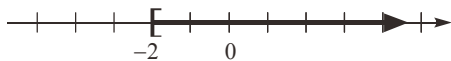
$$\begin{array}{ll}\sqrt{2(3)+3}-3=0 & \sqrt{2(-1)+3}-(-1)=0 \\ \sqrt{9}-3=0 & \sqrt{1}+1=0 \\ 3-3=0 & 1+1=0 \\ 0=0 \quad \text{T} & 2 \neq 0\end{array}$$

The solution set is $\{3\}$.

Chapter 1: Equations and Inequalities

Section 1.5

1. $x \geq -2$



2. False.

3. closed interval

4. multiplication properties (for inequalities)

5. True. This follows from the addition property for inequalities.

6. True. This follows from the addition property for inequalities.

7. True;. This follows from the multiplication property for inequalities.

8. False. Since both sides of the inequality are being divided by a negative number, the sense, or direction, of the inequality must be reversed.

That is, $\frac{a}{c} > \frac{b}{c}$.

9. True

10. False; either or both endpoints could be any real number.

11. d

12. c

13. Interval: $[0, 2]$
Inequality: $0 \leq x \leq 2$

14. Interval: $(-1, 2)$
Inequality: $-1 < x < 2$

15. Interval: $[2, \infty)$
Inequality: $x \geq 2$

16. Interval: $(-\infty, 0]$
Inequality: $x \leq 0$

17. Interval: $[0, 3)$
Inequality: $0 \leq x < 3$

18. Interval: $(-1, 1]$
Inequality: $-1 < x \leq 1$

19. a. $3 < 5$

$$3 + 3 < 5 + 3$$

$$6 < 8$$

b. $3 < 5$

$$3 - 5 < 5 - 5$$

$$-2 < 0$$

c. $3 < 5$

$$3(3) < 3(5)$$

$$9 < 15$$

d. $3 < 5$

$$-2(3) > -2(5)$$

$$-6 > -10$$

20. a. $2 > 1$

$$2 + 3 > 1 + 3$$

$$5 > 4$$

b. $2 > 1$

$$2 - 5 > 1 - 5$$

$$-3 > -4$$

c. $2 > 1$

$$3(2) > 3(1)$$

$$6 > 3$$

d. $2 > 1$

$$-2(2) < -2(1)$$

$$-4 < -2$$

21. a. $4 > -3$

$$4 + 3 > -3 + 3$$

$$7 > 0$$

b. $4 > -3$

$$4 - 5 > -3 - 5$$

$$-1 > -8$$

c. $4 > -3$

$$3(4) > 3(-3)$$

$$12 > -9$$

d. $4 > -3$

$$-2(4) < -2(-3)$$

$$-8 < 6$$

22. a. $-3 > -5$
 $-3 + 3 > -5 + 3$
 $0 > -2$

b. $-3 > -5$
 $-3 - 5 > -5 - 5$
 $-8 > -10$

c. $-3 > -5$
 $3(-3) > 3(-5)$
 $-9 > -15$

d. $-3 > -5$
 $-2(-3) < -2(-5)$
 $6 < 10$

23. a. $2x + 1 < 2$
 $2x + 1 + 3 < 2 + 3$
 $2x + 4 < 5$

b. $2x + 1 < 2$
 $2x + 1 - 5 < 2 - 5$
 $2x - 4 < -3$

c. $2x + 1 < 2$
 $3(2x + 1) < 3(2)$
 $6x + 3 < 6$

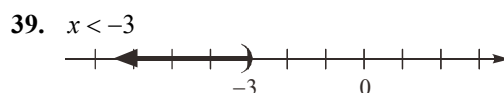
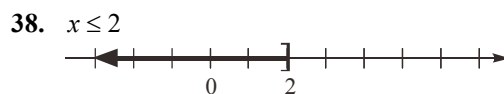
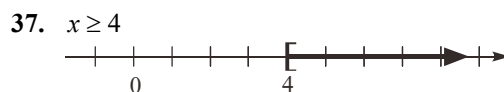
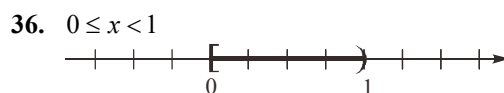
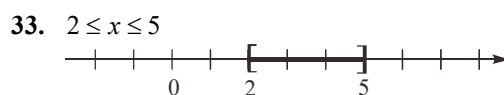
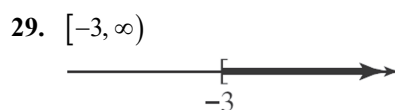
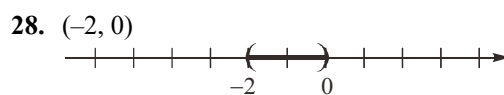
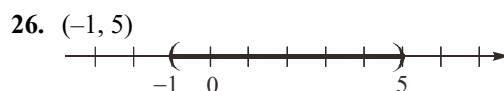
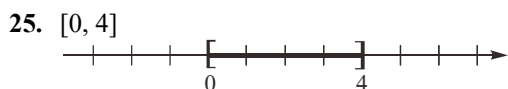
d. $2x + 1 < 2$
 $-2(2x + 1) > -2(2)$
 $-4x - 2 > -4$

24. a. $1 - 2x > 5$
 $1 - 2x + 3 > 5 + 3$
 $4 - 2x > 8$

b. $1 - 2x > 5$
 $1 - 2x - 5 > 5 - 5$
 $-4 - 2x > 0$

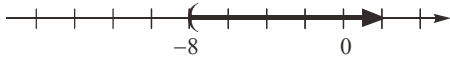
c. $1 - 2x > 5$
 $3(1 - 2x) > 3(5)$
 $3 - 6x > 15$

d. $1 - 2x > 5$
 $-2(1 - 2x) < -2(5)$
 $-2 + 4x < -10$



Chapter 1: Equations and Inequalities

40. $x > -8$



41. If $x < 5$, then $x - 5 < 0$.

42. If $x < -4$, then $x + 4 < 0$.

43. If $x > -4$, then $x + 4 > 0$.

44. If $x > 6$, then $x - 6 > 0$.

45. If $x \geq -4$, then $3x \geq -12$.

46. If $x \leq 3$, then $2x \leq 6$.

47. If $x > 6$, then $-2x < -12$.

48. If $x > -2$, then $-4x < 8$.

49. If $x \geq 5$, then $-4x \leq -20$.

50. If $x \leq -4$, then $-3x \geq 12$.

51. If $8x > 40$, then $x > 5$.

52. If $3x \leq 12$, then $x \leq 4$.

53. If $-\frac{1}{2}x \leq 3$, then $x \geq -6$.

54. If $-\frac{1}{4}x > 1$, then $x < -4$.

55. If $0 < 5 < x$, then $0 < \frac{1}{x} < \frac{1}{5}$

56. $0 < -4 < x$, then $\frac{1}{-4} \leq \frac{1}{x} < 0$

57. $-5 < x < 0$, then $\frac{1}{x} < \frac{1}{-5} < 0$

58. $0 < x < 10$, then $0 < \frac{1}{10} \leq \frac{1}{x}$

59. $x + 1 < 5$

$$x + 1 - 1 < 5 - 1$$

$$x < 4$$

$$\{x \mid x < 4\} \text{ or } (-\infty, 4)$$



60. $x - 6 < 1$

$$x - 6 + 6 < 1 + 6$$

$$x < 7$$

$$\text{The solution set is } \{x \mid x < 7\} \text{ or } (-\infty, 7).$$



61. $3 - 5x \leq -7$

$$-5x \leq -10$$

$$x \geq 2$$

$$\text{The solution set is } \{x \mid x \geq 2\} \text{ or } [2, \infty).$$

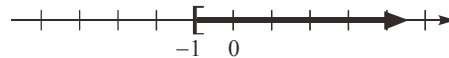


62. $2 - 3x \leq 5$

$$-3x \leq 3$$

$$x \geq -1$$

$$\text{The solution set is } \{x \mid x \geq -1\} \text{ or } [-1, \infty).$$



63. $3x - 7 > 2$

$$3x > 9$$

$$x > 3$$

$$\text{The solution set is } \{x \mid x > 3\} \text{ or } (3, \infty).$$



64. $2x + 5 > 1$

$$2x > -4$$

$$x > -2$$

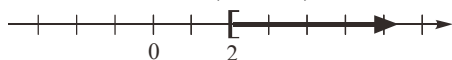
$$\text{The solution set is } \{x \mid x > -2\} \text{ or } (-2, \infty).$$



65. $3x - 1 \geq 3 + x$

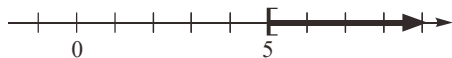
$2x \geq 4$

$x \geq 2$

The solution set is $\{x \mid x \geq 2\}$ or $[2, \infty)$.

66. $2x - 2 \geq 3 + x$

$x \geq 5$

The solution set is $\{x \mid x \geq 5\}$ or $[5, \infty)$.

67. $-2(x + 3) < 8$

$-2x - 6 < 8$

$-2x < 14$

$x > -7$

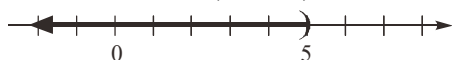
The solution set is $\{x \mid x > -7\}$ or $(-7, \infty)$.

68. $-3(1 - x) < 12$

$-3 + 3x < 12$

$3x < 15$

$x < 5$

The solution set is $\{x \mid x < 5\}$ or $(-\infty, 5)$.

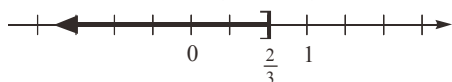
69. $4 - 3(1 - x) \leq 3$

$4 - 3 + 3x \leq 3$

$3x + 1 \leq 3$

$3x \leq 2$

$x \leq \frac{2}{3}$

The solution set is $\{x \mid x \leq \frac{2}{3}\}$ or $(-\infty, \frac{2}{3}]$.

70. $8 - 4(2 - x) \leq -2x$

$8 - 8 + 4x \leq -2x$

$4x \leq -2x$

$6x \leq 0$

$x \leq 0$

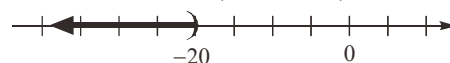
The solution set is $\{x \mid x \leq 0\}$ or $(-\infty, 0]$.

71. $\frac{1}{2}(x - 4) > x + 8$

$\frac{1}{2}x - 2 > x + 8$

$-\frac{1}{2}x > 10$

$x < -20$

The solution set is $\{x \mid x < -20\}$ or $(-\infty, -20)$.

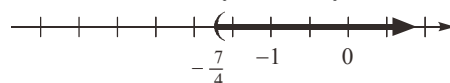
72. $3x + 4 > \frac{1}{3}(x - 2)$

$3x + 4 > \frac{1}{3}x - \frac{2}{3}$

$9x + 12 > x - 2$

$8x > -14$

$x > -\frac{7}{4}$

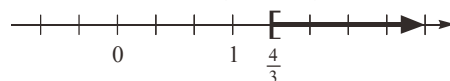
The solution set is $\{x \mid x > -\frac{7}{4}\}$ or $(-\frac{7}{4}, \infty)$.

73. $\frac{x}{2} \geq 1 - \frac{x}{4}$

$2x \geq 4 - x$

$3x \geq 4$

$x \geq \frac{4}{3}$

The solution set is $\{x \mid x \geq \frac{4}{3}\}$ or $[\frac{4}{3}, \infty)$.

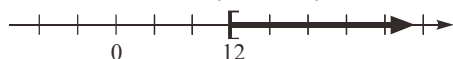
Chapter 1: Equations and Inequalities

74. $\frac{x}{3} \geq 2 + \frac{x}{6}$

$$2x \geq 12 + x$$

$$x \geq 12$$

The solution set is $\{x \mid x \geq 12\}$ or $[12, \infty)$.



75. $0 < 3x - 7 \leq 5$

$$7 < 3x \leq 12$$

$$\frac{7}{3} < x \leq 4$$

The solution set is $\left\{x \mid \frac{7}{3} < x \leq 4\right\}$ or $\left(\frac{7}{3}, 4\right]$.



76. $4 \leq 2x + 2 \leq 10$

$$2 \leq 2x \leq 8$$

$$1 \leq x \leq 4$$

The solution set is $\{x \mid 1 \leq x \leq 4\}$ or $[1, 4]$.

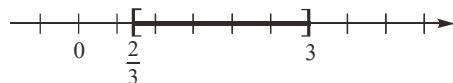


77. $-5 \leq 4 - 3x \leq 2$

$$-9 \leq -3x \leq -2$$

$$3 \geq x \geq \frac{2}{3}$$

The solution set is $\left\{x \mid \frac{2}{3} \leq x \leq 3\right\}$ or $\left[\frac{2}{3}, 3\right]$.



78. $-3 \leq 3 - 2x \leq 9$

$$-6 \leq -2x \leq 6$$

$$3 \geq x \geq -3$$

The solution set is $\{x \mid -3 \leq x \leq 3\}$ or $[-3, 3]$.



79. $-3 < \frac{2x-1}{4} < 0$

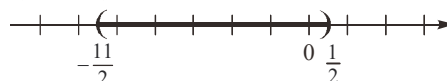
$$-12 < 2x - 1 < 0$$

$$-11 < 2x < 1$$

$$-\frac{11}{2} < x < \frac{1}{2}$$

The solution set is $\left\{x \mid -\frac{11}{2} < x < \frac{1}{2}\right\}$ or

$$\left(-\frac{11}{2}, \frac{1}{2}\right).$$



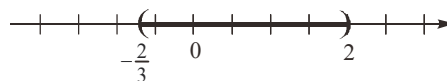
80. $0 < \frac{3x+2}{2} < 4$

$$0 < 3x + 2 < 8$$

$$-2 < 3x < 6$$

$$-\frac{2}{3} < x < 2$$

The solution set is $\left\{x \mid -\frac{2}{3} < x < 2\right\}$ or $\left(-\frac{2}{3}, 2\right)$.

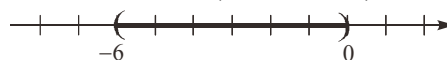


81. $1 < 1 - \frac{1}{2}x < 4$

$$0 < -\frac{1}{2}x < 3$$

$$0 > x > -6 \quad \text{or} \quad -6 < x < 0$$

The solution set is $\{x \mid -6 < x < 0\}$ or $(-6, 0)$.



82. $0 < 1 - \frac{1}{3}x < 1$

$$-1 < -\frac{1}{3}x < 0$$

$$3 > x > 0 \quad \text{or} \quad 0 < x < 3$$

The solution set is $\{x \mid 0 < x < 3\}$ or $(0, 3)$.



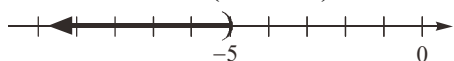
83. $(x+2)(x-3) > (x-1)(x+1)$

$$x^2 - x - 6 > x^2 - 1$$

$$-x - 6 > -1$$

$$-x > 5$$

$$x < -5$$

The solution set is $\{x \mid x < -5\}$ or $(-\infty, -5)$.

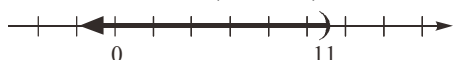
84. $(x-1)(x+1) > (x-3)(x+4)$

$$x^2 - 1 > x^2 + x - 12$$

$$-1 > x - 12$$

$$-x > -11$$

$$x < 11$$

The solution set is $\{x \mid x < 11\}$ or $(-\infty, 11)$.

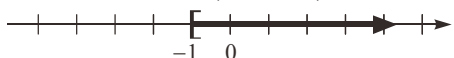
85. $x(4x+3) \leq (2x+1)^2$

$$4x^2 + 3x \leq 4x^2 + 4x + 1$$

$$3x \leq 4x + 1$$

$$-x \leq 1$$

$$x \geq -1$$

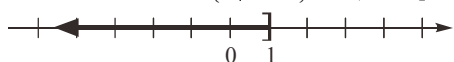
The solution set is $\{x \mid x \geq -1\}$ or $[-1, \infty)$.

86. $x(9x-5) \leq (3x-1)^2$

$$9x^2 - 5x \leq 9x^2 - 6x + 1$$

$$-5x \leq -6x + 1$$

$$x \leq 1$$

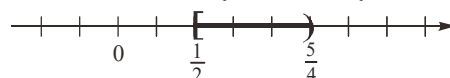
The solution set is $\{x \mid x \leq 1\}$ or $(-\infty, 1]$.

87. $\frac{1}{2} \leq \frac{x+1}{3} < \frac{3}{4}$

$$6 \leq 4x + 4 < 9$$

$$2 \leq 4x < 5$$

$$\frac{1}{2} \leq x < \frac{5}{4}$$

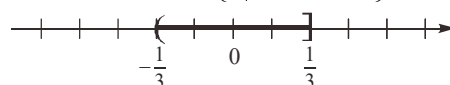
The solution set is $\left\{x \mid \frac{1}{2} \leq x < \frac{5}{4}\right\}$ or $\left[\frac{1}{2}, \frac{5}{4}\right)$.

