

***Mechanics of Machines 2nd edition Cleghorn
Solutions Manual***

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Instructor's Solutions Manual for

MECHANICS OF MACHINES

Second Edition

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William L. Cleghorn
Nikolai Dechev

P1.1

$$(a) \quad r_1 = 2.0 = s; \quad r_2 = 6.5 = p$$

$$r_3 = 3.0 = q; \quad r_4 = 7.0 = l$$

$$(i) \quad l < s + p + q, \quad \therefore \text{links can form a mechanism}$$

$$(ii) \quad s + l = 9.0, \quad p + q = 9.5$$

$$s + l < p + q, \quad \therefore \text{Class I four-bar kinematic chain}$$

$$\text{since } r_1 = s, \quad \text{drag link mechanism}$$

$$(b) \quad r_1 = 2.0 = s; \quad r_2 = 8.0 = p$$

$$r_3 = 3.0 = q; \quad r_4 = 9.0 = l$$

$$(i) \quad l < s + p + q, \quad \therefore \text{links can form a mechanism}$$

$$(ii) \quad s + l = 11.0, \quad p + q = 11.0$$

$$s + l = p + q, \quad \therefore \text{change point mechanism}$$

P1.1

CONTINUED

$$(c) \quad r_1 = 2.5 = l ; \quad r_2 = 1.0 = s$$

$$r_3 = 2.5 = p ; \quad r_4 = 2.0 = q$$

$$(i) \quad l < s + p + q, \quad \therefore \text{links can form a mechanism}$$

$$(ii) \quad s + l = 3.5, \quad p + q = 4.5$$

$$s + l < p + q, \quad \therefore \text{Class I four-bar kinematic chain}$$

since $r_2 = s$, crank rocker four-bar mechanism

$$(d) \quad r_1 = 2.5 = p ; \quad r_2 = 3.0 = l$$

$$r_3 = 1.0 = s ; \quad r_4 = 2.0 = q$$

$$(i) \quad l < s + p + q, \quad \therefore \text{links can form a mechanism}$$

$$(ii) \quad s + l = 4.0, \quad p + q = 4.5$$

$$s + l < p + q, \quad \therefore \text{Class I four-bar kinematic chain}$$

since $r_3 = s$, rocker-rocker four-bar mechanism

P1.1**CONTINUED**

$$(e) \quad r_1 = 2.0 = p; \quad r_2 = 4.5 = q$$

$$r_3 = 1.5 = s; \quad r_4 = 9.0 = l$$

$$(i) \quad l > s + p + q, \therefore \text{links cannot form a mechanism}$$

$$(f) \quad r_1 = 1.5 = s; \quad r_2 = 3.0 = p$$

$$r_3 = 2.5 = q; \quad r_4 = 6.0 = l$$

$$(i) \quad l < s + p + q, \therefore \text{links can form a mechanism}$$

$$(ii) \quad s + l = 7.5, \quad p + q = 5.5$$

$$s + l > p + q, \text{ rocker-rocker four-bar mechanism}$$

P1.2

$$r_1 = 1.0 ; r_3 = 2.5 ; r_4 = 2.0$$

- (a) to be a crank rocker mechanism, r_2 must be the shortest link (i.e., $r_2 < 1.0$)
 also, we require $s + l < p + q$ ($s = r_2$; $l = r_3 = 2.5$;
 $p = r_1 = 1.0$; $q = r_4 = 2.0$)

$$r_2 + 2.5 < 1.0 + 2.0$$

$$r_2 < 0.5$$

- (b) to be a drag link mechanism, r_1 must be the shortest link (i.e., $r_2 > 1.0$)
 r_2 may be p , q or l .

Case 1 : if r_2 is l ($r_1 = s, r_3 = p, r_4 = q$)

$$s + l < p + q$$

$$1.0 + r_2 < 2.5 + 2.0 , r_2 < 3.5$$

Case 2 : if $r_2 = p$ or q ($r_1 = s, r_3 = l, r_4 = q$ or p)

$$s + l < p + q$$

$$1.0 + 2.5 < r_2 + 2.0 , r_2 > 1.5$$

combining Cases 1 and 2: $1.5 < r_2 < 3.5$

P1.2

CONTINUED

(c) to be a change point mechanism, $s+l = p+q$

Case 1: if $r_2 = s$ ($r_1 = p, r_3 = l, r_4 = q$)

$$s+l = p+q$$

$$r_2 + 2.5 = 1.0 + 2.0, \quad r_2 = 0.5$$

Case 2: if $r_2 = l$ ($r_1 = s, r_3 = p, r_4 = q$)

$$s+l = p+q$$

$$1.0 + r_2 = 2.5 + 2.0, \quad r_2 = 3.5$$

Case 3: if $r_2 = p$ ($r_1 = s, r_3 = l, r_4 = q$)

$$s+l = p+q$$

$$1.0 + 2.5 = r_2 + 2.0, \quad r_2 = 1.5$$

combining Cases 1, 2 and 3: $r_2 = 0.5$ OR
 $r_2 = 1.5$ OR
 $r_2 = 3.5$

(d) to be a rocker-rocker mechanism, $s+l > p+q$

Case 1: if $r_2 = s$

$$s+l > p+q$$

$$r_2 + 2.5 > 1.0 + 2.0, \quad r_2 > 0.5 \quad (i)$$

also, because r_2 is the shortest link, $r_2 < 1.0$ (ii)

P1.2

CONTINUED

combining (i) and (ii) : $0.5 < r_2 < 1.0$ (I)

Case 2 : if $r_2 = l$

$$5 + l > p + q$$

$$1.0 + r_2 > 2.0 + 2.5, \quad r_2 > 3.5 \quad (\text{iii})$$

also, for assembly of the kinematic chain,

$$5 + p + q > l$$

$$1.0 + 2.0 + 2.5 > r_2, \quad r_2 < 5.5 \quad (\text{iv})$$

combining (iii) and (iv) : $3.5 < r_2 < 5.5$ (II)

Case 3 : if $r_2 = p$

$$5 + l > p + q$$

$$1.0 + 2.5 > r_2 + 2.0, \quad r_2 < 1.5 \quad (\text{v})$$

but for this case, r_2 cannot be the shortest link, and therefore $r_2 \geq 1.0$ (vi)

combining (v) and (vi) : $1.0 \leq r_2 < 1.5$ (III)

combining (I), (II) and (III)

$$0.5 < r_2 < 1.5 \quad \text{OR} \quad 3.5 < r_2 < 5.5$$

P1.3

$$r_1 = 1.0 ; r_2 = 3.0 ; r_3 = 2.5$$

- (a) to be a crank rocker mechanism, the input link must be the shortest link considering r_4 as the input link

$$s + l < p + q$$

$$r_4 + 3.0 < 1.0 + 2.5, \quad r_4 < 0.5$$

- (b) to be a drag link mechanism, the base link length, r_1 , must be the shortest link thus, r_4 may be p , q or l

Case 1: if $r_4 = l$

$$s + l < p + q$$

$$1.0 + r_4 < 3.0 + 2.5, \quad r_4 < 4.5$$

also, for r_4 to be the longest link, $r_4 > 3.0$

$$\therefore 3.0 < r_4 < 4.5$$

Cas 2: if $r_4 = p$ or q

$$s + l < p + q$$

$$1.0 + 3.0 < 2.5 + r_4, \quad r_4 > 1.5$$

also, for r_4 to be p or q , $r_4 \leq 3.0$

$$\therefore 1.5 < r_4 \leq 3.0$$

P1.3**CONTINUED**

(c) to be a change point mechanism: $s+l = p+q$

Case 1: $r_4 = s$

$$s+l = p+q$$

$$r_4 + 3.0 = 1.0 + 2.5, \quad r_4 = 0.5$$

Case 2: $r_4 = l$

$$s+l = p+q$$

$$1.0 + r_4 = 2.5 + 3.0, \quad r_4 = 4.5$$

Case 3: $r_4 = p$ or q

$$1.0 + 3.0 = r_4 + 2.5, \quad r_4 = 1.5$$

combining Cases 1, 2 and 3: $r_4 = 0.5$ OR
 $r_4 = 1.5$ OR
 $r_4 = 4.5$

(d) to be a rocker-rocker mechanism, $s+l > p+q$

Case 1: $r_4 = s$

$$r_4 + 3.0 > 1.0 + 2.5, \quad r_4 > 0.5$$

also, for r_4 to be the shortest link, $r_4 \leq 1.0$

$$\therefore 0.5 < r_4 \leq 1.0 \quad (I)$$

P1.3

CONTINUED

Case 2: $r_4 = p$ or q

$$s + l > p + q$$

$$1.0 + 3.0 > r_4 + 2.5, \quad r_4 < 1.5 \quad (i)$$

also, because r_4 cannot be the shortest link,
 $r_4 \geq 1.0 \quad (ii)$

$$\text{combining (i) and (ii): } 1.0 \leq r_4 < 1.5 \quad (II)$$

Case 3: $r_4 = l$

$$s + l > p + q$$

$$1.0 + r_4 > 2.5 + 3.0, \quad r_4 > 4.5 \quad (iii)$$

also, for assembly of the kinematic chain,

$$s + p + q > l$$

$$1.0 + 2.5 + 3.0 > r_4, \quad r_4 < 6.5 \quad (iv)$$

$$\text{combining (iii) and (iv): } 4.5 < r_4 < 6.5 \quad (III)$$

$$\text{combining (I), (II) and (III): } 0.5 < r_4 < 1.5 \text{ OR } 4.5 < r_4 < 6.5$$

P1.4

$$r_1 = 1.0 \text{ cm} ; \quad r_3 = 2.5 \text{ cm}$$

(a) for full rotation of link 2:

$$(i) \quad r_2 < r_3 \quad \text{and} \quad (ii) \quad |r_1| \leq r_3 - r_2$$

$$\therefore (i) \quad r_2 < 2.5 \text{ cm}$$

$$(ii) \quad |r_1| \leq r_3 - r_2$$

$$1.0 \leq 2.5 - r_2, \quad r_2 \leq 1.5 \text{ cm}$$

combining (i) & (ii), conclude $r_2 \leq 1.5 \text{ cm}$

(b) possible to assemble mechanism:

$$|r_1| \leq r_2 + r_3$$

$$1.0 \leq r_2 + 2.5, \quad r_2 \geq -1.5 \text{ cm}$$

from which we conclude $r_2 \geq 0$

P1.5

$$r_1 = 1.0 \text{ cm} ; \quad r_2 = 2.5 \text{ cm}$$

(a) for full rotation of link 2:

$$(i) \quad r_2 < r_3 \quad \text{and} \quad (ii) \quad |r_1| \leq r_3 - r_2$$

$$\therefore (i) \quad 2.5 < r_3 \quad , \quad r_3 > 2.5 \text{ cm}$$

$$(ii) \quad |r_1| \leq r_3 - r_2$$

$$1.0 \leq r_3 - 2.5 \quad , \quad r_3 \geq 3.5 \text{ cm}$$

combining (i) & (ii), conclude $r_3 \geq 3.5 \text{ cm}$

(b) possible to assemble mechanism:

$$|r_1| \leq r_2 + r_3$$

$$1.0 \leq 2.5 + r_3 \quad , \quad r_3 \geq -1.5 \text{ cm}$$

from which we conclude $r_3 \geq 0$

P1.6

(a) $r_3 = 3.5 \text{ cm}$

$$s = r_2 = 2.0 \text{ cm} ; p = r_4 = 3.5 \text{ cm}$$

$$l = r_1 = 7.0 \text{ cm} ; q = r_4 = 6.0 \text{ cm}$$

$$(s + l = 9.0 \text{ cm}) < (p + q = 9.5 \text{ cm})$$

$\therefore$ class I four-bar mechanism
since s is the input link, crank rocker

(b) $r_3 = 11.0 \text{ cm}$

$$s = r_2 = 2.0 \text{ cm} ; p = r_4 = 6.0 \text{ cm}$$

$$l = r_3 = 11.0 \text{ cm} ; q = r_1 = 7.0 \text{ cm}$$

$$(s + l = 13.0 \text{ cm}) = (p + q = 13.0 \text{ cm})$$

$\therefore$ change point

(c) $r_3 = 11.5 \text{ cm}$

$$s = r_2 = 2.0 \text{ cm} ; p = r_4 = 6.0 \text{ cm}$$

$$l = r_3 = 11.5 \text{ cm} ; q = r_1 = 7.0 \text{ cm}$$

$$(s + l = 13.5 \text{ cm}) > (p + q = 13.0 \text{ cm})$$

$\therefore$ class II four-bar mechanism
rocker-rocker

P1.7

crank rocker (a)

the input motion to the four-bar mechanism is the gear (i.e., crank) that executes full rotations

the output motion of the four-bar mechanism is the gear sector (i.e., rocker) that oscillates between two limit positions

the gear sector drives a smaller gear that is rigidly connected to the agitator that undergoes oscillatory motion and provides the washing action

P1.8

crank rocker (a)

the input motion to the four-bar mechanism executes full rotations

the output motion of the four-bar mechanism is the coupler point on the coupler that enters a perforation in the film. The coupler point then advances the film by one frame

P1.9

(a) $n = 6$

- turning pairs, 5 ;
- sliding pair, 1 ;
- rolling pair, 1 ($j_1 = 5 + 1 + 1 = 7$)
- 2 dot pairs, ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(6-1) - 2 \times 7 = 1
 \end{aligned}$$

(b) $n = 6$

- turning pairs, 5 ;
- sliding pairs, 2 ;
- rolling pairs, 0 ($j_1 = 5 + 2 + 0 = 7$)
- 2 dot pairs, 0 ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(6-1) - 2 \times 7 = 1
 \end{aligned}$$

P1.10

$$(a) \quad n = 4, \quad j_1 = 4, \quad j_2 = 0$$

$$\begin{aligned} m &= 3(n-1) - 2j_1 - j_2 \\ &= 3(4-1) - 2 \times 4 = 1 \end{aligned}$$

$$(b) \quad n = 6, \quad j_1 = 7, \quad j_2 = 0$$

$$m = 3(6-1) - 2 \times 7 = 1$$

P1.11

(a) $n = 6$

- turning pairs, 7 ;
- sliding pairs, 0 ;
- rolling pairs, 0 ($j_1 = 7 + 0 + 0 = 7$)
- 2 dof pairs, 0 ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(6-1) - 2 \times 7 = 1
 \end{aligned}$$

(b) $n = 8$

- turning pairs, 10 ;
- sliding pairs, 0 ;
- rolling pairs, 0 ($j_1 = 10 + 0 + 0 = 10$)
- 2 dof pairs, 0 ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(8-1) - 2 \times 10 = 1
 \end{aligned}$$

P1.12

(a) $n = 4$

- turning pairs, 0 ;
- sliding pairs, 5 ;
- rolling pairs, 0 ($j_1 = 0 + 5 + 0 = 5$)
- 2 dof pairs, 0 ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(4-1) - 2 \times 5 = -1
 \end{aligned}$$

- however, there are two redundant constraints (i.e., $N_R = 2$)
- relative rotations between links 2 and 3, and between links 3 and 4 are accounted for twice (once by sliding pairs between links 2 and 3, and between links 3 and 4; and then again by sliding pairs between links 1 and 2, 1 and 3, and 1 and 4)

$$\therefore m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.12

CONTINUED

(b) $n = 4$

- turning pairs, 0;
- sliding pairs, 3;
- rolling pairs, 0 ($j_1 = 0 + 3 + 0 = 3$)
- 2 dof pairs, 2 ($j_2 = 2$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(4-1) - 2 \times 3 - 2 = 1
 \end{aligned}$$

P1.13

(a) $n = 4$

- turning pairs, 3;
- sliding pairs, 0;
- rolling pairs, 2 ($j_1 = 3 + 0 + 2 = 5$)
- 2 dot pairs, 0 ($j_2 = 0$)

$$\begin{aligned}
 m &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(4-1) - 2 \times 5 - 0 = -1
 \end{aligned}$$

- however, there are two redundant constraints (i.e., $N_R = 2$)

- distances between base pivots of links 2 and 3, and between base pivots of links 3 and 4 are accounted for twice (once by rolling pairs between links 2 and 3, and between links 3 and 4; and then again by base pivots of link 2 and 3, and base pivots of links 3 and 4)

$$\therefore m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.13

CONTINUED

$$(b) \quad n = 8$$

- turning pairs, 9;
- sliding pairs, 0;
- rolling pairs, 2 ($j_1 = 9 + 0 + 2 = 11$)
- 2 dot pairs, 0 ($j_2 = 0$)

$$\begin{aligned} m &= 3(n-1) - 2j_1 - j_2 \\ &= 3(8-1) - 2 \times 11 = -1 \end{aligned}$$

- however, the above calculation incorporates two redundant constraints (same as for P1.5(a))

$$N_R = 2$$

$$\therefore m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.14

4 links, $n = 4$

4 turning pairs,

0 rolling pairs,

0 sliding pairs, $j_1 = 0 + 4 + 0 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$$m = 3(n - 1) - 2j_1 - j_2 + N_R = 1$$

P1.15

4 links, $n = 4$

4 turning pairs,

0 rolling pairs,

0 sliding pairs, $j_1 = 0 + 4 + 0 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$$m = 3(n - 1) - 2j_1 - j_2 + N_R = 1$$

P1.16

3 links, $n = 3$

1 turning pair,

1 rolling pair,

1 sliding pair, $j_1 = 1 + 1 + 1 = 3$

0 DOF pairs, $j_2 = 0$

one redundant constraints, $N_R = 1$

$$m = 3(n - 1) - 2j_1 - j_2 + N_R = 1$$

P1.17

4 links, $n = 4$

2 turning pairs,

1 rolling pairs,

1 sliding pairs, $j_1 = 2 + 1 + 1 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$$m = 3(n - 1) - 2j_1 - j_2 + N_R = 1$$

P1.18

$$(a) \quad n = 5; \quad j_1 = 6; \quad j_2 = 0$$

$$N_R = 1 \quad (\text{center-to-center distance between } O_5 \text{ \& } E \text{ accounted for length of link 5 AND lengths of } r_{O_4 O_5}, r_{O_4 D} \text{ \& } r_{DE})$$

$$\begin{aligned} m &= 3(n-1) - 2j_1 - j_2 + N_R \\ &= 3(5-1) - 2 \times 6 - 0 + 1 = 1 \end{aligned}$$

$$(b) \quad n = 6; \quad j_1 = 8; \quad j_2 = 0$$

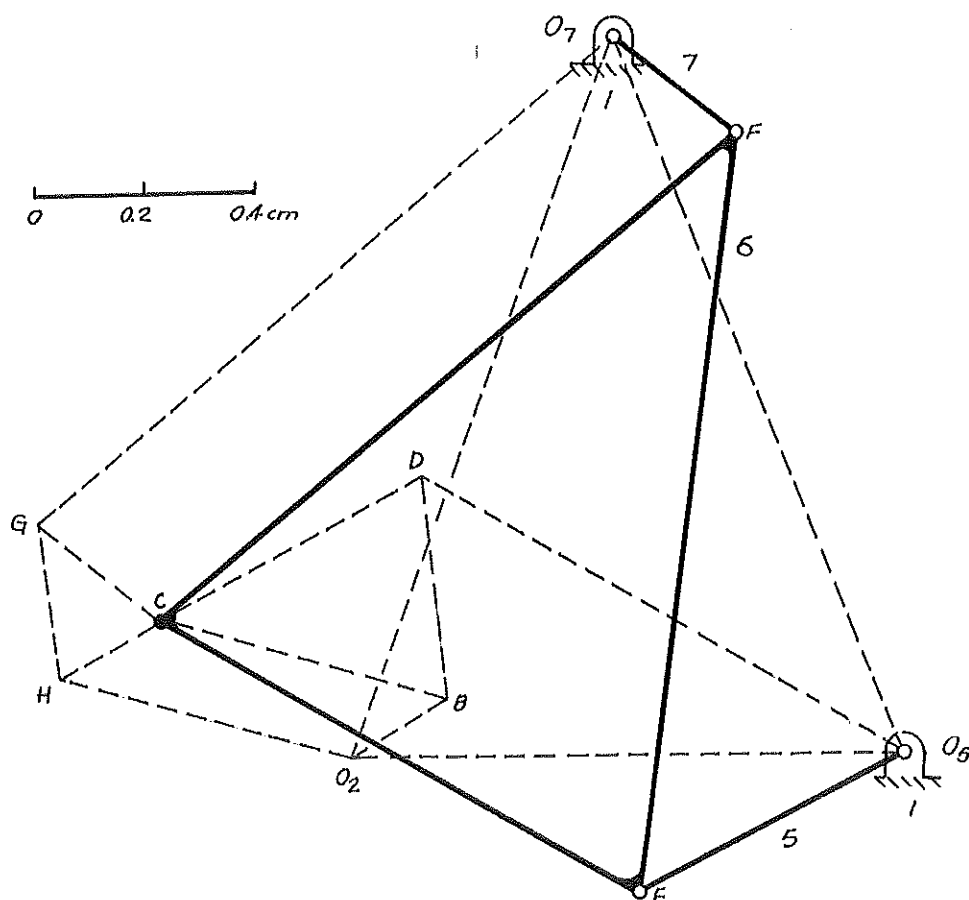
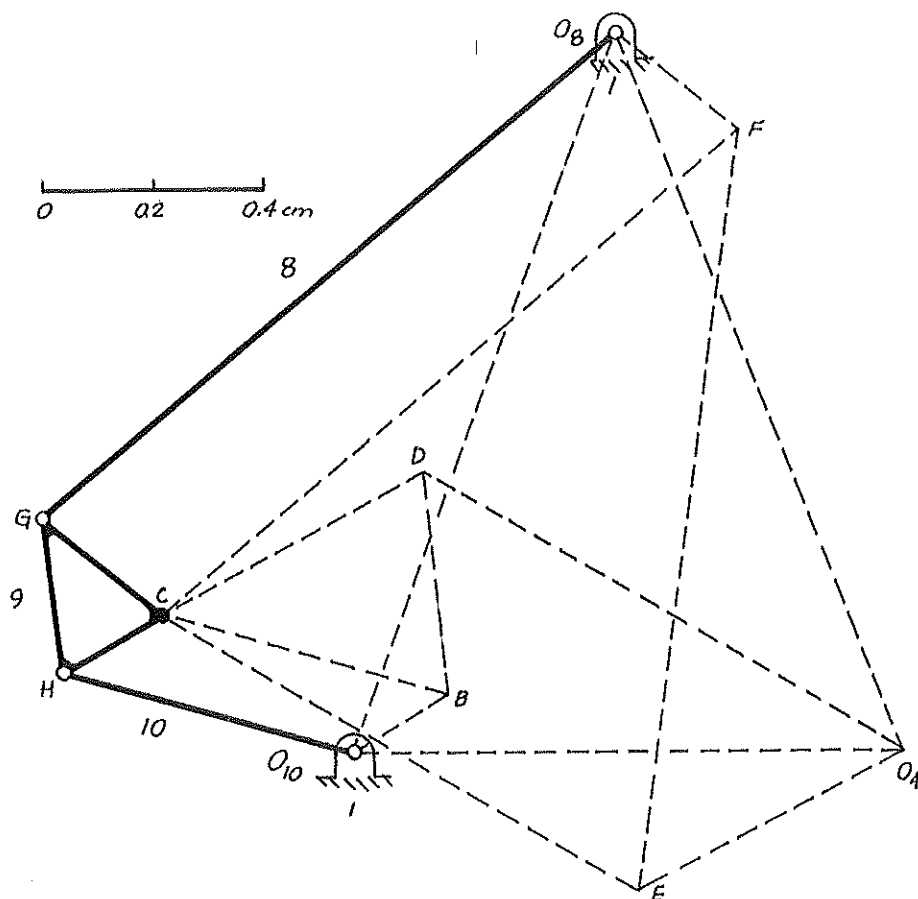
$$N_R = 2 \quad (\text{center-to-center distance between } O_5 \text{ \& } E \text{ accounted for length of link 5 AND lengths of } r_{O_4 O_5}, r_{O_4 D} \text{ \& } r_{DE};$$

similar reasoning for length $r_{O_6 F}$)

$$m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.19

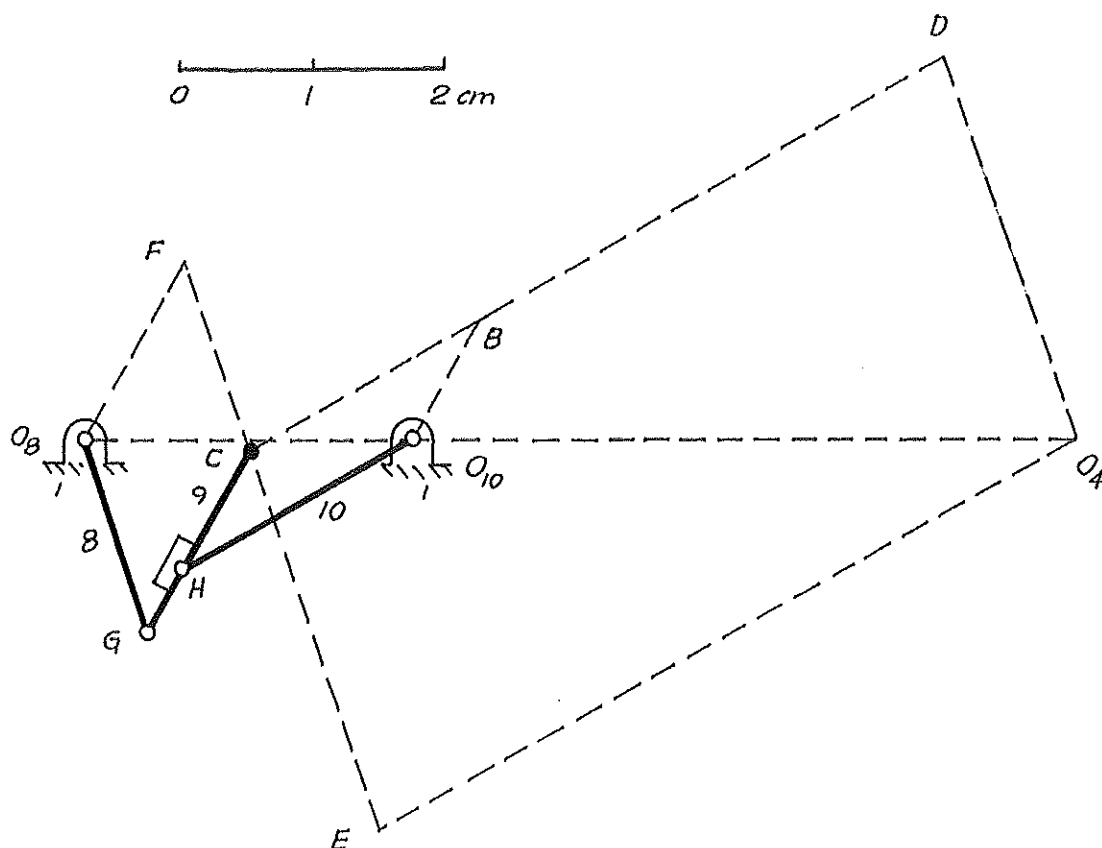
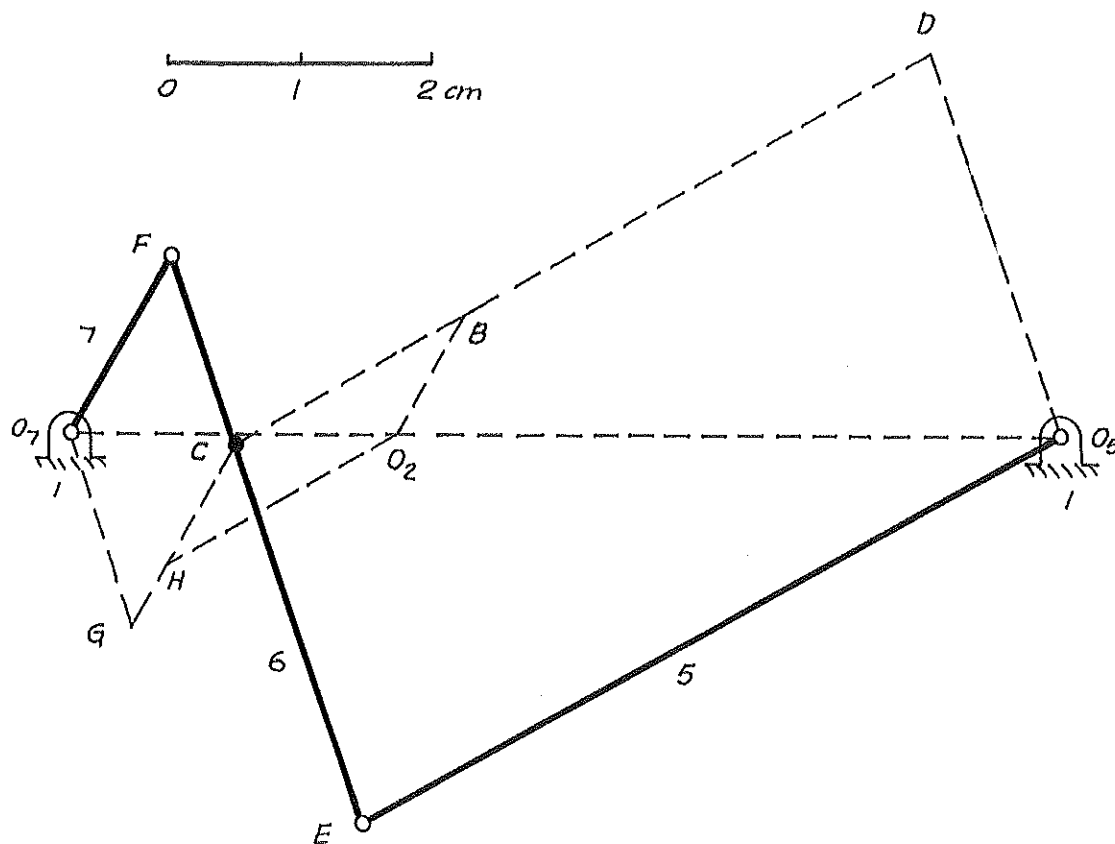
(a)



P1.19

CONTINUED

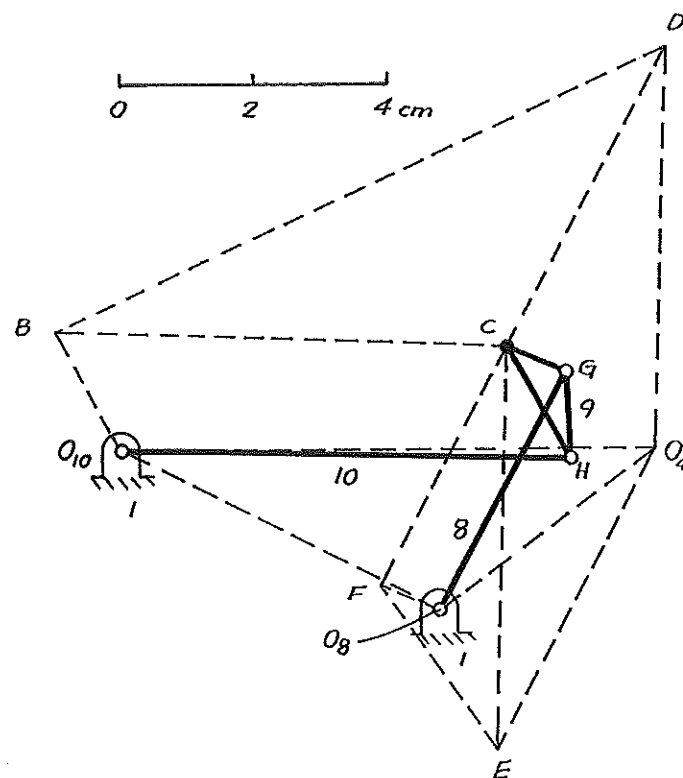
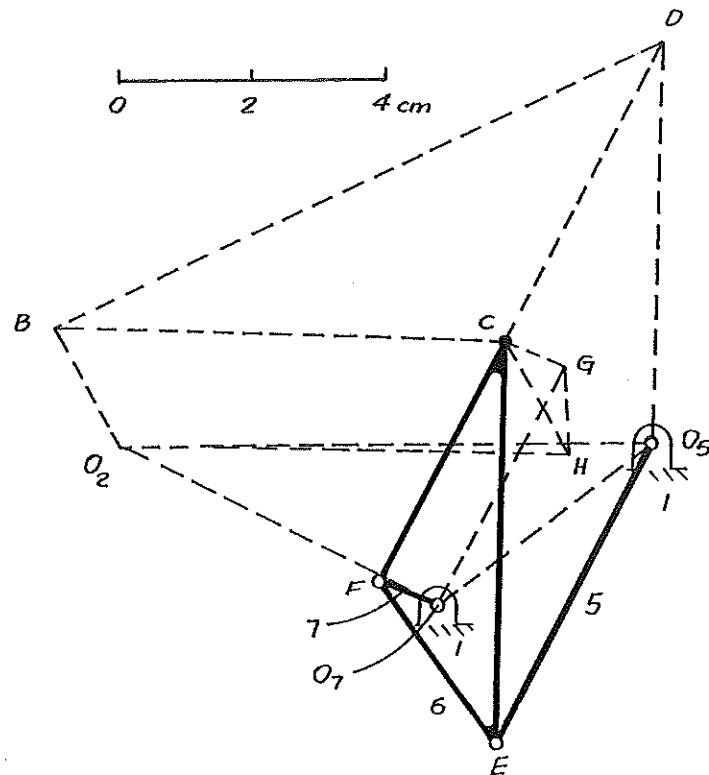
(b)



P1.19

CONTINUED

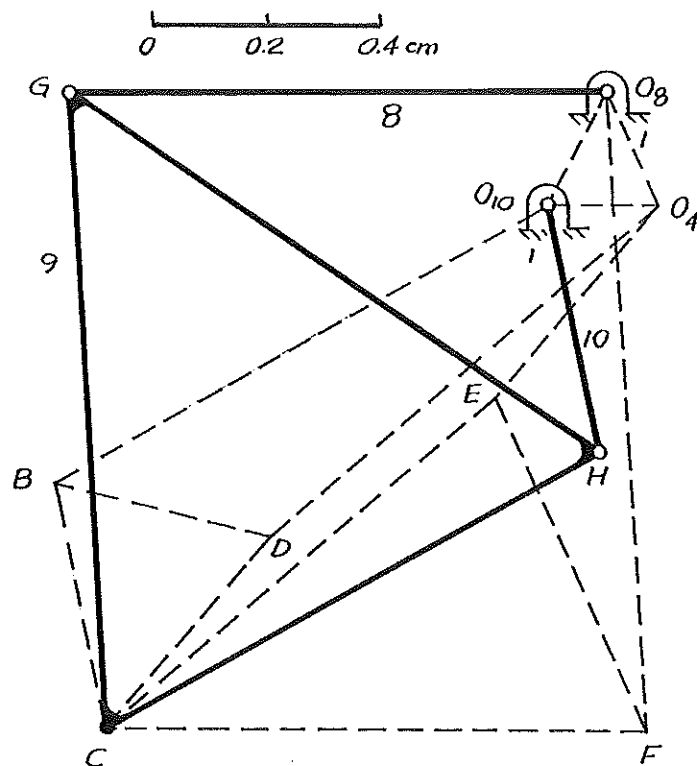
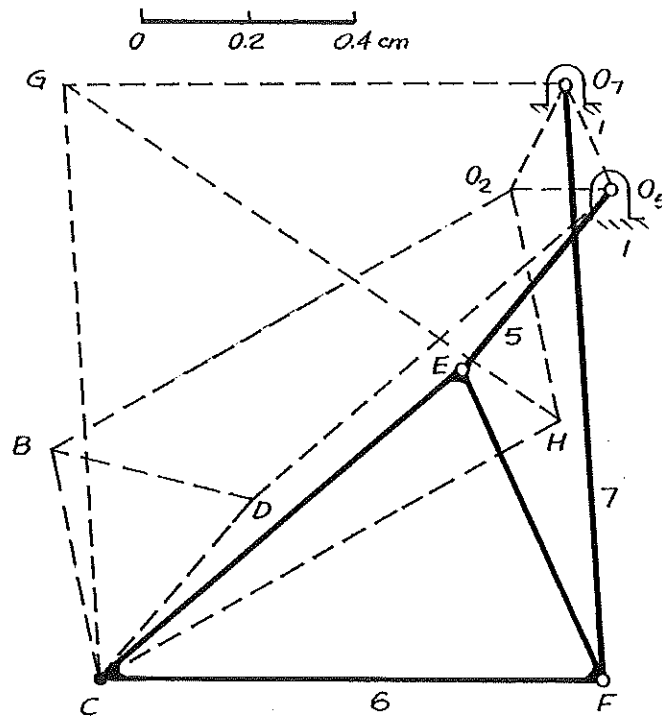
(c)