88. $\frac{1}{3} < \frac{x+1}{2} \leq \frac{2}{3}$

$$2 < 3x + 3 \leq 4$$

$$-1 < 3x \leq 1$$

$$-\frac{1}{3} < x \leq \frac{1}{3}$$

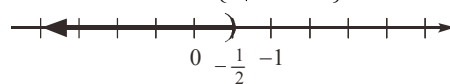
The solution set is $\left\{x \mid -\frac{1}{3} < x \leq \frac{1}{3}\right\}$ or $\left(-\frac{1}{3}, \frac{1}{3}\right]$.

89. $(4x+2)^{-1} < 0$

$$\frac{1}{4x+2} < 0$$

$$4x+2 < 0$$

$$x < -\frac{1}{2}$$

The solution set is $\left\{x \mid x < -\frac{1}{2}\right\}$ or $(-\infty, -\frac{1}{2})$.

90. $(2x-1)^{-1} > 0$

$$\frac{1}{2x-1} > 0$$

Since $\frac{1}{2x-1} > 0$, this means $2x-1 > 0$.

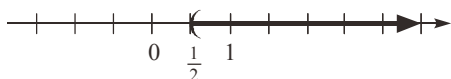
Therefore,

$$2x-1 > 0$$

$$x > \frac{1}{2}$$

Chapter 1: Equations and Inequalities

The solution set is $\left\{x \mid x > \frac{1}{2}\right\}$ or $\left(\frac{1}{2}, \infty\right)$.



91. $(1-4x)^{-1} \geq 7$

$$\frac{1}{1-4x} - 7 \geq 0$$

$$\frac{1-7(1-4x)}{1-4x} \geq 0$$

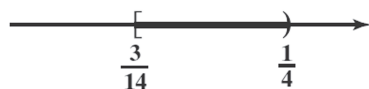
$$\frac{-6+28x}{1-4x} \geq 0$$

The zeros and values where the expression is undefined are $x = \frac{3}{14}$ and $x = \frac{1}{4}$

Interval	$(-\infty, \frac{3}{14})$	$(\frac{3}{14}, \frac{1}{4})$	$(\frac{1}{4}, \infty)$
Number Chosen	0	$\frac{22}{100}$	1
Value of f	-6	$\frac{4}{3}$	$-\frac{22}{3}$
Conclusion	Negative	Positive	Negative

We want to know where $f(x) \geq 0$, so the solution set is $\left\{x \mid x \geq \frac{3}{14} \text{ or } x < \frac{1}{4}\right\}$ or, using interval notation, $[\frac{3}{14}, \frac{1}{4})$. Note that $\frac{1}{4}$ is not in the solution set because $\frac{1}{4}$ is not in the domain of f .

The solution set is $[\frac{3}{14}, \frac{1}{4})$.



92. $2(3x+5)^{-1} \leq -3$

$$\frac{2}{(3x+5)} + 3 \leq 0$$

$$\frac{2+3(3x+5)}{2(3x+5)} \leq 0$$

$$\frac{17+9x}{(3x+5)} \leq 0$$

The zeros and values where the expression is undefined are $x = -\frac{17}{9}$ and $x = -\frac{5}{3}$

Interval	$(-\infty, -\frac{17}{9})$	$(-\frac{17}{9}, -\frac{5}{3})$	$(-\frac{5}{3}, \infty)$
Number Chosen	-2	-1.7	0
Value of f	1	-17	$\frac{17}{5}$
Conclusion	Positive	Negative	Positive

We want to know where $f(x) \leq 0$, so the solution set is $\left\{x \mid x \geq -\frac{17}{9} \text{ or } x < -\frac{5}{3}\right\}$ or, using interval notation, $[-\frac{17}{9}, -\frac{5}{3})$. Note that $-\frac{5}{3}$ is not in the solution set because $\frac{2}{7}$ is not in the domain of f .

The solution set is $[-\frac{17}{9}, -\frac{5}{3})$.



93. $0 < \frac{2}{x} < \frac{3}{5}$

$$0 < \frac{2}{x} \text{ and } \frac{2}{x} < \frac{3}{5}$$

Since $\frac{2}{x} > 0$, this means that $x > 0$. Therefore,

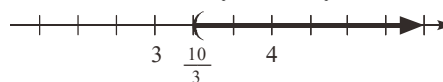
$$\frac{2}{x} < \frac{3}{5}$$

$$5x\left(\frac{2}{x}\right) < 5x\left(\frac{3}{5}\right)$$

$$10 < 3x$$

$$\frac{10}{3} < x$$

The solution set is $\left\{x \mid x > \frac{10}{3}\right\}$ or $\left(\frac{10}{3}, \infty\right)$.



94. $0 < \frac{4}{x} < \frac{2}{3}$

$$0 < \frac{4}{x} \text{ and } \frac{4}{x} < \frac{2}{3}$$

Since $\frac{4}{x} > 0$, this means that $x > 0$. Therefore,

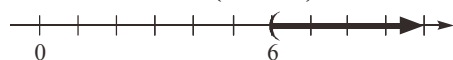
$$\frac{4}{x} < \frac{2}{3}$$

$$3x\left(\frac{4}{x}\right) < 3x\left(\frac{2}{3}\right)$$

$$12 < 2x$$

$$6 < x$$

The solution set is $\{x \mid x > 6\}$ or $(6, \infty)$.



$$95. \quad 0 < (2x-4)^{-1} < \frac{1}{2}$$

$$0 < \frac{1}{2x-4} < \frac{1}{2}$$

$$0 < \frac{1}{2x-4} \quad \text{and} \quad \frac{1}{2x-4} < \frac{1}{2}$$

Since $\frac{1}{2x-4} > 0$, this means that $2x-4 > 0$.

Therefore,

$$\frac{1}{2x-4} < \frac{1}{2}$$

$$\frac{1}{2(x-2)} < \frac{1}{2}$$

$$2(x-2)\left(\frac{1}{2(x-2)}\right) < 2(x-2)\left(\frac{1}{2}\right)$$

$$1 < x-2$$

$$3 < x$$

The solution set is $\{x \mid x > 3\}$ or $(3, \infty)$.



$$96. \quad 0 < (3x+6)^{-1} < \frac{1}{3}$$

$$0 < \frac{1}{3x+6} < \frac{1}{3}$$

$$0 < \frac{1}{3x+6} \quad \text{and} \quad \frac{1}{3x+6} < \frac{1}{3}$$

Since $\frac{1}{3x+6} > 0$, this means that $3x+6 > 0$.

Therefore,

$$\frac{1}{3x+6} < \frac{1}{3}$$

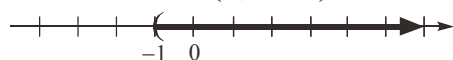
$$\frac{1}{3(x+2)} < \frac{1}{3}$$

$$3(x+2)\left(\frac{1}{3(x+2)}\right) < 3(x+2)\left(\frac{1}{3}\right)$$

$$1 < x+2$$

$$-1 < x$$

The solution set is $\{x \mid x > -1\}$ or $(-1, \infty)$.



$$97. \quad \text{If } -1 < x < 1, \text{ then}$$

$$-1+4 < x+4 < 1+4$$

$$3 < x+4 < 5$$

$$\text{So, } a = 3 \text{ and } b = 5.$$

$$98. \quad \text{If } -3 < x < 2, \text{ then}$$

$$-3-6 < x-6 < 2-6$$

$$-9 < x-6 < -4$$

$$\text{So, } a = -9 \text{ and } b = -4.$$

$$99. \quad \text{If } 2 < x < 3, \text{ then}$$

$$-4(2) < -4(x) < -4(3)$$

$$-12 < -4x < -8$$

$$\text{So, } a = -12 \text{ and } b = -8.$$

$$100. \quad \text{If } -4 < x < 0, \text{ then}$$

$$\frac{1}{2}(-4) < \frac{1}{2}(x) < \frac{1}{2}(0)$$

$$-2 < \frac{1}{2}x < 0$$

$$\text{So, } a = -2 \text{ and } b = 0.$$

$$101. \quad \text{If } 0 < x < 4, \text{ then}$$

$$2(0) < 2(x) < 2(4)$$

$$0 < 2x < 8$$

$$0+3 < 2x+3 < 8+3$$

$$3 < 2x+3 < 11$$

$$\text{So, } a = 3 \text{ and } b = 11.$$

$$102. \quad \text{If } -3 < x < 3, \text{ then}$$

$$-2(-3) > -2(x) > -2(3)$$

$$6 > -2x > -6$$

$$6+1 > -2x+1 > -6+1$$

$$7 > 1-2x > -5$$

$$-5 < 1-2x < 7$$

$$\text{So, } a = -5 \text{ and } b = 7.$$

$$103. \quad \text{If } -3 < x < 0, \text{ then}$$

$$-3+4 < x+4 < 0+4$$

$$1 < x+4 < 4$$

$$1 > \frac{1}{x+4} > \frac{1}{4}$$

$$\frac{1}{4} < \frac{1}{x+4} < 1$$

$$\text{So, } a = \frac{1}{4} \text{ and } b = 1.$$

Chapter 1: Equations and Inequalities

104. If $2 < x < 4$, then
 $2 - 6 < x - 6 < 4 - 6$

$$-4 < x - 6 < -2$$

$$-\frac{1}{4} > \frac{1}{x-6} > -\frac{1}{2}$$

$$-\frac{1}{2} < \frac{1}{x-6} < -\frac{1}{4}$$

So, $a = -\frac{1}{2}$ and $b = -\frac{1}{4}$.

105. If $6 < 3x < 12$, then

$$\frac{6}{3} < \frac{3x}{3} < \frac{12}{3}$$

$$2 < x < 4$$

$$2^2 < x^2 < 4^2$$

$$4 < x^2 < 16$$

So, $a = 4$ and $b = 16$.

106. If $0 < 2x < 6$, then

$$\frac{0}{2} < \frac{2x}{2} < \frac{6}{2}$$

$$0 < x < 3$$

$$0^2 < x^2 < 3^2$$

$$0 < x^2 < 9$$

So, $a = 0$ and $b = 9$.

107. $\sqrt{3x+6}$

We need $3x+6 \geq 0$

$$3x \geq -6$$

$$x \geq -2$$

To the domain is $\{x|x \geq -2\}$ or $[-2, \infty)$.

108. $\sqrt{8+2x}$

We need $8+2x \geq 0$

$$2x \geq -8$$

$$x \geq -4$$

To the domain is $\{x|x \geq -4\}$ or $[-4, \infty)$.

109. $21 < \text{young adult's age} < 30$

110. $40 \leq \text{middle-aged} < 60$

111. a. Let $x = \text{age at death}$.
 $x - 30 \geq 52.2$

$$x \geq 82.2$$

Therefore, the average life expectancy for a 30-year-old male in 2023 will be greater than or equal to 82.2 years.

b. Let $x = \text{age at death}$.
 $x - 30 \geq 55.8$

$$x \geq 85.8$$

Therefore, the average life expectancy for a 30-year-old female in 2023 will be greater than or equal to 85.8 years.

c. By the given information, a female can expect to live $85.8 - 82.2 = 3.6$ years longer.

112. $V = 20T$

$$80^\circ \leq T \leq 120^\circ$$

$$80^\circ \leq \frac{V}{20} \leq 120^\circ$$

$$1600 \leq V \leq 2400$$

The volume ranges from 1600 to 2400 cubic centimeters, inclusive.

113. Let P represent the selling price and C represent the commission.

Calculating the commission:

$$C = 45,000 + 0.25(P - 900,000)$$

$$= 45,000 + 0.25P - 225,000$$

$$= 0.25P - 180,000$$

Calculate the commission range, given the price range:

$$900,000 \leq P \leq 1,100,000$$

$$0.25(900,000) \leq 0.25P \leq 0.25(1,100,000)$$

$$225,000 \leq 0.25P \leq 275,000$$

$$225,000 - 180,000 \leq 0.25P - 180,000 \leq 275,000 - 180,000$$

$$45,000 \leq C \leq 95,000$$

The agent's commission ranges from \$45,000 to \$95,000, inclusive.

$$\frac{45,000}{900,000} = 0.05 = 5\% \text{ to } \frac{95,000}{1,100,000} = 0.086 = 8.6\%$$

inclusive.

As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive.

114. Let C represent the commission.

Calculate the commission range:

$$25 + 0.4(200) \leq C \leq 25 + 0.4(3000)$$

$$105 \leq C \leq 1225$$

The commissions are at least \$105 and at most \$1225.

- 115.** Let W = weekly wages and T = tax withheld. Calculating the withholding tax range, given the range of weekly wages:

$$500 \leq W \leq 700$$

$$500 - 312.5 \leq W - 312.5 \leq 700 - 312.5$$

$$187.50 \leq W - 312.5 \leq 387.5$$

$$0.12(187.5) \leq 0.12(W - 312.5) \leq 0.12(387.5)$$

$$22.50 \leq 0.12(W - 312.5) \leq 46.5$$

$$22.5 + 1100 \leq 0.12(W - 312.5) + 1100 \leq 46.5 + 1100$$

$$1122.50 \leq T \leq 1146.50$$

The amount withheld varies from \$1122.50 to \$1146.50, inclusive.

- 116.** Let x represent the length of time you should exercise on the last two days.

$$25 + 35 + 0 + 40 + 15 + x = 150$$

$$115 + x = 150$$

$$x = 35$$

You will stay within the guidelines by exercising from 35 minutes total on the last two days.

- 117.** Let x represent the amount paid for international minutes and y represent the number of international minutes. The range of the bills is \$69.50 to \$140.75. The rate plan is \$60. Thus the range of costs of the international minutes is: $9.50 \leq x \leq 80.75$. The cost per min is \$0.25.

$$\frac{9.50}{0.25} \leq y \leq \frac{80.75}{0.25}$$

$$38 \leq y \leq 323$$

The minutes varies from 38 to 323 minutes, inclusive.

- 118.** Let C represent the amount paid for the fares. The range of the fare is \$20.93 to \$40.44. of miles. The number of miles is 23.

$$20.93 \leq 23x \leq 40.44$$

$$0.91 \leq x \leq 1.76$$

The cost per mile varies from \$0.91 per mile to \$1.76 per mile, inclusive.

- 119.** You have already consumed 40 grams of fat. Let C represent the number of cookies. Then we have the following equation:

$$40 + 8C \leq 64$$

$$8C \leq 24$$

$$C \leq 3$$

You may eat up to 3 cookies and keep the total fat content of your meal not more than 64g.

- 120.** You have already consumed 730 calories. Let x represent the number of apple sauce orders you can eat. Then we have the following equation:

$$730 + 50x \leq 830$$

$$50x \leq 100$$

$$x \leq 2$$

You may eat up to 2 orders of apple sauce and keep your calories below or equal to 830.

- 121. a.** Let T represent the score on the last test and G represent the course grade. Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + T}{5}$$

$$G = \frac{326 + T}{5}$$

$$5G = 326 + T$$

$$T = 5G - 326$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$400 \leq 5G < 450$$

$$74 \leq 5G - 326 < 124$$

$$74 \leq T < 124$$

To get a grade of B, you need at least a 74 on the fifth test.

- b.** Let T represent the score on the last test and G represent the course grade. Calculating the course grade and solving for the last test:

$$G = \frac{68 + 82 + 87 + 89 + 2T}{6}$$

$$G = \frac{326 + 2T}{6}$$

$$G = \frac{163 + T}{3}$$

$$T = 3G - 163$$

Calculating the range of scores on the last test, given the grade range:

$$80 \leq G < 90$$

$$240 \leq 3G < 270$$

$$77 \leq 3G - 163 < 107$$

$$77 \leq T < 107$$

To get a grade of B, you need at least a 77 on the fifth test.

Chapter 1: Equations and Inequalities

- 122.** Let T represent the test scores of the people in the top 2.5%.

$$T > 1.96(12) + 100 = 123.52$$

People in the top 2.5% will have test scores greater than 123.52. That is, $T > 123.52$ or $(123.52, \infty)$.

- 123.** Since $a < b$,

$$\begin{aligned} \frac{a}{2} < \frac{b}{2} \quad \text{and} \quad \frac{a}{2} < \frac{b}{2} \\ \frac{a}{2} + \frac{a}{2} < \frac{a}{2} + \frac{b}{2} \quad \text{and} \quad \frac{a}{2} + \frac{b}{2} < \frac{b}{2} + \frac{b}{2} \\ a < \frac{a+b}{2} \quad \text{and} \quad \frac{a+b}{2} < b \end{aligned}$$

$$\text{So, } a < \frac{a+b}{2} < b.$$

- 124.** From problem 123, $a < \frac{a+b}{2} < b$, so

$$d\left(a, \frac{a+b}{2}\right) = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} \quad \text{and}$$

$$d\left(b, \frac{a+b}{2}\right) = b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}.$$

Therefore, $\frac{a+b}{2}$ is equidistant from a and b .