P1.19

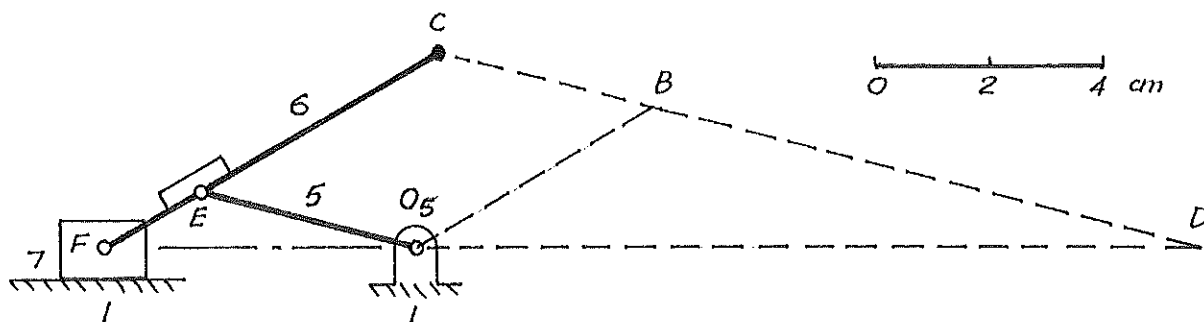
CONTINUED

(d)

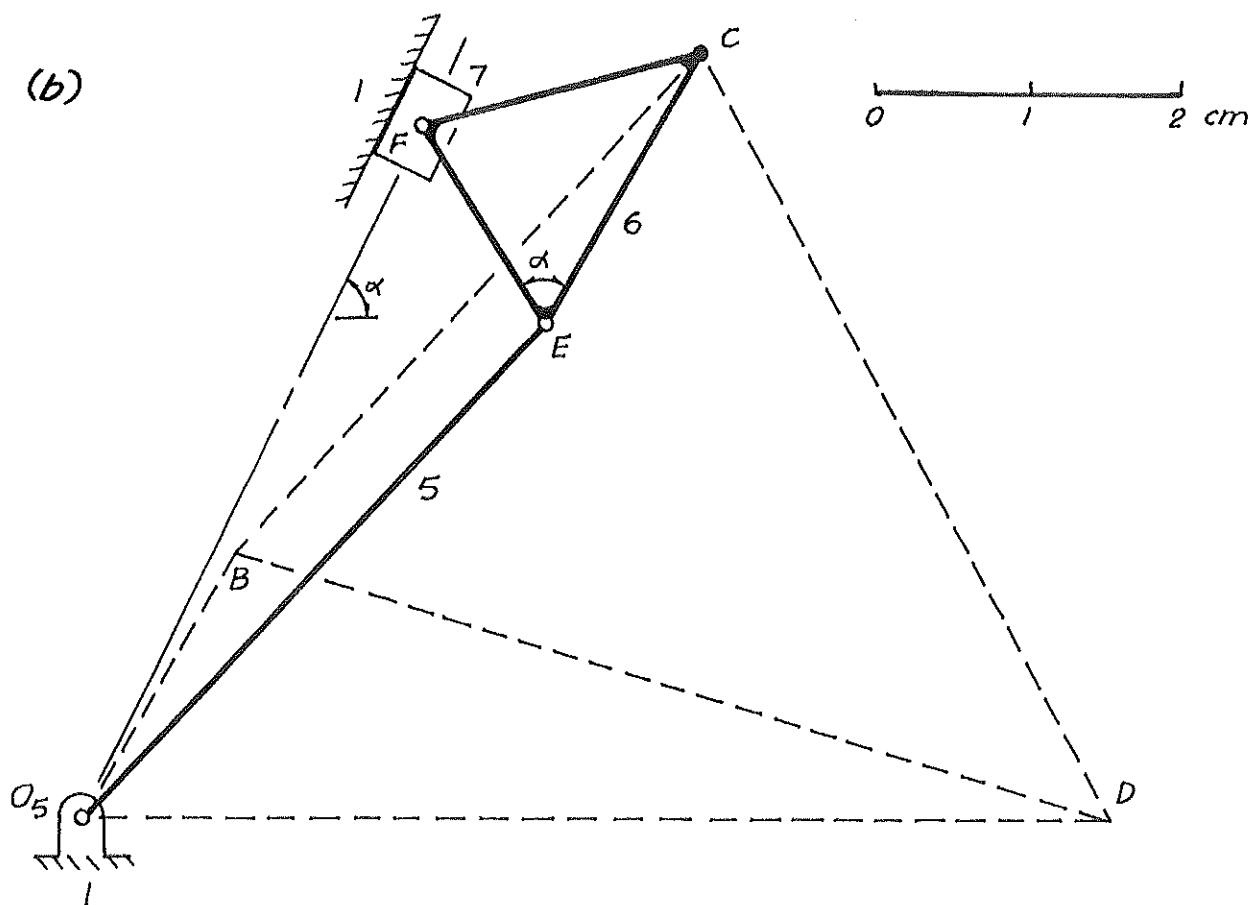


P1.20

(a)



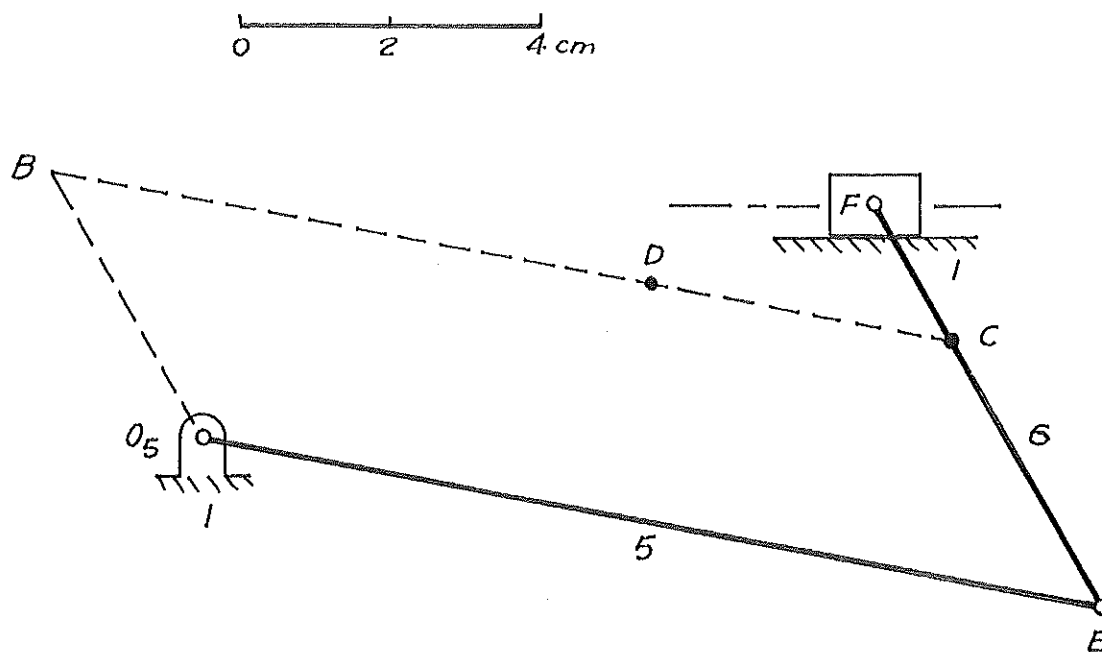
(b)



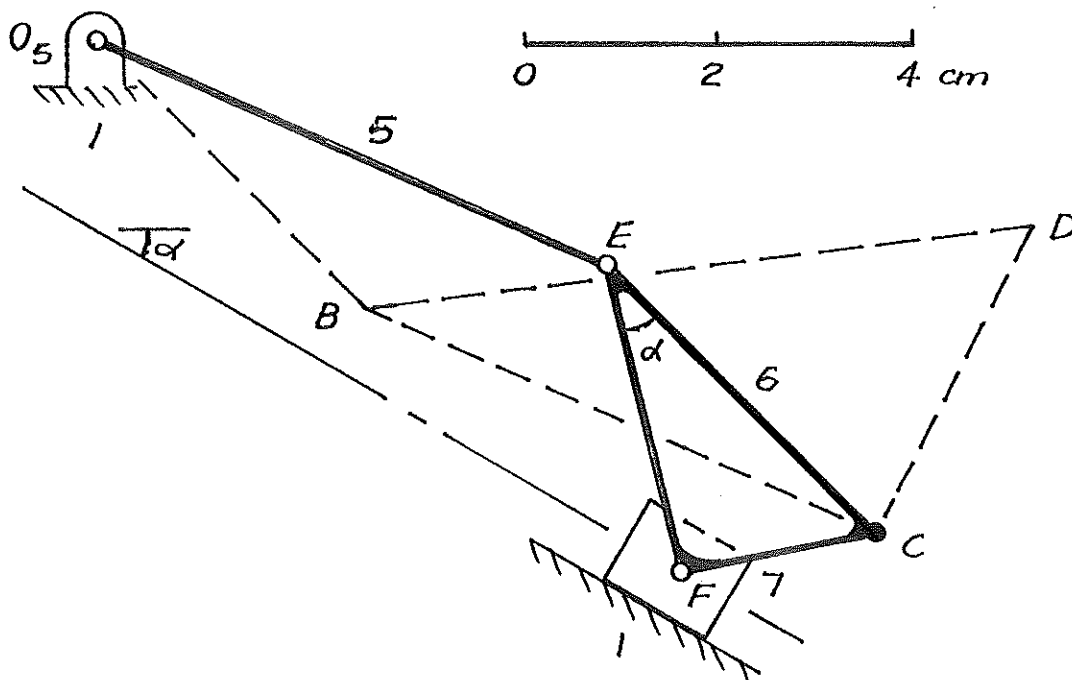
P1.20

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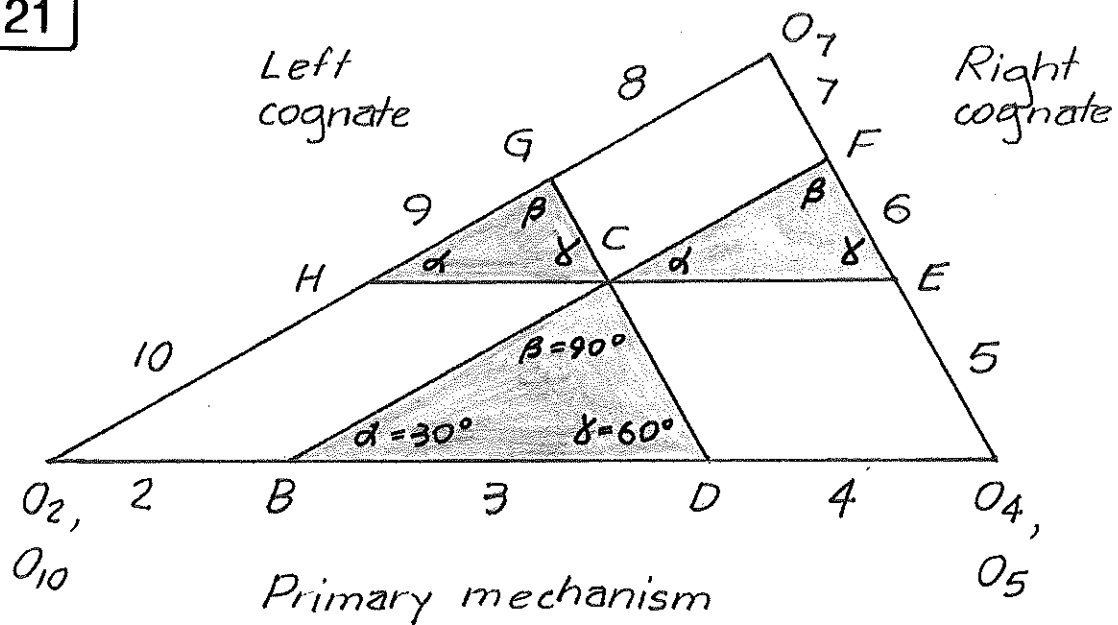
(c)



(d)



P1.21



Left cognate:

base link : $r_{O_8 O_{10}} = 11.8 \text{ cm}$;

link 8 : $r_{O_8 G} = 6.00 \text{ cm}$; link 9 : $r_{G H} = 4.33 \text{ cm}$;

link 10 : $r_{O_{10} H} = 7.79 \text{ cm}$

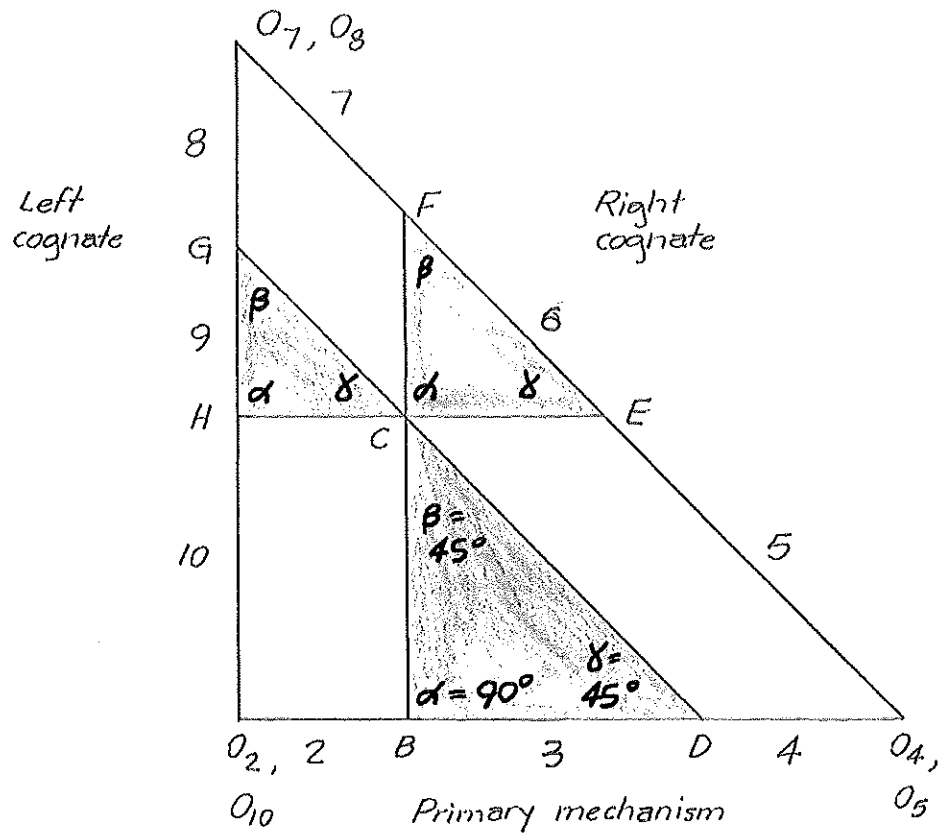
Right cognate:

base link : $r_{O_5 O_7} = 6.50 \text{ cm}$;

link 5 : $r_{O_5 E} = 4.50 \text{ cm}$; link 6 : $r_{E F} = 3.00 \text{ cm}$;

link 7 : $r_{O_7 F} = 2.50 \text{ cm}$

P1.22



Left cognate:

base link: $r_{O_8 O_{10}} = 12.0 \text{ cm}$;

link 8: $r_{O_8 G} = 6.00 \text{ cm}$; link 9: $r_{GH} = 5.00 \text{ cm}$;

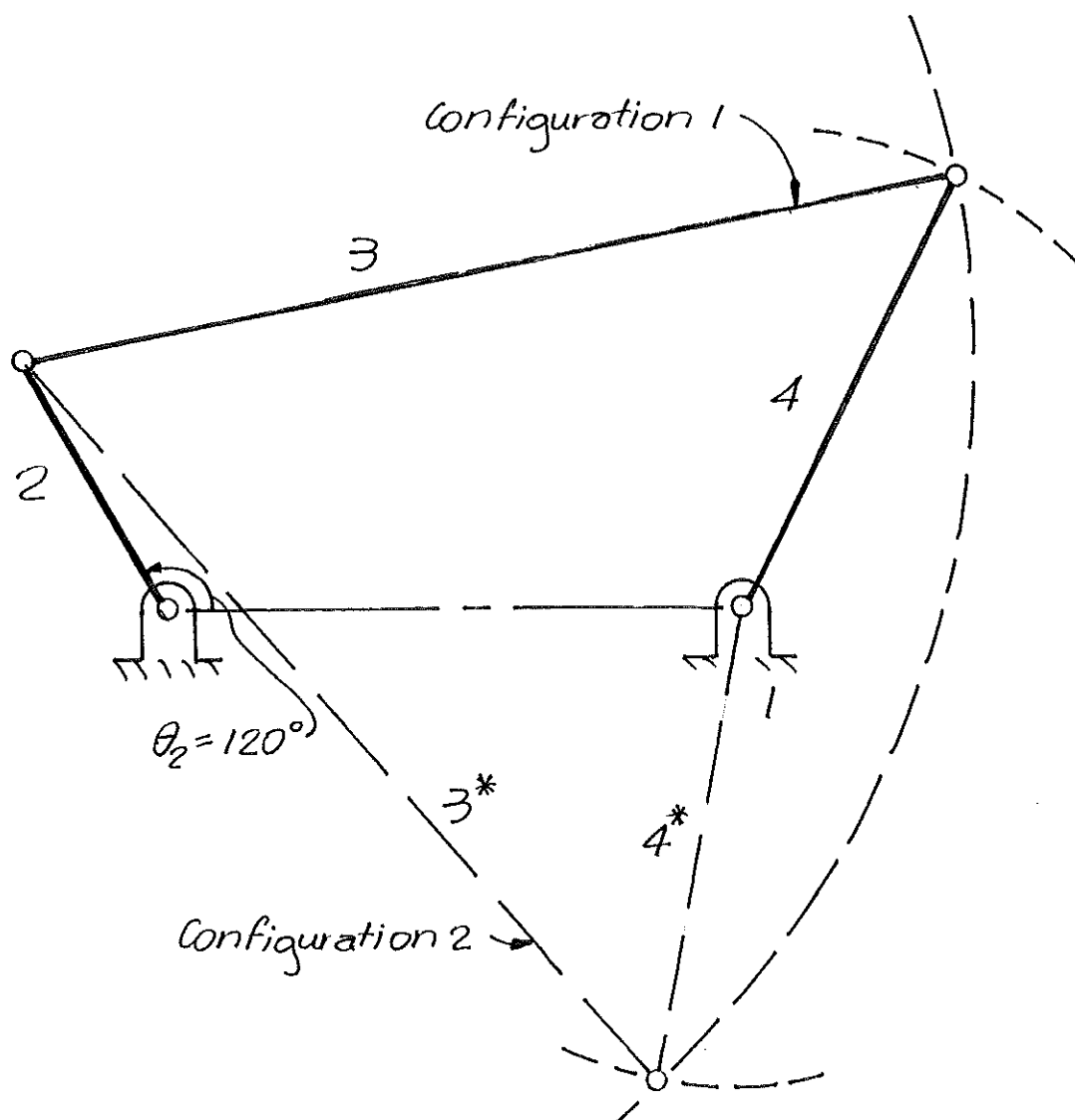
link 10: $r_{O_{10} H} = 9.00 \text{ cm}$

Right cognate:

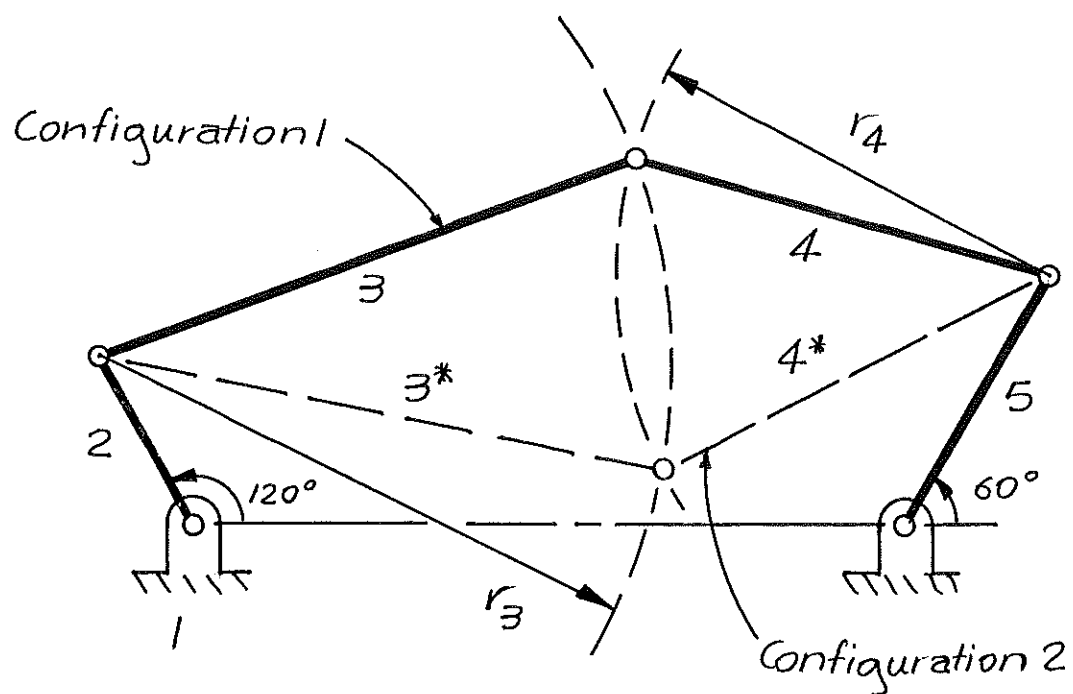
base link: $r_{O_5 O_7} = 17.0 \text{ cm}$;

link 5: $r_{O_5 E} = 12.7 \text{ cm}$; link 6: $r_{EF} = 8.49 \text{ cm}$;

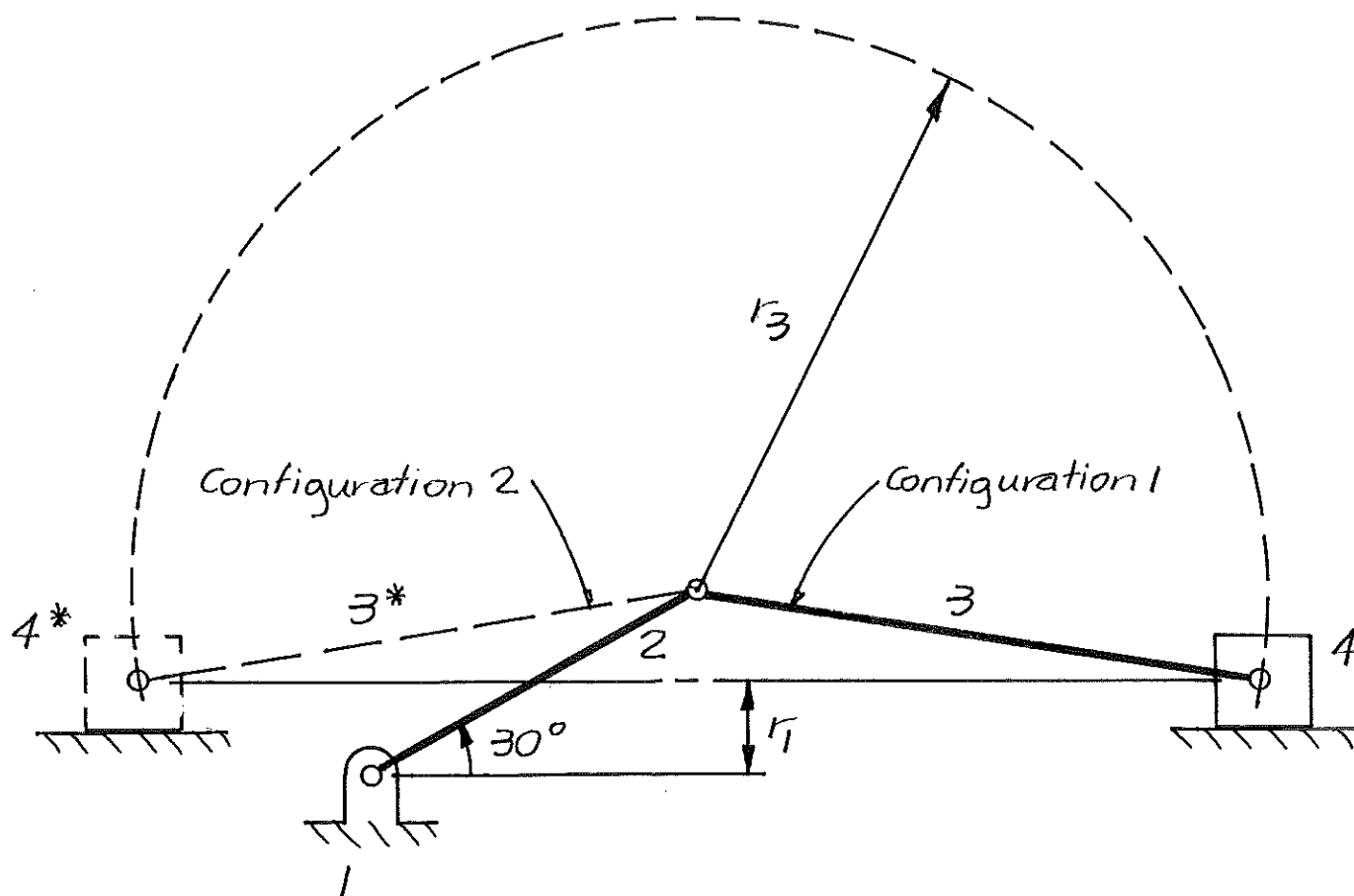
link 7: $r_{O_7 F} = 7.07 \text{ cm}$

P2.1

P2.2



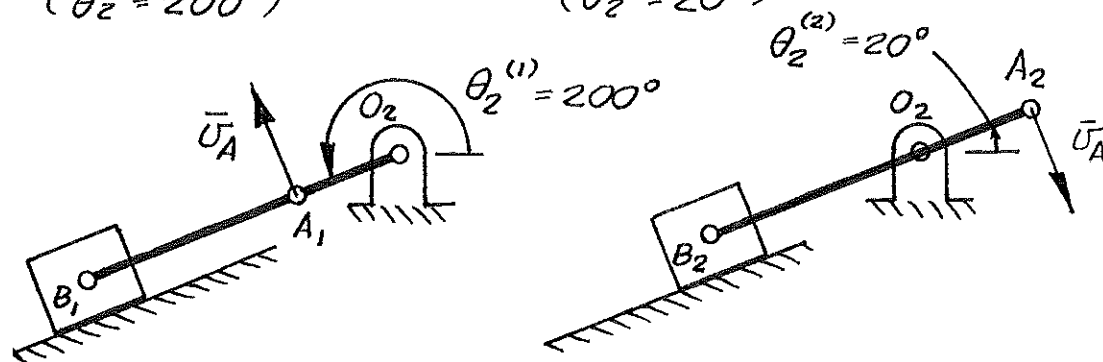
P2.3



P2.4

(a) limit position 1:
($\theta_2^{(1)} = 200^\circ$)

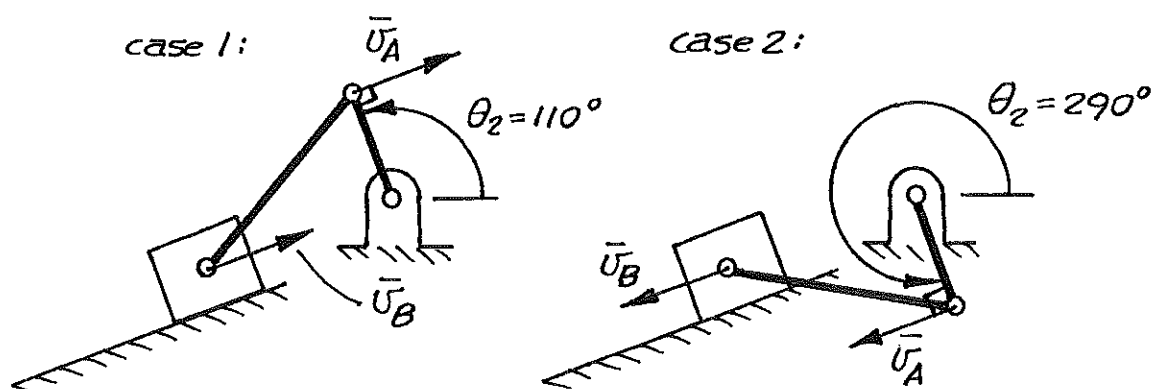
limit position 2:
($\theta_2^{(2)} = 20^\circ$)



for both limit geometries

$$|\dot{\theta}_3| = \frac{v_A}{r_{AB}} = \frac{r_{AO_2} |\dot{\theta}_2|}{r_{AB}} = \frac{3.0 \times 100}{6.0} = 50 \text{ rpm}$$

(b) rotational speed of link 3 is zero when $\bar{v}_A$ and $\bar{v}_B$ have the same direction

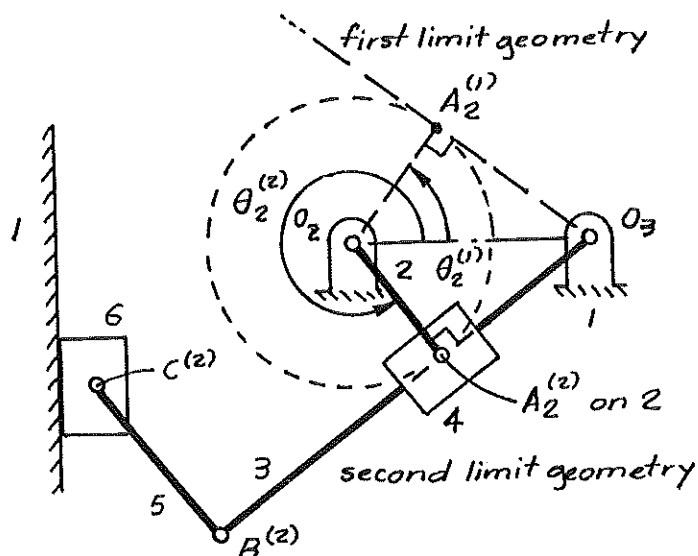


for both cases
$$\underline{v_B} = r_{AO_2} |\dot{\theta}_2| = 3.0 \times 100 \times \frac{2\pi}{60}$$

$$= \underline{31.4 \frac{\text{cm}}{\text{sec}}}$$

P2.5

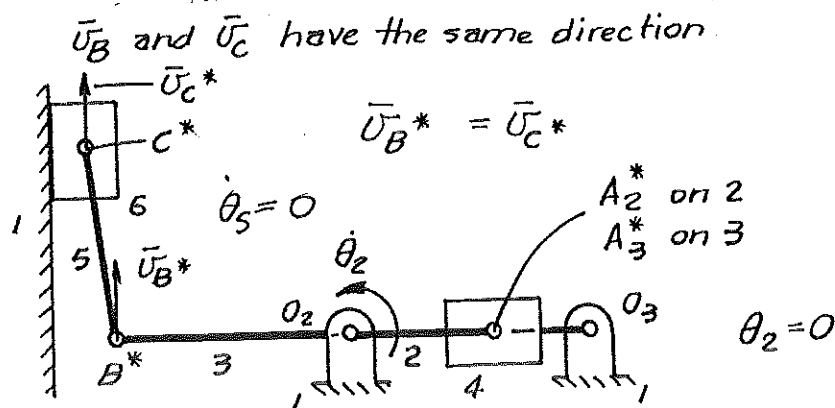
- (a) geometries for which velocity of link 6 is zero (i.e., limit positions)



$$\theta_2^{(1)} = \cos^{-1} (r_{A_2O_2} / r_{O_2O_3}) = \cos^{-1} (3.0 / 5.0) = 53.1^\circ$$

$$\theta_2^{(2)} = 360^\circ - \theta_2^{(1)} = 306.9^\circ$$

(b)



$$U_{A_2}^* = r_{A_2O_2} |\dot{\theta}_2| = r_{A_3}^* O_3 |\dot{\theta}_3|, |\dot{\theta}_3| = \frac{r_{A_2O_2}}{r_{A_3}^* O_3} |\dot{\theta}_2|$$

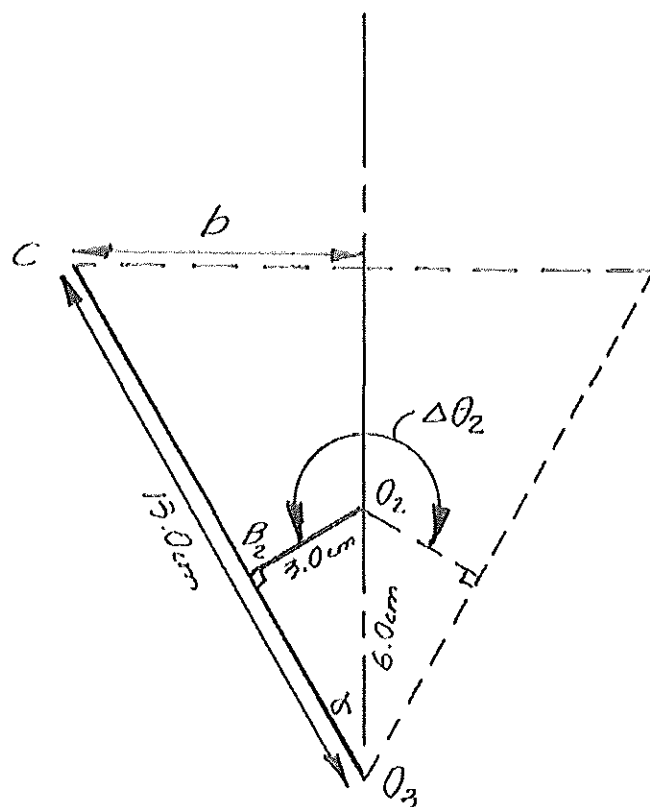
$$|\bar{U}_C^*| = |\bar{U}_B^*| = r_{B^*O_3} |\dot{\theta}_3| = r_{B^*O_3} \frac{r_{A_2O_2}}{r_{A_3}^* O_3} |\dot{\theta}_2|$$

$$= 10.0 \times \frac{3.0}{5.0 - 3.0} \times 10.0 = 150 \frac{\text{cm}}{\text{sec}}$$

$$\text{similarly at } \theta_2 = 180^\circ, |\bar{U}_C^*| = |\bar{U}_B^*| = 37.5 \frac{\text{cm}}{\text{sec}}$$

P2.6

limit geometries:



$$\alpha = \sin^{-1} \frac{3.0}{6.0} = 30^\circ$$

$$b = 13.0 \times \sin 30^\circ = 6.5 \text{ cm}$$

(a) stroke = $2b = 13.0 \text{ cm}$

(b) $\theta_2^{(1)} = 180^\circ + \alpha = \underline{210^\circ}$

$\theta_2^{(2)} = 360^\circ - \alpha = \underline{330^\circ}$

(c) $\theta_2 = \underline{90^\circ}$ or $\underline{270^\circ}$ when link 5 is in pure translation

also $\theta_2 = \underline{210^\circ}$ or $\underline{330^\circ}$ when $\dot{\theta}_3 = 0$ & $\dot{\theta}_5 = 0$