- 125.** If $0 < a < b$, then

$$\begin{aligned} ab > a^2 > 0 \quad \text{and} \quad b^2 > ab > 0 \\ (\sqrt{ab})^2 > a^2 \quad \text{and} \quad b^2 > (\sqrt{ab})^2 \\ \sqrt{ab} > a \quad \text{and} \quad b > \sqrt{ab} \end{aligned}$$

Therefore, $a < \sqrt{ab} < b$.

- 126.** Show that $\sqrt{ab} < \frac{a+b}{2}$.

$$\begin{aligned} \frac{a+b}{2} - \sqrt{ab} &= \frac{1}{2}(a - 2\sqrt{ab} + b) \\ &= \frac{1}{2}(\sqrt{a} - \sqrt{b})^2 > 0, \text{ since } a \neq b. \end{aligned}$$

Therefore, $\sqrt{ab} < \frac{a+b}{2}$.

- 127.** For $0 < a < b$, $\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

$$h \cdot \frac{1}{h} = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$

$$1 = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$

$$h = \frac{2ab}{a+b}$$

$$\begin{aligned} h - a &= \frac{2ab}{a+b} - a = \frac{2ab - a(a+b)}{a+b} \\ &= \frac{2ab - a^2 - ab}{a+b} = \frac{ab - a^2}{a+b} \\ &= \frac{a(b-a)}{a+b} > 0 \end{aligned}$$

Therefore, $h > a$.

$$\begin{aligned} b - h &= b - \frac{2ab}{a+b} = \frac{b(a+b) - 2ab}{a+b} \\ &= \frac{ab + b^2 - 2ab}{a+b} = \frac{b^2 - ab}{a+b} \\ &= \frac{b(b-a)}{a+b} > 0 \end{aligned}$$

Therefore, $h < b$, and we have $a < h < b$.

- 128.** Show that $h = \frac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

$$\frac{h}{2} = \frac{ab}{a+b}$$

$$h = 2 \cdot \frac{ab}{a+b} = \frac{(\sqrt{ab})^2}{\left(\frac{1}{2}(a+b)\right)}$$

- 129.** $x - 4 < 2x - 3 \leq \frac{x+5}{3}$

$$3x - 12 < 6x - 9 \leq x + 5$$

$$3x - 12 < 6x - 9 \quad \text{and} \quad 6x - 9 \leq x + 5$$

$$-3 < 3x \quad 5x \leq 14$$

$$-1 < x \quad x \leq \frac{14}{5}$$

Section 1.6: Equations and Inequalities Involving Absolute Value

This is equivalent to $-1 < x \leq \frac{14}{5}$. The solution set, in interval notation, is $\left(-1, \frac{14}{5}\right]$.

- 130.** The largest value of $2x^2 - 3$ occurs at the largest value for $|x|$.

$$2 \leq 5 - x \leq 9$$

$$-3 \leq -x \leq 4$$

$$-4 \leq x \leq 3$$

$$3 \geq x \geq -4 \text{ or } -4 \leq x \leq 3$$

The largest value for $2x^2 - 3$ is

$$2(-4)^2 - 3 = 32 - 3 = 29.$$

- 131.** Answers will vary

- 132.** Answers will vary. One possibility:

$$\text{No solution: } 4x + 6 \leq 2(x - 5) + 2x$$

$$\text{One solution: } 3x + 5 \leq 2(x + 3) + 1 \leq 3(x + 2) - 1$$

- 133.** Since $x^2 \geq 0$, we have

$$x^2 + 1 \geq 0 + 1$$

$$x^2 + 1 \geq 1$$

Therefore, the expression $x^2 + 1$ can never be less than -5 .

- 134.** Answers will vary.

9. $|3x| = 15$

$$3x = 15 \text{ or } 3x = -15$$

$$x = 5 \text{ or } x = -5$$

The solution set is $\{-5, 5\}$.

10. $|3x| = 12$

$$3x = 12 \text{ or } 3x = -12$$

$$x = 4 \text{ or } x = -4$$

The solution set is $\{-4, 4\}$.

11. $|2x + 3| = 5$

$$2x + 3 = 5 \text{ or } 2x + 3 = -5$$

$$2x = 2 \text{ or } 2x = -8$$

$$x = 1 \text{ or } x = -4$$

The solution set is $\{-4, 1\}$.

12. $|3x - 1| = 2$

$$3x - 1 = 2 \text{ or } 3x - 1 = -2$$

$$3x = 3 \text{ or } 3x = -1$$

$$x = 1 \text{ or } x = -\frac{1}{3}$$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

13. $|1 - 4t| + 8 = 13$

$$|1 - 4t| = 5$$

$$1 - 4t = 5 \text{ or } 1 - 4t = -5$$

$$-4t = 4 \text{ or } -4t = -6$$

$$t = -1 \text{ or } t = \frac{3}{2}$$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

14. $|1 - 2z| + 6 = 9$

$$|1 - 2z| = 3$$

$$1 - 2z = 3 \text{ or } 1 - 2z = -3$$

$$-2z = 2 \text{ or } -2z = -4$$

$$z = -1 \text{ or } z = 2$$

The solution set is $\{-1, 2\}$.

15. $|-2x| = |8|$

$$|-2x| = 8$$

$$-2x = 8 \text{ or } -2x = -8$$

$$x = -4 \text{ or } x = 4$$

The solution set is $\{-4, 4\}$.

Section 1.6

1. $|-2| = 2$

2. True

3. $\{-5, 5\}$

4. $\{x | -5 < x < 5\}$

5. True

6. True

7. d

8. a

Chapter 1: Equations and Inequalities

16. $|-x| = |1|$

$$|-x| = 1$$

$$-x = 1 \quad \text{or} \quad -x = -1$$

The solution set is $\{-1, 1\}$.

17. $|-2|x = 4$

$$2x = 4$$

$$x = 2$$

The solution set is $\{2\}$.

18. $|3|x = 9$

$$3x = 9$$

$$x = 3$$

The solution set is $\{3\}$.

19. $\frac{8}{7}|x| = 3$

$$|x| = \frac{21}{8}$$

$$x = \frac{21}{8} \quad \text{or} \quad x = -\frac{21}{8}$$

The solution set is $\left\{-\frac{21}{8}, \frac{21}{8}\right\}$.

20. $\frac{3}{4}|x| = 9$

$$|x| = 12$$

$$x = 12 \quad \text{or} \quad x = -12$$

The solution set is $\{-12, 12\}$.

21. $\left|\frac{x}{3} + \frac{2}{5}\right| = 2$

$$\frac{x}{3} + \frac{2}{5} = 2 \quad \text{or} \quad \frac{x}{3} + \frac{2}{5} = -2$$

$$5x + 6 = 30 \quad \text{or} \quad 5x + 6 = -30$$

$$5x = 24 \quad \text{or} \quad 5x = -36$$

$$x = \frac{24}{5} \quad \text{or} \quad x = -\frac{36}{5}$$

The solution set is $\left\{-\frac{36}{5}, \frac{24}{5}\right\}$.

22. $\left|\frac{x}{2} - \frac{1}{3}\right| = 1$

$$\frac{x}{2} - \frac{1}{3} = 1 \quad \text{or} \quad \frac{x}{2} - \frac{1}{3} = -1$$

$$3x - 2 = 6 \quad \text{or} \quad 3x - 2 = -6$$

$$3x = 8 \quad \text{or} \quad 3x = -4$$

$$x = \frac{8}{3} \quad \text{or} \quad x = -\frac{4}{3}$$

The solution set is $\left\{-\frac{4}{3}, \frac{8}{3}\right\}$.

23. $|u - 2| = -\frac{1}{2}$

No solution, since absolute value always yields a non-negative number.

24. $|2 - v| = -1$

No solution, since absolute value always yields a non-negative number.

25. $5 - |4x| = 4$

$$-|4x| = -1$$

$$|4x| = 1$$

$$4x = 1 \quad \text{or} \quad 4x = -1$$

$$x = \frac{1}{4} \quad \text{or} \quad x = -\frac{1}{4}$$

The solution set is $\left\{-\frac{1}{4}, \frac{1}{4}\right\}$.

26. $5 - \left|\frac{1}{2}x\right| = 3$

$$-\left|\frac{1}{2}x\right| = -2$$

$$\left|\frac{1}{2}x\right| = 2$$

$$\frac{1}{2}x = 2 \quad \text{or} \quad \frac{1}{2}x = -2$$

$$x = 4 \quad \text{or} \quad x = -4$$

The solution set is $\{-4, 4\}$.

27. $|x^2 - 9| = 0$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

The solution set is $\{-3, 3\}$.

Section 1.6: Equations and Inequalities Involving Absolute Value

28. $|x^2 - 16| = 0$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

The solution set is $\{-4, 4\}$.

29. $|x^2 - 2x| = 3$

$$x^2 - 2x = 3 \quad \text{or} \quad x^2 - 2x = -3$$

$$x^2 - 2x - 3 = 0 \quad \text{or} \quad x^2 - 2x + 3 = 0$$

$$(x-3)(x+1) = 0 \quad \text{or} \quad x = \frac{2 \pm \sqrt{4-12}}{2}$$

$$x = 3 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = \frac{2 \pm \sqrt{-8}}{2} \quad \text{no real sol.}$$

The solution set is $\{-1, 3\}$.

30. $|x^2 + x| = 12$

$$x^2 + x = 12 \quad \text{or} \quad x^2 + x = -12$$

$$x^2 + x - 12 = 0 \quad \text{or} \quad x^2 + x + 12 = 0$$

$$(x-3)(x+4) = 0 \quad \text{or} \quad x = \frac{-1 \pm \sqrt{1-48}}{2}$$

$$x = 3 \quad \text{or} \quad x = -4 \quad \text{or} \quad x = \frac{1 \pm \sqrt{-47}}{2} \quad \text{no real sol.}$$

The solution set is $\{-4, 3\}$.

31. $|x^2 + x - 1| = 1$

$$x^2 + x - 1 = 1 \quad \text{or} \quad x^2 + x - 1 = -1$$

$$x^2 + x - 2 = 0 \quad \text{or} \quad x^2 + x = 0$$

$$(x-1)(x+2) = 0 \quad \text{or} \quad x(x+1) = 0$$

$$x = 1, x = -2 \quad \text{or} \quad x = 0, x = -1$$

The solution set is $\{-2, -1, 0, 1\}$.

32. $|x^2 + 3x - 2| = 2$

$$x^2 + 3x - 2 = 2 \quad \text{or} \quad x^2 + 3x - 2 = -2$$

$$x^2 + 3x = 4 \quad \text{or} \quad x^2 + 3x = 0$$

$$x^2 + 3x - 4 = 0 \quad \text{or} \quad x(x+3) = 0$$

$$(x+4)(x-1) = 0 \quad \text{or} \quad x = 0, x = -3$$

$$x = -4, x = 1$$

The solution set is $\{-4, -3, 0, 1\}$.

33. $\left| \frac{5x-3}{3x-5} \right| = 2$

$$\frac{5x-3}{3x-5} = 2 \quad \text{or} \quad \frac{5x-3}{3x-5} = -2$$

$$5x-3 = 2(3x-5) \quad \text{or} \quad 5x-3 = -2(3x-5)$$

$$5x-3 = 6x-10 \quad \text{or} \quad 5x-3 = -6x+10$$

$$-x = -7 \quad \text{or} \quad 11x = 13$$

$$x = 7 \quad \text{or} \quad x = \frac{13}{11}$$

Neither of these values cause the denominator to equal zero, so the solution set is $\left\{\frac{13}{11}, 7\right\}$.

34. $\left| \frac{2x+1}{3x+4} \right| = 1$

$$\frac{2x+1}{3x+4} = 1 \quad \text{or} \quad \frac{2x+1}{3x+4} = -1$$

$$2x+1 = 1(3x+4) \quad \text{or} \quad 2x+1 = -1(3x+4)$$

$$2x+1 = 3x+4 \quad \text{or} \quad 2x+1 = -3x-4$$

$$-x = 3 \quad \text{or} \quad 5x = -5$$

$$x = -3 \quad \text{or} \quad x = -1$$

Neither of these values cause the denominator to equal zero, so the solution set is $\{-3, -1\}$.

35. $|x^2 + 3x| = |x^2 - 2x|$

$$x^2 + 3x = x^2 - 2x \quad \text{or} \quad x^2 + 3x = -(x^2 - 2x)$$

$$3x = -2x \quad \text{or} \quad x^2 + 3x = -x^2 + 2x$$

$$5x = 0 \quad \text{or} \quad 2x^2 + x = 0$$

$$x = 0 \quad \text{or} \quad x(2x+1) = 0$$

$$x = 0 \quad \text{or} \quad x = 0 \quad \text{or} \quad x = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, 0\right\}$.

36. $|x^2 - 2x| = |x^2 + 6x|$

$$x^2 - 2x = x^2 + 6x \quad \text{or} \quad x^2 - 2x = -(x^2 + 6x)$$

$$-2x = 6x \quad \text{or} \quad x^2 - 2x = -x^2 - 6x$$

$$-8x = 0 \quad \text{or} \quad 2x^2 + 4x = 0$$

$$x = 0 \quad \text{or} \quad 2x(x+2) = 0$$

$$x = 0 \quad \text{or} \quad x = 0 \quad \text{or} \quad x = -2$$

The solution set is $\{-2, 0\}$.

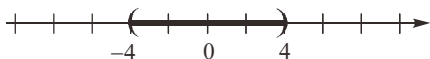
Chapter 1: Equations and Inequalities

37. $|2x| < 8$

$$-8 < 2x < 8$$

$$-4 < x < 4$$

$$\{x | -4 < x < 4\} \text{ or } (-4, 4)$$

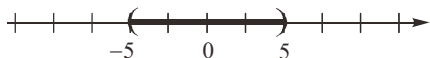


38. $|3x| < 15$

$$-15 < 3x < 15$$

$$-5 < x < 5$$

$$\{x | -5 < x < 5\} \text{ or } (-5, 5)$$

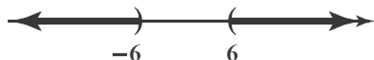


39. $|7x| > 42$

$$7x < -42 \text{ or } 7x > 42$$

$$x < -6 \text{ or } x > 6$$

$$\{x | x < -6 \text{ or } x > 6\} \text{ or } (-\infty, -6) \cup (6, \infty)$$

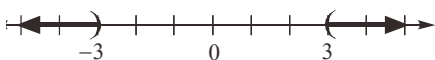


40. $|2x| > 6$

$$2x < -6 \text{ or } 2x > 6$$

$$x < -3 \text{ or } x > 3$$

$$\{x | x < -3 \text{ or } x > 3\} \text{ or } (-\infty, -3) \cup (3, \infty)$$



41. $|x-2| + 2 < 3$

$$|x-2| < 1$$

$$-1 < x-2 < 1$$

$$1 < x < 3$$

$$\{x | 1 < x < 3\} \text{ or } (1, 3)$$



42. $|x+4| + 3 < 5$

$$|x+4| < 2$$

$$-2 < x+4 < 2$$

$$-6 < x < -2$$

$$\{x | -6 < x < -2\} \text{ or } (-6, -2)$$



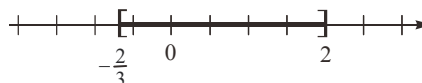
43. $|3t-2| \leq 4$

$$-4 \leq 3t-2 \leq 4$$

$$-2 \leq 3t \leq 6$$

$$-\frac{2}{3} \leq t \leq 2$$

$$\left\{t \mid -\frac{2}{3} \leq t \leq 2\right\} \text{ or } \left[-\frac{2}{3}, 2\right]$$



44. $|2u+5| \leq 7$

$$-7 \leq 2u+5 \leq 7$$

$$-12 \leq 2u \leq 2$$

$$-6 \leq u \leq 1$$

$$\{u | -6 \leq u \leq 1\} \text{ or } [-6, 1]$$



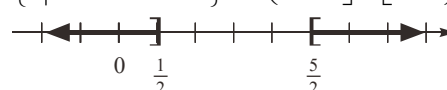
45. $|2x-3| \geq 2$

$$2x-3 \leq -2 \text{ or } 2x-3 \geq 2$$

$$2x \leq 1 \text{ or } 2x \geq 5$$

$$x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}$$

$$\left\{x \mid x \leq \frac{1}{2} \text{ or } x \geq \frac{5}{2}\right\} \text{ or } \left(-\infty, \frac{1}{2}\right] \cup \left[\frac{5}{2}, \infty\right)$$