P2.6

CONTINUED

(d) motion to left requires 120° rotation of link 2

$$|\dot{\theta}_2| = 8.0 \frac{\text{rad}}{\text{sec}} \times \frac{180^\circ}{\pi \text{ rad}} = 458^\circ/\text{sec}$$

$$\therefore \text{time taken} : \frac{120^\circ}{458^\circ/\text{sec}} = 0.262 \text{ sec}$$

$$\begin{aligned} \therefore \text{average speed to left} &= \frac{\text{stroke}}{\text{time taken}} \\ &= \frac{13.0 \text{ cm}}{0.262 \text{ sec}} = \underline{49.7 \frac{\text{cm}}{\text{sec}}} \end{aligned}$$

(e) motion to right requires 240° rotation of link 2

$$\text{time taken} \frac{240^\circ}{458^\circ/\text{sec}} = 0.524 \text{ sec}$$

$$\begin{aligned} \therefore \text{average speed to the right} &= \frac{\text{stroke}}{\text{time taken}} \\ &= \frac{13.0 \text{ cm}}{0.524 \text{ sec}} = \underline{24.8 \frac{\text{cm}}{\text{sec}}} \end{aligned}$$

P2.7

$$r_1 = 8.0 \text{ cm} ; r_2 = 2.0 \text{ cm} ; r_3 = 4.0 \text{ cm} ; r_4 = 7.0 \text{ cm}$$

$$\dot{\theta}_2 = 40.0 \frac{\text{rad}}{\text{sec}} \text{ CW}$$

$$\alpha_1 = \cos^{-1} \left[\frac{r_1^2 + (r_2 + r_3)^2 - r_4^2}{2(r_2 + r_3)r_1} \right] = 57.9^\circ$$

$$\alpha_2 = \cos^{-1} \left[\frac{r_1^2 + (r_3 - r_2)^2 - r_4^2}{2(r_3 - r_2)r_1} \right] = 53.6^\circ$$

$$\beta_1 = \cos^{-1} \left[\frac{r_1^2 - (r_2 + r_3)^2 + r_4^2}{2r_1r_4} \right] = 46.6^\circ$$

$$\beta_2 = \cos^{-1} \left[\frac{r_1^2 - (r_3 - r_2)^2 + r_4^2}{2r_1r_4} \right] = 13.3^\circ$$

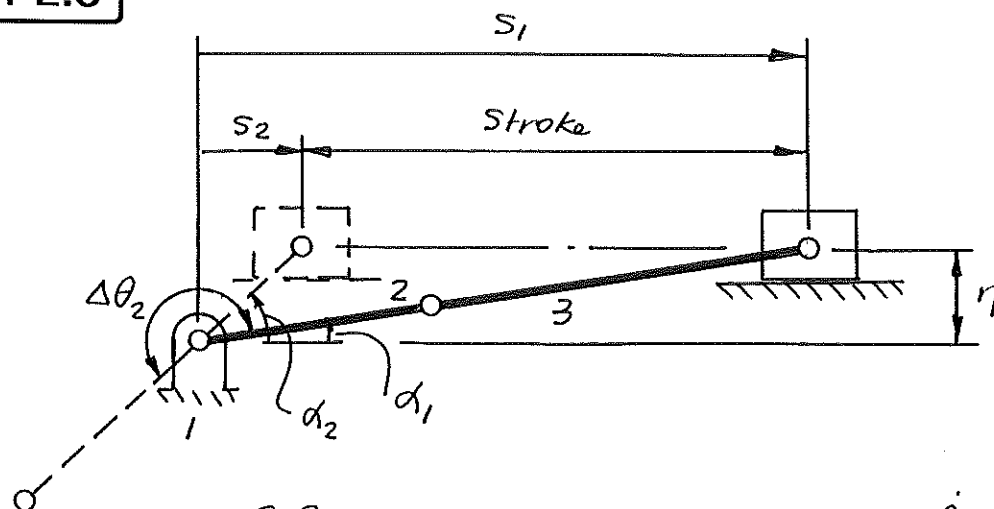
$$\Delta\theta_2 = 180^\circ + \alpha_2 - \alpha_1 = 175.7^\circ ; \Delta\theta_4 = \beta_1 - \beta_2 = 33.3^\circ$$

$$(a) \quad \underline{(\dot{\theta}_{4 \text{ avg}})_{\text{CW}}} = \frac{\Delta\theta_4}{(\Delta\theta_2 / \dot{\theta}_2)} = \underline{7.58 \frac{\text{rad}}{\text{sec}}}$$

$$(b) \quad \underline{(\dot{\theta}_{4 \text{ avg}})_{\text{CCW}}} = \frac{\Delta\theta_4}{\left(\frac{2\pi - \Delta\theta_2}{\dot{\theta}_2} \right)} = \underline{7.22 \frac{\text{rad}}{\text{sec}}}$$

$$(c) \quad \underline{T_R} = \frac{2\pi - \Delta\theta_2}{\Delta\theta_2} = \underline{1.05}$$

P2.8



$$r_1 = 2.0 \text{ cm}; r_2 = 5.0 \text{ cm}, r_3 = 8.0 \text{ cm}, \dot{\theta}_2 = 30 \frac{\text{rad}}{\text{sec}} \text{ CCW}$$

$$\alpha_1 = \sin^{-1} \left(\frac{r_1}{r_2 + r_3} \right) = \sin^{-1} \left(\frac{2.0}{5.0 + 8.0} \right) = 8.8^\circ$$

$$\alpha_2 = \sin^{-1} \left(\frac{r_1}{r_3 - r_2} \right) = 41.8^\circ$$

$$\Delta \theta_2 = 180^\circ + \alpha_2 - \alpha_1 = 213^\circ = 3.72 \text{ rad}$$

$$s_1 = \left[(r_2 + r_3)^2 - r_1^2 \right]^{1/2} = \left[(5.0 + 8.0)^2 - 2.0^2 \right]^{1/2} = 12.85 \text{ cm}$$

$$s_2 = \left[(r_3 - r_2)^2 - r_1^2 \right]^{1/2} = 2.24 \text{ cm}$$

$$\text{Stroke} = s_1 - s_2 = 12.85 - 2.24 = 10.61 \text{ cm}$$

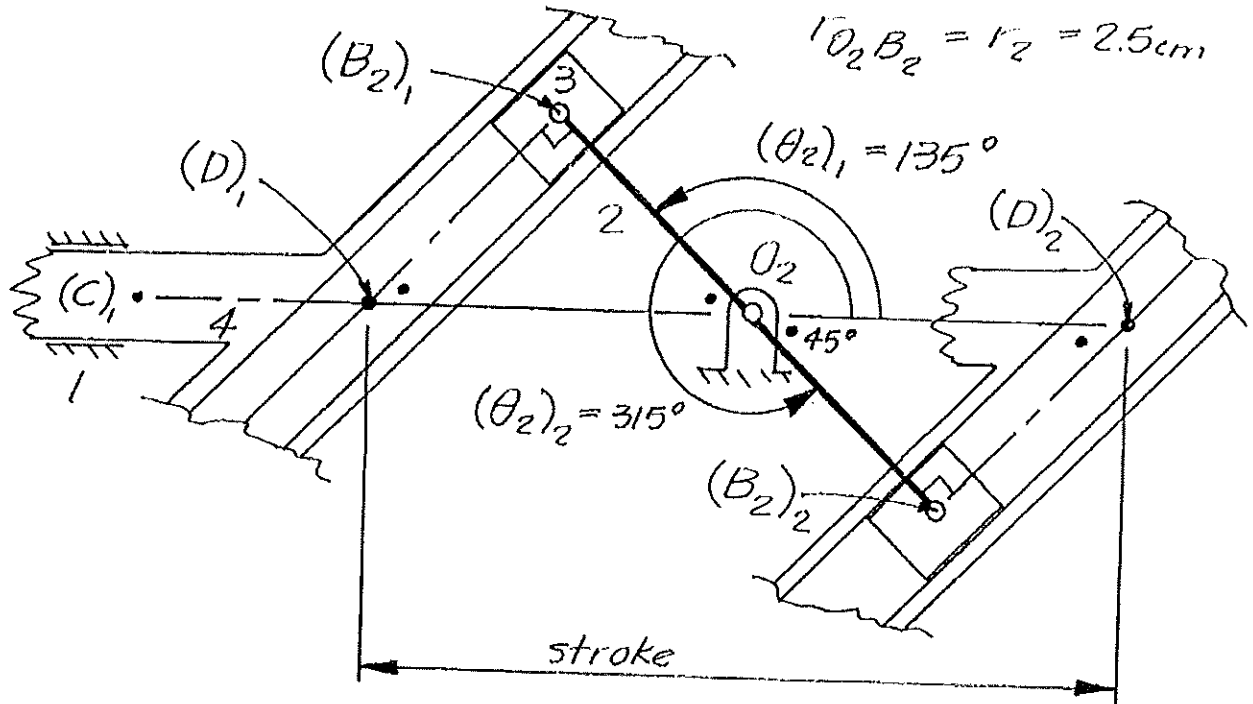
$$(a) \quad \underline{(U_{4, \text{avg}})_{\text{right}}} = \frac{\text{Stroke}}{\left(\frac{2\pi - \Delta \theta_2}{\dot{\theta}_2} \right)} = 124 \frac{\text{cm}}{\text{sec}}$$

$$(b) \quad \underline{(U_{4, \text{avg}})_{\text{left}}} = \frac{\text{Stroke}}{\Delta \theta_2 / \dot{\theta}_2} = 85.6 \frac{\text{cm}}{\text{sec}}$$

$$(c) \quad \underline{T_R} = \frac{\Delta \theta_2}{2\pi - \Delta \theta_2} = 1.45$$

P2.9

(a) limit geometries of mechanism
(i.e., $v_C = 0$)



values of θ_2 when point C is stationary:
 135° & 315°

(b) stroke of C = stroke of D

$$= 2 \left\{ 2 \left[r_2 \cos 45^\circ \right] \right\} = \underline{7.07 \text{ cm}}$$

P2.9

CONTINUED

(c) time to move between limit positions :

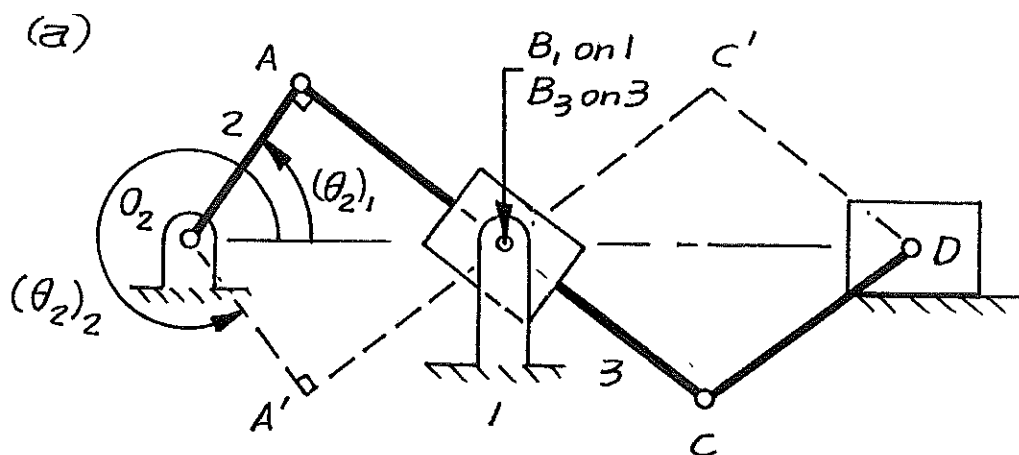
$$\Delta t = \frac{\Delta \theta_2}{\dot{\theta}_2} = \frac{\pi \text{ rad}}{8 \frac{\text{rad}}{\text{sec}}} = 0.393 \text{ sec}$$

$$\therefore \underline{\text{average speed to the left}} = \frac{\text{stroke}}{\Delta t}$$

$$= \frac{7.07 \text{ cm}}{0.393 \text{ sec}} = 18.0 \frac{\text{cm}}{\text{sec}}$$

P2.10

$$r_{O_2A} = 3.0 \text{ cm}; \quad r_{O_2B_1} = 5.0 \text{ cm}; \quad r_{AC} = 8.0 \text{ cm}; \quad r_{CD} = 4.0 \text{ cm}$$



$\dot{\theta}_3 = 0$ when link 2 is perpendicular to link 3

$$\begin{aligned}
 (\theta_2)_1 &= \tan^{-1} \left\{ \frac{r_{AB_3}}{r_{O_2A}} \right\} = \tan^{-1} \left\{ \frac{[(r_{O_2B_3})^2 - (r_{O_2A})^2]^{\frac{1}{2}}}{r_{O_2A}} \right\} \\
 &= \tan^{-1} \left\{ \frac{[5.0^2 - 3.0^2]^{\frac{1}{2}}}{3.0} \right\} = 53.1^\circ
 \end{aligned}$$

$$(\theta_2)_2 = 360^\circ - (\theta_2)_1 = 306.9^\circ$$

(b) $\dot{\theta}_D = 0$ when mechanism is in limit positions (i.e., $\theta_2 = 0, 180^\circ$)

P2.10

CONTINUED

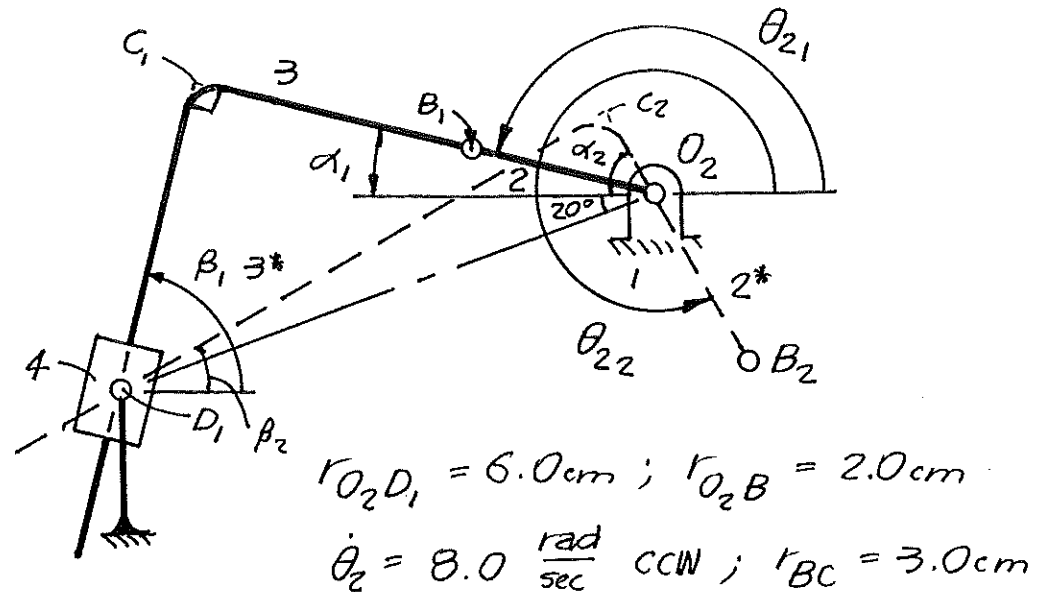
$$(c) \quad |a_{B_1 B_3}^c| = |2 \dot{r}_{B_1 B_3} \dot{\theta}_3|$$

$$a_{B_1 B_3}^c = 0 \quad \text{when} \quad \dot{r}_{B_1 B_3} = 0 \quad \text{OR} \\ \dot{\theta}_3 = 0.$$

$\therefore$ the corresponding values of θ_2 are
 $0, 53.1^\circ, 180^\circ, 306.9^\circ$

P2.11

(a)



$$\alpha_1 + 20^\circ = \cos^{-1} \left(\frac{5.0}{6.0} \right) = 33.6^\circ,$$

$$\therefore \theta_{21} = 180^\circ - 33.6^\circ + 20^\circ = \underline{166.4^\circ}$$

$$\alpha_2 + 20^\circ = \cos^{-1} \left(\frac{1.0}{6.0} \right) = 80.4^\circ$$

$$\therefore \theta_{22} = 360^\circ - 80.4^\circ + 20^\circ = \underline{299.6^\circ}$$

(b) triangle $D_1 C_1 O_2$:

$$(\beta_1 - 20^\circ) + \underbrace{\alpha_1 + 20^\circ}_{33.6^\circ} + 90^\circ = 180^\circ ; \beta_1 = 76.4^\circ$$

triangle $D_1 C_2 O_2$:

$$(\beta_2 - 20^\circ) + \underbrace{\alpha_2 + 20^\circ}_{80.4^\circ} + 90^\circ = 180^\circ ; \beta_2 = 29.6^\circ$$

P2.11

CONTINUED

(b) between θ_{21} & θ_{22} link 4 rotates CW

$$\Delta t = \frac{(\theta_{22} - \theta_{21})}{\dot{\theta}_2} = \frac{299.6^\circ - 166.4^\circ}{8.0} \times \frac{\pi}{180}$$

$$= 0.291 \text{ sec}$$

$$(\dot{\theta}_4, \text{avg})_{\text{CW}} = \left| \frac{\beta_2 - \beta_1}{\Delta t} \right| = \left| \frac{29.6^\circ - 76.4^\circ}{0.291 \text{ sec}} \times \frac{\pi}{180} \right|$$