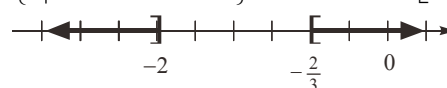
46. $|3x+4| \geq 2$

$$3x+4 \leq -2 \text{ or } 3x+4 \geq 2$$

$$3x \leq -6 \text{ or } 3x \geq -2$$

$$x \leq -2 \text{ or } x \geq -\frac{2}{3}$$

$$\left\{x \mid x \leq -2 \text{ or } x \geq -\frac{2}{3}\right\} \text{ or } (-\infty, -2] \cup \left[-\frac{2}{3}, \infty\right)$$



Section 1.6: Equations and Inequalities Involving Absolute Value

47. $|1-4x|-7 < -2$

$$|1-4x| < 5$$

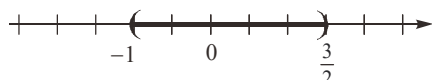
$$-5 < 1-4x < 5$$

$$-6 < -4x < 4$$

$$\frac{-6}{-4} > x > \frac{4}{-4}$$

$$\frac{3}{2} > x > -1 \quad \text{or} \quad -1 < x < \frac{3}{2}$$

$$\left\{x \mid -1 < x < \frac{3}{2}\right\} \text{ or } \left(-1, \frac{3}{2}\right)$$



48. $|1-2x|-4 < -1$

$$|1-2x| < 3$$

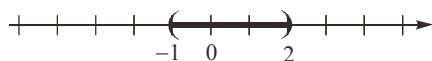
$$-3 < 1-2x < 3$$

$$-4 < -2x < 2$$

$$\frac{-4}{-2} > x > \frac{2}{-2}$$

$$2 > x > -1 \quad \text{or} \quad -1 < x < 2$$

$$\{x \mid -1 < x < 2\} \text{ or } (-1, 2)$$



49. $|5-2x| > -7$

$$5-2x < -7 \quad \text{or} \quad 5-2x > 7$$

$$-2x < -12 \quad \text{or} \quad -2x > 2$$

$$x > 6 \quad \text{or} \quad x < -1$$

$$\{x \mid x < -1 \text{ or } x > 6\} \text{ or } (-\infty, -1) \cup (6, \infty)$$



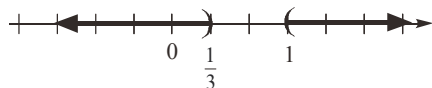
50. $|2-3x| > 1$

$$2-3x < -1 \quad \text{or} \quad 2-3x > 1$$

$$-3x < -3 \quad \text{or} \quad -3x > -1$$

$$x > 1 \quad \text{or} \quad x < \frac{1}{3}$$

$$\left\{x \mid x < \frac{1}{3} \text{ or } x > 1\right\} \text{ or } \left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$$

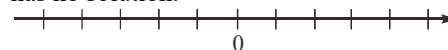


51. $|-4x|+|-5| \leq 1$

$$|-4x|+5 \leq 1$$

$$|-4x| \leq -4$$

This is impossible since absolute value always yields a non-negative number. The inequality has no solution.



52. $|-x|-|4| \leq 2$

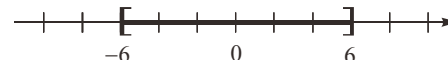
$$|-x|-4 \leq 2$$

$$|-x| \leq 6$$

$$-6 \leq -x \leq 6$$

$$6 \geq x \geq -6$$

$$\{x \mid -6 \leq x \leq 6\} \text{ or } [-6, 6]$$



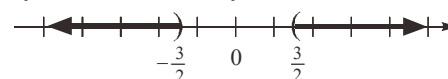
53. $|-2x| > |-3|$

$$|2x| > 3$$

$$2x < -3 \quad \text{or} \quad 2x > 3$$

$$x < -\frac{3}{2} \quad \text{or} \quad x > \frac{3}{2}$$

$$\left\{x \mid x < -\frac{3}{2} \text{ or } x > \frac{3}{2}\right\} \text{ or } \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$



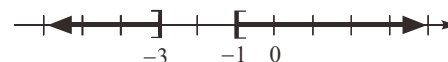
54. $|-x-2| \geq 1$

$$-x-2 \leq -1 \quad \text{or} \quad -x-2 \geq 1$$

$$-x \leq 1 \quad \text{or} \quad -x \geq 3$$

$$x \geq -1 \quad \text{or} \quad x \leq -3$$

$$\{x \mid x \leq -3 \text{ or } x \geq -1\} \text{ or } (-\infty, -3] \cup [-1, \infty)$$



55. $-3|2x-5| \geq -21$

$$|2x-5| \leq 7$$

$$-7 \leq 2x-5 \leq 7$$

$$-2 \leq 2x \leq 12$$

$$-1 \leq x \leq 6$$

$$\{x \mid -1 \leq x \leq 6\} \text{ or } [-1, 6]$$



Chapter 1: Equations and Inequalities

56. $-|1-2x| \geq -3$

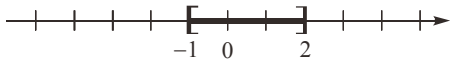
$$|1-2x| \leq 3$$

$$-3 \leq 1-2x \leq 3$$

$$-4 \leq -2x \leq 2$$

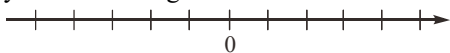
$$2 \geq x \geq -1$$

$$\{x | -1 \leq x \leq 2\} \text{ or } [-1, 2]$$



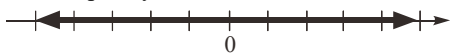
57. $|9x| < -5$

This is impossible since absolute value always yields a non-negative number. No solution.



58. $|3x| \geq 0$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty, \infty)$.



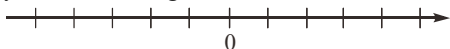
59. $|5x| \geq -1$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty, \infty)$.



60. $|6x| < -2$

This is impossible since absolute value always yields a non-negative number. No solution.



61. $\left| \frac{2x+3}{3} - \frac{1}{2} \right| < 1$

$$-1 < \frac{2x+3}{3} - \frac{1}{2} < 1$$

$$6(-1) < 6\left(\frac{2x+3}{3} - \frac{1}{2}\right) < 6(1)$$

$$-6 < 2(2x+3) - 3 < 6$$

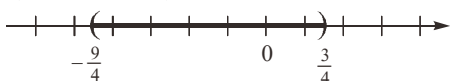
$$-6 < 4x + 6 - 3 < 6$$

$$-6 < 4x + 3 < 6$$

$$-9 < 4x < 3$$

$$-\frac{9}{4} < x < \frac{3}{4}$$

$$\left\{x \mid -\frac{9}{4} < x < \frac{3}{4}\right\} \text{ or } \left(-\frac{9}{4}, \frac{3}{4}\right)$$



62. $3 - |x+1| < \frac{1}{2}$

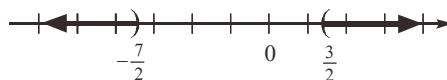
$$-|x+1| < -\frac{5}{2}$$

$$|x+1| > \frac{5}{2}$$

$$x+1 < -\frac{5}{2} \text{ or } x+1 > \frac{5}{2}$$

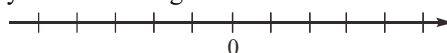
$$x < -\frac{7}{2} \text{ or } x > \frac{3}{2}$$

$$\left\{x \mid x < -\frac{7}{2} \text{ or } x > \frac{3}{2}\right\} \text{ or } \left(-\infty, -\frac{7}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$



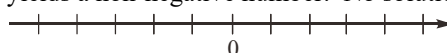
63. $|8-4x| \leq -13$

This is impossible since absolute value always yields a non-negative number. No solution.



64. $|7x+4| \leq -9$

This is impossible since absolute value always yields a non-negative number. No solution.



65. $\left| \frac{7-2x}{3} \right| \leq 0$

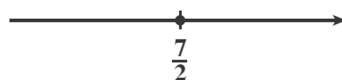
Since the absolute value cannot be negative, the only possible solution would be:

$$\frac{7-2x}{3} = 0$$

$$7-2x = 0$$

$$-2x = -7$$

$$x = \frac{7}{2}$$



66. $-\left| \frac{4x-15}{6} \right| \geq 0$

$$\left| \frac{4x-15}{6} \right| \leq 0$$

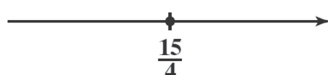
Since the absolute value cannot be negative, the only possible solution would be:

$$\frac{4x-15}{6} = 0$$

$$4x-15=0$$

$$4x=15$$

$$x = \frac{15}{4}$$



$$67. \left| \frac{3x-7}{5} \right| > 4$$

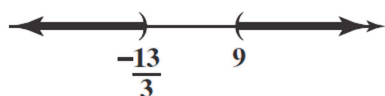
$$\frac{3x-7}{5} < -4 \quad \text{or} \quad \frac{3x-7}{5} > 4$$

$$3x-7 < -20 \quad \text{or} \quad 3x-7 > 20$$

$$3x < 13 \quad \text{or} \quad 3x > 27$$

$$x < \frac{13}{3} \quad \text{or} \quad x > 9$$

$$\left\{ x \mid x < \frac{13}{3} \text{ or } x > 9 \right\} \text{ or } \left(-\infty, \frac{13}{3} \right) \cup (9, \infty)$$



$$68. \left| \frac{5-2x}{9} \right| \geq 8$$

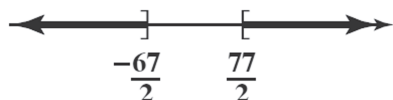
$$\frac{5-2x}{9} \leq -8 \quad \text{or} \quad \frac{5-2x}{9} \geq 8$$

$$5-2x \leq -72 \quad \text{or} \quad 5-2x \geq 72$$

$$-2x \leq -77 \quad \text{or} \quad -2x \geq 67$$

$$x \geq \frac{77}{2} \quad \text{or} \quad x \leq -\frac{67}{2}$$

$$\left\{ x \mid x \leq -\frac{67}{2} \text{ or } x \geq \frac{77}{2} \right\} \text{ or } \left(-\infty, -\frac{67}{2} \right] \cup \left[\frac{77}{2}, \infty \right)$$



$$69. 5 + |x-1| > \frac{1}{2}$$

$$|x-1| > -\frac{9}{2}$$

Absolute value yields a non-negative number, so this inequality is true for all real numbers, $(-\infty, \infty)$.



$$70. \left| \frac{2x-3}{2} + \frac{1}{3} \right| > 1$$

$$\frac{2x-3}{2} + \frac{1}{3} < -1 \quad \text{or} \quad \frac{2x-3}{2} + \frac{1}{3} > 1$$

$$6\left(\frac{2x-3}{2} + \frac{1}{3}\right) < 6(-1) \quad \text{or} \quad 6\left(\frac{2x-3}{2} + \frac{1}{3}\right) > 6(1)$$

$$3(2x-3) + 2 < -6 \quad \text{or} \quad 3(2x-3) + 2 > 6$$

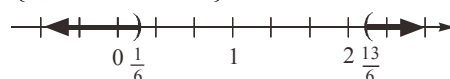
$$6x-9+2 < -6 \quad \text{or} \quad 6x-9+2 > 6$$

$$6x-7 < -6 \quad \text{or} \quad 6x-7 > 6$$

$$6x < 1 \quad \text{or} \quad 6x > 13$$

$$x < \frac{1}{6} \quad \text{or} \quad x > \frac{13}{6}$$

$$\left\{ x \mid x < \frac{1}{6} \text{ or } x > \frac{13}{6} \right\} \text{ or } \left(-\infty, \frac{1}{6} \right) \cup \left(\frac{13}{6}, \infty \right)$$



71. A temperature x that differs from 98.6°F by at least 1.5°F .

$$|x - 98.6| \geq 1.5$$

$$x - 98.6 \leq -1.5 \quad \text{or} \quad x - 98.6 \geq 1.5$$

$$x \leq 97.1 \quad \text{or} \quad x \geq 100.1$$

The temperatures that are considered unhealthy are those that are less than 97.1°F or greater than 100.1°F , inclusive.

72. The length L must be within 0.0025 of 5.375 inches.

$$|L - 5.375| \leq 0.0025$$

$$-0.0025 \leq L - 5.375 \leq 0.0025$$

$$5.3725 \leq L \leq 5.3775$$

The lengths must be between 5.3725 and 5.3775 inches, inclusive.

73. The percentage must be within 3.9 percentage points of 44 percent. The inequality that represents this would be:

$$|x - 44| \leq 3.9$$

$$-3.9 \leq x - 44 \leq 3.9$$

$$40.1 \leq x \leq 47.9$$

The actual percentage is likely to fall between 40.1% and 47.9%, inclusive.

74. The speed x varies from 707 mph by up to 55 mph.

$$\text{a. } |x - 707| \leq 55$$

Chapter 1: Equations and Inequalities

b. $-55 \leq x - 707 \leq 55$

$$-55 \leq x - 707 \leq 55$$

$$652 \leq x \leq 762$$

The speed of sound is between 652 and 762 miles per hour, depending on conditions.

75. x differs from 3 by less than $\frac{1}{2}$.

$$|x - 3| < \frac{1}{2}$$

$$-\frac{1}{2} < x - 3 < \frac{1}{2}$$

$$\frac{5}{2} < x < \frac{7}{2}$$

$$\left\{ x \mid \frac{5}{2} < x < \frac{7}{2} \right\}$$

76. x differs from -4 by less than 1

$$|x - (-4)| < 1$$

$$|x + 4| < 1$$

$$-1 < x + 4 < 1$$

$$-5 < x < -3$$

$$\{x \mid -5 < x < -3\}$$

77. x differs from -3 by more than 2.

$$|x - (-3)| > 2$$

$$|x + 3| > 2$$

$$x + 3 < -2 \text{ or } x + 3 > 2$$

$$x < -5 \text{ or } x > -1$$

$$\{x \mid x < -5 \text{ or } x > -1\}$$

78. x differs from 2 by more than 3.

$$|x - 2| > 3$$

$$x - 2 < -3 \text{ or } x - 2 > 3$$

$$x < -1 \text{ or } x > 5$$

$$\{x \mid x < -1 \text{ or } x > 5\}$$

79. $|x - 1| < 3$

$$-3 < x - 1 < 3$$

$$-3 + 5 < (x - 1) + 5 < 3 + 5$$

$$2 < x + 4 < 8$$

$$a = 2, b = 8$$

80. $|x + 2| < 5$

$$-5 < x + 2 < 5$$

$$-5 - 4 < (x + 2) - 4 < 5 - 4$$

$$-9 < x - 2 < 1$$

$$a = -9, b = 1$$

81. $|x + 4| \leq 2$

$$-2 \leq x + 4 \leq 2$$

$$-6 \leq x \leq -2$$

$$-12 \leq 2x \leq -4$$

$$-15 \leq 2x - 3 \leq -7$$

$$a = -15, b = -7$$

82. $|x - 3| \leq 1$

$$-1 \leq x - 3 \leq 1$$

$$2 \leq x \leq 4$$

$$6 \leq 3x \leq 12$$

$$7 \leq 3x + 1 \leq 13$$

$$a = 7, b = 13$$

83. $|x - 2| \leq 7$

$$-7 \leq x - 2 \leq 7$$

$$-5 \leq x \leq 9$$

$$-15 \leq x - 10 \leq -1$$

$$-\frac{1}{15} \geq \frac{1}{x - 10} \geq -1$$

$$-1 \leq \frac{1}{x - 10} \leq -\frac{1}{15}$$

$$a = -1, b = -\frac{1}{15}$$

84. $|x + 1| \leq 3$

$$-3 \leq x + 1 \leq 3$$

$$-4 \leq x \leq 2$$

$$1 \leq x + 5 \leq 7$$

$$1 \geq \frac{1}{x + 5} \geq \frac{1}{7}$$

$$\frac{1}{7} \leq \frac{1}{x + 5} \leq 1$$

$$a = \frac{1}{7}, b = 1$$

85. Given that $a > 0$, $b > 0$, and $\sqrt{a} < \sqrt{b}$, show that $a < b$.

Note that $b - a = (\sqrt{b} + \sqrt{a})(\sqrt{b} - \sqrt{a})$.

Since $\sqrt{a} < \sqrt{b}$ means $\sqrt{b} - \sqrt{a} > 0$, we have

$$b - a = (\sqrt{b} + \sqrt{a})(\sqrt{b} - \sqrt{a}) > 0.$$

Therefore, $b - a > 0$ which means $a < b$.