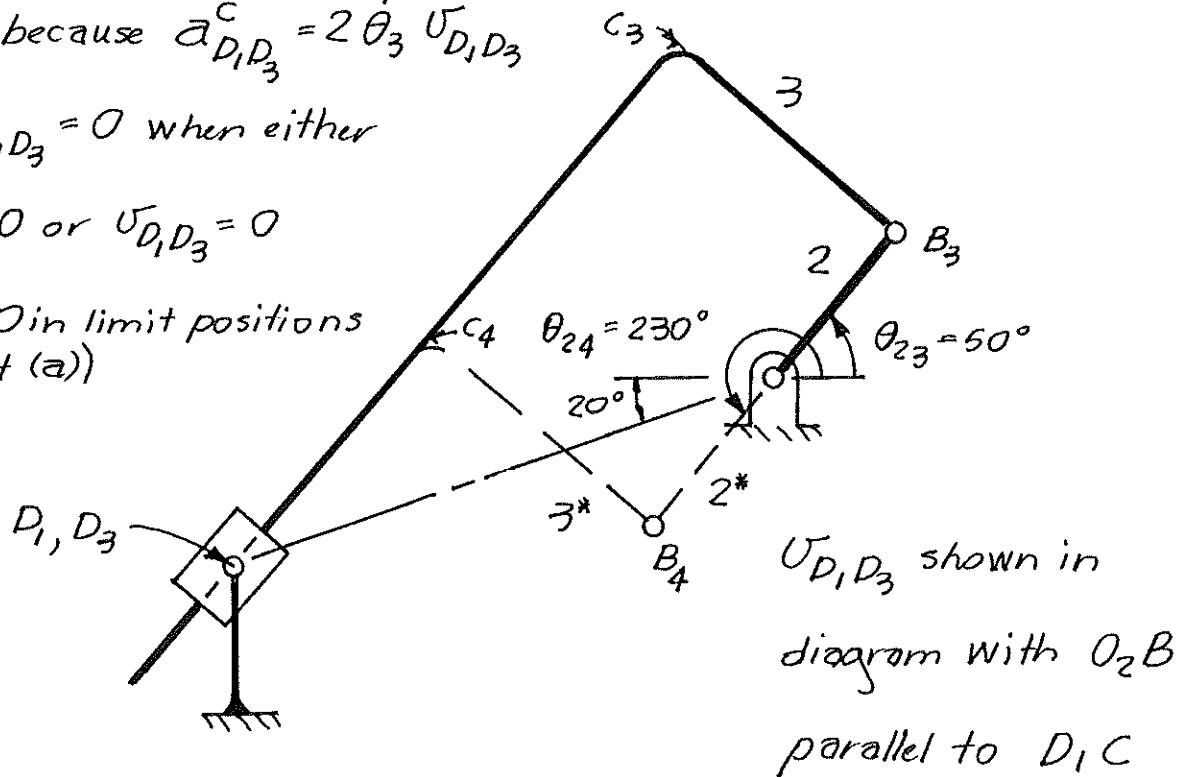
$$= \left| -2.81 \frac{\text{rad}}{\text{sec}} \right|; \quad \underline{(\dot{\theta}_4, \text{avg})_{\text{CW}} = 2.81 \frac{\text{rad}}{\text{sec}}}$$

(c) because $\ddot{a}_{D_1 D_3}^C = 2 \dot{\theta}_3 \dot{v}_{D_1 D_3}$

$\therefore \ddot{a}_{D_1 D_3}^C = 0$ when either

$\dot{\theta}_3 = 0$ or $\dot{v}_{D_1 D_3} = 0$

$\dot{\theta}_3 = 0$ in limit positions
(part (a))



complete set of values when $\ddot{a}_{D_1 D_3}^C = 0$:

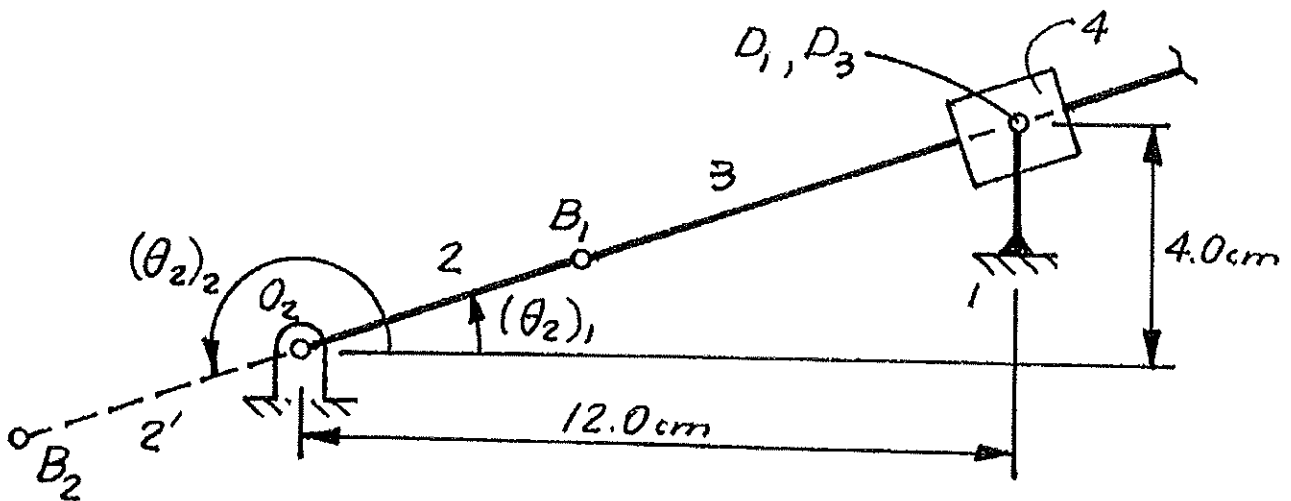
$$\underline{\theta_2 = 50^\circ, 166.4^\circ, 230^\circ, 299.6^\circ}$$

P2.12

(a) since $a_{D_1 D_3}^C = 2 \dot{r}_{D_1 D_3} \dot{\theta}_3$, then

$$a_{D_1 D_3}^C = 0 \text{ when:}$$

(i) $\dot{r}_{D_1 D_3} = 0$ (i.e., limit positions)



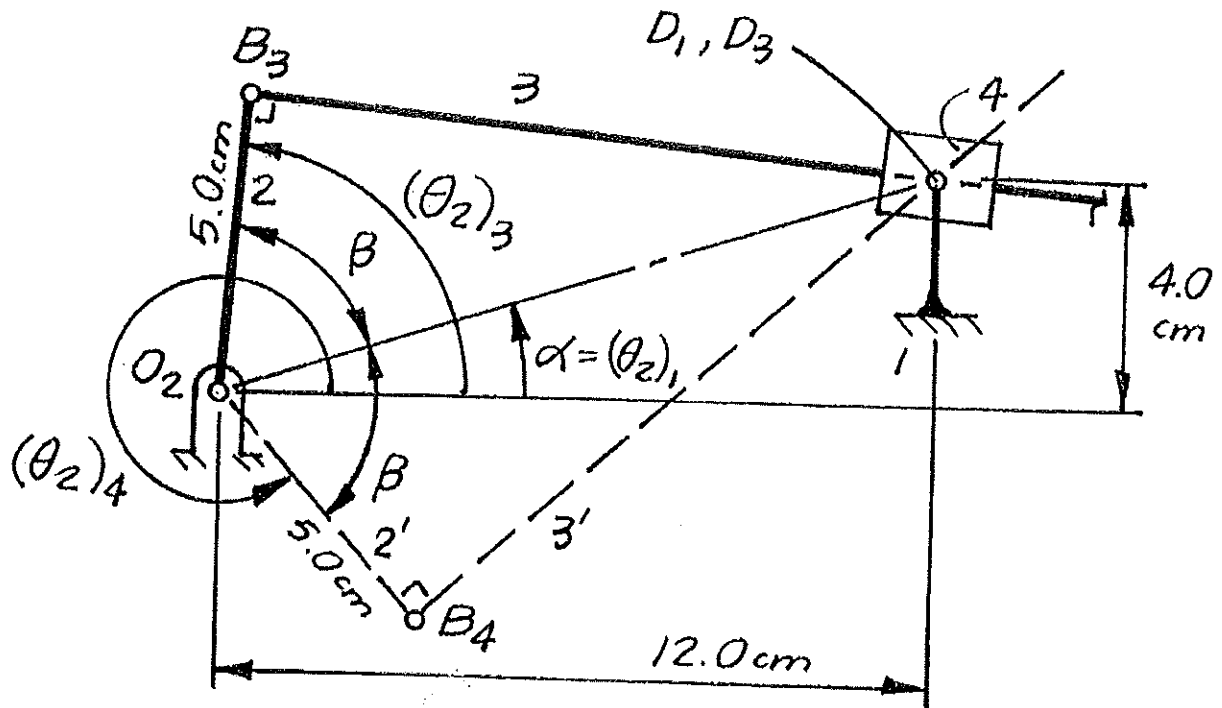
$$(\theta_2)_1 = \tan^{-1} \left(\frac{4.0}{12.0} \right) = 18.4^\circ$$

$$(\theta_2)_2 = 180^\circ + (\theta_2)_1 = 198.4^\circ$$

and ...

P2.12

CONTINUED

(ii) $\dot{\theta}_3 = 0$ (i.e., pure translation of link 3)

$$r_{O_2 D_1} = (12.0^2 + 4.0^2)^{1/2} = 12.65 \text{ cm}$$

$$\beta = \cos^{-1} \left(\frac{r_{O_2 B_3}}{r_{O_2 D_1}} \right) = 66.7^\circ$$

$$(\theta_2)_3 = \alpha + \beta = 85.1^\circ$$

$$(\theta_2)_4 = 360^\circ - \beta + \alpha = 311.7^\circ$$

complete set of values of θ_2 when $a_{D_1 D_3}^C = 0$:

$$\underline{18.4^\circ, 85.2^\circ, 198.4^\circ, 311.7^\circ}$$

P2.13

$$r_1 = 10 \text{ cm} ; r_2 = 4.0 \text{ cm}$$

$$r_3 = 12 \text{ cm} ; r_4 = 8.0 \text{ cm}$$

$$\underline{\phi_{\min}} = \cos^{-1} \left[\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4} \right] = \underline{26.4^\circ}$$

$$\underline{\phi_{\max}} = \cos^{-1} \left[\frac{r_3^2 + r_4^2 - (r_1 + r_2)^2}{2r_3r_4} \right] = \underline{86.4^\circ}$$

P2.14

$$r_1 = 2.0 \text{ cm} ; \quad r_2 = 4.0 \text{ cm} ; \quad r_3 = 12 \text{ cm}$$

$$\underline{\phi_{\min}} = \cos^{-1} \left[\frac{r_2 - r_1}{r_3} \right] = \underline{80.4^\circ}$$

$$\underline{\phi_{\max}} = \cos^{-1} \left[\frac{-r_2 - r_1}{r_3} \right] = \underline{120^\circ}$$

P2.15

$$r_1 = 10 \text{ cm} ; r_2 = 2.0 \text{ cm}$$

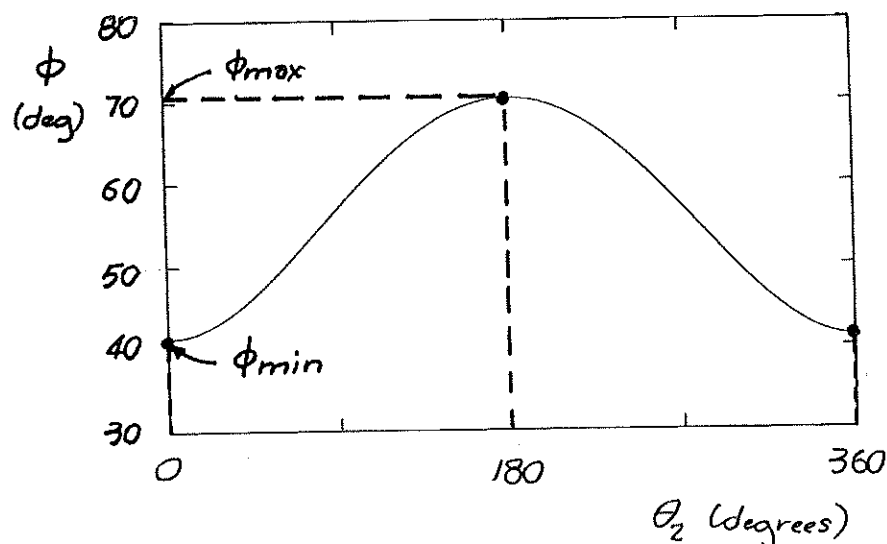
$$r_3 = 12 \text{ cm} ; r_4 = 8.0 \text{ cm}$$

$$r_{BO_4}^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \theta_2$$

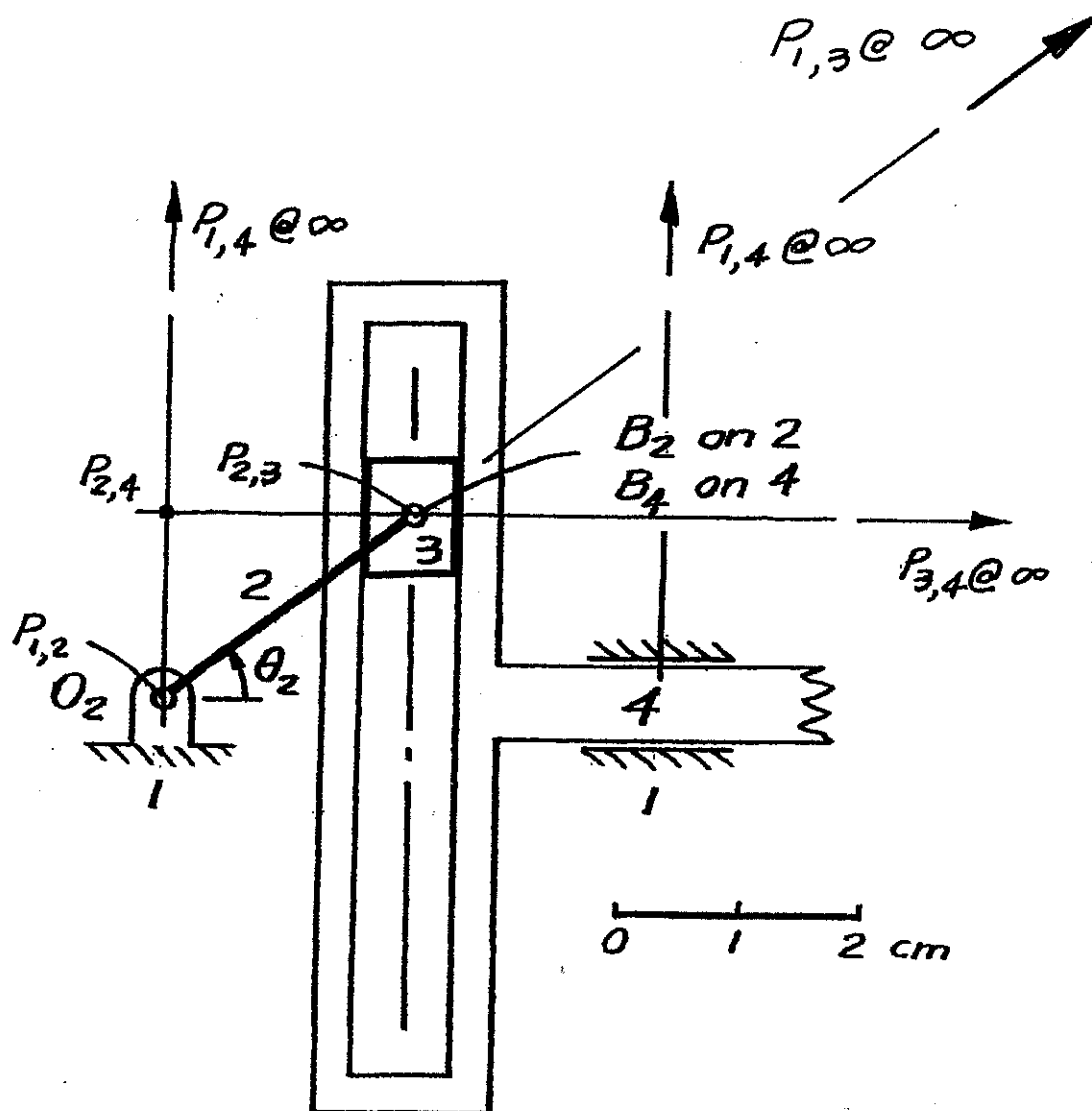
$$\phi = \cos^{-1} \left[\frac{r_3^2 + r_4^2 - r_{BO_4}^2}{2r_3r_4} \right]$$

$$\phi_{\min} = \cos^{-1} \left[\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4} \right] = 41.4^\circ$$

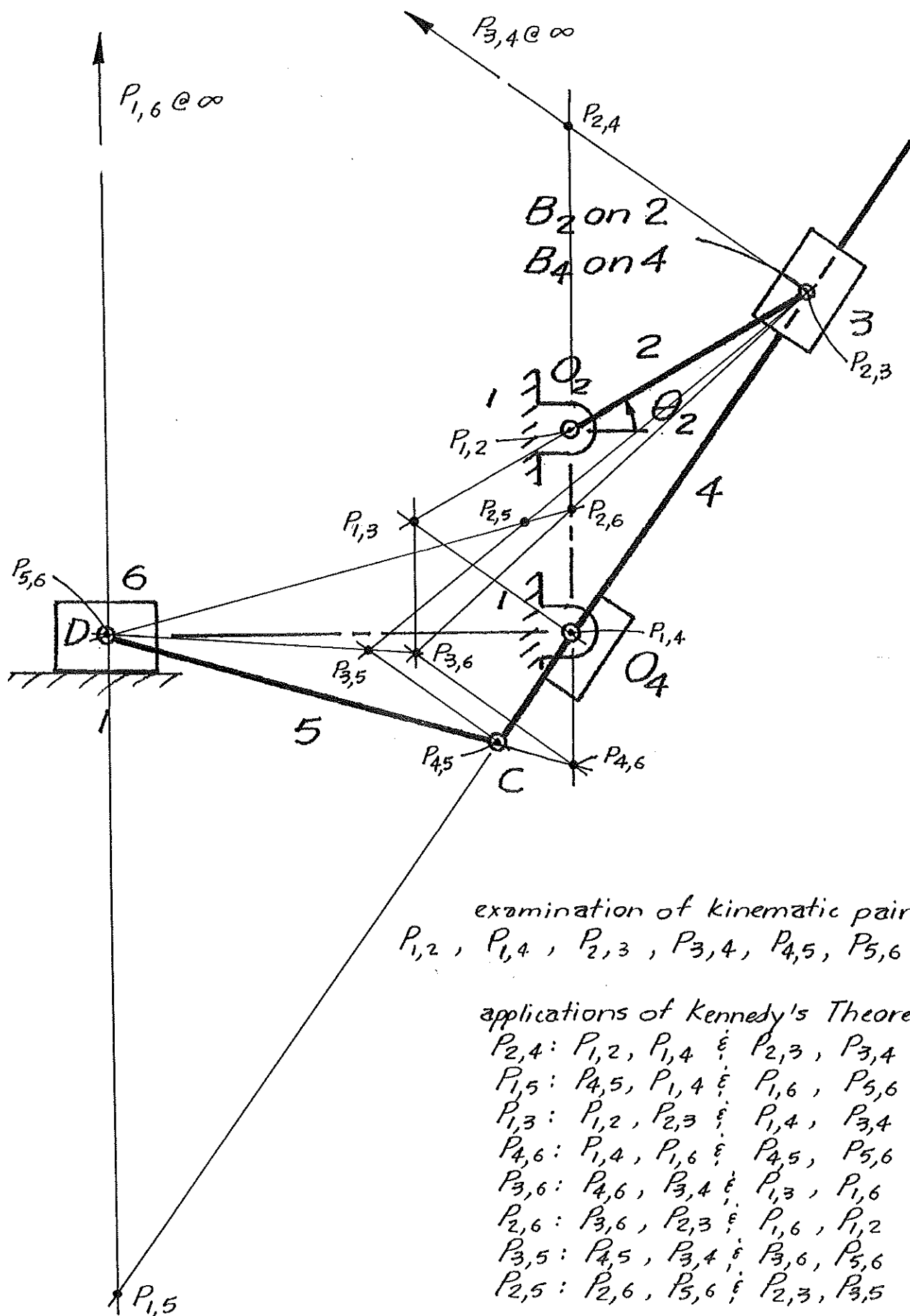
$$\phi_{\max} = \cos^{-1} \left[\frac{r_3^2 + r_4^2 - (r_1 + r_2)^2}{2r_3r_4} \right] = 70.5^\circ$$