86. Show that $a \leq |a|$.

We know $0 \leq |a|$. So if $a < 0$, then we have

Section 1.6: Equations and Inequalities Involving Absolute Value

$a < 0 \leq |a|$ which means $a \leq |a|$. Now, if $a \geq 0$, then $|a| = a$. So $a \leq |a|$.

87. Prove $|a+b| \leq |a|+|b|$.

Note that $|a+b|^2 = |a+b| \cdot |a+b|$.

Case 1: If $a+b \geq 0$, then $|a+b| = a+b$, so

$$\begin{aligned} |a+b| \cdot |a+b| &= (a+b)(a+b) \\ &= a^2 + 2ab + b^2 \\ &\leq |a|^2 + 2|a| \cdot |b| + |b|^2 \\ &= (|a|+|b|)^2 \text{ by problem 86} \end{aligned}$$

$$\begin{aligned} \text{Thus, } (|a+b|)^2 &\leq (|a|+|b|)^2 \\ |a+b| &\leq |a|+|b|. \end{aligned}$$

Case 2: If $a+b < 0$, then $|a+b| = -(a+b)$, so

$$\begin{aligned} |a+b| \cdot |a+b| &= (-(a+b))(-(a+b)) \\ &= (a+b)(a+b) \\ &= a^2 + 2ab + b^2 \\ &\leq |a|^2 + 2|a| \cdot |b| + |b|^2 \\ &= (|a|+|b|)^2 \text{ by problem 86} \end{aligned}$$

$$\begin{aligned} \text{Thus, } (|a+b|)^2 &\leq (|a|+|b|)^2 \\ |a+b| &\leq |a|+|b|. \end{aligned}$$

88. Prove $|a-b| \geq |a|-|b|$.

$|a| = |(a-b)+b| \leq |a-b|+|b|$ by the Triangle

Inequality, so $|a| \leq |a-b|+|b|$ which means

$|a|-|b| \leq |a-b|$. Therefore, $|a-b| \geq |a|-|b|$.

89. Given that $a > 0$,

$$\begin{aligned} x^2 &< a \\ x^2 - a &< 0 \\ (x+\sqrt{a})(x-\sqrt{a}) &< 0 \end{aligned}$$

If $x < -\sqrt{a}$, then $x+\sqrt{a} < 0$ and $x-\sqrt{a} < -2\sqrt{a} < 0$. Therefore, $(x+\sqrt{a})(x-\sqrt{a}) > 0$, which is a contradiction.

If $-\sqrt{a} < x < \sqrt{a}$, then $0 < x+\sqrt{a} < 2\sqrt{a}$ and $-2\sqrt{a} < x-\sqrt{a} < 0$.

Therefore, $(x+\sqrt{a})(x-\sqrt{a}) < 0$.

If $x > \sqrt{a}$, then $x+\sqrt{a} > 2\sqrt{a} > 0$ and $x-\sqrt{a} > 0$. Therefore, $(x+\sqrt{a})(x-\sqrt{a}) > 0$, which is a contradiction. So the solution set for $x^2 < a$ is $\{x | -\sqrt{a} < x < \sqrt{a}\}$.

90. Given that $a > 0$,

$$\begin{aligned} x^2 &> a \\ x^2 - a &> 0 \\ (x+\sqrt{a})(x-\sqrt{a}) &> 0 \end{aligned}$$

If $x < -\sqrt{a}$, then $x+\sqrt{a} < 0$ and $x-\sqrt{a} < -2\sqrt{a} < 0$.

Therefore, $(x+\sqrt{a})(x-\sqrt{a}) > 0$.

If $-\sqrt{a} < x < \sqrt{a}$, then $0 < x+\sqrt{a} < 2\sqrt{a}$ and $-2\sqrt{a} < x-\sqrt{a} < 0$. Therefore, $(x+\sqrt{a})(x-\sqrt{a}) < 0$, which is a contradiction.

If $x > \sqrt{a}$, then $x+\sqrt{a} > 2\sqrt{a} > 0$ and $x-\sqrt{a} > 0$. Therefore, $(x+\sqrt{a})(x-\sqrt{a}) > 0$.

So the solution set for $x^2 > a$ is $\{x | x < -\sqrt{a} \text{ or } x > \sqrt{a}\}$.

91. $x^2 < 1$

$$\begin{aligned} -\sqrt{1} &< x < \sqrt{1} \\ -1 &< x < 1 \end{aligned}$$

The solution set is $\{x | -1 < x < 1\}$.

92. $x^2 < 4$

$$\begin{aligned} -\sqrt{4} &< x < \sqrt{4} \\ -2 &< x < 2 \end{aligned}$$

The solution set is $\{x | -2 < x < 2\}$.

93. $x^2 \geq 9$

$$\begin{aligned} x &\leq -\sqrt{9} \text{ or } x \geq \sqrt{9} \\ x &\leq -3 \text{ or } x \geq 3 \end{aligned}$$

The solution set is $\{x | x \leq -3 \text{ or } x \geq 3\}$.

94. $x^2 \geq 1$

$$\begin{aligned} x &\leq -\sqrt{1} \text{ or } x \geq \sqrt{1} \\ x &\leq -1 \text{ or } x \geq 1 \end{aligned}$$

The solution set is $\{x | x \leq -1 \text{ or } x \geq 1\}$.

Chapter 1: Equations and Inequalities

95. $x^2 \leq 16$
 $-\sqrt{16} \leq x \leq \sqrt{16}$
 $-4 \leq x \leq 4$

The solution set is $\{x | -4 \leq x \leq 4\}$.

96. $x^2 \leq 9$
 $-\sqrt{9} \leq x \leq \sqrt{9}$
 $-3 \leq x \leq 3$

The solution set is $\{x | -3 \leq x \leq 3\}$.

97. $x^2 > 4$
 $x < -\sqrt{4}$ or $x > \sqrt{4}$
 $x < -2$ or $x > 2$

The solution set is $\{x | x < -2$ or $x > 2\}$.

98. $x^2 \geq 16$
 $x \leq -\sqrt{16}$ or $x \geq \sqrt{16}$
 $x \leq -4$ or $x \geq 4$

The solution set is $\{x | x < -4$ or $x > 4\}$.

100. $|x + |3x - 2|| = 2$
 $x + |3x - 2| = 2$ or $x + |3x - 2| = -2$
 $x + |3x - 2| = 2$
 $|3x - 2| = 2 - x$
 $3x - 2 = 2 - x$ or $3x - 2 = -(2 - x)$
 $4x = 4$ or $3x - 2 = -2 + x$
 $x = 1$ or $2x = 0$
 $x = 1$ or $x = 0$

or

$x + |3x - 2| = -2$
 $|3x - 2| = -2 - x$
 $3x - 2 = -2 - x$ or $3x - 2 = -(-2 - x)$
 $4x = 0$ or $3x - 2 = 2 + x$
 $x = 0$ or $2x = 4$
 $x = 0$ or $x = 2$

The value 2 is extraneous. The solution set is $\{0, 1\}$.

101. $|2x - 5| = x + 13$ or $2x - 5 = -(x + 13)$
 $2x - 5 = x + 13$ or $2x - 5 = -x - 13$
 $x = 18$ or $3x = -8$
 $x = -\frac{8}{3}$

99. $|3x - |2x + 1|| = 4$
 $3x - |2x + 1| = 4$ or $3x - |2x + 1| = -4$
 $3x - |2x + 1| = 4$
 $3x - 4 = |2x + 1|$
 $2x + 1 = 3x - 4$ or $2x + 1 = -(3x - 4)$
 $-x = -5$ or $2x + 1 = -3x + 4$
 $x = 5$ or $5x = 3$
 $x = 5$ or $x = \frac{3}{5}$

or

$3x - |2x + 1| = -4$
 $3x + 4 = |2x + 1|$
 $2x + 1 = 3x + 4$ or $2x + 1 = -(3x + 4)$
 $-x = 3$ or $2x + 1 = -3x - 4$
 $x = -3$ or $5x = -5$
 $x = -3$ or $x = -1$

The values $\frac{3}{5}$ and -3 are extraneous.

The solution set is $\{-1, 5\}$.

$4 - 3y = -2$ or $4 - 3y = 2$
 $-3y = -6$ or $-3y = -2$
 $y = 2$ or $y = \frac{2}{3}$

The value of $\frac{y}{x}$ is largest using $x = 18$ and $y = 2$,

so $\frac{y}{x} = \frac{2}{18} = \frac{1}{9}$.

102. Since $|x| \geq 0$ for all real numbers, then

$\frac{|x|}{1 + |x|} \geq 0$ and $1 + \frac{|x|}{1 + |x|} \geq 1$. This means

$1 \leq 1 + \frac{|x|}{1 + |x|} \leq \frac{3}{2}$

$0 \leq \frac{1 + |x| - 1}{1 + |x|} \leq \frac{1}{2}$

$-1 \leq \frac{-1}{1 + |x|} \leq -\frac{1}{2}$

$1 \geq \frac{-1}{1 + |x|} \geq \frac{1}{2}$

$1 \leq 1 + |x| \leq 2$

$0 \leq |x| \leq 1$

Section 1.7: Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications

Therefore $-1 \leq x \leq 1$. The solution set in interval notation is $[-1, 1]$.

103 – 105. Answers will vary.

Section 1.7

1. mathematical modeling
2. interest
3. uniform motion
4. False; the amount charged for the use of principal is the interest.
5. True; this is the uniform motion formula.
6. a
7. b
8. c
9. Let A represent the area of the circle and r the radius. The area of a circle is the product of π times the square of the radius: $A = \pi r^2$
10. Let C represent the circumference of a circle and r the radius. The circumference of a circle is the product of π times twice the radius:
 $C = 2\pi r$
11. Let A represent the area of the square and s the length of a side. The area of the square is the square of the length of a side: $A = s^2$
12. Let P represent the perimeter of a square and s the length of a side. The perimeter of a square is four times the length of a side: $P = 4s$
13. Let F represent the force, m the mass, and a the acceleration. Force equals the product of the mass times the acceleration: $F = ma$
14. Let P represent the pressure, F the force, and A the area. Pressure is the force per unit area:
 $P = \frac{F}{A}$
15. Let W represent the work, F the force, and d the distance. Work equals force times distance:
 $W = Fd$
16. Let K represent the kinetic energy, m the mass, and v the velocity. Kinetic energy is one-half the product of the mass and the square of the velocity: $K = \frac{1}{2}mv^2$
17. C = total variable cost in dollars, x = number of dishwashers manufactured: $C = 150x$
18. R = total revenue in dollars, x = number of dishwashers sold: $R = 250x$
19. Let x represent the amount of money invested in bonds. Then $5000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$500, we have:
$$0.15x + 0.07(5000 - x) = 500$$
$$(100)(0.15x + 0.07(5000 - x)) = (500)(100)$$
$$15x + 7(5000 - x) = 50,000$$
$$15x + 35,000 - 7x = 50,000$$
$$8x + 35,000 = 50,000$$
$$8x = 15,000$$
$$x = 1875$$

\$1875 should be invested in bonds at 15% and \$3125 should be invested in CD's at 7%.
20. Let x represent the amount of money invested in bonds. Then $5000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$700, we have:
$$0.15x + 0.07(5000 - x) = 700$$
$$(100)(0.15x + 0.07(5000 - x)) = (700)(100)$$
$$15x + 7(5000 - x) = 70,000$$
$$15x + 35,000 - 7x = 70,000$$
$$8x + 35,000 = 70,000$$
$$8x = 35,000$$
$$x = 4375$$

\$4375 should be invested in bonds at 15% and \$625 should be invested in CD's at 7%.
21. Let x represent the amount of money loaned at 4%. Then $12,000 - x$ represents the amount of money loaned at 5.5%. Since the total interest is to be \$600, we have:

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 0.04x + 0.055(12,000 - x) &= 600 \\
 (100)(0.04x + 0.055(12,000 - x)) &= (600)(100) \\
 4x + 5.5(12,000 - x) &= 60,000 \\
 4x + 66,000 - 5.5x &= 60,000 \\
 -1.5x + 66,000 &= 60,000 \\
 -1.5x &= -6,000 \\
 x &= 4000
 \end{aligned}$$

\$4000 is loaned at 4%.

22. Let R represent the interest rate for the loan. Aditi will owe \$16,350 at the end of two years. This would be \$1350 in interest for the two years, or \$675 per year. We have:

$$\begin{aligned}
 R(15000) &= 675 \\
 R &= \frac{675}{15000} \\
 &= 0.045 \text{ or } 4.5\%
 \end{aligned}$$

Her interest rate would be 4.5%.

23. Let x represent the number of pounds of Earl Gray tea. Then $100 - x$ represents the number of pounds of Orange Pekoe tea.

$$6x + 4(100 - x) = 5.50(100)$$

$$6x + 400 - 4x = 550$$

$$2x + 400 = 550$$

$$2x = 150$$

$$x = 75$$

75 pounds of Earl Gray tea must be blended with 25 pounds of Orange Pekoe.

24. Let x represent the number of pounds of the first kind of coffee. Then $100 - x$ represents the number of pounds of the second kind of coffee.

$$2.75x + 5(100 - x) = 4.10(100)$$

$$2.75x + 500 - 5x = 410$$

$$-2.25x + 500 = 410$$

$$-2.25x = -90$$

$$x = 40$$

40 pounds of the first kind of coffee must be blended with 60 pounds of the second kind of coffee.

25. Let x represent the number of pounds of cashews. Then $x + 60$ represents the number of pounds in the mixture.

$$9x + 4.50(60) = 7.75(x + 60)$$

$$9x + 270 = 7.75x + 465$$

$$1.25x = 195$$

$$x = 156$$

156 pounds of cashews must be added to the 60 pounds of almonds.

26. Let x represent the number of caramels in the box. Then $30 - x$ represents the number of cremes in the box.

$$\text{Revenue} - \text{Cost} = \text{Profit}$$

$$12.50 - (0.25x + 0.45(30 - x)) = 3.00$$

$$12.50 - (0.25x + 13.5 - 0.45x) = 3.00$$

$$12.50 - (13.5 - 0.20x) = 3.00$$

$$12.50 - 13.50 + 0.20x = 3.00$$

$$-1.00 + 0.20x = 3.00$$

$$0.20x = 4.00$$

$$x = 20$$

The box should contain 20 caramels and 10 cremes.

27. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$16 - r$	$\frac{20}{60} = \frac{1}{3}$	$\frac{16 - r}{3}$
Downstream	$16 + r$	$\frac{15}{60} = \frac{1}{4}$	$\frac{16 + r}{4}$

Since the distance is the same in each direction:

$$\frac{16 - r}{3} = \frac{16 + r}{4}$$

$$4(16 - r) = 3(16 + r)$$

$$64 - 4r = 48 + 3r$$

$$16 = 7r$$

$$r = \frac{16}{7} \approx 2.286$$

The speed of the current is approximately 2.286 miles per hour.

28. Let r represent the speed of the motorboat.

	Rate	Time	Distance
Upstream	$r - 3$	5	$5(r - 3)$
Downstream	$r + 3$	2.5	$2.5(r + 3)$

The distance is the same in each direction:

$$5(r - 3) = 2.5(r + 3)$$

$$5r - 15 = 2.5r + 7.5$$

$$2.5r = 22.5$$

$$r = 9$$

The speed of the motorboat is 9 miles per hour.

Section 1.7: Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications

29. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15 - r$	$\frac{10}{15 - r}$	10
Downstream	$15 + r$	$\frac{10}{15 + r}$	10

Since the total time is 1.5 hours, we have:

$$\begin{aligned}\frac{10}{15 - r} + \frac{10}{15 + r} &= 1.5 \\ 10(15 + r) + 10(15 - r) &= 1.5(15 - r)(15 + r) \\ 150 + 10r + 150 - 10r &= 1.5(225 - r^2) \\ 300 &= 1.5(225 - r^2) \\ 200 &= 225 - r^2 \\ r^2 - 25 &= 0 \\ (r - 5)(r + 5) &= 0 \\ r &= 5 \text{ or } r = -5\end{aligned}$$

Speed must be positive, so disregard $r = -5$.
The speed of the current is 5 miles per hour.

30. Let r represent the rate of the slower car. Then $r + 10$ represents the rate of the faster car.

	Rate	Time	Distance
Slower car	r	3.5	$3.5r$
Faster car	$r + 10$	3	$3(r + 10)$

$$\begin{aligned}3.5r &= 3(r + 10) \\ 3.5r &= 3r + 30 \\ 0.5r &= 30 \\ r &= 60\end{aligned}$$

The slower car travels at a rate of 60 miles per hour. The faster car travels at a rate of 70 miles per hour. The distance is $(70)(3) = 210$ miles.