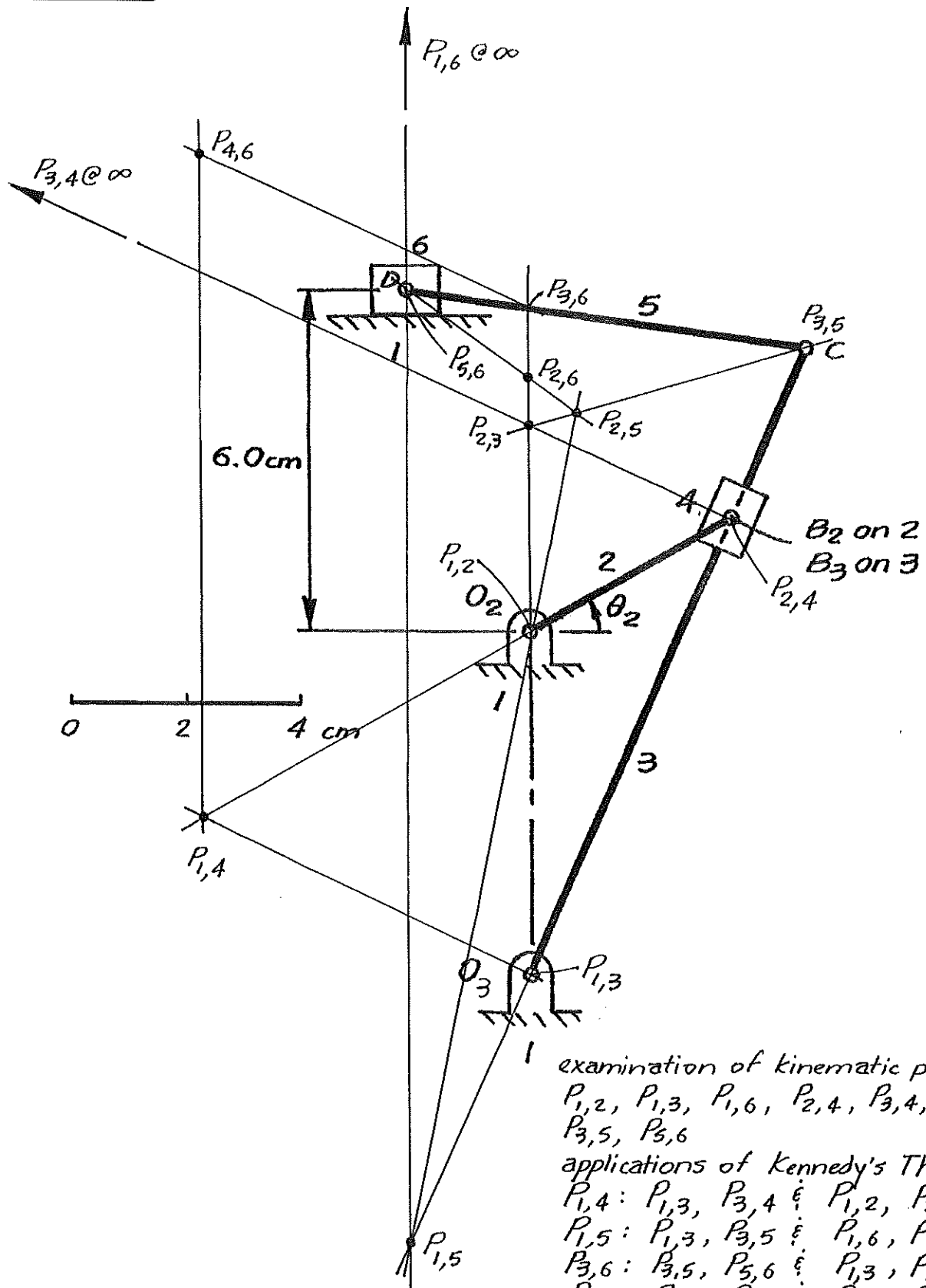
P2.16



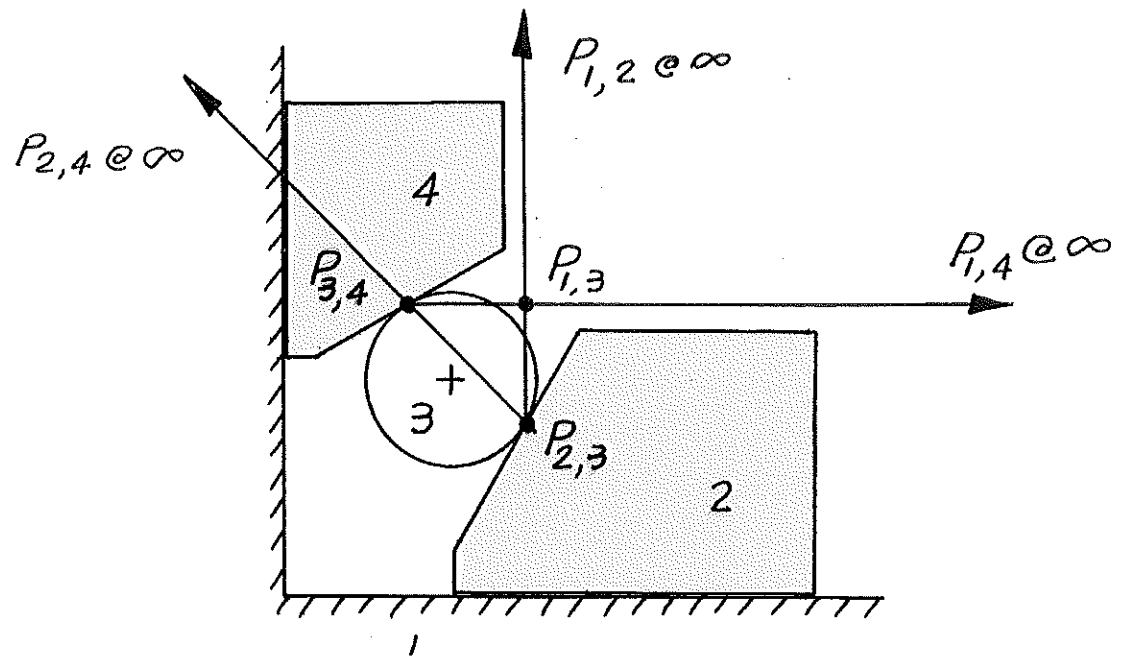
P2.17



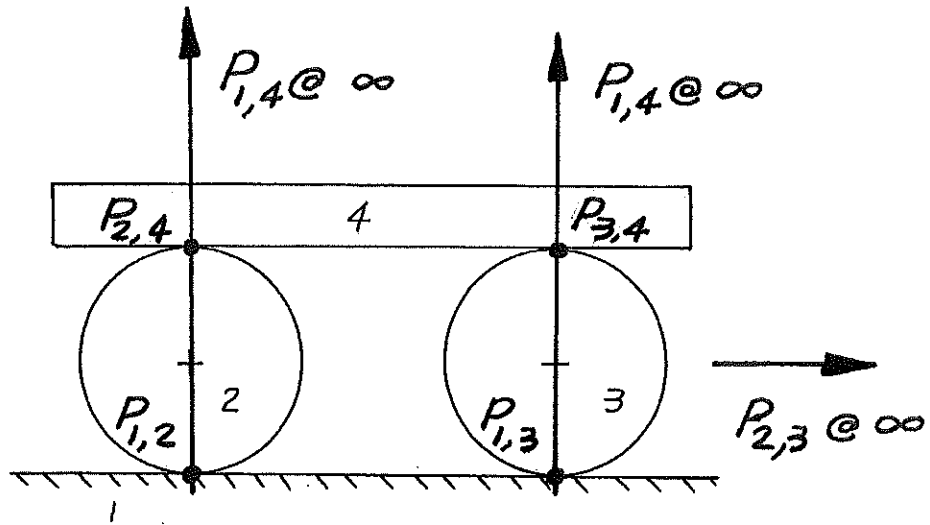
P2.18



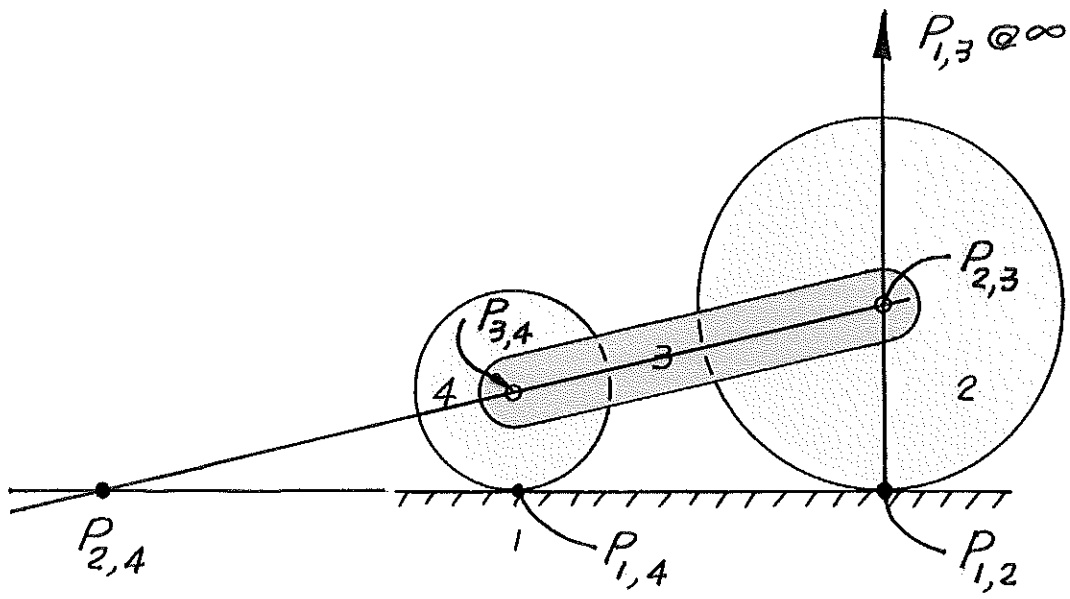
P2.19



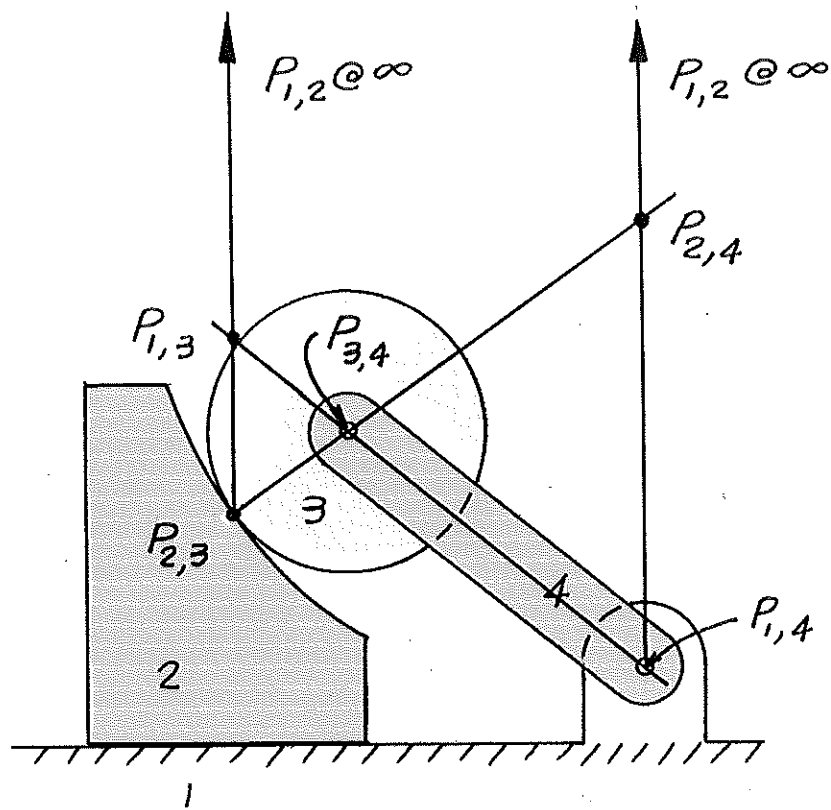
P2.20



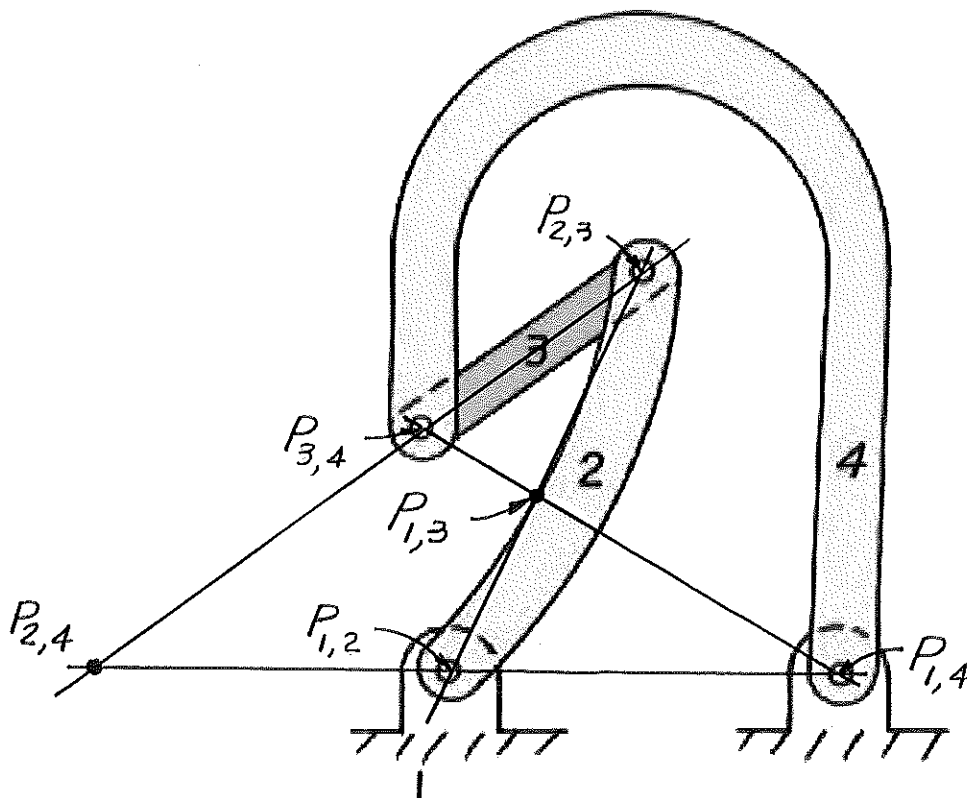
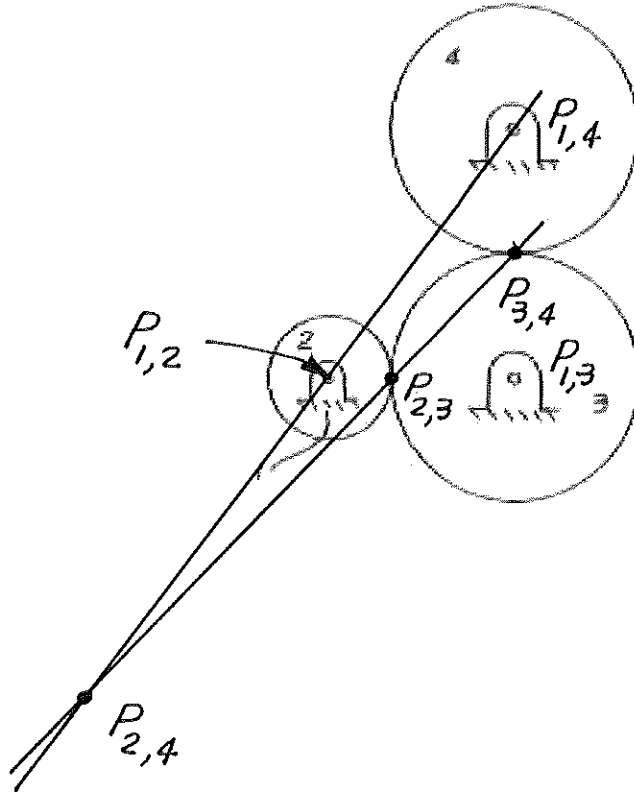
P2.21