31. Let r represent Karen's normal walking speed.

	Rate	Time	Distance
With walkway	$r + 2.5$	$\frac{50}{r + 2.5}$	50
Against walkway	$r - 2.5$	$\frac{50}{r - 2.5}$	50

Since the total time is 48 seconds:

$$\begin{aligned}\frac{50}{r + 2.5} + \frac{50}{r - 2.5} &= 48 \\ 50(r - 2.5) + 50(r + 2.5) &= 48(r - 2.5)(r + 2.5) \\ 50r - 125 + 50r + 125 &= 48(r^2 - 6.25) \\ 100r &= 48r^2 - 300 \\ 0 &= 48r^2 - 100r - 300 \\ 0 &= 12r^2 - 25r - 75\end{aligned}$$

$$\begin{aligned}r &= \frac{-(-25) \pm \sqrt{(-25)^2 - 4(12)(-75)}}{2(12)} \\ &= \frac{25 \pm \sqrt{4225}}{24}\end{aligned}$$

$$r \approx 3.75 \text{ or } r \approx -1.67$$

Speed must be positive, so disregard $r \approx -1.67$.
Karen's normal walking speed is approximately 3.75 feet per second.

32. Let r represent the speed of the airport walkway.

	Rate	Time	Distance
Walking with	$1.5 + r$	$\frac{280}{1.5 + r}$	280
Standing still	r	$\frac{280}{r}$	280

Walking with the walkway takes 60 seconds less time than standing still on the walkway:

$$\begin{aligned}\frac{280}{1.5 + r} &= \frac{280}{r} - 60 \\ 280r &= 280(1.5 + r) - 60r(r + 1.5) \\ 280r &= 420 + 280r - 60r^2 - 90r \\ 60r^2 + 90r - 420 &= 0 \\ 2r^2 + 3r - 14 &= 0 \\ (2r + 7)(r - 2) &= 0 \\ 2r + 7 = 0 &\quad \text{or} \quad r - 2 = 0 \\ r = -\frac{7}{2} &\quad \text{or} \quad r = 2\end{aligned}$$

Speed must be positive, so disregard $r = -\frac{7}{2}$.

The speed of the airport walkway is 2 meters per second.

33. Let w represent the width of a regulation doubles tennis court. Then $2w + 6$ represents the length.

The area is 2808 square feet:

$$\begin{aligned}w(2w + 6) &= 2808 \\ 2w^2 + 6w &= 2808 \\ 2w^2 + 6w - 2808 &= 0 \\ w^2 + 3w - 1404 &= 0 \\ (w + 39)(w - 36) &= 0 \\ w + 39 = 0 &\quad \text{or} \quad w - 36 = 0 \\ w = -39 &\quad \text{or} \quad w = 36\end{aligned}$$

The width must be positive, so disregard $w = -39$.
The width of a regulation doubles tennis court is 36 feet and the length is $2(36) + 6 = 78$ feet.

Chapter 1: Equations and Inequalities

34. l = length of the court and w = width of the court

The width of the court is to be two feet less than half of its length. Thus, $w = \frac{1}{2}l - 2$. The area of the court is 880 square feet, so:

$$\left(\frac{1}{2}l - 2\right)(l) = 880$$

$$\frac{1}{2}l^2 - 2l = 880$$

$$\frac{1}{2}l^2 - 2l - 880 = 0$$

$$l^2 - 4l - 1760 = 0$$

$$(l + 40)(l - 44) = 0$$

$$l = -40 \text{ or } l = 44$$

Since the length cannot be negative then the length is 44 ft.

The area is 880, so:

$$44w = 880$$

$$w = \frac{880}{44}$$

$$w = 20$$

The dimensions of the court are 44 ft by 20 feet.

35. Let t represent the time it takes to do the job together.

	Time to do job	Part of job done in one minute
Trent	30	$\frac{1}{30}$
Lois	20	$\frac{1}{20}$
Together	t	$\frac{1}{t}$

$$\frac{1}{30} + \frac{1}{20} = \frac{1}{t}$$

$$2t + 3t = 60$$

$$5t = 60$$

$$t = 12$$

Working together, the job can be done in 12 minutes.

36. Let t represent the time it takes April to do the job working alone.

	Time to do job	Part of job done in one hour
Patrice	10	$\frac{1}{10}$
April	t	$\frac{1}{t}$
Together	6	$\frac{1}{6}$

$$\frac{1}{10} + \frac{1}{t} = \frac{1}{6}$$

$$3t + 30 = 5t$$

$$2t = 30$$

$$t = 15$$

April would take 15 hours to paint the rooms.

37. l = length of the garden
 w = width of the garden

- a. The length of the garden is to be twice its width. Thus, $l = 2w$.

The dimensions of the fence are $l + 4$ and $w + 4$.

The perimeter is 46 feet, so:

$$2(l + 4) + 2(w + 4) = 46$$

$$2(2w + 4) + 2(w + 4) = 46$$

$$4w + 8 + 2w + 8 = 46$$

$$6w + 16 = 46$$

$$6w = 30$$

$$w = 5$$

The dimensions of the garden are 5 feet by 10 feet.

- b. Area = $l \cdot w = 5 \cdot 10 = 50$ square feet
- c. If the dimensions of the garden are the same, then the length and width of the fence are also the same $(l + 4)$. The perimeter is 46 feet, so:

$$2(l + 4) + 2(l + 4) = 46$$

$$2l + 8 + 2l + 8 = 46$$

$$4l + 16 = 46$$

$$4l = 30$$

$$l = 7.5$$

The dimensions of the garden are 7.5 feet by 7.5 feet.

- d. Area = $l \cdot w = 7.5(7.5) = 56.25$ square feet.

38. l = length of the pond
 w = width of the pond

- a. The pond is to be a square. Thus, $l = w$.
The dimensions of the fenced area are $w + 6$

Section 1.7: Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications

on each side. The perimeter is 100 feet, so:

$$4(w+6) = 100$$

$$4w + 24 = 100$$

$$4w = 76$$

$$w = 19$$

The dimensions of the pond are 19 feet by 19 feet.

- b. The length of the pond is to be three times the width. Thus, $l = 3w$. The dimensions of the fenced area are $w + 6$ and $l + 6$. The perimeter is 100 feet, so:

$$2(w+6) + 2(l+6) = 100$$

$$2(w+6) + 2(3w+6) = 100$$

$$2w + 12 + 6w + 12 = 100$$

$$8w + 24 = 100$$

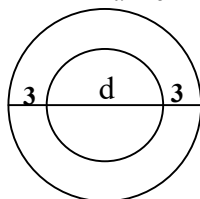
$$8w = 76$$

$$w = 9.5$$

$$l = 3(9.5) = 28.5$$

The dimensions of the pond are 9.5 feet by 28.5 feet.

- c. If the pond is circular, the diameter is d and the diameter of the circle with the pond and the deck is $d + 6$.



The perimeter is 100 feet, so:

$$\pi(d+6) = 100$$

$$\pi d + 6\pi = 100$$

$$\pi d = 100 - 6\pi$$

$$d = \frac{100}{\pi} - 6 \approx 25.83$$

The diameter of the pond is 25.83 feet.

- d. $\text{Area}_{\text{square}} = l \cdot w = 19(19) = 361 \text{ ft}^2$.

$$\text{Area}_{\text{rectangle}} = l \cdot w = 28.5(9.5) = 270.75 \text{ ft}^2$$

$$\text{Area}_{\text{circle}} = \pi r^2 = \pi \left(\frac{25.83}{2} \right)^2 \approx 524 \text{ ft}^2$$

The circular pond has the largest area.

39. Let t represent the time it takes for the defensive back to catch the tight end.

	Time to run 100 yards	Time	Rate	Distance
Tight End	12 sec	t	$\frac{100}{12} = \frac{25}{3}$	$\frac{25}{3}t$
Def. Back	10 sec	t	$\frac{100}{10} = 10$	$10t$

Since the defensive back has to run 5 yards farther, we have:

$$\frac{25}{3}t + 5 = 10t$$

$$25t + 15 = 30t$$

$$15 = 5t$$

$$t = 3 \rightarrow 10t = 30$$

The defensive back will catch the tight end at the 45 yard line ($15 + 30 = 45$).

40. Let x represent the number of highway miles traveled. Then $30,000 - x$ represents the number of city miles traveled.

$$\frac{x}{40} + \frac{30,000 - x}{25} = 900$$

$$200 \left(\frac{x}{40} + \frac{30,000 - x}{25} \right) = 200(900)$$

$$5x + 240,000 - 8x = 180,000$$

$$-3x + 240,000 = 180,000$$

$$-3x = -60,000$$

$$x = 20,000$$

There is allowed to claim 20,000 miles as a business expense.

41. Let x represent the number of gallons of pure water. Then $x + 1$ represents the number of gallons in the 60% solution.

$$(\%)(\text{gallons}) + (\%)(\text{gallons}) = (\%)(\text{gallons})$$

$$0(x) + 1(1) = 0.60(x + 1)$$

$$1 = 0.6x + 0.6$$

$$0.4 = 0.6x$$

$$x = \frac{4}{6} = \frac{2}{3}$$

$\frac{2}{3}$ gallon of pure water should be added.

42. Let x represent the number of liters to be drained and replaced with pure antifreeze.

Chapter 1: Equations and Inequalities

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$1(x) + 0.40(15 - x) = 0.60(15)$$

$$x + 6 - 0.40x = 9$$

$$0.60x = 3$$

$$x = 5$$

5 liters should be drained and replaced with pure antifreeze.

43. Let x represent the number of ounces of water to be evaporated; the amount of salt remains the same. Therefore, we get

$$0.04(32) = 0.06(32 - x)$$

$$1.28 = 1.92 - 0.06x$$

$$0.06x = 0.64$$

$$x = \frac{0.64}{0.06} = \frac{64}{6} = \frac{32}{3} = 10\frac{2}{3}$$

$10\frac{2}{3} \approx 10.67$ ounces of water need to be evaporated.

44. Let x represent the number of gallons of water to be evaporated; the amount of salt remains the same.

$$0.03(240) = 0.05(240 - x)$$

$$7.2 = 12 - 0.05x$$

$$0.05x = 4.8$$

$$x = \frac{4.8}{0.05} = 96$$

96 gallons of water need to be evaporated.

45. Let x represent the number of grams of pure gold. Then $60 - x$ represents the number of grams of 12 karat gold to be used.

$$x + \frac{1}{2}(60 - x) = \frac{2}{3}(60)$$

$$x + 30 - 0.5x = 40$$

$$0.5x = 10$$

$$x = 20$$

20 grams of pure gold should be mixed with 40 grams of 12 karat gold.

46. Let x represent the number of atoms of oxygen. $2x$ represents the number of atoms of hydrogen. $x + 1$ represents the number of atoms of carbon. $x + 2x + x + 1 = 45$

$$4x = 44$$

$$x = 11$$

There are 11 atoms of oxygen and 22 atoms of hydrogen in the sugar molecule.

47. Let t represent the time it takes for Mike to catch up with Dan. Since the distances are the same, we have:

$$\frac{1}{6}t = \frac{1}{9}(t + 1)$$

$$3t = 2t + 2$$

$$t = 2$$

Mike will pass Dan after 2 minutes, which is a distance of $\frac{1}{3}$ mile.

48. Let t represent the time of flight with the wind. The distance is the same in each direction:

$$330t = 270(5 - t)$$

$$330t = 1350 - 270t$$

$$600t = 1350$$

$$t = 2.25$$

The distance the plane can fly and still return safely is $330(2.25) = 742.5$ miles.

49. Let t represent the time the auxiliary pump needs to run. Since the two pumps are emptying one tanker, we have:

$$\frac{3}{4} + \frac{t}{9} = 1$$

$$27 + 4t = 36$$

$$4t = 9$$

$$t = \frac{9}{4} = 2.25$$

The auxiliary pump must run for 2.25 hours. It must be started at 9:45 a.m.

50. Let x represent the number of pounds of pure cement. Then $x + 20$ represents the number of pounds in the 40% mixture.

$$x + 0.25(20) = 0.40(x + 20)$$

$$x + 5 = 0.4x + 8$$

$$0.6x = 3$$

$$x = \frac{30}{6} = 5$$

5 pounds of pure cement should be added.

51. Let t represent the time for the tub to fill with the faucets on and the stopper removed. Since one tub is being filled, we have:

Section 1.7: Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications

$$\frac{t}{15} + \left(-\frac{t}{20}\right) = 1$$

$$4t - 3t = 60$$

$$t = 60$$

60 minutes is required to fill the tub.

52. Let t be the time the 5 horsepower pump needs to run to finish emptying the pool. Since the two pumps are emptying one pool, we have:

$$\frac{t+2}{5} + \frac{2}{8} = 1$$

$$4(2+t) + 5 = 20$$

$$8 + 4t + 5 = 20$$

$$4t = 7$$

$$t = 1.75$$

The 5 horsepower pump must run for an additional 1.75 hours or 1 hour and 45 minutes to empty the pool.

53. Let t represent the time spent running. Then $5-t$ represents the time spent biking.

	Rate	Time	Distance
Run	6	t	$6t$
Bike	25	$5-t$	$25(5-t)$

The total distance is 87 miles:

$$6t + 25(5-t) = 87$$

$$6t + 125 - 25t = 87$$

$$-19t + 125 = 87$$

$$-19t = -38$$

$$t = 2$$

The time spent running is 2 hours, so the distance of the run is $6(2) = 12$ miles. The distance of the bicycle race is $25(5-2) = 75$ miles.

54. Let r represent the speed of the eastbound cyclist. Then $r+5$ represents the speed of the westbound cyclist.

	Rate	Time	Distance
Eastbound	r	6	$6r$
Westbound	$r+5$	6	$6(r+5)$

The total distance is 246 miles:

$$6r + 6(r+5) = 246$$

$$6r + 6r + 30 = 246$$

$$12r + 30 = 246$$

$$12r = 216$$

$$r = 18$$

The speed of the eastbound cyclist is 18 miles per hour, and the speed of the westbound cyclist is $18+5 = 23$ miles per hour.

55. Burke's rate is $\frac{100}{12}$ meters/sec. In 9.81 seconds, Burke will run $\frac{100}{12}(9.81) = 81.75$ meters. Bolt would win by $100-81.75=18.25$ meters.

56. $A = 2\pi r^2 + 2\pi r h$. Since $A = 58.9\pi$ square inches and $h = 6.4$ inches,

$$2\pi r^2 + 2\pi r(6.4) = 58.9\pi$$

$$2\pi r^2 + 12.8\pi r - 58.9\pi = 0$$

$$2r^2 + 12.8r - 58.9 = 0$$

$$r = \frac{-12.8 \pm \sqrt{(12.8)^2 - 4(2)(-58.9)}}{2(2)}$$

$$= \frac{-12.8 \pm \sqrt{635.04}}{4}$$

$$r = 3.1 \text{ or } r = -9.5$$

The radius of the coffee can is 3.1 inches.

57. Let the individual times to complete the project be E for Elaine, B for Brian, and D for either daughter. Using the respective rates gives

$$\frac{1}{E} + \frac{1}{B} = \frac{1}{2}, \frac{1}{E} + \frac{1}{D} + \frac{1}{D} = \frac{1}{2} \text{ (or } \frac{1}{E} + \frac{2}{D} = \frac{1}{2}),$$

$$\text{and } \frac{1}{B} + \frac{1}{D} = \frac{1}{4}. \text{ From the first two equations,}$$

$$\frac{1}{B} = \frac{2}{D}. \text{ Substituting into the third equation}$$

$$\text{gives } \frac{2}{D} + \frac{1}{D} = \frac{1}{4} \text{ or } \frac{3}{D} = \frac{1}{4} \rightarrow D = 12 \text{ hours.}$$

$$\text{Then } \frac{1}{E} + \frac{2}{12} = \frac{1}{2} \rightarrow E = 3 \text{ hours and}$$

$$\frac{1}{B} + \frac{1}{12} = \frac{1}{4} \rightarrow B = 6 \text{ hours. The combined rate}$$

$$\text{of Elaine, Brian, and one of their daughters is } \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{7}{12} \text{ project per hour, so it will take}$$

$$\text{them } \frac{12}{7} \text{ hours to complete the project.}$$

58. If x = liters of original solution, then there were originally $0.2x$ liters of salt and 0.8 liters of pure water. Over time, the solution loses $0.25(0.8x) = 0.2x$ liters of pure water. She adds 20 liters of salt so the total amount of salt is

Chapter 1: Equations and Inequalities

$0.2x + 20$ liters. She also adds 10 liters of pure water, so the total amount of pure water is $0.8x - 0.2x + 10 = 0.6x + 10$ liters. The resulting concentration is $33\frac{1}{3}\%$ which means

$$\frac{0.2x + 20}{0.2x + 20 + 0.6x + 10} = \frac{1}{3} \text{ or } \frac{0.2x + 20}{0.8x + 30} = \frac{1}{3} \text{ or } 0.6x + 60 = 0.8x + 30 \rightarrow x = 150.$$

There were initially 150 liters of solution in the vat.

59. The speed of the train relative to the man is $30 - 4 = 26$ miles per hour. The time is

$$5 \text{ sec} = \frac{5}{60} \text{ min} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}.$$

$$d = rt$$

$$= 26 \left(\frac{1}{720} \right)$$

$$= \frac{26}{720} \text{ miles}$$

$$= \frac{26}{720} \cdot 5280 \approx 190.67 \text{ feet}$$

The freight train is about 190.67 feet long.

60. Answers will vary.
61. Let x be the original selling price of the shirt.
 Profit = Revenue - Cost
 $4 = x - 0.40x - 20 \rightarrow 24 = 0.60x \rightarrow x = 40$
 The original price should be \$40 to ensure a profit of \$4 after the sale.

If the sale is 50% off, the profit is:
 $40 - 0.50(40) - 20 = 40 - 20 - 20 = 0$
 At 50% off there will be no profit.

62. Let t_1 and t_2 represent the times for the two segments of the trip. Since Atlanta is halfway between Chicago and Miami, the distances are equal.
- $$45t_1 = 55t_2$$
- $$t_1 = \frac{55}{45}t_2$$
- $$t_1 = \frac{11}{9}t_2$$

Computing the average speed:

$$\begin{aligned} \text{Avg Speed} &= \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2} \\ &= \frac{45\left(\frac{11}{9}t_2\right) + 55t_2}{\frac{11}{9}t_2 + t_2} = \frac{55t_2 + 55t_2}{\left(\frac{11t_2 + 9t_2}{9}\right)} \\ &= \frac{110t_2}{\left(\frac{20t_2}{9}\right)} = \frac{990t_2}{20t_2} \\ &= \frac{99}{2} = 49.5 \text{ miles per hour} \end{aligned}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

63. The time traveled with the tail wind was:

$$t = \frac{919}{550} \approx 1.67091 \text{ hours}.$$

Since they were 20 minutes $\left(\frac{1}{3} \text{ hour}\right)$ early, the time in still air would have been:

$$1.67091 \text{ hrs} + 20 \text{ min} = (1.67091 + 0.33333) \text{ hrs} \approx 2.00424 \text{ hrs}$$

Thus, with no wind, the ground speed is

$$\frac{919}{2.00424} \approx 458.53. \text{ Therefore, the tail wind is } 550 - 458.53 = 91.47 \text{ knots}.$$

64. It is impossible to mix two solutions with a lower concentration and end up with a new solution with a higher concentration.

Algebraic Solution:

Let x = the number of liters of 25% solution.

$$(\%)(\text{liters}) + (\%)(\text{liters}) = (\%)(\text{liters})$$

$$0.25x + 0.48(20) = 0.58(20 + x)$$

$$0.25x + 9.6 = 10.6 + 0.58x$$

$$-0.33x = 1$$

$$x \approx -3.03 \text{ liters}$$

(not possible)

Chapter 1 Review

1. $2 - \frac{x}{3} = 8$

$6 - x = 24$

$x = -18$

The solution set is $\{-18\}$.

2. $-2(5 - 3x) + 8 = 4 + 5x$

$-10 + 6x + 8 = 4 + 5x$

$6x - 2 = 4 + 5x$

$x = 6$

The solution set is $\{6\}$.

3. $\frac{x}{x-1} = \frac{6}{5}$

$5x = 6x - 6$

$6 = x$

Since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

4. $(2x + 7)^2 = 20$

$2x + 7 = \pm\sqrt{20}$

$2x + 7 = \pm 2\sqrt{5}$

$2x = -7 \pm 2\sqrt{5}$

$x = \frac{-7 \pm 2\sqrt{5}}{2}$

The solution set is $\left\{\frac{-7 - 2\sqrt{5}}{2}, \frac{-7 + 2\sqrt{5}}{2}\right\}$

5. $x(1 - x) = 6$

$x - x^2 = 6$

$0 = x^2 - x + 6$

$b^2 - 4ac = (-1)^2 - 4(1)(6)$

$= 1 - 24 = -23$

Therefore, there are no real solutions.

6. $x(1 + x) = 6$

$x + x^2 = 6$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \text{ or } x = 2$

The solution set is $\{-3, 2\}$.

7. $\frac{1}{2}\left(x - \frac{1}{3}\right) = \frac{3}{4} - \frac{x}{6}$

$(12)\left(\frac{1}{2}\right)\left(x - \frac{1}{3}\right) = \left(\frac{3}{4} - \frac{x}{6}\right)(12)$

$6x - 2 = 9 - 2x$

$8x = 11$

$x = \frac{11}{8}$

The solution set is $\left\{\frac{11}{8}\right\}$.

8. $\frac{1 - 3x}{4} = \frac{x + 6}{3} + \frac{1}{2}$

$(12)\left(\frac{1 - 3x}{4}\right) = \left(\frac{x + 6}{3} + \frac{1}{2}\right)(12)$

$3(1 - 3x) = 4(x + 6) + 6$

$3 - 9x = 4x + 24 + 6$

$-13x = 27$

$x = -\frac{27}{13}$

The solution set is $\left\{-\frac{27}{13}\right\}$.

9. $(x - 1)(2x + 3) = 3$

$2x^2 + x - 3 = 3$

$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$x = \frac{3}{2} \text{ or } x = -2$

The solution set is $\left\{-2, \frac{3}{2}\right\}$.

10. $2x + 3 = 4x^2$

$0 = 4x^2 - 2x - 3$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-3)}}{2(4)}$

$= \frac{2 \pm \sqrt{52}}{8} = \frac{2 \pm 2\sqrt{13}}{8} = \frac{1 \pm \sqrt{13}}{4}$

The solution set is $\left\{\frac{1 - \sqrt{13}}{4}, \frac{1 + \sqrt{13}}{4}\right\}$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 11. \quad & \sqrt[3]{x^2 - 1} = 2 \\
 & (\sqrt[3]{x^2 - 1})^3 = (2)^3 \\
 & x^2 - 1 = 8 \\
 & x^2 = 9 \\
 & x = \pm 3
 \end{aligned}$$

Check $x = -3$:

$$\begin{aligned}
 \sqrt[3]{(-3)^2 - 1} &= 2 \\
 \sqrt[3]{9 - 1} &= 2 \\
 \sqrt[3]{8} &= 2 \\
 2 &= 2
 \end{aligned}$$

Check $x = 3$:

$$\begin{aligned}
 \sqrt[3]{(3)^2 - 1} &= 2 \\
 \sqrt[3]{9 - 1} &= 2 \\
 \sqrt[3]{8} &= 2 \\
 2 &= 2
 \end{aligned}$$

The solution set is $\{-3, 3\}$.

$$\begin{aligned}
 12. \quad & \sqrt{1 + x^3} = 3 \\
 & (\sqrt{1 + x^3})^2 = (3)^2 \\
 & 1 + x^3 = 9 \\
 & x^3 = 8 \\
 & x = \sqrt[3]{8} = 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Check } x = 2: \quad & \sqrt{1 + (2)^3} = 3 \\
 & \sqrt{9} = 3 \\
 & 3 = 3
 \end{aligned}$$

The solution set is $\{2\}$.

$$\begin{aligned}
 13. \quad & x(x+1) + 2 = 0 \\
 & x^2 + x + 2 = 0 \\
 & x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(2)}}{2(1)} = \frac{-1 \pm \sqrt{-7}}{2}
 \end{aligned}$$

No real solution.

$$\begin{aligned}
 14. \quad & x^4 - 5x^2 + 4 = 0 \\
 & (x^2 - 4)(x^2 - 1) = 0 \\
 & x^2 - 4 = 0 \text{ or } x^2 - 1 = 0 \\
 & x = \pm 2 \text{ or } x = \pm 1
 \end{aligned}$$

The solution set is $\{-2, -1, 1, 2\}$.

$$\begin{aligned}
 15. \quad & \sqrt{2x - 3} + x = 3 \\
 & \sqrt{2x - 3} = 3 - x \\
 & 2x - 3 = 9 - 6x + x^2 \\
 & x^2 - 8x + 12 = 0 \\
 & (x - 2)(x - 6) = 0 \\
 & x = 2 \text{ or } x = 6
 \end{aligned}$$

$$\text{Check } x = 2: \sqrt{2(2) - 3} + 2 = \sqrt{1} + 2 = 3$$

$$\text{Check } x = 6: \sqrt{2(6) - 3} + 6 = \sqrt{9} + 6 = 9 \neq 3$$

The solution set is $\{2\}$.

$$\begin{aligned}
 16. \quad & \sqrt[4]{2x + 3} = 2 \\
 & (\sqrt[4]{2x + 3})^4 = 2^4 \\
 & 2x + 3 = 16 \\
 & 2x = 13 \\
 & x = \frac{13}{2}
 \end{aligned}$$

$$\text{Check } x = \frac{13}{2}:$$

$$\sqrt[4]{2\left(\frac{13}{2}\right) + 3} = \sqrt[4]{13 + 3} = \sqrt[4]{16} = 2$$

The solution set is $\left\{\frac{13}{2}\right\}$.

$$\begin{aligned}
 17. \quad & \sqrt{4x^2 + x - 6} = \sqrt{x - 1} \\
 & (\sqrt{4x^2 + x - 6})^2 = (\sqrt{x - 1})^2 \\
 & 4x^2 + x - 6 = x - 1 \\
 & 4x^2 = 5 \\
 & x^2 = \frac{5}{4}
 \end{aligned}$$

$$x = \pm \sqrt{\frac{5}{4}} = \pm \frac{\sqrt{5}}{2}$$

$$\text{Check } x = \frac{\sqrt{5}}{2}:$$

$$\begin{aligned}
 \sqrt{4\left(\frac{\sqrt{5}}{2}\right)^2 + \frac{\sqrt{5}}{2} - 6} &= \sqrt{\left(\frac{\sqrt{5}}{2}\right) - 1} \\
 0.34356 &= 0.34356
 \end{aligned}$$

$$\text{Check } x = -\frac{\sqrt{5}}{2}:$$

$$\sqrt{4\left(-\frac{\sqrt{5}}{2}\right)^2 - \frac{\sqrt{5}}{2} - 6} = \sqrt{\left(-\frac{\sqrt{5}}{2}\right) - 1}$$

The second solution is not possible because it makes the radicand negative.

The solution set is $\left\{\frac{\sqrt{5}}{2}\right\}$.

$$\begin{aligned}
 18. \quad & \sqrt{2x-1} - \sqrt{x-5} = 3 \\
 & \sqrt{2x-1} = 3 + \sqrt{x-5} \\
 & (\sqrt{2x-1})^2 = (3 + \sqrt{x-5})^2 \\
 & 2x-1 = 9 + 6\sqrt{x-5} + x-5 \\
 & x-5 = 6\sqrt{x-5} \\
 & (x-5)^2 = (6\sqrt{x-5})^2 \\
 & x^2 - 10x + 25 = 36(x-5) \\
 & x^2 - 10x + 25 = 36x - 180 \\
 & x^2 - 46x + 205 = 0 \\
 & (x-41)(x-5) = 0 \\
 & x = 41 \text{ or } x = 5
 \end{aligned}$$

Check $x = 41$:

$$\sqrt{2(41)-1} - \sqrt{41-5} = \sqrt{81} - \sqrt{36} = 9 - 6 = 3$$

Check $x = 5$:

$$\sqrt{2(5)-1} - \sqrt{5-5} = \sqrt{9} - \sqrt{0} = 3 - 0 = 3$$

The solution set is $\{5, 41\}$.

$$\begin{aligned}
 19. \quad & 2x^{1/2} - 3 = 0 \\
 & 2x^{1/2} = 3 \\
 & (2x^{1/2})^2 = 3^2 \\
 & 4x = 9 \\
 & x = \frac{9}{4}
 \end{aligned}$$

Check $x = \frac{9}{4}$:

$$2\left(\frac{9}{4}\right)^{1/2} - 3 = 2\left(\frac{3}{2}\right) - 3 = 3 - 3 = 0$$

The solution set is $\left\{\frac{9}{4}\right\}$.

$$\begin{aligned}
 20. \quad & x^{-6} - 7x^{-3} - 8 = 0 \\
 & \text{Let } u = x^{-3} \text{ so that } u^2 = x^{-6}. \\
 & u^2 - 7u - 8 = 0 \\
 & (u-8)(u+1) = 0 \\
 & u = 8 \quad \text{or} \quad u = -1 \\
 & x^{-3} = 8 \quad \text{or} \quad x^{-3} = -1 \\
 & (x^{-3})^{-1/3} = (8)^{-1/3} \quad \text{or} \quad (x^{-3})^{-1/3} = (-1)^{-1/3} \\
 & x = \frac{1}{2} \quad \text{or} \quad x = -1
 \end{aligned}$$

$$\text{Check } \frac{1}{2}: \left(\frac{1}{2}\right)^{-6} - 7\left(\frac{1}{2}\right)^{-3} - 8 = 64 - 56 - 8 = 0$$

$$\text{Check } -1: (-1)^{-6} - 7(-1)^{-3} - 8 = 1 + 7 - 8 = 0$$

The solution set is $\left\{-1, \frac{1}{2}\right\}$.

$$\begin{aligned}
 21. \quad & x^2 + m^2 = 2mx + (nx)^2 \\
 & x^2 + m^2 = 2mx + n^2x^2 \\
 & x^2 - n^2x^2 - 2mx + m^2 = 0 \\
 & (1-n^2)x^2 - 2mx + m^2 = 0 \\
 & x = \frac{-(-2m) \pm \sqrt{(-2m)^2 - 4(1-n^2)m^2}}{2(1-n^2)} \\
 & = \frac{2m \pm \sqrt{4m^2 - 4m^2 + 4m^2n^2}}{2(1-n^2)} \\
 & = \frac{2m \pm \sqrt{4m^2n^2}}{2(1-n^2)} = \frac{2m \pm 2mn}{2(1-n^2)} \\
 & = \frac{2m(1 \pm n)}{2(1-n^2)} = \frac{m(1 \pm n)}{1-n^2} \\
 & x = \frac{m(1+n)}{1-n^2} = \frac{m(1+n)}{(1+n)(1-n)} = \frac{m}{1-n} \\
 & \text{or} \\
 & x = \frac{m(1-n)}{1-n^2} = \frac{m(1-n)}{(1+n)(1-n)} = \frac{m}{1+n}
 \end{aligned}$$

The solution set is $\left\{\frac{m}{1-n}, \frac{m}{1+n}\right\}$, $n \neq 1$, $n \neq -1$.

$$\begin{aligned}
 22. \quad & 10a^2x^2 - 2abx - 36b^2 = 0 \\
 & 5a^2x^2 - abx - 18b^2 = 0 \\
 & (5ax + 9b)(ax - 2b) = 0 \\
 & 5ax + 9b = 0 \quad \text{or} \quad ax - 2b = 0 \\
 & 5ax = -9b \quad \quad \quad ax = 2b \\
 & x = -\frac{9b}{5a} \quad \quad \quad x = \frac{2b}{a}
 \end{aligned}$$

The solution set is $\left\{-\frac{9b}{5a}, \frac{2b}{a}\right\}$, $a \neq 0$.