P2.22

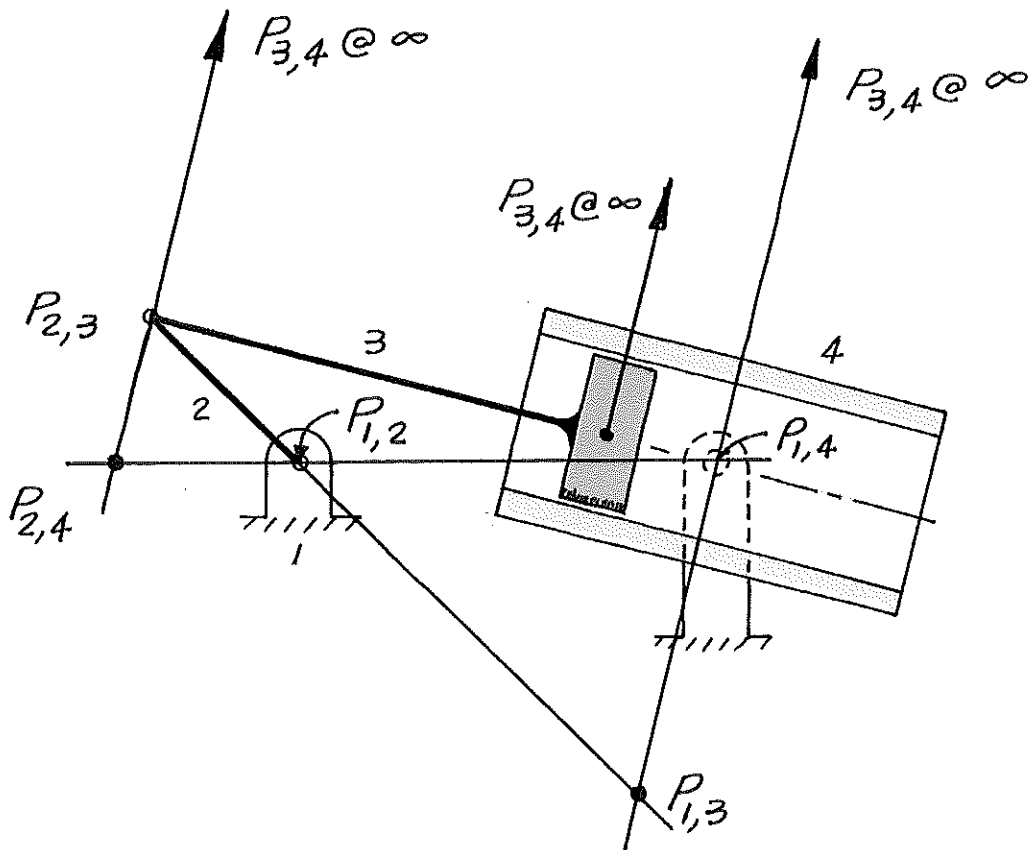
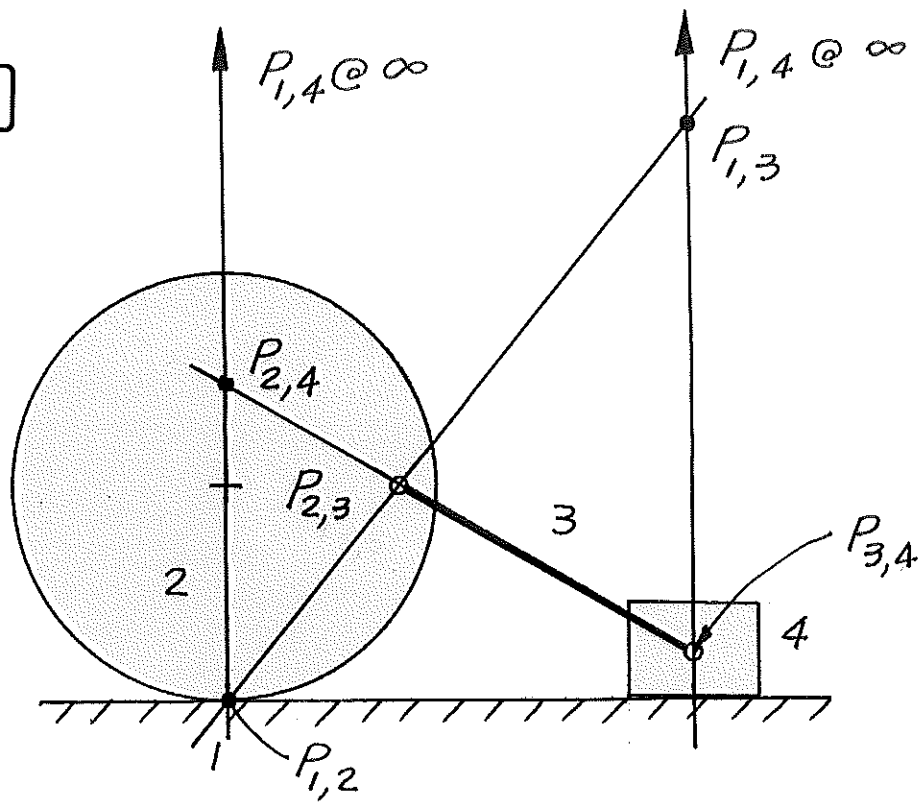


P2.23

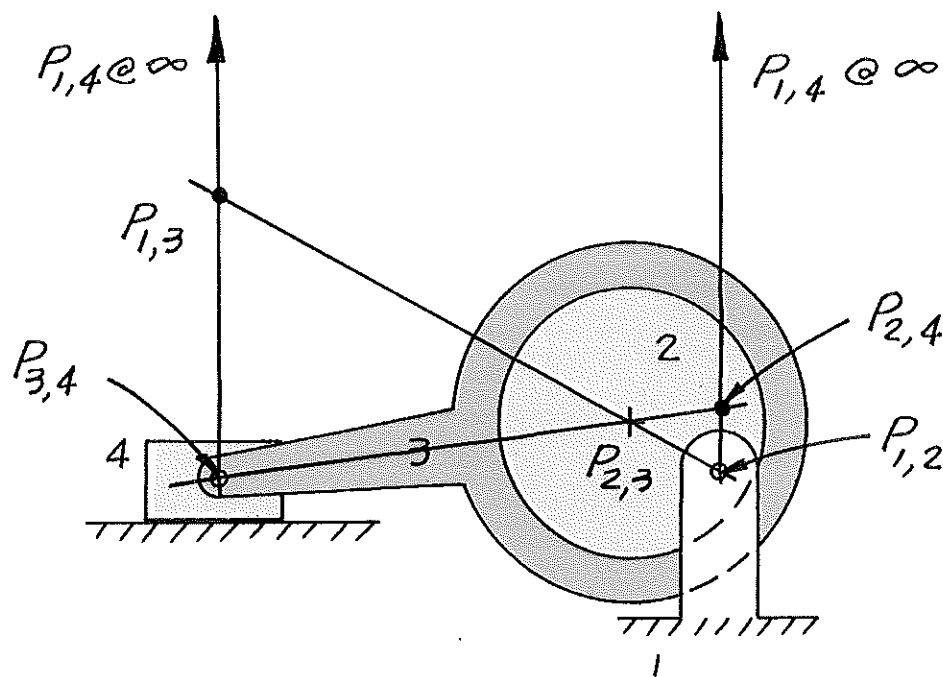
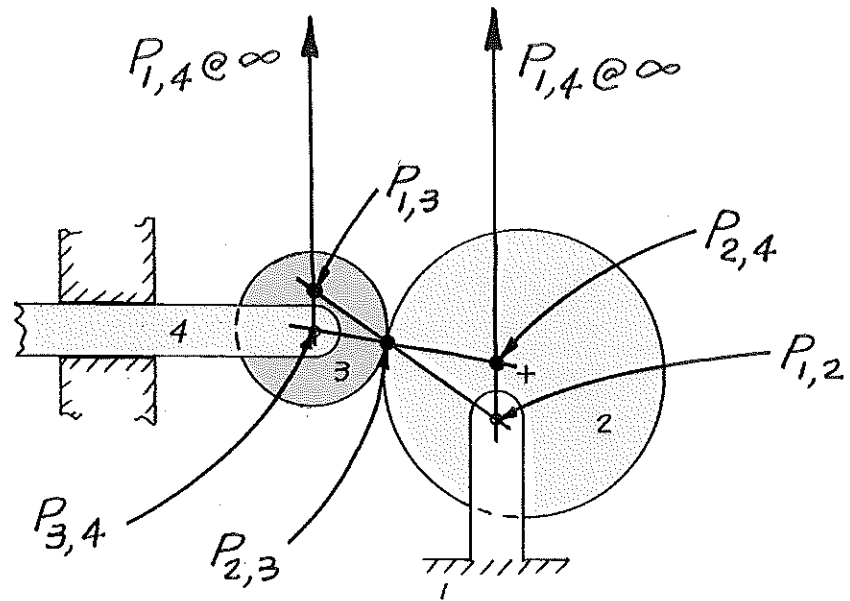


P2.24

2.29



P2.25



P2.26

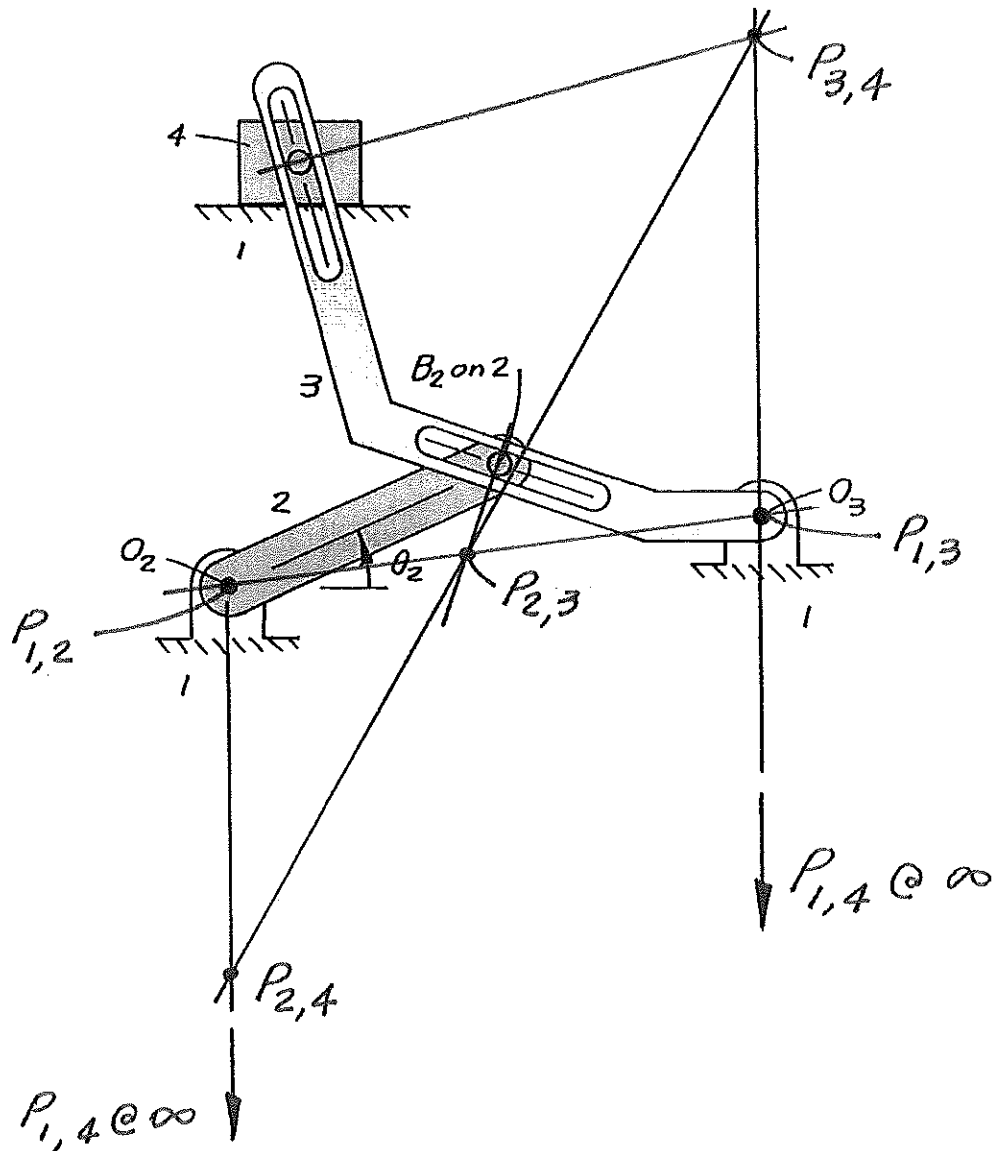
(a) $n = 4$

$$\left. \begin{array}{l} \text{turning pairs: } 2 \\ \text{sliding pairs: } 1 \end{array} \right\} j_1 = 2 + 1 = 3$$

two degree of freedom pairs: 2, $j_2 = 2$

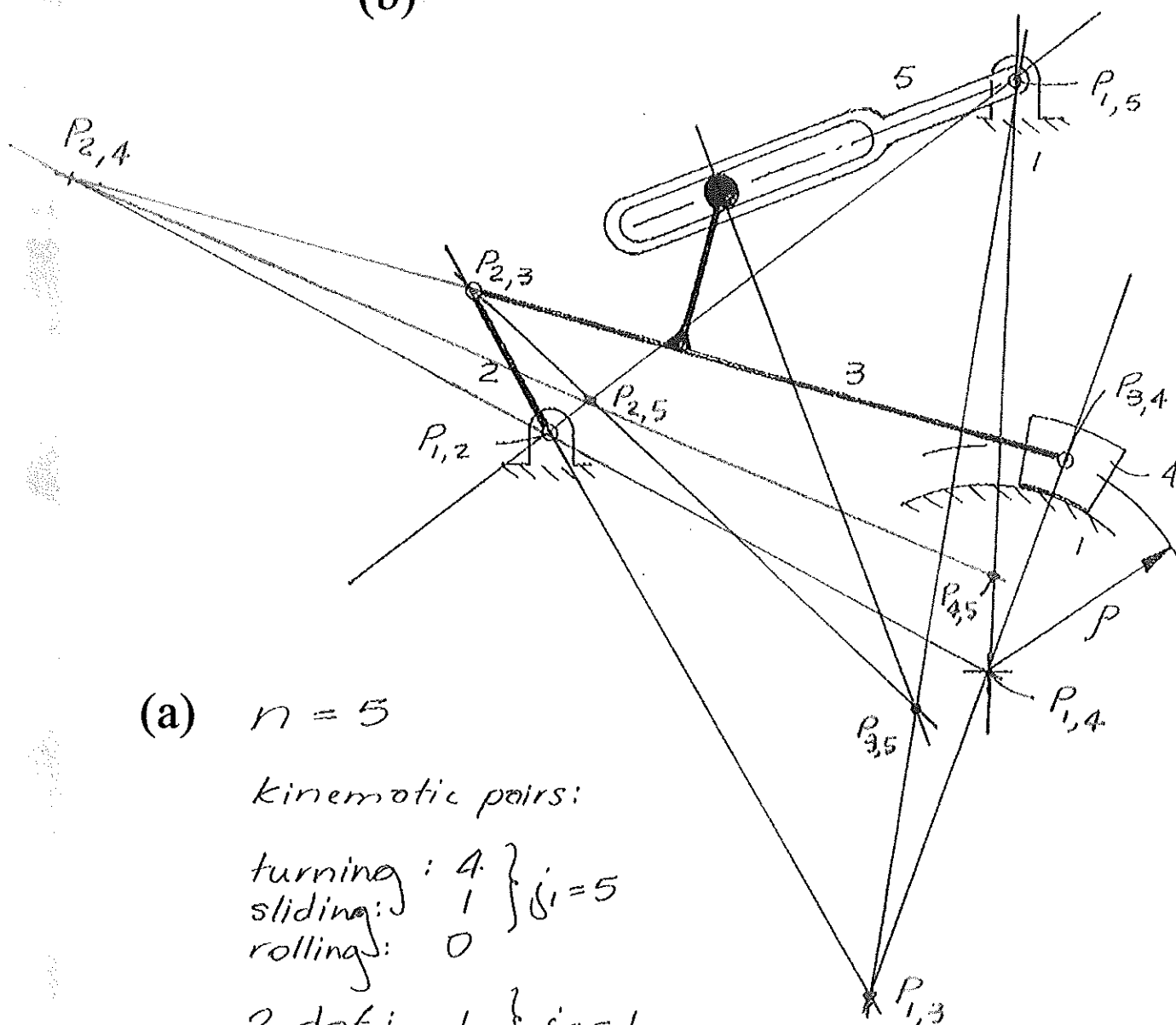
$$m = 3(n-1) - 2j_1 - j_2 = 3(4-1) - 2 \times 3 - 2 = 1$$

(b)



P2.27

(b)

(a) $n = 5$

kinematic pairs:

turning:	4	} $j_1 = 5$
sliding:	1	
rolling:	0	

2 dof:	1	} $j_2 = 1$
--------	---	-------------

mobility:

$$\begin{aligned}
 \underline{m} &= 3(n-1) - 2j_1 - j_2 \\
 &= 3(5-1) - 2 \times 5 - 1 = \underline{1}
 \end{aligned}$$

P2.28

(a) kinematic pairs :

$$\left. \begin{array}{l} \bullet \text{ sliding: } 1 \\ \bullet \text{ turning: } 3 \\ \bullet \text{ rolling: } 0 \end{array} \right\} j_1 = 1 + 3 + 0 = 4$$

$$\bullet \text{ 2 dof: } 3 \quad ; \quad j_2 = 3$$

$$\text{no. of links: } 5 ; n = 5$$

$$\begin{aligned} \text{mobility: } m &= 3(n-1) - 2j_1 - j_2 \\ &= 3(5-1) - 2 \times 4 - 3 = 1 \end{aligned}$$

(b) see diagram

P2.29

(a) no. of links : $n = 5$

one degree of freedom pairs :

4 turning

1 sliding

0 rolling $\therefore j_1 = 4 + 1 + 0 = 5$

two degree of freedom pair : 1 , $j_2 = 1$

mobility :

$$\underline{m} = 3(n-1) - 2j_1 - j_2 = \underline{1}$$

(b) no. of instantaneous centers : $\frac{n(n-1)}{2} = 10$

(7 instantaneous centers are given)

$\therefore$ no. of instantaneous centers to be found:
 $10 - 7 = 3$

- by inspection, instantaneous centers to be found : $P_{1,4}$, $P_{2,5}$, $P_{4,5}$

- see diagram

P2.29

CONTINUED