Chapter 1: Equations and Inequalities

$$\begin{aligned}
 23. \quad & \sqrt{x^2+3x+7} - \sqrt{x^2-3x+9} + 2 = 0 \\
 & \sqrt{x^2+3x+7} = \sqrt{x^2-3x+9} - 2 \\
 & (\sqrt{x^2+3x+7})^2 = (\sqrt{x^2-3x+9} - 2)^2 \\
 & x^2 + 3x + 7 = x^2 - 3x + 9 - 4\sqrt{x^2-3x+9} + 4 \\
 & 6x - 6 = -4\sqrt{x^2-3x+9} \\
 & (6(x-1))^2 = (-4\sqrt{x^2-3x+9})^2 \\
 & 36(x^2 - 2x + 1) = 16(x^2 - 3x + 9) \\
 & 36x^2 - 72x + 36 = 16x^2 - 48x + 144 \\
 & 20x^2 - 24x - 108 = 0 \\
 & 5x^2 - 6x - 27 = 0 \\
 & (5x+9)(x-3) = 0 \\
 & x = -\frac{9}{5} \quad \text{or} \quad x = 3
 \end{aligned}$$

Check $x = -\frac{9}{5}$:

$$\begin{aligned}
 & \sqrt{\left(-\frac{9}{5}\right)^2 + 3\left(-\frac{9}{5}\right) + 7} - \sqrt{\left(-\frac{9}{5}\right)^2 - 3\left(-\frac{9}{5}\right) + 9} + 2 \\
 &= \sqrt{\frac{81}{25} - \frac{27}{5} + 7} - \sqrt{\frac{81}{25} + \frac{27}{5} + 9} + 2 \\
 &= \sqrt{\frac{81-135+175}{25}} - \sqrt{\frac{81+135+225}{25}} + 2 \\
 &= \sqrt{\frac{121}{25}} - \sqrt{\frac{441}{25}} + 2 = \frac{11}{5} - \frac{21}{5} + 2 = 0
 \end{aligned}$$

Check $x = 3$:

$$\begin{aligned}
 & \sqrt{(3)^2 + 3(3) + 7} - \sqrt{(3)^2 - 3(3) + 9} + 2 \\
 &= \sqrt{9+9+7} - \sqrt{9-9+9} + 2 \\
 &= \sqrt{25} - \sqrt{9} + 2 = 2 + 2 \\
 &= 4 \neq 0
 \end{aligned}$$

The solution set is $\left\{-\frac{9}{5}\right\}$.

$$\begin{aligned}
 24. \quad & |2x+3| = 7 \\
 & 2x+3 = 7 \quad \text{or} \quad 2x+3 = -7 \\
 & 2x = 4 \quad \text{or} \quad 2x = -10 \\
 & x = 2 \quad \text{or} \quad x = -5 \\
 & \text{The solution set is } \{-5, 2\}.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & |2-3x| + 2 = 9 \\
 & |2-3x| = 7 \\
 & 2-3x = 7 \quad \text{or} \quad 2-3x = -7 \\
 & -3x = 5 \quad \text{or} \quad -3x = -9 \\
 & x = -\frac{5}{3} \quad \text{or} \quad x = 3
 \end{aligned}$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

$$\begin{aligned}
 26. \quad & 2x^3 = 3x^2 \\
 & 2x^3 - 3x^2 = 0 \\
 & x^2(2x-3) = 0 \\
 & x^2 = 0 \quad \text{or} \quad 2x-3 = 0 \\
 & x = 0 \quad \text{or} \quad x = \frac{3}{2}
 \end{aligned}$$

The solution set is $\left\{0, \frac{3}{2}\right\}$.

$$\begin{aligned}
 27. \quad & 2x^3 + 5x^2 - 8x - 20 = 0 \\
 & x^2(2x+5) - 4(2x+5) = 0 \\
 & (2x+5)(x^2-4) = 0 \\
 & 2x+5 = 0 \quad \text{or} \quad x^2-4 = 0 \\
 & 2x = -5 \quad \text{or} \quad x^2 = 4 \\
 & x = -\frac{5}{2} \quad \text{or} \quad x = \pm 2
 \end{aligned}$$

The solution set is $\left\{-\frac{5}{2}, -2, 2\right\}$.

$$\begin{aligned}
 28. \quad & \frac{2x-3}{5} + 2 \leq \frac{x}{2} \\
 & 2(2x-3) + 10(2) \leq 5x \\
 & 4x - 6 + 20 \leq 5x \\
 & 14 \leq x \\
 & x \geq 14 \\
 & \{x | x \geq 14\} \quad \text{or} \quad [14, \infty)
 \end{aligned}$$



$$29. -9 \leq \frac{2x+3}{-4} \leq 7$$

$$36 \geq 2x+3 \geq -28$$

$$33 \geq 2x \geq -31$$

$$\frac{33}{2} \geq x \geq -\frac{31}{2}$$

$$-\frac{31}{2} \leq x \leq \frac{33}{2}$$

$$\left\{x \mid -\frac{31}{2} \leq x \leq \frac{33}{2}\right\} \text{ or } \left[-\frac{31}{2}, \frac{33}{2}\right]$$



$$30. 2 < \frac{3-3x}{12} < 6$$

$$24 < 3-3x < 72$$

$$21 < -3x < 69$$

$$-7 > x > -23$$

$$\{x \mid -23 < x < -7\} \text{ or } (-23, -7)$$



$$31. |3x+4| < \frac{1}{2}$$

$$-\frac{1}{2} < 3x+4 < \frac{1}{2}$$

$$-\frac{9}{2} < 3x < -\frac{7}{2}$$

$$-\frac{3}{2} < x < -\frac{7}{6}$$

$$\left\{x \mid -\frac{3}{2} < x < -\frac{7}{6}\right\} \text{ or } \left(-\frac{3}{2}, -\frac{7}{6}\right)$$



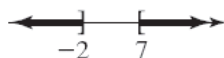
$$32. |2x-5| \geq 9$$

$$2x-5 \leq -9 \text{ or } 2x-5 \geq 9$$

$$2x \leq -4 \text{ or } 2x \geq 14$$

$$x \leq -2 \text{ or } x \geq 7$$

$$\{x \mid x \leq -2 \text{ or } x \geq 7\} \text{ or } (-\infty, -2] \cup [7, \infty)$$



$$33. 2 + |2-3x| \leq 4$$

$$|2-3x| \leq 2$$

$$-2 \leq 2-3x \leq 2$$

$$-4 \leq -3x \leq 0$$

$$\frac{4}{3} \geq x \geq 0$$

$$\left\{x \mid 0 \leq x \leq \frac{4}{3}\right\} \text{ or } \left[0, \frac{4}{3}\right]$$



$$34. 1 - |2-3x| < -4$$

$$-|2-3x| < -5$$

$$|2-3x| > 5$$

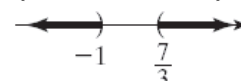
$$2-3x < -5 \text{ or } 2-3x > 5$$

$$7 < 3x \text{ or } -3 > 3x$$

$$\frac{7}{3} < x \text{ or } -1 > x$$

$$x < -1 \text{ or } x > \frac{7}{3}$$

$$\left\{x \mid x < -1 \text{ or } x > \frac{7}{3}\right\} \text{ or } (-\infty, -1) \cup \left(\frac{7}{3}, \infty\right)$$



$$35. (6+3i) - (2-4i) = (6-2) + (3-(-4))i = 4+7i$$

$$36. 4(3-i) + 3(-5+2i) = 12-4i-15+6i = -3+2i$$

$$37. \frac{3}{3+i} = \frac{3}{3+i} \cdot \frac{3-i}{3-i} = \frac{9-3i}{9-3i+3i-i^2} \\ = \frac{9-3i}{10} = \frac{9}{10} - \frac{3}{10}i$$

$$38. i^{50} = i^{48} \cdot i^2 = (i^4)^{12} \cdot i^2 = 1^{12}(-1) = -1$$

$$39. (2+3i)^3 = (2+3i)^2(2+3i) \\ = (4+12i+9i^2)(2+3i) \\ = (-5+12i)(2+3i) \\ = -10-15i+24i+36i^2 \\ = -46+9i$$

Chapter 1: Equations and Inequalities

40. $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1,$

$b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3$

$$x = \frac{-1 \pm \sqrt{-3}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The solution set is $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}.$

41. $2x^2 + x - 2 = 0$

$a = 2, b = 1, c = -2,$

$b^2 - 4ac = 1^2 - 4(2)(-2) = 1 + 16 = 17$

$$x = \frac{-1 \pm \sqrt{17}}{2(2)} = \frac{-1 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{-1 - \sqrt{17}}{4}, \frac{-1 + \sqrt{17}}{4}\right\}.$

42. $x^2 + 3 = x$

$x^2 - x + 3 = 0$

$a = 1, b = -1, c = 3,$

$b^2 - 4ac = (-1)^2 - 4(1)(3) = 1 - 12 = -11$

$$x = \frac{-(-1) \pm \sqrt{-11}}{2(1)} = \frac{1 \pm \sqrt{11}i}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}i$$

The solution set is $\left\{\frac{1}{2} - \frac{\sqrt{11}}{2}i, \frac{1}{2} + \frac{\sqrt{11}}{2}i\right\}.$

43. $x(1 - x) = 6$

$-x^2 + x - 6 = 0$

$a = -1, b = 1, c = -6,$

$b^2 - 4ac = 1^2 - 4(-1)(-6) = 1 - 24 = -23$

$$x = \frac{-1 \pm \sqrt{-23}}{2(-1)} = \frac{-1 \pm \sqrt{23}i}{-2} = \frac{1}{2} \pm \frac{\sqrt{23}}{2}i$$

The solution set is $\left\{\frac{1}{2} - \frac{\sqrt{23}}{2}i, \frac{1}{2} + \frac{\sqrt{23}}{2}i\right\}.$

44. $c = 50,000 + 95x$

45. Let x represent the amount of money invested in bonds. Then $70,000 - x$ represents the amount of money invested in CD's.

Since the total interest is to be \$5000, we have:

$0.08x + 0.05(70,000 - x) = 5000$

$(100)(0.08x + 0.05(70,000 - x)) = (5000)(100)$

$8x + 350,000 - 5x = 500,000$

$3x + 350,000 = 500,000$

$3x = 150,000$

$x = 50,000$

\$50,000 should be invested in bonds at 8% and \$20,000 should be invested in CD's at 5%.

46. Using $s = vt$, we have $t = 3$ and $v = 1100$.

Finding the distance s in feet:

$s = 1100(3) = 3300$

The storm is 3300 feet away.

47. $1600 \leq I \leq 3600$

$1600 \leq \frac{900}{x^2} \leq 3600$

$\frac{1}{1600} \geq \frac{x^2}{900} \geq \frac{1}{3600}$

$\frac{9}{16} \geq x^2 \geq \frac{1}{4}$

$\frac{3}{4} \geq x \geq \frac{1}{2}$

The range of distances is from 0.5 meters to 0.75 meters, inclusive.

48. Let s represent the distance the plane can travel.

	With wind	Against wind
Rate	$250 + 30 = 280$	$250 - 30 = 220$
Time	$\frac{(s/2)}{280}$	$\frac{(s/2)}{220}$
Dist.	$\frac{s}{2}$	$\frac{s}{2}$

Since the total time is at most 5 hours, we have:

$\frac{(s/2)}{280} + \frac{(s/2)}{220} \leq 5$

$\frac{s}{560} + \frac{s}{440} \leq 5$

$11s + 14s \leq 5(6160)$

$25s \leq 30,800$

$s \leq 1232$

The plane can travel at most 1232 miles or 616 miles one way and return 616 miles.

49. Let t represent the time it takes the helicopter to reach the raft.

	Raft	Helicopter
Rate	5	90
Time	t	t
Dist.	$5t$	$90t$

Since the total distance is 150 miles, we have:

$$5t + 90t = 150$$

$$95t = 150$$

$$t \approx 1.58 \text{ hours} \approx 1 \text{ hour and 35 minutes}$$

The helicopter will reach the raft in about 1 hour and 35 minutes.

50. Given that $s = 1280 - 32t - 16t^2$,

- a. The object hits the ground when $s = 0$.

$$0 = 1280 - 32t - 16t^2$$

$$t^2 + 2t - 80 = 0$$

$$(t+10)(t-8) = 0$$

$$t = -10, t = 8$$

The object hits the ground after 8 seconds.

- b. After 4 seconds, the object's height is

$$s = 1280 - 32(4) - 16(4)^2 = 896 \text{ feet.}$$

51. Let t represent the time it takes Clarissa to complete the job by herself.

	Clarissa	Shawna
Time to do job alone	t	$t+5$
Part of job done in 1 day	$\frac{1}{t}$	$\frac{1}{t+5}$
Time on job (days)	6	6
Part of job done by each person	$\frac{6}{t}$	$\frac{6}{t+5}$

Since the two people paint one house, we have:

$$\frac{6}{t} + \frac{6}{t+5} = 1$$

$$6(t+5) + 6t = t(t+5)$$

$$6t + 30 + 6t = t^2 + 5t$$

$$t^2 - 7t - 30 = 0$$

$$(t-10)(t+3) = 0$$

$$t = 10 \text{ or } t = -3$$

It takes Clarissa 10 days to paint the house when working by herself.

52. Let t represent the time it takes the smaller pump to empty the tank.

	Small Pump	Large Pump
Time to do job alone	t	$t-4$
Part of job done in 1 hr	$\frac{1}{t}$	$\frac{1}{t-4}$
Time on job (hrs)	5	5
Part of job done by each pump	$\frac{5}{t}$	$\frac{5}{t-4}$

Since the two pumps empty one tank, we have:

$$\frac{5}{t} + \frac{5}{t-4} = 1$$

$$5(t-4) + 5t = t(t-4)$$

$$5t - 20 + 5t = t^2 - 4t$$

$$t^2 - 14t + 20 = 0$$

We can solve this equation for t by using the quadratic formula:

$$t = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(20)}}{2(1)}$$

$$= \frac{14 \pm \sqrt{116}}{2} = \frac{14 \pm 2\sqrt{29}}{2}$$

$$= 7 \pm \sqrt{29} \approx 7 + 5.385$$

$$t = 12.385 \text{ or } t = 1.615 \text{ (not feasible)}$$

It takes the small pump approximately 12.385 hours (12 hr 23 min) to empty the tank.

53. Let x represent the amount of water added.

% salt	Tot. amt.	amt. of salt
10%	64	$(0.10)(64)$
0%	x	$(0.00)(x)$
2%	$64+x$	$(0.02)(64+x)$

$$(0.10)(64) + (0.00)(x) = (0.02)(64+x)$$

$$6.4 = 1.28 + 0.02x$$

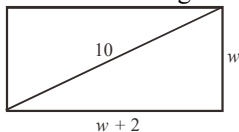
$$5.12 = 0.02x$$

$$x = 256$$

256 ounces of water must be added.

Chapter 1: Equations and Inequalities

54. Consider the diagram



By the Pythagorean Theorem we have

$$w^2 + (w+2)^2 = (10)^2$$

$$w^2 + w^2 + 4w + 4 = 100$$

$$2w^2 + 4w - 96 = 0$$

$$w^2 + 2w - 48 = 0$$

$$(w+8)(w-6) = 0$$

$$w = -8 \text{ or } w = 6$$

The width is 6 inches and the length is $6 + 2 = 8$ inches.

55. Let x represent the amount of the 15% solution added.

% acid	tot. amt.	amt. of acid
40%	60	$(0.40)(60)$
15%	x	$(0.15)(x)$
25%	$60 + x$	$(0.25)(60 + x)$

$$(0.40)(60) + (0.15)(x) = (0.25)(60 + x)$$

$$24 + 0.15x = 15 + 0.25x$$

$$9 = 0.1x$$

$$x = 90$$

90 cubic centimeters of the 15% solution must be added, producing 150 cubic centimeters of the 25% solution.

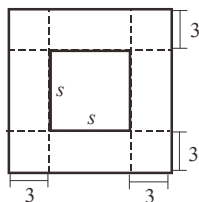
56. a. Consider the following diagram:

$$4(s+6) = 50$$

$$4s + 24 = 50$$

$$4s = 26$$

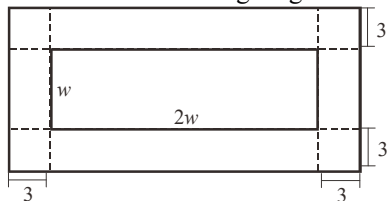
$$s = 6.5$$



The painting is 6.5 inches by 6.5 inches.

$s + 6 = 12.5$, so the frame is 12.5 inches by 12.5 inches.

- b. Consider the following diagram:



$$2(2w+6) + 2(w+6) = 50$$

$$4w + 12 + 2w + 12 = 50$$

$$6w = 26$$

$$w = \frac{26}{6} = 4\frac{1}{3}$$

$$l = 2w = 8\frac{2}{3}$$

The painting is $8\frac{2}{3}$ inches by $4\frac{1}{3}$ inches.

The frame is $14\frac{2}{3}$ inches by $10\frac{1}{3}$ inches.

57. Let x represent the amount Scott receives. Then $\frac{3}{4}x$ represents the amount Alice receives and $\frac{1}{2}x$ represents the amount Tricia receives. The total amount is \$900,000, so we have:

$$x + \frac{3}{4}x + \frac{1}{2}x = 900,000$$

$$4\left(x + \frac{3}{4}x + \frac{1}{2}x\right) = 4(900,000)$$

$$4x + 3x + 2x = 3,600,000$$

$$9x = 3,600,000$$

$$x = 400,000$$

$$\text{So, } \frac{3}{4}x = \frac{3}{4}(400,000) = 300,000 \text{ and}$$

$$\frac{1}{2}x = \frac{1}{2}(400,000) = 200,000.$$

Scott receives \$400,000, Alice receives \$300,000, and Tricia receives \$200,000.

58. Let t represent the time it takes the older machine to complete the job by itself.

	Old copier	New copier
Time to do job alone	t	$t-1$
Part of job done in 1 hr	$\frac{1}{t}$	$\frac{1}{t-1}$
Time on job (hrs)	1.2	1.2
Part of job done by each copier	$\frac{1.2}{t}$	$\frac{1.2}{t-1}$

Since the two copiers complete one job, we have: