

***Finite Mathematics 8th edition Waner Solutions  
Manual***

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# **Finite Mathematics Solutions**



# Finite Mathematics Solutions

EIGHTH  
EDITION

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Australia • Brazil • Canada • Mexico • Singapore • United Kingdom • United States

***Finite Mathematics Solutions,***  
**Eighth Edition**  
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# Chapter 0 Precalculus Review

## Solutions Section 0.1

### Section 0.1

$$\begin{aligned} 1. & 2(4 + (-1))(2 \cdot -4) \\ & = 2(3)(-8) = (6)(-8) = -48 \end{aligned}$$

$$\begin{aligned} 2. & 3 + ([4 - 2] \cdot 9) \\ & = 3 + (2 \times 9) = 3 + 18 = 21 \end{aligned}$$

$$\begin{aligned} 3. & 20/(3 * 4) - 1 \\ & = \frac{20}{12} - 1 = \frac{5}{3} - 1 = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 4. & 2 - (3 * 4)/10 \\ & = 2 - \frac{12}{10} = 2 - \frac{6}{5} = \frac{4}{5} \end{aligned}$$

$$\begin{aligned} 5. & \frac{3 + ([3 + (-5)])}{3 - 2 \times 2} \\ & = \frac{3 + (-2)}{3 - 4} = \frac{1}{-1} = -1 \end{aligned}$$

$$\begin{aligned} 6. & \frac{12 - (1 - 4)}{2(5 - 1) \cdot 2 - 1} \\ & = \frac{12 - (-3)}{16 - 1} = \frac{15}{15} = 1 \end{aligned}$$

$$\begin{aligned} 7. & (2 - 5 * (-1))/1 - 2 * (-1) \\ & = \frac{2 - 5 \cdot (-1)}{1} - 2 \cdot (-1) \\ & = \frac{2 + 5}{1} + 2 = 7 + 2 = 9 \end{aligned}$$

$$\begin{aligned} 8. & 2 - 5 * (-1)/(1 - 2 * (-1)) \\ & = 2 - \frac{5 \cdot (-1)}{1 - 2 \cdot (-1)} \\ & = 2 - \frac{-5}{1 + 2} + 2 = 2 + \frac{5}{3} = \frac{11}{3} \end{aligned}$$

$$\begin{aligned} 9. & 2 \cdot (-1)^2/2 = \frac{2 \times (-1)^2}{2} \\ & = \frac{2 \times 1}{2} = \frac{2}{2} = 1 \end{aligned}$$

$$10. 2 + 4 \cdot 3^2 = 2 + 4 \times 9 = 2 + 36 = 38$$

$$\begin{aligned} 11. & 2 \cdot 4^2 + 1 \\ & = 2 \times 16 + 1 = 32 + 1 = 33 \end{aligned}$$

$$\begin{aligned} 12. & 1 - 3 \cdot (-2)^2 \times 2 \\ & = 1 - 3 \times 4 \times 2 = 1 - 24 = -23 \end{aligned}$$

$$\begin{aligned} 13. & 3^2 + 2^2 + 1 = 9 + 4 + 1 = 14 \end{aligned}$$

$$\begin{aligned} 14. & 2^{(2^2 - 2)} \\ & = 2^{(2^2 - 2)} = 2^{4 - 2} = 2^2 = 4 \end{aligned}$$

$$\begin{aligned} 15. & \frac{3 - 2(-3)^2}{-6(4 - 1)^2} \\ & = \frac{3 - 2 \times 9}{-6(3)^2} = \frac{3 - 18}{-6 \times 9} \\ & = \frac{-15}{-54} = \frac{5}{18} \end{aligned}$$

$$\begin{aligned} 16. & \frac{1 - 2(1 - 4)^2}{2(5 - 1)^2 \cdot 2} \\ & = \frac{1 - 2(-3)^2}{2(4)^2 \cdot 2} = \frac{1 - 2 \times 9}{2 \times 16 \times 2} \\ & = \frac{1 - 18}{64} = -\frac{17}{64} \end{aligned}$$

$$\begin{aligned} 17. & 10 * (1 + 1/10)^3 \\ & = 10 \left(1 + \frac{1}{10}\right)^3 = 10(1.1)^3 \\ & = 10 \times 1.331 = 13.31 \end{aligned}$$

$$\begin{aligned} 18. & 121 / (1 + 1/10)^2 \\ & = \frac{121}{\left(1 + \frac{1}{10}\right)^2} = \frac{121}{1.1^2} \\ & = \frac{121}{1.21} = 100 \end{aligned}$$

## Solutions Section 0.1

$$\begin{aligned} 19. & 3 \left[ \frac{-2 \cdot 3^2}{-(4-1)^2} \right] \\ &= 3 \left[ \frac{-2 \times 9}{-3^2} \right] = 3 \left[ \frac{-18}{-9} \right] \\ &= 3 \times 2 = 6 \end{aligned}$$

$$\begin{aligned} 20. & - \left[ \frac{8(1-4)^2}{-9(5-1)^2} \right] \\ &= - \left[ \frac{8 \times 9}{-9 \times 16} \right] = - \left( \frac{72}{-144} \right) \\ &= - \left( -\frac{1}{2} \right) = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 21. & 3 \left[ 1 - \left( -\frac{1}{2} \right)^2 \right]^2 + 1 \\ &= 3 \left[ 1 - \frac{1}{4} \right]^2 + 1 \\ &= 3 \left[ \frac{3}{4} \right]^2 + 1 = 3 \left[ \frac{9}{16} \right] + 1 \\ &= \frac{27}{16} + 1 = \frac{43}{16} \end{aligned}$$

$$\begin{aligned} 22. & 3 \left[ \frac{1}{9} - \left( \frac{2}{3} \right)^2 \right]^2 + 1 \\ &= 3 \left[ \frac{1}{9} - \frac{4}{9} \right]^2 + 1 \\ &= 3 \left[ \frac{-3}{9} \right]^2 + 1 = 3 \left[ \frac{-1}{3} \right]^2 + 1 \\ &= 3 \frac{1}{9} + 1 = \frac{3}{9} + 1 = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} 23. & (1/2)^2 - 1/2^2 \\ &= \left[ \frac{1}{2} \right]^2 - \frac{1}{2^2} = \frac{1}{4} - \frac{1}{4} = 0 \end{aligned}$$

$$\begin{aligned} 24. & 2/(1^2) - (2/1)^2 \\ &= \frac{2}{1^2} - \left[ \frac{2}{1} \right]^2 = \frac{2}{1} - \frac{4}{1} = -2 \end{aligned}$$

$$25. 3 \times (2-5) = 3 * (2-5)$$

$$26. 4 + \frac{5}{9} = 4+5/9 \text{ or } 4+(5/9)$$

$$27. \frac{3}{2-5} = 3/(2-5)$$

$$28. \frac{4-1}{3} = (4-1)/3$$

$$29. \frac{3-1}{8+6} = (3-1)/(8+6)$$

$$30. 3 + \frac{3}{2-9} = 3+3/(2-9)$$

**Note**  $3-1/8-6$  is wrong, as it corresponds to  $3 - \frac{1}{8} - 6$ .

$$31. 3 - \frac{4+7}{8} = 3-(4+7)/8$$

$$32. \frac{4 \times 2}{\left( \frac{2}{3} \right)} = 4*2/(2/3) \text{ or } (4*2)/(2/3)$$

$$33. \frac{2}{3+x} - xy^2 = 2/(3+x) - x*y^2$$

$$34. 3 + \frac{3+x}{xy} = 3+(3+x)/(x*y)$$

$$\begin{aligned} 35. & 3.1x^3 - 4x^{-2} - \frac{60}{x^2-1} \\ &= 3.1x^3 - 4x^{(-2)} - 60/(x^2-1) \end{aligned}$$

$$\begin{aligned} 36. & 2.1x^{-3} - x^{-1} + \frac{x^2-3}{2} \\ &= 2.1x^{(-3)} - x^{(-1)} + (x^2-3)/2 \end{aligned}$$

## Solutions Section 0.1

$$37. \frac{\left(\frac{2}{3}\right)}{5}$$

$$= (2/3)/5$$

$$38. \frac{2}{\left(\frac{3}{5}\right)}$$

$$= 2/(3/5)$$

$$39. 3^{4-5} \times 6$$

$$= 3^{(4-5)} * 6$$

Note that the entire exponent is in parentheses.

$$40. \frac{2}{3+5^{7-9}}$$

$$= 2/(3+5^{(7-9)})$$

Note that the entire exponent is in parentheses.

$$41. 3\left(1 + \frac{4}{100}\right)^{-3}$$

$$= 3 * (1+4/100)^{(-3)}$$

$$42. 3\left(\frac{1+4}{100}\right)^{-3}$$

$$= 3((1+4)/100)^{(-3)}$$

$$43. 3^{2x-1} + 4^x - 1$$

$$= 3^{(2*x-1)} + 4^x - 1$$

$$44. 2^{x^2} - (2^{2x})^2$$

$$= 2^{(x^2)} - (2^{(2*x)})^2$$

$$45. 2^{2x^2-x+1}$$

$$= 2^{(2x^2-x+1)}$$

Note that the entire exponent is in parentheses.

$$46. 2^{2x^2-x} + 1$$

$$= 2^{(2x^2-x)} + 1$$

Note that the entire exponent is in parentheses.

$$47. \frac{4e^{-2x}}{2-3e^{-2x}}$$

$$= 4 * e^{(-2*x)} / (2-3e^{(-2*x)})$$

$$\text{or } 4(*e^{(-2*x)}) / (2-3e^{(-2*x)})$$

$$\text{or } (4*e^{(-2*x)}) / (2-3e^{(-2*x)})$$

$$48. \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$$

$$= (e^{(2*x)} + e^{(-2*x)}) / (e^{(2*x)} - e^{(-2*x)})$$

$$49. 3\left(1 - \left(-\frac{1}{2}\right)^2\right)^2 + 1$$

$$= 3(1 - (-1/2)^2)^2 + 1$$

$$50. 3\left(\frac{1}{9} - \left(\frac{2}{3}\right)^2\right)^2 + 1$$

$$= 3(1/9 - (2/3)^2)^2 + 1$$

## Solutions Section 0.2

### Section 0.2

$$1. 3^3 = 27$$

$$2. (-2)^3 = -8$$

$$3. -(2 \cdot 3)^2 = -(2^2 \cdot 3^2) = -(4 \cdot 9) = -36$$

or

$$-(2 \cdot 3)^2 = -6^2 = -36$$

$$4. (4 \cdot 2)^2 = 4^2 \cdot 2^2 = 16 \cdot 4 = 64 \text{ or}$$

$$(4 \cdot 2)^2 = 8^2 = 64$$

$$5. \left(\frac{-2}{3}\right)^2 = \frac{(-2)^2}{3^2} = \frac{4}{9}$$

$$6. \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

$$7. (-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$$

$$8. -2^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$$

$$9. \left(\frac{1}{4}\right)^{-2} = \frac{1}{(1/4)^2} = \frac{1}{1/16} = 16$$

$$10. \left(\frac{-2}{3}\right)^{-2} = \frac{1}{(-2/3)^2} = \frac{1}{4/9} = \frac{9}{4}$$

$$11. 2 \cdot 3^0 = 2 \cdot 1 = 2$$

$$12. 3 \cdot (-2)^0 = 3 \cdot 1 = 3$$

$$13. 2^3 2^2 = 2^{3+2} = 2^5 = 32$$

or

$$2^3 2^2 = 8 \cdot 4 = 32$$

$$14. 3^2 3 = 3^2 3^1 = 3^{2+1} = 3^3 = 27 \text{ or}$$

$$3^2 3 = 9 \cdot 3 = 27$$

$$15. 2^2 2^{-1} 2^4 2^{-4} = 2^{2-1+4-4} = 2^1 = 2$$

$$16. 5^2 5^{-3} 5^2 5^{-2} = 5^{2-3+2-2} = 5^{-1} = \frac{1}{5}$$

$$17. x^3 x^2 = x^{3+2} = x^5$$

$$18. x^4 x^{-1} = x^{4-1} = x^3$$

$$19. -x^2 x^{-3} y = -x^{2-3} y = -x^{-1} y = -\frac{y}{x}$$

$$20. -x y^{-1} x^{-1} = -x^{1-1} y^{-1} = -x^0 y^{-1}$$

$$= -\frac{1}{y}$$

$$21. \frac{x^3}{x^4} = x^{3-4} = x^{-1} = \frac{1}{x}$$

$$22. \frac{y^5}{y^3} = y^{5-3} = y^2$$

$$23. \frac{x^2 y^2}{x^{-1} y} = x^{2-(-1)} y^{2-1} = x^3 y$$

$$24. \frac{x^{-1} y}{x^2 y^2} = x^{-1-2} y^{1-2} = x^{-3} y^{-1} = \frac{1}{x^3 y}$$

## Solutions Section 0.2

$$\begin{aligned} 25. \frac{(xy^{-1}z^3)^2}{x^2yz^2} &= \frac{x^2(y^{-1})^2(z^3)^2}{x^2yz^2} \\ &= \frac{x^2y^{-2}z^6}{x^2yz^2} = x^{2-2}y^{-2-1}z^{6-2} \\ &= x^0y^{-3}z^4 = \frac{z^4}{y^3} \end{aligned}$$

$$\begin{aligned} 26. \frac{x^2yz^2}{(xyz^{-1})^{-1}} &= \frac{x^2yz^2}{x^{-1}y^{-1}(z^{-1})^{-1}} \\ &= \frac{x^2yz^2}{x^{-1}y^{-1}z} = x^{2-(-1)}y^{1-(-1)}z^{2-1} \\ &= x^3y^2z^1 \end{aligned}$$

$$\begin{aligned} 27. \left(\frac{xy^{-2}z}{x^{-1}z}\right)^3 &= \frac{(xy^{-2}z)^3}{(x^{-1}z)^3} \\ &= \frac{x^3y^{-6}z^3}{x^{-3}z^3} = x^{3-(-3)}y^{-6}z^{3-3} \\ &= x^6y^{-6}z^0 = \frac{x^6}{y^6} \end{aligned}$$

$$\begin{aligned} 28. \left(\frac{x^2y^{-1}z^0}{xyz}\right)^2 &= \frac{(x^2y^{-1}z^0)^2}{(xyz)^2} \\ &= \frac{x^4y^{-2}}{x^2y^2z^2} = x^{4-2}y^{-2-2}z^{-2} \\ &= x^2y^{-4}z^{-2} = \frac{x^2}{y^4z^2} \end{aligned}$$

$$\begin{aligned} 29. \left(\frac{x^{-1}y^{-2}z^2}{xy}\right)^{-2} &= (x^{-1-1}y^{-2-1}z^2)^{-2} \\ &= (x^{-2}y^{-3}z^2)^{-2} = x^{4}y^6z^{-4} \\ &= \frac{x^4y^6}{z^4} \end{aligned}$$

$$\begin{aligned} 30. \left(\frac{xy^{-2}}{x^2y^{-1}z}\right)^{-3} &= (x^{1-2}y^{-2+1}z^{-1})^{-3} \\ &= (x^{-1}y^{-1}z^{-1})^{-3} = x^3y^3z^3 \end{aligned}$$

$$31. \sqrt{4} = 2$$

$$32. \sqrt{5} \approx 2.236$$

$$33. \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$$

$$34. \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$$

$$35. \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

$$36. \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

$$37. \frac{\sqrt{4}}{5} = \frac{2}{5}$$

$$38. \frac{6}{\sqrt{25}} = \frac{6}{5}$$

$$39. \sqrt{9} + \sqrt{16} = 3 + 4 = 7$$

$$40. \sqrt{25} - \sqrt{16} = 5 - 4 = 1$$

$$41. \sqrt{9+16} = \sqrt{25} = 5$$

$$42. \sqrt{25-16} = \sqrt{9} = 3$$

$$43. \sqrt[3]{8-27} = \sqrt[3]{-19} \approx -2.668$$

$$44. \sqrt[4]{81-16} = \sqrt[4]{65} \approx 2.839$$

$$45. \sqrt[3]{27/8} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$$

$$46. \sqrt[3]{8 \times 64} = \sqrt[3]{8} \cdot \sqrt[3]{64} = 2 \cdot 4 = 8$$

## Solutions Section 0.2

$$47. \sqrt{(-2)^2} = \sqrt{4} = 2$$

$$48. \sqrt{(-1)^2} = \sqrt{1} = 1$$

$$49. \sqrt{\frac{1}{4}(1+15)} = \sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$$

$$50. \sqrt{\frac{1}{9}(3+33)} = \sqrt{\frac{36}{9}} = \frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3} = 2$$

$$51. \sqrt{a^2b^2} = \sqrt{a^2}\sqrt{b^2} = ab$$

$$52. \sqrt{\frac{a^2}{b^2}} = \frac{\sqrt{a^2}}{\sqrt{b^2}} = \frac{a}{b}$$

$$53. \sqrt{(x+9)^2} = x+9$$

$(x+9 > 0 \text{ because } x \text{ is positive.})$

$$54. (\sqrt{x+9})^2 = x+9$$

$$55. \sqrt[3]{x^3(a^3+b^3)} = \sqrt[3]{x^3} \sqrt[3]{a^3+b^3}$$

$$= x \sqrt[3]{a^3+b^3} \quad (\text{Not } x(a+b))$$

$$56. \sqrt[4]{\frac{x^4}{a^4b^4}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{a^4}\sqrt[4]{b^4}} = \frac{x}{ab}$$

$$57. \sqrt{\frac{4xy^3}{x^2y}} = \sqrt{\frac{4y^2}{x}} = \frac{\sqrt{4}\sqrt{y^2}}{\sqrt{x}} = \frac{2y}{\sqrt{x}}$$

$$58. \sqrt{\frac{4(x^2+y^2)}{c^2}} = \frac{\sqrt{4}\sqrt{x^2+y^2}}{\sqrt{c^2}}$$

$$= \frac{2\sqrt{x^2+y^2}}{c} \quad (\text{Not } 2(x+y)/c)$$

## Solutions Section 0.3

### Section 0.3

$$1. 3x^{-4} = \frac{3}{x^4}$$

$$2. \frac{1}{2}x^{-4} = \frac{1}{2x^4}$$

$$3. \frac{3}{4}x^{-2/3} = \frac{3}{4x^{2/3}}$$

$$4. \frac{4}{5}y^{-3/4} = \frac{4}{5y^{3/4}}$$

$$5. 1 - \frac{0.3}{x^{-2}} - \frac{6}{5}x^{-1} = 1 - 0.3x^2 - \frac{6}{5x}$$

$$6. \frac{1}{3x^{-4}} + \frac{0.1x^{-2}}{3} = \frac{x^4}{3} + \frac{0.1}{3x^2}$$

$$7. 2^{2/3} = \sqrt[3]{2^2}$$

$$8. 3^{4/5} = \sqrt[5]{3^4}$$

$$9. x^{4/3} = \sqrt[3]{x^4} = \sqrt[3]{x^3} \sqrt[3]{x} = x \sqrt[3]{x}$$

$$10. y^{7/4} = \sqrt[4]{y^7} = \sqrt[4]{y^4} \sqrt[4]{y^3} = |y| \sqrt[4]{y^3}$$

$$11. (x^{1/2}y^{1/3})^{1/5} = \sqrt[5]{\sqrt{x} \sqrt[3]{y}}$$

$$12. x^{-1/3}y^{3/2} = \frac{y^{3/2}}{x^{1/3}} = \frac{\sqrt{y^3}}{\sqrt[3]{x}} = \frac{|y|\sqrt{y}}{\sqrt[3]{x}}$$

$$13. -\frac{3}{2}x^{-1/4} = -\frac{3}{2x^{1/4}} = -\frac{3}{2\sqrt[4]{x}}$$

$$14. \frac{4}{5}x^{3/2} = \frac{4\sqrt{x^3}}{5} = \frac{4|x|\sqrt{x}}{5}$$

$$\begin{aligned} 15. 0.2x^{-2/3} + \frac{3}{7x^{-1/2}} &= \frac{0.2}{x^{2/3}} + \frac{3x^{1/2}}{7} \\ &= \frac{0.2}{\sqrt[3]{x^2}} + \frac{3\sqrt{x}}{7} \end{aligned}$$

$$\begin{aligned} 16. \frac{3.1}{x^{-4/3}} - \frac{11}{7}x^{-1/7} &= 3.1x^{4/3} - \frac{11}{7x^{1/7}} \\ &= 3.1\sqrt[3]{x^4} - \frac{11}{7\sqrt[7]{x}} = 3.1x\sqrt[3]{x} - \frac{11}{7\sqrt[7]{x}} \end{aligned}$$

$$\begin{aligned} 17. \frac{3}{4(1-x)^{5/2}} &= \frac{3}{4\sqrt{(1-x)^5}} \\ &= \frac{3}{4\sqrt{(1-x)^4}\sqrt{1-x}} \\ &= \frac{3}{4(1-x)^2\sqrt{1-x}} \end{aligned}$$

$$\begin{aligned} 18. \frac{9}{4(1-x)^{-7/3}} &= \frac{9(1-x)^{7/3}}{4} \\ &= \frac{9\sqrt[3]{(1-x)^7}}{4} = \frac{9\sqrt[3]{(1-x)^6}\sqrt[3]{1-x}}{4} \\ &= \frac{9(1-x)^2\sqrt[3]{1-x}}{4} \end{aligned}$$

$$19. \sqrt{3} = 3^{1/2}$$

$$20. \sqrt{8} = 8^{1/2}$$

$$21. \sqrt{x^3} = x^{3/2}$$

$$22. \sqrt[3]{x^2} = x^{2/3}$$

## Solutions Section 0.3

$$23. \sqrt[3]{xy^2} = (xy^2)^{1/3}$$

$$24. \sqrt{x^2y} = (x^2y)^{1/2}$$

$$25. \frac{x^2}{\sqrt{x}} = \frac{x^2}{x^{1/2}} = x^{2-1/2} = x^{3/2}$$

$$26. \frac{x}{\sqrt{x}} = \frac{x}{x^{1/2}} = x^{1-1/2} = x^{1/2}$$

$$27. \frac{3}{5x^2} = \frac{3}{5}x^{-2}$$

$$28. \frac{2}{5x^{-3}} = \frac{2}{5}x^3$$

$$29. \frac{3x^{-1.2}}{2} - \frac{1}{3x^{2.1}} = \frac{3}{2}x^{-1.2} - \frac{1}{3}x^{-2.1}$$

$$30. \frac{2}{3x^{-1.2}} - \frac{x^{2.1}}{3} = \frac{2}{3}x^{1.2} - \frac{1}{3}x^{2.1}$$

$$\begin{aligned} 31. \frac{2x}{3} - \frac{x^{0.1}}{2} + \frac{4}{3x^{1.1}} \\ = \frac{2}{3}x - \frac{1}{2}x^{0.1} + \frac{4}{3}x^{-1.1} \end{aligned}$$

$$\begin{aligned} 32. \frac{4x^2}{3} + \frac{x^{3/2}}{6} - \frac{2}{3x^2} \\ = \frac{4}{3}x^2 + \frac{1}{6}x^{3/2} - \frac{2}{3}x^{-2} \end{aligned}$$

$$\begin{aligned} 33. \frac{3\sqrt{x}}{4} - \frac{5}{3\sqrt{x}} + \frac{4}{3x\sqrt{x}} \\ = \frac{3x^{1/2}}{4} - \frac{5}{3x^{1/2}} + \frac{4}{3x \cdot x^{1/2}} \\ = \frac{3}{4}x^{1/2} - \frac{5}{3}x^{-1/2} + \frac{4}{3}x^{-3/2} \end{aligned}$$

$$\begin{aligned} 34. \frac{3}{5\sqrt{x}} - \frac{5\sqrt{x}}{8} + \frac{7}{2\sqrt[3]{x}} \\ = \frac{3}{5x^{1/2}} - \frac{5x^{1/2}}{8} + \frac{7}{2x^{1/3}} \\ = \frac{3}{5}x^{-1/2} - \frac{5}{8}x^{1/2} + \frac{7}{2}x^{-1/3} \end{aligned}$$

$$\begin{aligned} 35. \frac{3\sqrt[5]{x^2}}{4} - \frac{7}{2\sqrt{x^3}} = \frac{3x^{2/5}}{4} - \frac{7}{2x^{3/2}} \\ = \frac{3}{4}x^{2/5} - \frac{7}{2}x^{-3/2} \end{aligned}$$

$$\begin{aligned} 36. \frac{1}{8x\sqrt{x}} - \frac{2}{3\sqrt[5]{x^3}} = \frac{1}{8x^{3/2}} - \frac{2}{3x^{3/5}} \\ = \frac{1}{8}x^{-3/2} - \frac{2}{3}x^{-3/5} \end{aligned}$$

$$\begin{aligned} 37. \frac{1}{(x^2+1)^3} - \frac{3}{4\sqrt[3]{x^2+1}} \\ = \frac{1}{(x^2+1)^3} - \frac{3}{(x^2+1)^{1/3}} \\ = (x^2+1)^{-3} - \frac{3}{4}(x^2+1)^{-1/3} \end{aligned}$$

$$\begin{aligned} 38. \frac{2}{3(x^2+1)^{-3}} - \frac{3\sqrt[3]{(x^2+1)^7}}{4} \\ = \frac{2}{3}(x^2+1)^3 - \frac{3}{4}(x^2+1)^{7/3} \end{aligned}$$

$$39. 4^{-1/2}4^{7/2} = 4^{-1/2+7/2} = 4^{6/2} = 4^3 = 64$$

$$40. \frac{2^{1/a}}{2^{2/a}} = 2^{1/a-2/a} = 2^{-1/a} = \frac{1}{2^{1/a}}$$

$$41. 3^{2/3}3^{-1/6} = 3^{2/3-1/6} = 3^{1/2} = \sqrt{3}$$

$$\begin{aligned} 42. 2^{1/3}2^{-1}2^{2/3}2^{-1/3} &= 2^{1/3-1+2/3-1/3} \\ &= 2^{-1/3} = \frac{1}{2^{1/3}} \end{aligned}$$

### Solutions Section 0.3

$$43. \frac{x^{3/2}}{x^{5/2}} = x^{3/2-5/2} = x^{-1} = \frac{1}{x}$$

$$44. \frac{y^{5/4}}{y^{3/4}} = y^{5/4-3/4} = y^{1/2} = \sqrt{y}$$

$$45. \frac{x^{1/2}y^2}{x^{-1/2}y} = x^{1/2-(-1/2)}y^{2-1} = xy$$

$$46. \frac{x^{-1/2}y}{x^2y^{3/2}} = x^{-1/2-2}y^{1-3/2} = x^{-5/2}y^{-1/2}$$

$$\begin{aligned} 47. \left(\frac{x}{y}\right)^{1/3} \left(\frac{y}{x}\right)^{2/3} &= \frac{x^{1/3}}{y^{1/3}} \cdot \frac{y^{2/3}}{x^{2/3}} \\ &= x^{1/3-2/3}y^{-1/3+2/3} \\ &= x^{-1/3}y^{1/3} \text{ or } \left(\frac{y}{x}\right)^{1/3} \end{aligned}$$

$$\begin{aligned} 48. \left(\frac{x}{y}\right)^{-1/3} \left(\frac{y}{x}\right)^{1/3} &= \frac{x^{-1/3}}{y^{-1/3}} \cdot \frac{y^{1/3}}{x^{1/3}} \\ &= x^{-1/3-1/3}y^{1/3+1/3} \\ &= x^{-2/3}y^{2/3} \text{ or } \left(\frac{y}{x}\right)^{2/3} \end{aligned}$$

$$\begin{aligned} 49. x^2 - 16 &= 0 \Rightarrow x^2 = 16 \\ &\Rightarrow x = \pm\sqrt{16} \Rightarrow x = \pm 4 \end{aligned}$$

$$\begin{aligned} 50. x^2 - 1 &= 0 \Rightarrow x^2 = 1 \\ &\Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1 \end{aligned}$$

$$\begin{aligned} 51. x^2 - \frac{4}{9} &= 0 \Rightarrow x^2 = \frac{4}{9} \\ &\Rightarrow x = \pm\sqrt{\frac{4}{9}} \Rightarrow x = \pm\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 52. x^2 - \frac{1}{10} &= 0 \Rightarrow x^2 = \frac{1}{10} \\ &\Rightarrow x = \pm\sqrt{\frac{1}{10}} \Rightarrow x = \pm\frac{1}{\sqrt{10}} \end{aligned}$$

$$\begin{aligned} 53. x^2 - (1 + 2x)^2 &= 0 \Rightarrow x^2 = (1 + 2x)^2 \\ &\Rightarrow x = \pm 1 + 2x \\ \text{If } x &= 1 + 2x, \text{ then } -x = 1 \Rightarrow x = -1. \\ \text{If } x &= -(1 + 2x), \text{ then } -1 \Rightarrow x = -\frac{1}{3}. \\ \text{So, } x &= 1 \text{ or } -\frac{1}{3}. \end{aligned}$$

$$\begin{aligned} 54. x^2 - (2 - 3x)^2 &= 0 \Rightarrow x^2 = (2 - 3x)^2 \\ &\Rightarrow x = \pm 2 - 3x \\ \text{If } x &= 2 - 3x, \text{ then } 4x = 2 \Rightarrow x = \frac{1}{2}. \\ \text{If } x &= -(2 - 3x), \text{ then } -2x = -2 \Rightarrow x = 1. \\ \text{So, } x &= 1 \text{ or } \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} 55. x^5 + 32 &= 0 \Rightarrow x^5 = -32 \\ &\Rightarrow x = \sqrt[5]{-32} = -2 \end{aligned}$$

$$\begin{aligned} 56. x^4 - 81 &= 0 \Rightarrow x^4 = 81 \\ &\Rightarrow x = \pm\sqrt[4]{81} = \pm 3 \end{aligned}$$

$$\begin{aligned} 57. x^{1/2} - 4 &= 0 \Rightarrow x^{1/2} = 4 \\ &\Rightarrow x = 4^2 = 16 \end{aligned}$$

$$\begin{aligned} 58. x^{1/3} - 2 &= 0 \Rightarrow x^{1/3} = 2 \\ &\Rightarrow x = 2^3 = 8 \end{aligned}$$

$$\begin{aligned} 59. 1 - \frac{1}{x^2} &= 0 \Rightarrow 1 = \frac{1}{x^2} \\ &\Rightarrow x^2 = 1 \Rightarrow x = \pm\sqrt{1} = \pm 1 \end{aligned}$$

$$\begin{aligned} 60. \frac{2}{x^3} - \frac{6}{x^4} &= 0 \Rightarrow \frac{2}{x^3} = \frac{6}{x^4} \\ &\Rightarrow 2x^4 = 6x^3 \Rightarrow 2x = 6 \Rightarrow x = 3 \end{aligned}$$

$$\begin{aligned} 61. (x - 4)^{-1/3} &= 2 \Rightarrow x - 4 = 2^{-3} = \frac{1}{8} \\ &\Rightarrow x = 4 + \frac{1}{8} = \frac{33}{8} \end{aligned}$$

$$\begin{aligned} 62. (x - 4)^{2/3} + 1 &= 5 \Rightarrow (x - 4)^{2/3} = 4 \\ &\Rightarrow x - 4 = \pm 4^{3/2} = \pm 8 \\ &\Rightarrow x = 4 \pm 8 = -4 \text{ or } 12 \end{aligned}$$

## Solutions Section 0.4

### Section 0.4

$$1. x(4x + 6) = 4x^2 + 6x$$

$$2. (4y - 2)y = 4y^2 - 2y$$

$$3. (2x - y)y = 2xy - y^2$$

$$4. x(3x + y) = 3x^2 + xy$$

$$5. (x + 1)(x - 3) = x^2 + x - 3x - 3 \\ = x^2 - 2x - 3$$

$$6. (y + 3)(y + 4) = y^2 + 3y + 4y + 12 \\ = y^2 + 7y + 12$$

$$7. (2y + 3)(y + 5) = 2y^2 + 3y + 10y + 15 \\ = 2y^2 + 13y + 15$$

$$8. (2x - 2)(3x - 4) = 6x^2 - 6x - 8x + 8 \\ = 6x^2 - 14x + 8$$

$$9. (2x - 3)^2 = 4x^2 - 12x + 9$$

$$10. (3x + 1)^2 = 9x^2 + 6x + 1$$

$$11. \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$$

$$12. \left(y - \frac{1}{y}\right)^2 = y^2 - 2 + \frac{1}{y^2}$$

$$13. (2x - 3)(2x + 3) = (2x)^2 - 3^2 = 4x^2 - 9$$

$$14. (4 + 2x)(4 - 2x) = 4^2 - (2x)^2 = 16 - 4x^2$$

$$15. \left(y - \frac{1}{y}\right)\left(y + \frac{1}{y}\right) = y^2 - \left(\frac{1}{y}\right)^2 = y^2 - \frac{1}{y^2}$$

$$16. (x - x^2)(x + x^2) = x^2 - (x^2)^2 = x^2 - x^4$$

$$17. (x^2 + x - 1)(2x + 4) \\ = x^2(2x + 4) + x(2x + 4) - 1(2x + 4) \\ = 2x^3 + 4x^2 + 2x^2 + 4x - 2x - 4 \\ = 2x^3 + 6x^2 + 2x - 4$$

$$18. (3x + 1)(2x^2 - x + 1) \\ = 3x(2x^2 - x + 1) + 1(2x^2 - x + 1) \\ = 6x^3 - 3x^2 + 3x + 2x^2 - x + 1 \\ = 6x^3 - x^2 + 2x + 1$$

$$19. (x^2 - 2x + 1)^2 = (x^2 - 2x + 1)(x^2 - 2x + 1) \\ = x^2(x^2 - 2x + 1) - 2x(x^2 - 2x + 1) + 1(x^2 - 2x + 1) \\ = x^4 - 2x^3 + x^2 - 2x^3 + 4x^2 - 2x + x^2 - 2x + 1 \\ = x^4 - 4x^3 + 6x^2 - 4x + 1$$

$$20. (x + y - xy)^2 = (x + y - xy)(x + y - xy) \\ = x(x + y - xy) + y(x + y - xy) - xy(x + y - xy) \\ = x^2 + xy - x^2y + xy + y^2 - xy^2 - x^2y - xy^2 + x^2y^2 \\ = x^2 + 2xy + y^2 - 2x^2y - 2xy^2 + x^2y^2$$

$$21. (y^3 + 2y^2 + y)(y^2 + 2y - 1) \\ = y^3(y^2 + 2y - 1) + 2y^2(y^2 + 2y - 1) + y(y^2 + 2y - 1) \\ = y^5 + 2y^4 - y^3 + 2y^4 + 4y^3 - 2y^2 + y^3 + 2y^2 - y \\ = y^5 + 4y^4 + 4y^3 - y$$

## Solutions Section 0.4

$$\begin{aligned}
 22. & (x^3 - 2x^2 + 4)(3x^2 - x + 2) \\
 &= x^3(3x^2 - x + 2) - 2x^2(3x^2 - x + 2) + 4(3x^2 - x + 2) \\
 &= 3x^5 - x^4 + 2x^3 - 6x^4 + 2x^3 - 4x^2 + 12x^2 - 4x + 8 \\
 &= 3x^5 - 7x^4 + 4x^3 + 8x^2 - 4x + 8
 \end{aligned}$$

$$\begin{aligned}
 23. & (x+1)(x+2) + (x+1)(x+3) \\
 &= (x+1)(x+2+x+3) \\
 &= (x+1)(2x+5)
 \end{aligned}$$

$$\begin{aligned}
 24. & (x+1)(x+2)^2 + (x+1)^2(x+2) \\
 &= (x+1)(x+2)(x+2+x+1) \\
 &= (x+1)(x+2)(2x+3)
 \end{aligned}$$

$$\begin{aligned}
 25. & (x^2+1)^5(x+3)^4 + (x^2+1)^6(x+3)^3 \\
 &= (x^2+1)^5(x+3)^3(x+3+x^2+1) \\
 &= (x^2+1)^5(x+3)^3(x^2+x+4)
 \end{aligned}$$

$$\begin{aligned}
 26. & 10x(x^2+1)^4(x^3+1)^5 + 15x^2(x^2+1)^5(x^3+1)^4 \\
 &= 5x(x^2+1)^4(x^3+1)^4[2(x^3+1) + 3x(x^2+1)] \\
 &= 5x(x^2+1)^4(x^3+1)^4(5x^3+3x+2)
 \end{aligned}$$

$$\begin{aligned}
 27. & (x^3+1)\sqrt{x+1} - (x^3+1)^2\sqrt{x+1} \\
 &= (x^3+1)\sqrt{x+1} \cdot [1 - (x^3+1)] \\
 &= -x^3(x^3+1)\sqrt{x+1}
 \end{aligned}$$

$$\begin{aligned}
 28. & (x^2+1)\sqrt{x+1} - \sqrt{(x+1)^3} \\
 &= \sqrt{x+1} \cdot [x^2+1 - \sqrt{(x+1)^2}] \\
 &= \sqrt{x+1} \cdot [x^2+1 - (x+1)] \\
 &= (x^2-x)\sqrt{x+1} \\
 &= x(x-1)\sqrt{x+1}
 \end{aligned}$$

$$\begin{aligned}
 29. & \sqrt{(x+1)^3} + \sqrt{(x+1)^5} \\
 &= \sqrt{(x+1)^3} \cdot [1 + \sqrt{(x+1)^2}] \\
 &= \sqrt{(x+1)^3} \cdot (1+x+1) \\
 &= (x+2)\sqrt{(x+1)^3}
 \end{aligned}$$

$$\begin{aligned}
 30. & (x^2+1)\sqrt[3]{(x+1)^4} - \sqrt[3]{(x+1)^7} \\
 &= \sqrt[3]{(x+1)^4} \cdot [x^2+1 - \sqrt[3]{(x+1)^3}] \\
 &= \sqrt[3]{(x+1)^4} \cdot [x^2+1 - (x+1)] \\
 &= (x^2-x)\sqrt[3]{(x+1)^4} \\
 &= x(x-1)\sqrt[3]{(x+1)^4}
 \end{aligned}$$

31.  $a = 2, b = 6, c = 5$ , so that the discriminant is

$$b^2 - 4ac = 6^2 - 4(2)(5) = 36 - 40 = -4$$

As the discriminant is negative, the expression does not factor at all.

32.  $a = 4, b = -6, c = 2$ , so that the discriminant is

$$b^2 - 4ac = (-6)^2 - 4(4)(2) = 36 - 32 = 4 = 2^2$$

As the discriminant is a perfect square, the expression factors over the integers.

33.  $a = -3, b = -2, c = 3$ , so that the discriminant is

$$b^2 - 4ac = (-2)^2 - 4(-3)(3) = 4 + 36 = 40$$

As the discriminant is positive but not a perfect square, the expression factors, but not over the integers.

34.  $a = 1, b = -4, c = -7$ , so that the discriminant is

$$b^2 - 4ac = (-4)^2 - 4(1)(-7) = 16 + 28 = 44$$

As the discriminant is positive but not a perfect square, the expression factors, but not over the integers.

35.  $a = 8, b = 12, c = 4$ , so that the discriminant is

$$b^2 - 4ac = 12^2 - 4(8)(4) = 144 - 128 = 16 = 4^2$$

As the discriminant is a perfect square, the expression factors over the integers.

## Solutions Section 0.4

36.  $a = 1, b = 2, c = 19$ , so that the discriminant is

$$b^2 - 4ac = 2^2 - 4(1)(19) = 4 - 76 = -72$$

As the discriminant is negative, the expression does not factor at all.

37.  $a = 40, b = -64, c = 24$ , so that the discriminant is

$$b^2 - 4ac = (-64)^2 - 4(40)(24) = 4,096 - 3,840 = 256 = 16^2$$

As the discriminant is a perfect square, the expression factors over the integers.

38.  $a = -10, b = -32, c = -32$ , so that the discriminant is

$$b^2 - 4ac = (-32)^2 - 4(-10)(-32) = 1,024 - 1,280 = -256$$

As the discriminant is negative, the expression does not factor at all.

39.  $a = 6, b = -22, c = 16$ , so that the discriminant is

$$b^2 - 4ac = (-22)^2 - 4(6)(16) = 484 - 384 = 100 = 10^2$$

As the discriminant is a perfect square, the expression factors over the integers.

40.  $a = 48, b = 32, c = 4$ , so that the discriminant is

$$b^2 - 4ac = 32^2 - 4(48)(4) = 1,024 - 768 = 256 = 16^2$$

As the discriminant is a perfect square, the expression factors over the integers.

41. **a.**  $2x + 3x^2 = x(2 + 3x)$

**b.**  $x(2 + 3x) = 0$

$$x = 0 \text{ or } 2 + 3x = 0$$

$$x = 0 \text{ or } -2/3$$

42. **a.**  $y^2 - 4y = y(y - 4)$

**b.**  $y(y - 4) = 0$

$$y = 0 \text{ or } y - 4 = 0$$

$$y = 0 \text{ or } 4$$

43. **a.**  $6x^3 - 2x^2 = 2x^2(3x - 1)$

**b.**  $2x^2(3x - 1) = 0$

$$x^2 = 0 \text{ or } 3x - 1 = 0$$

$$x = 0 \text{ or } 1/3$$

44. **a.**  $3y^3 - 9y^2 = 3y^2(y - 3)$

**b.**  $3y^2(y - 3) = 0$

$$y^2 = 0 \text{ or } y - 3 = 0$$

$$y = 0 \text{ or } 3$$

45. **a.**  $x^2 - 8x + 7 = (x - 1)(x - 7)$

**b.**  $(x - 1)(x - 7) = 0$

$$x - 1 = 0 \text{ or } x - 7 = 0$$

$$x = 1 \text{ or } 7$$

46. **a.**  $y^2 + 6y + 8 = (y + 2)(y + 4)$

**b.**  $(y + 2)(y + 4) = 0$

$$y + 2 = 0 \text{ or } y + 4 = 0$$

$$y = -2 \text{ or } -4$$

47. **a.**  $x^2 + x - 12 = (x - 3)(x + 4)$

**b.**  $(x - 3)(x + 4) = 0$

$$x - 3 = 0 \text{ or } x + 4 = 0$$

$$x = 3 \text{ or } -4$$

48. **a.**  $y^2 + y - 6 = (y - 2)(y + 3)$

**b.**  $(y - 2)(y + 3) = 0$

$$y - 2 = 0 \text{ or } y + 3 = 0$$

$$y = 2 \text{ or } -3$$

49. **a.**  $2x^2 - 3x - 2 = (2x + 1)(x - 2)$

**b.**  $(2x + 1)(x - 2) = 0$

$$2x + 1 = 0 \text{ or } x - 2 = 0$$

$$x = -1/2 \text{ or } 2$$

50. **a.**  $3y^2 - 8y - 3 = (3y + 1)(y - 3)$

**b.**  $(3y + 1)(y - 3) = 0$

$$3y + 1 = 0 \text{ or } y - 3 = 0$$

$$y = -1/3 \text{ or } 3$$

## Solutions Section 0.4

**51. a.**  $6x^2 + 13x + 6 = (2x + 3)(3x + 2)$

**b.**  $(2x + 3)(3x + 2) = 0$

$2x + 3 = 0$  or  $3x + 2 = 0$

$x = -3/2$  or  $-2/3$

**52. a.**  $6y^2 + 17y + 12 = (3y + 4)(2y + 3)$

**b.**  $(3y + 4)(2y + 3) = 0$

$3y + 4 = 0$  or  $2y + 3 = 0$

$y = -4/3$  or  $-3/2$

**53. a.**  $12x^2 + x - 6 = (3x - 2)(4x + 3)$

**b.**  $(3x - 2)(4x + 3) = 0$

$3x - 2 = 0$  or  $4x + 3 = 0$

$x = 2/3$  or  $-3/4$

**54. a.**  $20y^2 + 7y - 3 = (4y - 1)(5y + 3)$

**b.**  $(4y - 1)(5y + 3) = 0$

$4y - 1 = 0$  or  $5y + 3 = 0$

$y = 1/4$  or  $-3/5$

**55. a.**  $x^2 + 4xy + 4y^2 = (x + 2y)^2$

**b.**  $(x + 2y)^2 = 0$

$x + 2y = 0$

$x = -2y$

**56. a.**  $4y^2 - 4xy + x^2 = (2y - x)^2$

**b.**  $(2y - x)^2 = 0$

$2y - x = 0$

$y = x/2$

**57. a.**  $x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$

$= (x + 1)(x - 1)(x + 2)(x - 2)$

**b.**  $(x + 1)(x - 1)(x + 2)(x - 2) = 0$

$x + 1 = 0$  or  $x - 1 = 0$  or

$x + 2 = 0$  or  $x - 2 = 0$

$x = \pm 1$  or  $\pm 2$

**58. a.**  $y^4 + 2y^2 - 3 = (y^2 - 1)(y^2 + 3)$

$= (y + 1)(y - 1)(y^2 + 3)$

**b.**  $(y + 1)(y - 1)(y^2 + 3) = 0$

$y + 1 = 0$  or  $y - 1 = 0$  or  $y^2 + 3 = 0$

$y = \pm 1$

(Notice that  $y^2 + 3 = 0$  has no real solutions.)

**59. a.**  $x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

**b.**  $(x - \sqrt{3})(x + \sqrt{3}) = 0$

$x - \sqrt{3} = 0$  or  $x + \sqrt{3} = 0$

$x = \pm \sqrt{3}$

**60. a.**  $y^2 - 7 = (y - \sqrt{7})(y + \sqrt{7})$

**b.**  $(y - \sqrt{7})(y + \sqrt{7}) = 0$

$y - \sqrt{7} = 0$  or  $y + \sqrt{7} = 0$

$y = \pm \sqrt{7}$

## Section 0.5

$$1. \frac{x-4}{x+1} \cdot \frac{2x+1}{x-1} = \frac{(x-4)(2x+1)}{(x+1)(x-1)} = \frac{2x^2-7x-4}{x^2-1}$$

$$2. \frac{2x-3}{x-2} \cdot \frac{x+3}{x+1} = \frac{(2x-3)(x+3)}{(x-2)(x+1)} = \frac{2x^2+3x-9}{x^2-x-2}$$

$$3. \frac{x-4}{x+1} + \frac{2x+1}{x-1} = \frac{(x-4)(x-1) + (x+1)(2x+1)}{(x+1)(x-1)} = \frac{3x^2-2x+5}{x^2-1}$$

$$4. \frac{2x-3}{x-2} + \frac{x+3}{x+1} = \frac{(2x-3)(x+1) + (x-2)(x+3)}{(x-2)(x+1)} = \frac{3x^2-9}{x^2-x-2}$$

$$5. \frac{x^2}{x+1} - \frac{x-1}{x+1} = \frac{x^2-(x-1)}{x+1} = \frac{x^2-x+1}{x+1}$$

$$6. \frac{x^2-1}{x-2} - \frac{1}{x-1} = \frac{(x^2-1)(x-1) - (x-2)}{(x-2)(x-1)} = \frac{x^3-x^2-2x+3}{x^2-3x+2}$$

$$7. \frac{1}{\left(\frac{x}{x-1}\right)} + x - 1 = \frac{x-1}{x} + x - 1 = \frac{x-1+x(x-1)}{x} = \frac{x^2-1}{x}$$

$$8. \frac{2}{\left(\frac{x-2}{x^2}\right)} - \frac{1}{x-2} = \frac{2x^2}{x-2} - \frac{1}{x-2} = \frac{2x^2-1}{x-2}$$

$$9. \frac{1}{x} \left[ \frac{x-3}{xy} + \frac{1}{y} \right] = \frac{1}{x} \left[ \frac{x-3+x}{xy} \right] = \frac{2x-3}{x^2y}$$

$$10. \frac{y^2}{x} \left[ \frac{2x-3}{y} + \frac{x}{y} \right] = \frac{y^2}{x} \left[ \frac{2x-3+x}{xy} \right] = \frac{y^2(3x-3)}{xy} = \frac{y(3x-3)}{x} = \frac{3xy-3y}{x}$$

$$11. \frac{(x+1)^2(x+2)^3 - (x+1)^3(x+2)^2}{(x+2)^6} = \frac{(x+1)^2(x+2)^2[(x+2) - (x+1)]}{(x+2)^6} = \frac{(x+1)^2}{(x+2)^4}$$

$$12. \frac{6x(x^2+1)^2(x^3+2)^3 - 9x^2(x^2+1)^3(x^3+2)^2}{(x^3+2)^6} \\ = \frac{3x(x^2+1)^2(x^3+2)^2[2(x^3+2) - 3x(x^2+1)]}{(x^3+2)^6} = \frac{3x(x^2+1)^2(-x^3-3x+4)}{(x^3+2)^4}$$

$$13. \frac{(x^2-1)\sqrt{x^2+1} - \frac{x^4}{\sqrt{x^2+1}}}{x^2+1} = \frac{(x^2-1)(x^2+1) - x^4}{(x^2+1)\sqrt{x^2+1}} = \frac{-1}{\sqrt{(x^2+1)^3}}$$

$$14. \frac{x\sqrt{x^3-1} - \frac{3x^4}{\sqrt{x^3-1}}}{x^3-1} = \frac{x(x^3-1) - 3x^4}{(x^3-1)\sqrt{x^3-1}} = \frac{-2x^4-x}{\sqrt{(x^3-1)^3}} = \frac{-x(2x^3+1)}{\sqrt{(x^3-1)^3}}$$

### Solutions Section 0.5

$$15. \frac{\frac{1}{(x+y)^2} - \frac{1}{x^2}}{y} = \frac{x^2 - (x+y)^2}{yx^2(x+y)^2} = \frac{x^2 - x^2 - 2xy - y^2}{yx^2(x+y)^2} = \frac{-y(2x+y)}{yx^2(x+y)^2} = \frac{-(2x+y)}{x^2(x+y)^2}$$

$$16. \frac{\frac{1}{(x+y)^3} - \frac{1}{x^3}}{y} = \frac{x^2 - (x+y)^2}{yx^2(x+y)^2} = \frac{x^3 - x^3 - 3x^2y - 3xy^2 - y^3}{yx^3(x+y)^3} \\ = \frac{-y(3x^2 + 3xy + y^2)}{yx^3(x+y)^3} = \frac{-(3x^2 + 3xy + y^2)}{x^3(x+y)^3}$$

## Solutions Section 0.6

### Section 0.6

1.  $x + 1 = 0 \Rightarrow x = 0 - 1 \Rightarrow x = -1$

2.  $x - 3 = 1 \Rightarrow x = 1 + 3 \Rightarrow x = 4$

3.  $-x + 5 = 0 \Rightarrow -x = -5 \Rightarrow x = 5$

4.  $2x + 4 = 1 \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

5.  $4x - 5 = 8 \Rightarrow 4x = 13 \Rightarrow x = \frac{13}{4}$

6.  $\frac{3}{4}x + 1 = 0 \Rightarrow \frac{3}{4}x = -1 \Rightarrow x = -\frac{4}{3}$

7.  $7x + 55 = 98 \Rightarrow 7x = 43 \Rightarrow x = -\frac{43}{7}$

8.  $3x + 1 = x \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$

9.  $x + 1 = 2x + 2 \Rightarrow -x = 1 \Rightarrow x = -1$

10.  $x + 1 = 3x + 1 \Rightarrow -2x = 0 \Rightarrow x = 0$

11.  $ax + b = c \Rightarrow ax = c - b \Rightarrow x = \frac{c - b}{a}$

12.  $x - 1 = cx + d \Rightarrow (1 - c)x = d + 1 \Rightarrow x = \frac{d + 1}{1 - c}$

13.  $2x^2 + 7x - 4 = 0 \Rightarrow (2x - 1)(x + 4) = 0 \Rightarrow x = -4, \frac{1}{2}$

14.  $x^2 + x + 1 = 0 \Rightarrow \Delta = b^2 - 4ac = -3 < 0$ , so this equation has no real solutions.

15.  $x^2 - x + 1 = 0 \Rightarrow \Delta = b^2 - 4ac = -3 < 0$ , so this equation has no real solutions.

16.  $2x^2 - 4x + 3 = 0 \Rightarrow \Delta = b^2 - 4ac = -8 < 0$ , so this equation has no real solutions.

17.  $2x^2 - 5 = 0 \Rightarrow x^2 = \frac{5}{2} \Rightarrow x = \pm\sqrt{\frac{5}{2}}$

18.  $3x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

19.  $-x^2 - 2x - 1 = 0 \Rightarrow -(x + 1)^2 = 0 \Rightarrow x = -1$

20.  $2x^2 - x - 3 = 0 \Rightarrow (2x - 3)(x + 1) = 0 \Rightarrow x = \frac{3}{2}, -1$

21.  $\frac{1}{2}x^2 - x - \frac{3}{2} = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x + 1)(x - 3) = 0 \Rightarrow x = -1, 3$

22.  $-\frac{1}{2}x^2 - \frac{1}{2}x + 1 = 0 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$

## Solutions Section 0.6

$$23. x^2 - x = 1 \Rightarrow x^2 - x - 1 = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{5}}{2}$$

$$24. 16x^2 = -24x - 9 \Rightarrow 16x^2 + 24x + 9 = 0(4x + 3)^2 = 0 \Rightarrow x = -\frac{3}{4}$$

$$25. x = 2 - \frac{1}{x} \Rightarrow x^2 = 2x - 1 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$$

$$26. x + 4 = \frac{1}{x - 2} \Rightarrow (x + 4)(x - 2) = 1 \Rightarrow x^2 + 2x - 8 = 1 \Rightarrow x^2 + 2x - 9 = 0 \\ \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{40}}{2} = -1 \pm \sqrt{10}$$

$$27. x^4 - 10x^2 + 9 = 0 \Rightarrow (x^2 - 1)(x^2 - 9) = 0 \Rightarrow x^2 = 1 \text{ or } x^2 = 9 \Rightarrow x = \pm 1, \pm 3$$

$$28. x^4 - 2x^2 + 1 = 0 \Rightarrow (x^2 - 1)^2 = 0 \\ \Rightarrow x = \pm 1$$

$$29. x^4 + x^2 - 1 = 0 \Rightarrow x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x^2 = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow x = \pm \sqrt{\frac{-1 \pm \sqrt{5}}{2}}$$

$$30. x^3 + 2x^2 + x = 0 \Rightarrow x(x^2 + 2x + 1) = 0 \Rightarrow x(x + 1)^2 = 0 \Rightarrow x = 0, -1$$

$$31. x^3 + 16x^2 + 11x + 6 = 0 \Rightarrow (x + 1)(x + 2)(x + 3) = 0 \Rightarrow x = -1, -2, -3$$

$$32. x^3 - 6x^2 + 12x - 8 = 0 \Rightarrow (x - 2)^3 = 0 \Rightarrow x = 2$$

$$33. x^3 + 4x^2 + 4x + 3 = 0 \Rightarrow (x + 3)(x^2 + x + 1) = 0 \Rightarrow x = -3 \\ (\text{For } x^2 + x + 1 = 0, \Delta = b^2 - 4ac = -3 < 0, \text{ so there are no real solutions to this quadratic equation.})$$

$$34. y^3 + 64 = 0 \Rightarrow y^3 = -64 \Rightarrow y = \sqrt[3]{-64} = -4$$

$$35. x^3 - 1 = 0 \Rightarrow x^3 = 1 \Rightarrow x = \sqrt[3]{1} = 1$$

$$36. x^3 - 27 = 0 \\ \Rightarrow x^3 = 27 \Rightarrow x = \sqrt[3]{27} = 3$$

$$37. y^3 + 3y^2 + 3y + 2 = 0 \Rightarrow (y + 2)(y^2 + y + 1) = 0 \Rightarrow y = -2 \\ (\text{For } y^2 + y + 1 = 0, \Delta = b^2 - 4ac = -3 < 0, \text{ so there are no real solutions to this quadratic equation.})$$

$$38. y^3 - 2y^2 - 2y - 3 = 0 \Rightarrow (y - 3)(y^2 + y + 1) = 0 \Rightarrow y = 3 \\ (\text{For } y^2 + y + 1 = 0, \Delta = b^2 - 4ac = -3 < 0, \text{ so there are no real solutions to this quadratic equation.})$$

$$39. x^3 - x^2 - 5x + 5 = 0 \Rightarrow (x - 1)(x^2 - 5) = 0 \Rightarrow x = 1, \pm \sqrt{5}$$

$$40. x^3 - x^2 - 3x + 3 = 0 \Rightarrow (x - 1)(x^2 - 3) = 0 \Rightarrow x = 1, \pm \sqrt{3}$$

### Solutions Section 0.6

41.  $2x^6 - x^4 - 2x^2 + 1 = 0 \Rightarrow (2x^2 - 1)(x^4 - 1) = 0 \Rightarrow (2x^2 - 1)(x^2 - 1)(x^2 + 1) = 0$   
(Think of the cubic you get by substituting  $y$  for  $x^2$ )  
 $\Rightarrow x = \pm 1, \pm \frac{1}{\sqrt{2}}$

42.  $3x^6 - x^4 - 12x^2 + 4 = 0 \Rightarrow (3x^2 - 1)(x^4 - 4) = 0$   
 $\Rightarrow (3x^2 - 1)(x^2 - 2)(x^2 + 2) = 0$   
(Think of the cubic you get by substituting  $y$  for  $x^2$ )  
 $\Rightarrow x = \pm\sqrt{2}, \pm \frac{1}{\sqrt{3}}$

43.  $(x^2 + 3x + 2)(x^2 - 5x + 6) = 0 \Rightarrow (x + 2)(x + 1)(x - 2)(x - 3) = 0$   
 $\Rightarrow x = -2, -1, 2, 3$

44.  $(x^2 - 4x + 4)^2(x^2 + 6x + 5)^3 = 0 \Rightarrow (x - 2)^4(x + 1)^3(x + 5)^3 = 0$   
 $\Rightarrow x = -5, -1, 2$

## Section 0.7

1.  $x^4 - 3x^3 = 0$ ,  $x^3(x - 3) = 0$ ,  $x = 0, 3$

2.  $x^6 - 9x^4 = 0$ ,  $x^4(x^2 - 9) = 0$ ,  $x = 0, \pm 3$

3.  $x^4 - 4x^2 = -4$ ,  $x^4 - 4x^2 + 4 = 0$ ,  $(x^2 - 2)^2 = 0$ ,  $x = \pm\sqrt{2}$

4.  $x^4 - x^2 = 6$ ,  $x^4 - x^2 - 6 = 0$ ,  $(x^2 - 3)(x^2 + 2) = 0$ ,  $x = \pm\sqrt{3}$

5.  $(x + 1)(x + 2) + (x + 1)(x + 3) = 0$ ,  $(x + 1)(x + 2 + x + 3) = 0$ ,  
 $(x + 1)(2x + 5) = 0$ ,  $x = -1, -5/2$

6.  $(x + 1)(x + 2)^2 + (x + 1)^2(x + 2) = 0$ ,  $(x + 1)(x + 2)(x + 2 + x + 1) = 0$ ,  
 $(x + 1)(x + 2)(2x + 3) = 0$ ,  $x = -1, -2, -3/2$

7.  $(x^2 + 1)^5(x + 3)^4 + (x^2 + 1)^6(x + 3)^3 = 0$ ,  
 $(x^2 + 1)^5(x + 3)^3(x + 3 + x^2 + 1) = 0$ ,  
 $(x^2 + 1)^5(x + 3)^3(x^2 + x + 4) = 0$ ,  $x = -3$

(Neither  $x^2 + 1 = 0$  nor  $x^2 + x + 4 = 0$  has a real solution.)

8.  $10x(x^2 + 1)^4(x^3 + 1)^5 - 10x^2(x^2 + 1)^5(x^3 + 1)^4 = 0$ ,  
 $10x(x^2 + 1)^4(x^3 + 1)^4[x^3 + 1 - x(x^2 + 1)] = 0$ ,  
 $10x(x^2 + 1)^4(x^3 + 1)^4(1 - x) = 0$ ,  $x = -1, 0, 1$

9.  $(x^3 + 1)\sqrt{x + 1} - (x^3 + 1)^2\sqrt{x + 1} = 0$ ,  
 $(x^3 + 1)\sqrt{x + 1} [1 - (x^3 + 1)] = 0$ ,  
 $-x^3(x^3 + 1)\sqrt{x + 1} = 0$ ,  $x = 0, -1$

10.  $(x^2 + 1)\sqrt{x + 1} - \sqrt{(x + 1)^3} = 0$ ,  
 $\sqrt{x + 1} [x^2 + 1 - (x + 1)] = 0$ ,  
 $(x^2 - x)\sqrt{x + 1} = 0$ ,  $x(x - 1)\sqrt{x + 1} = 0$ ,  $x = -1, 0, 1$

11.  $\sqrt{(x + 1)^3} + \sqrt{(x + 1)^5} = 0$ ,  
 $\sqrt{(x + 1)^3} (1 + x + 1) = 0$ ,  
 $(x + 2)\sqrt{(x + 1)^3} = 0$ ,  $x = -1$

( $x = -2$  is not a solution because  $\sqrt{(x + 1)^3}$  is not defined for  $x = -2$ .)

12.  $(x^2 + 1)\sqrt[3]{(x + 1)^4} - \sqrt[3]{(x + 1)^7} = 0$ ,  
 $\sqrt[3]{(x + 1)^4} [x^2 + 1 - (x + 1)] = 0$ ,  
 $(x^2 - x)\sqrt[3]{(x + 1)^4} = 0$ ,  $x(x - 1)\sqrt[3]{(x + 1)^4} = 0$ ,  
 $x = -1, 0, 1$

13.  $(x + 1)^2(2x + 3) - (x + 1)(2x + 3)^2 = 0$ ,  
 $(x + 1)(2x + 3)(x + 1 - 2x - 3) = 0$ ,  
 $(x + 1)(2x + 3)(-x - 2) = 0$ ,  
 $x = -2, -3/2, -1$

## Solutions Section 0.7

$$\begin{aligned}
 14. \quad & (x^2 - 1)^2(x + 2)^3 - (x^2 - 1)^3(x + 2)^2 = 0, \\
 & (x^2 - 1)^2(x + 2)^2(x + 2 - x^2 + 1) = 0, \\
 & -(x^2 - 1)^2(x + 2)^2(x^2 - x - 3) = 0, \\
 & x = -2, -1, 1, (1 \pm \sqrt{13})/2
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{(x + 1)^2(x + 2)^3 - (x + 1)^3(x + 2)^2}{(x + 2)^6} = 0, \\
 & \frac{(x + 1)^2(x + 2)^2[(x + 2) - (x + 1)]}{(x + 2)^6} = 0, \\
 & \frac{(x + 1)^2}{(x + 2)^4} = 0, (x + 1)^2 = 0, x = -1
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{6x(x^2 + 1)^2(x^2 + 2)^4 - 8x(x^2 + 1)^3(x^2 + 2)^3}{(x^2 + 2)^8} = 0, \\
 & \frac{2x(x^2 + 1)^2(x^2 + 2)^3[3(x^2 + 2) - 4(x^2 + 1)]}{(x^2 + 2)^8} = 0, \\
 & \frac{-2x(x^2 + 1)^2(x^2 - 2)}{(x^2 + 2)^5} = 0, \\
 & -2x(x^2 + 1)^2(x^2 - 2) = 0, x = 0, \pm\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{2(x^2 - 1)\sqrt{x^2 + 1} - \frac{x^4}{\sqrt{x^2 + 1}}}{x^2 + 1} = 0, \frac{2(x^2 - 1)(x^2 + 1) - x^4}{(x^2 + 1)\sqrt{x^2 + 1}} = 0, \\
 & \frac{x^4 - 2}{(x^2 + 1)\sqrt{x^2 + 1}} = 0, x^4 - 2 = 0, x = \pm\sqrt[4]{2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{4x\sqrt{x^3 - 1} - \frac{3x^4}{\sqrt{x^3 - 1}}}{x^3 - 1} = 0, \frac{4x(x^3 - 1) - 3x^4}{(x^3 - 1)\sqrt{x^3 - 1}} = 0, \\
 & \frac{x^4 - 4x}{(x^3 - 1)\sqrt{x^3 - 1}} = 0, x^4 - 4x = 0, x(x^3 - 4) = 0, x = 0, \pm\sqrt[3]{4}
 \end{aligned}$$

$$19. \quad x - \frac{1}{x} = 0, x^2 - 1 = 0, x = \pm 1$$

$$20. \quad 1 - \frac{4}{x^2} = 0, x^2 - 4 = 0, x = \pm 2$$

$$21. \quad \frac{1}{x} - \frac{9}{x^3} = 0, x^2 - 9 = 0, x = \pm 3$$

$$22. \quad \frac{1}{x^2} - \frac{1}{x + 1} = 0, x + 1 - x^2 = 0, x^2 - x - 1 = 0, x = (1 \pm \sqrt{5})/2$$

$$\begin{aligned}
 23. \quad & \frac{x - 4}{x + 1} - \frac{x}{x - 1} = 0, \frac{(x - 4)(x - 1) - x(x + 1)}{(x + 1)(x - 1)} = 0, \\
 & \frac{-6x + 4}{(x + 1)(x - 1)} = 0, -6x + 4 = 0, x = 2/3
 \end{aligned}$$

### Solutions Section 0.7

$$24. \frac{2x-3}{x-1} - \frac{2x+3}{x+1} = 0, \frac{(2x-3)(x+1) - (2x+3)(x-1)}{(x-1)(x+1)} = 0,$$

$$\frac{-2x}{(x-1)(x+1)} = 0, -2x = 0, x = 0$$

$$25. \frac{x+4}{x+1} + \frac{x+4}{3x} = 0, \frac{3x(x+4) + (x+1)(x+4)}{3x(x+1)} = 0,$$

$$\frac{(x+4)(3x+x+1)}{3x(x+1)} = 0, \frac{(x+4)(4x+1)}{3x(x+1)} = 0,$$

$$(x+4)(4x+1) = 0, x = -4, -1/4$$

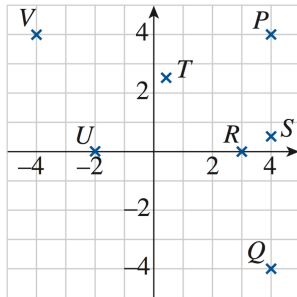
$$26. \frac{2x-3}{x} - \frac{2x-3}{x+1} = 0, \frac{(2x-3)(x+1) - x(2x-3)}{x(x+1)} = 0,$$

$$\frac{(2x-3)(x+1-x)}{x(x+1)} = 0, \frac{(2x-3)}{x(x+1)} = 0, 2x-3 = 0, x = 3/2$$

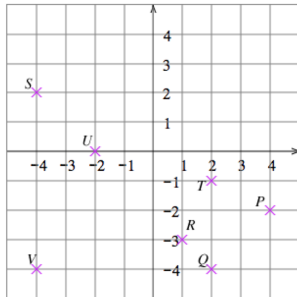
## Solutions Section 0.8

### Section 0.8

1.  $P(0, 2), Q(4, -2), R(-2, 3), S(-3.5, -1.5), T(-2.5, 0), U(2, 2.5)$
2.  $P(-2, 2), Q(3.5, 2), R(0, -3), S(-3.5, -1.5), T(2.5, 0), U(-2, 2.5)$
- 3.



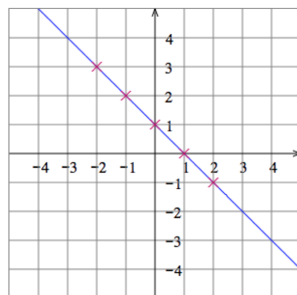
4.



5. Solve the equation  $x + y = 1$  for  $y$  to get  $y = 1 - x$ . Then plot some points:

|             |    |    |   |   |    |
|-------------|----|----|---|---|----|
| $x$         | -2 | -1 | 0 | 1 | 2  |
| $y = 1 - x$ | 3  | 2  | 1 | 0 | -1 |

Graph:

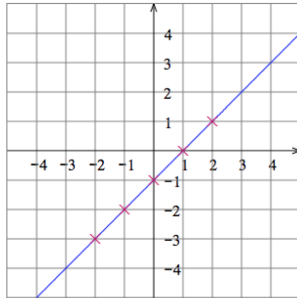


6. Solve the equation  $y - x = -1$  for  $y$  to get  $y = -1 + x$ . Then plot some points:

|              |    |    |    |   |   |
|--------------|----|----|----|---|---|
| $x$          | -2 | -1 | 0  | 1 | 2 |
| $y = -1 + x$ | -3 | -2 | -1 | 0 | 1 |

## Solutions Section 0.8

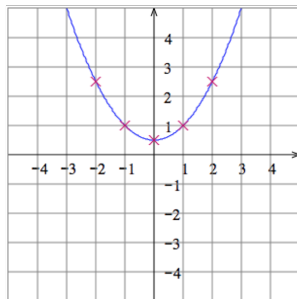
Graph:



7. Solve the equation  $2y - x^2 = 1$  for  $y$  to get  $y = \frac{1 + x^2}{2}$ . Then plot some points:

|                         |     |    |     |   |     |
|-------------------------|-----|----|-----|---|-----|
| $x$                     | -2  | -1 | 0   | 1 | 2   |
| $y = \frac{1 + x^2}{2}$ | 2.5 | 1  | 0.5 | 1 | 2.5 |

Graph:



8. Solve the equation  $2y + \sqrt{x} = 1$  for  $y$  to get  $y = \frac{1 - \sqrt{x}}{2}$ . Then plot some points:

|                              |     |   |      |    |      |
|------------------------------|-----|---|------|----|------|
| $x$                          | 0   | 1 | 4    | 9  | 16   |
| $y = \frac{1 - \sqrt{x}}{2}$ | 0.5 | 0 | -0.5 | -1 | -1.5 |

Graph:

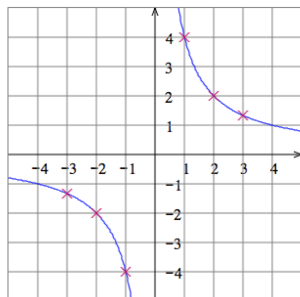


## Solutions Section 0.8

9. Solve the equation  $xy = 4$  for  $y$  to get  $y = \frac{4}{x}$ . Then plot some points:

|                   |                |    |    |   |   |               |
|-------------------|----------------|----|----|---|---|---------------|
| $x$               | -3             | -2 | -1 | 1 | 2 | 3             |
| $y = \frac{4}{x}$ | $-\frac{4}{3}$ | -2 | -4 | 4 | 2 | $\frac{4}{3}$ |

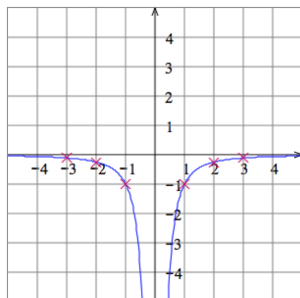
Graph:



10. Solve the equation  $x^2y = -1$  for  $y$  to get  $y = -\frac{1}{x^2}$ . Then plot some points:

|                      |                |                |    |    |                |                |
|----------------------|----------------|----------------|----|----|----------------|----------------|
| $x$                  | -3             | -2             | -1 | 1  | 2              | 3              |
| $y = -\frac{1}{x^2}$ | $-\frac{1}{9}$ | $-\frac{1}{4}$ | -1 | -1 | $-\frac{1}{4}$ | $-\frac{1}{9}$ |

Graph:

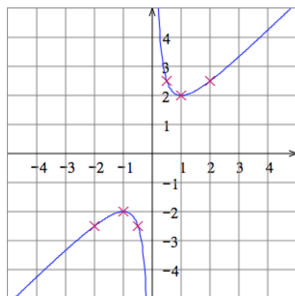


11. Solve the equation  $xy = x^2 + 1$  for  $y$  to get  $y = x + \frac{1}{x}$ . Then plot some points:

|                       |                |    |                |               |   |               |
|-----------------------|----------------|----|----------------|---------------|---|---------------|
| $x$                   | -2             | -1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 2             |
| $y = x + \frac{1}{x}$ | $-\frac{5}{2}$ | -2 | $-\frac{5}{2}$ | $\frac{5}{2}$ | 2 | $\frac{5}{2}$ |

## Solutions Section 0.8

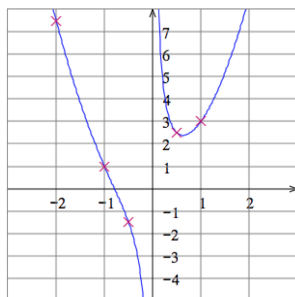
Graph:



12. Solve the equation  $xy = 2x^3 + 1$  for  $y$  to get  $y = 2x^2 + \frac{1}{x}$ . Then plot some points:

| $x$                      | -2             | -1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 2              |
|--------------------------|----------------|----|----------------|---------------|---|----------------|
| $y = 2x^2 + \frac{1}{x}$ | $\frac{15}{2}$ | 1  | $-\frac{3}{2}$ | $\frac{5}{2}$ | 3 | $\frac{17}{2}$ |

Graph:



13.  $\sqrt{(2-1)^2 + (-2+1)^2} = \sqrt{2}$

14.  $\sqrt{(6-1)^2 + (1-0)^2} = \sqrt{26}$

15.  $\sqrt{(0-a)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

16.  $\sqrt{(b-a)^2 + (b-0)^2} = \sqrt{2(b-a)^2}$  or  $\sqrt{2}|b-a|$

17. Set the two distances equal and solve:

$$\begin{aligned}\sqrt{(1-0)^2 + (k-0)^2} &= \sqrt{(1-2)^2 + (k-1)^2} \\ \Rightarrow \sqrt{1+k^2} &= \sqrt{2-2k+k^2} \\ \Rightarrow 1+k^2 &= 2-2k+k^2 \\ \Rightarrow 2k &= 1 \Rightarrow k = \frac{1}{2}\end{aligned}$$

18. Set the two distances equal and solve:

$$\begin{aligned}\sqrt{(k+1)^2 + (k-0)^2} &= \sqrt{(k-0)^2 + (k-2)^2} \\ \Rightarrow \sqrt{2k^2+2k+1} &= \sqrt{2k^2-4k+4} \\ \Rightarrow 2k^2+2k+1 &= 2k^2-4k+4 \\ \Rightarrow 6k &= 3 \Rightarrow k = \frac{1}{2}\end{aligned}$$

### Solutions Section 0.8

19. Circle with center  $(0, 0)$  and radius 3.

20. The single point  $(0, 0)$ .

## Solutions Section 0.9

### Section 0.9

1.

|                         |                     |                 |                  |                     |                                  |                                         |
|-------------------------|---------------------|-----------------|------------------|---------------------|----------------------------------|-----------------------------------------|
| <b>Exponential Form</b> | $10^2 = 100$        | $4^3 = 64$      | $4^4 = 256$      | $0.45^0 = 1$        | $8^{1/2} = 2\sqrt{2}$            | $4^{-3} = \frac{1}{64}$                 |
| <b>Logarithmic Form</b> | $\log_{10} 100 = 2$ | $\log_4 64 = 3$ | $\log_4 256 = 4$ | $\log_{0.45} 1 = 0$ | $\log_8 2\sqrt{2} = \frac{1}{2}$ | $\log_4 \left(\frac{1}{64}\right) = -3$ |

2.

|                         |                        |                                 |                                         |
|-------------------------|------------------------|---------------------------------|-----------------------------------------|
| <b>Exponential Form</b> | $10^1 = 10$            | $5^5 = 3,125$                   | $6^3 = 216$                             |
| <b>Logarithmic Form</b> | $\log_{10} 10 = 1$     | $\log_5 3,125 = 5$              | $\log_6 216 = 3$                        |
| <b>Exponential Form</b> | $0.5^3 = 0.125$        | $4^{1/4} = \sqrt{2}$            | $3^{-4} = \frac{1}{81}$                 |
| <b>Logarithmic Form</b> | $\log_{0.5} 0.125 = 3$ | $\log_4 \sqrt{2} = \frac{1}{4}$ | $\log_3 \left(\frac{1}{81}\right) = -4$ |

3.

|                         |                            |                                  |                                        |
|-------------------------|----------------------------|----------------------------------|----------------------------------------|
| <b>Exponential Form</b> | $0.3^2 = 0.09$             | $\left(\frac{1}{2}\right)^0 = 1$ | $10^{-3} = 0.001$                      |
| <b>Logarithmic Form</b> | $\log_{0.3} 0.09 = 2$      | $\log_{1/2} 1 = 0$               | $\log_{10} 0.001 = -3$                 |
| <b>Exponential Form</b> | $9^{-2} = \frac{1}{81}$    | $2^{10} = 1,024$                 | $64^{-1/3} = \frac{1}{4}$              |
| <b>Logarithmic Form</b> | $\log_9 \frac{1}{81} = -2$ | $\log_2 1,024 = 10$              | $\log_{64} \frac{1}{4} = -\frac{1}{3}$ |

4.

|                         |                           |                                        |                          |
|-------------------------|---------------------------|----------------------------------------|--------------------------|
| <b>Exponential Form</b> | $2^{-1} = \frac{1}{2}$    | $10^5 = 100,000$                       | $10^{-5} = 0.00001$      |
| <b>Logarithmic Form</b> | $\log_2 \frac{1}{2} = -1$ | $\log_{10} 100,000 = 5$                | $\log_{10} 0.00001 = -5$ |
| <b>Exponential Form</b> | $2^{11} = 2,048$          | $16^{-1/4} = \frac{1}{2}$              | $0.25^2 = 0.0625$        |
| <b>Logarithmic Form</b> | $\log_2 2,048 = 11$       | $\log_{16} \frac{1}{2} = -\frac{1}{4}$ | $\log_{0.25} 0.0625 = 2$ |

5.  $\log_4 16$  = the power to which we need to raise 4 in order to get 16. Because  $4^{\boxed{2}} = 16$ , this power is 2, so  $\log_4 16 = 2$ .

6.  $\log_5 125$  = the power to which we need to raise 5 in order to get 125. Because  $5^{\boxed{3}} = 125$ , this power is 3, so  $\log_5 125 = 3$ .

7.  $\log_5 \frac{1}{25}$  = the power to which we need to raise 5 in order to get  $\frac{1}{25}$ . Because  $5^{\boxed{-2}} = \frac{1}{25}$ , this power is  $-2$ , so  $\log_5 \frac{1}{25} = -2$ .

8.  $\log_4 \frac{1}{64}$  = the power to which we need to raise 4 in order to get  $\frac{1}{64}$ . Because  $4^{\boxed{-3}} = \frac{1}{64}$ , this power is  $-3$ , so  $\log_4 \frac{1}{64} = -3$ .

## Solutions Section 0.9

9.  $\log 100,000 = \log_{10} 100,000$  = the power to which we need to raise 10 in order to get 100,000. Because  $10^{\boxed{5}} = 100,000$ , this power is 5, so  $\log 100,000 = 5$ .

10.  $\log 1,000 = \log_{10} 1,000$  = the power to which we need to raise 10 in order to get 1,000. Because  $10^{\boxed{3}} = 1,000$ , this power is 3, so  $\log 1,000 = 3$ .

11.  $\log_{16} 16$  = the power to which we need to raise 16 in order to get 16. Because  $16^{\boxed{1}} = 16$ , this power is 1, so  $\log_{16} 16 = 1$ .

12.  $\log_{1/2} \frac{1}{2}$  = the power to which we need to raise  $\frac{1}{2}$  in order to get  $\frac{1}{2}$ . Because  $\left(\frac{1}{2}\right)^{\boxed{1}} = \frac{1}{2}$ , this power is 1, so  $\log_{1/2} \frac{1}{2} = 1$ .

13.  $\log_4 \frac{1}{16}$  = the power to which we need to raise 4 in order to get  $\frac{1}{16}$ . Because  $4^{\boxed{-2}} = \frac{1}{16}$ , this power is -2, so  $\log_4 \frac{1}{16} = -2$ .

14.  $\log_2 \frac{1}{8}$  = the power to which we need to raise 2 in order to get  $\frac{1}{8}$ . Because  $2^{\boxed{-3}} = \frac{1}{8}$ , this power is -3, so  $\log_2 \frac{1}{8} = -3$ .

15.  $\log_2 \sqrt{2}$  = the power to which we need to raise 2 in order to get  $\sqrt{2}$ . Because  $2^{\boxed{1/2}} = \sqrt{2}$ , this power is  $\frac{1}{2}$ , so  $\log_2 \sqrt{2} = \frac{1}{2}$ .

16.  $\log_4 \sqrt{2}$  = the power to which we need to raise 4 in order to get  $\sqrt{2}$ . Because  $4^{\boxed{1/4}} = \sqrt{2}$ , this power is  $\frac{1}{4}$ , so  $\log_4 \sqrt{2} = \frac{1}{4}$ .

17. By Identity (1),  $\log_b 3 + \log_b 4 = \log_b (3 \times 4) = \log_b \boxed{12}$ .

18. By Identity (2),  $\log_b 3 - \log_b 4 = \log_b \boxed{\frac{3}{4}}$ .

19.  $\log_b 2 - \log_b 5 - \log_b 4 = \log_b 2 - (\log_b 5 + \log_b 4) = \log_b 2 - (\log_b (5 \times 4))$  (Identity 1)  
 $= \log_b \left(\frac{2}{5 \times 4}\right)$  (Identity 2)  $= \log_b \boxed{\frac{1}{10}}$

20.  $\log_b 3 + \log_b 2 - \log_b 7 = \log_b (3 \times 2) - \log_b 7$  (Identity 1)  $= \log_b \left(\frac{3 \times 2}{7}\right)$  (Identity 2)  $= \log_b \boxed{\frac{6}{7}}$

21.  $\log_b 3 - 3 \log_b 2 = \log_b 3 - \log_b 2^3$  (Identity 3)  $= \log_b \left(\frac{3}{2^3}\right)$  (Identity 2)  $= \log_b \boxed{\frac{3}{8}}$

22.  $3 \log_b 2 + 2 \log_b 3 = \log_b 2^3 + \log_b 3^2$  (Identity 3)  $= \log_b (2^3 \times 3^2)$  (Identity 1)  $= \log_b \boxed{72}$

23.  $4 \log_b x + 5 \log_b y = \log_b x^4 + \log_b y^5$  (Identity 3)  $= \log_b \boxed{x^4 y^5}$  (Identity 1)

## Solutions Section 0.9

$$24. 3 \log_b p - 2 \log_b q = \log_b p^3 - \log_b q^2 \text{ (Identity 3)} = \log_b \boxed{\frac{p^3}{q^2}} \text{ (Identity 2)}$$

$$25. 2 \log_b x + 3 \log_b y - 4 \log_b z = \log_b x^2 + \log_b y^3 - \log_b z^4 \text{ (Identity 3)} \\ = \log_b (x^2 y^3) - \log_b z^4 \text{ (Identity 1)} = \log_b \boxed{\frac{x^2 y^3}{z^4}} \text{ (Identity 2)}$$

$$26. 4 \log_b p - 3 \log_b q - 2 \log_b r = \log_b p^4 - \log_b q^3 - \log_b r^2 \text{ (Identity 3)} \\ = \log_b p^4 - (\log_b q^3 + \log_b r^2) = \log_b p^4 - \log_b (q^3 r^2) \text{ (Identity 1)} = \log_b \boxed{\frac{p^4}{q^3 r^2}} \text{ (Identity 2)}$$

$$27. x \log_b 2 - 2 \log_b x = \log_b 2^x - \log_b x^2 \text{ (Identity 3)} = \log_b \boxed{\frac{2^x}{x^2}} \text{ (Identity 2)}$$

$$28. p \log_b q + q \log_b p = \log_b q^p + \log_b p^q \text{ (Identity 3)} = \log_b \boxed{q^p p^q} \text{ (Identity 1)}$$

$$29. \log 21 = \log(3 \times 7) = \log 3 + \log 7 = b + c \text{ (Identity 1)}$$

$$30. \log 14 = \log(2 \times 7) = \log 2 + \log 7 = a + c \text{ (Identity 1)}$$

$$31. \log 42 = \log(2 \times 3 \times 7) = \log 2 + \log 3 + \log 7 = a + b + c \text{ (Identity 1)}$$

$$32. \log 28 = \log(2 \times 2 \times 7) = \log 2 + \log 2 + \log 7 = 2a + c \text{ (Identity 1)}$$

$$33. \log\left(\frac{1}{7}\right) = -\log 7 = -c \text{ (Identity 5)}$$

$$34. \log\left(\frac{1}{3}\right) = -\log 3 = -b \text{ (Identity 5)}$$

$$35. \log\left(\frac{2}{3}\right) = \log 2 - \log 3 = a - b \text{ (Identity 2)}$$

$$36. \log\left(\frac{7}{2}\right) = \log 7 - \log 2 = c - a \text{ (Identity 2)}$$

$$37. \log\left(\frac{4}{7}\right) = \log 4 - \log 7 \text{ (Identity 2)} = \log 2 + \log 2 - \log 7 \text{ (Identity 1)} = 2a - c$$

$$38. \log\left(\frac{2}{9}\right) = \log 2 - \log 9 \text{ (Identity 2)} = \log 2 - (\log 3 + \log 3) \text{ (Identity 1)} = a - 2b$$

$$39. \log 16 = \log 2^4 = 4 \log 2 = 4a \text{ (Identity 3)}$$

$$40. \log 81 = \log 3^4 = 4 \log 3 = 4b \text{ (Identity 3)}$$

$$41. \log 0.03 = \log\left(\frac{3}{10^2}\right) = \log 3 - \log 10^2 = \log 3 - 2 \log 10 = b - 2$$

$$42. \log 7,000 = \log(7 \times 10^3) = \log 7 + \log 10^3 = \log 7 + 3 \log 10 = c + 3$$

## Solutions Section 0.9

43.  $\log 5 = \log\left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - a$  (Identity 2)

44.  $\log 25 = \log\left(\frac{100}{4}\right) = \log 100 - \log 4 = \log 10^2 - \log 2^2 = 2 \log 10 - 2 \log 2 = 2 - 2a$

45.  $\log \sqrt{7} = \log(7^{1/2}) = \frac{1}{2} \log 7 = \frac{c}{2}$

46.  $\log\left(\frac{2}{\sqrt{3}}\right) = \log(2) - \log \sqrt{3} = \log(2) - \log(3^{1/2}) = \log 2 - \frac{1}{2} \log 3 = a - \frac{b}{2}$

47.  $4 = 2^x$  is exactly the equation we solve in order to calculate  $\log_2 4$ ; answer: 2. Alternatively, write the equation  $4 = 2^x$  in logarithmic form to get  $x = \log_2 4 = 2$ .

48.  $81 = 3^x$  is exactly the equation we solve in order to calculate  $\log_3 81$ ; answer: 4. Alternatively, write the equation  $81 = 3^x$  in logarithmic form to get  $x = \log_3 81 = 4$ .

49. Take the base 3 logarithm of both sides of the given equation  $27 = 3^{2x-1}$  to get

$$\log_3 27 = \log_3 3^{2x-1}$$

$$3 = (2x - 1) \log_3 3 \quad \text{By Identity 3}$$

$$3 = 2x - 1 \quad \text{By Identity 4}$$

$$x = \frac{3+1}{2} = 2 \quad \text{Solve for } x.$$

50. Take the base 4 logarithm of both sides of the given equation  $4^{2-3x} = 256$  to get

$$\log_4 4^{2-3x} = \log_4 256$$

$$(2 - 3x) \log_4 4 = 4 \quad \text{By Identity 3}$$

$$2 - 3x = 4 \quad \text{By Identity 4}$$

$$x = \frac{2-4}{3} = -\frac{2}{3} \quad \text{Solve for } x.$$

51. Take the base 5 logarithm of both sides of the given equation  $5^{-x+1} = \frac{1}{125}$  to get

$$\log_5 5^{-x+1} = \log_5 \left(\frac{1}{125}\right)$$

$$(-x + 1) \log_5 5 = -\log_5 125 \quad \text{By Identities 3 and 5}$$

$$-x + 1 = -3 \quad \text{By Identity 4}$$

$$x = 1 + 3 = 4 \quad \text{Solve for } x.$$

52. Take the base 3 logarithm of both sides of the given equation  $3^{x^2-12} = \frac{1}{27}$  to get

$$\log_3 3^{x^2-12} = \log_3 \left(\frac{1}{27}\right)$$

$$(x^2 - 12) \log_3 3 = -\log_3 27 \quad \text{By Identities 3 and 5}$$

$$x^2 - 12 = -3 \quad \text{By Identity 4}$$

$$x = \pm \sqrt{9} = \pm 3 \quad \text{Solve for } x.$$

53. First divide both sides of the given equation by 50 to get  $2.4 = 2^{3t}$ . Take the common logarithm of both sides:

$$\log 2.4 = \log(2^{3t})$$

## Solutions Section 0.9

$$\log 2.4 = 3t \log 2 \quad \text{By Identity 3}$$

$$3t = \frac{\log 2.4}{\log 2} \quad \text{Divide.}$$

$$t = \frac{\log 2.4}{3 \log 2} \approx 0.4210 \quad \text{Solve for } t.$$

If instead you take the base-2 logarithm, the answer is represented as  $\frac{\log_2(2.4)}{3}$ .

**54.** First divide both sides of the given equation by 500 to get  $2 = 1.1^{2t}$ . Take the common logarithm of both sides:

$$\log 2 = \log(1.1^{2t})$$

$$\log 2 = 2t \log 1.1 \quad \text{By Identity 3}$$

$$2t = \frac{\log 2}{\log 1.1} \quad \text{Divide.}$$

$$t = \frac{\log 2}{2 \log 1.1} \approx 3.6363 \quad \text{Solve for } t.$$

**55.** First divide both sides of the given equation by 300 to get  $\frac{10}{3} = 1.3^{4t-1}$ . Take the common logarithm of both sides:

$$\log\left(\frac{10}{3}\right) = \log(1.3^{4t-1})$$

$$\log 10 - \log 3 = (4t - 1) \log 1.3 \quad \text{By Identities 2 and 3}$$

$$4t - 1 = \frac{\log 10 - \log 3}{\log 1.3} \quad \text{Divide.}$$

$$t = \frac{1}{4} \left( \frac{\log 10 - \log 3}{\log 1.3} + 1 \right) \approx 1.3972 \quad \text{Solve for } t.$$

**56.** First divide both sides of the given equation by 700 to get  $\frac{100}{7} = 1.04^{3t+1}$ . Take the common logarithm of both sides:

$$\log\left(\frac{100}{7}\right) = \log(1.04^{3t+1})$$

$$\log 100 - \log 7 = (3t + 1) \log 1.04 \quad \text{By Identities 2 and 3}$$

$$3t + 1 = \frac{\log 100 - \log 7}{\log 1.04} \quad \text{Divide.}$$

$$t = \frac{1}{3} \left( \frac{\log 100 - \log 7}{\log 1.04} - 1 \right) \approx 22.2675 \quad \text{Solve for } t.$$

# Chapter 1 Functions and Applications

## Solutions Section 1.1

### Section 1.1

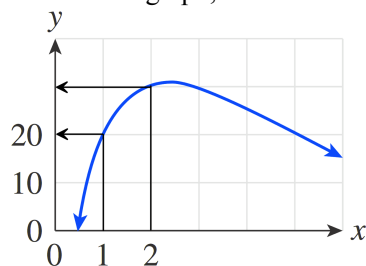
1. Using the table:     **a.**  $f(0) = 2$      **b.**  $f(2) = -0.5$

2. Using the table:     **a.**  $f(-1) = 4$      **b.**  $f(1) = -1$

3. Using the table:     **a.**  $f(2) - f(-2) = -0.5 - 2 = -2.5$      **b.**  $f(-1)f(-2) = (4)(2) = 8$   
                              **c.**  $-2f(-1) = -2(4) = -8$

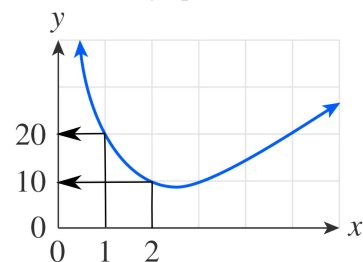
4. Using the table:     **a.**  $f(1) - f(-1) = -1 - 4 = -5$      **b.**  $f(1)f(-2) = (-1)(2) = -2$   
                              **c.**  $3f(-2) = 3(2) = 6$

5. From the graph, we estimate:     **a.**  $f(1) = 20$      **b.**  $f(2) = 30$



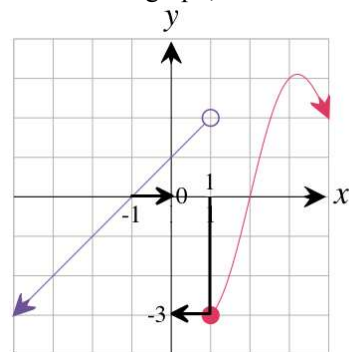
In a similar way, we find:     **c.**  $f(3) = 30$      **d.**  $f(5) = 20$      **e.**  $f(3) - f(2) = 30 - 30 = 0$   
                              **f.**  $f(3 - 2) = f(1) = 20$

6. From the graph, we estimate:     **a.**  $f(1) = 20$      **b.**  $f(2) = 10$



In a similar way, we find:     **c.**  $f(3) = 10$      **d.**  $f(5) = 20$      **e.**  $f(3) - f(2) = 10 - 10 = 0$   
                              **f.**  $f(3 - 2) = f(1) = 20$

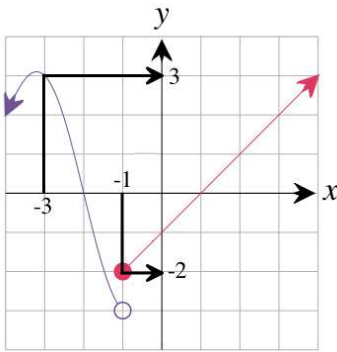
7. From the graph, we estimate:     **a.**  $f(-1) = 0$      **b.**  $f(1) = -3$  since the solid dot is on  $(1, -3)$ .



In a similar way, we estimate **c.**  $f(3) = 3$  **d.** Since  $f(3) = 3$  and  $f(1) = -3$ ,  $\frac{f(3) - f(1)}{3 - 1} = \frac{3 - (-3)}{3 - 1} = 3$ .

## Solutions Section 1.1

8. From the graph, we estimate:     **a.**  $f(-3) = 3$      **b.**  $f(-1) = -2$  since the solid dot is on  $(-1, -2)$ .



In a similar way, we estimate **c.**  $f(1) = 0$

- d.** Since  $f(3) = 2$  and  $f(1) = 0$ ,  $\frac{f(3) - f(1)}{3 - 1} = \frac{2 - 0}{3 - 1} = 1$ .

9.  $f(x) = x - \frac{1}{x^2}$ , with its natural domain.

The natural domain consists of all  $x$  for which  $f(x)$  makes sense: all real numbers other than 0.

- a.** Since 4 is in the natural domain,  $f(4)$  is defined, and  $f(4) = 4 - \frac{1}{4^2} = 4 - \frac{1}{16} = \frac{63}{16}$ .  
**b.** Since 0 is not in the natural domain,  $f(0)$  is not defined.  
**c.** Since  $-1$  is in the natural domain,  $f(-1) = -1 - \frac{1}{(-1)^2} = -1 - \frac{1}{1} = -2$ .

10.  $f(x) = \frac{2}{x} - x^2$ , with domain  $[2, \infty)$

- a.** Since 4 is in  $[2, \infty)$ ,  $f(4)$  is defined, and  $f(4) = \frac{2}{4} - 4^2 = \frac{1}{2} - 16 = -\frac{31}{2}$ .  
**b.** Since 0 is not in  $[2, \infty)$ ,  $f(0)$  is not defined.     **c.** Since 1 is not in  $[2, \infty)$ ,  $f(1)$  is not defined.

11.  $f(x) = \sqrt{x + 10}$ , with domain  $[-10, 0)$

- a.** Since 0 is not in  $[-10, 0)$ ,  $f(0)$  is not defined.     **b.** Since 9 is not in  $[-10, 0)$ ,  $f(9)$  is not defined.  
**c.** Since  $-10$  is in  $[-10, 0)$ ,  $f(-10)$  is defined, and  $f(-10) = \sqrt{-10 + 10} = \sqrt{0} = 0$

12.  $f(x) = \sqrt{9 - x^2}$ , with domain  $(-3, 3)$

- a.** Since 0 is in  $(-3, 3)$ ,  $f(0)$  is defined, and  $f(0) = \sqrt{9 - 0} = 3$ .  
**b.** Since 3 is not in  $(-3, 3)$ ,  $f(3)$  is not defined.     **c.** Since  $-3$  is not in  $(-3, 3)$ ,  $f(-3)$  is not defined.

13.  $f(x) = 4x - 3$

- a.**  $f(-1) = 4(-1) - 3 = -4 - 3 = -7$      **b.**  $f(0) = 4(0) - 3 = 0 - 3 = -3$   
**c.**  $f(1) = 4(1) - 3 = 4 - 3 = 1$      **d.** Substitute  $y$  for  $x$  to obtain  $f(y) = 4y - 3$   
**e.** Substitute  $(a + b)$  for  $x$  to obtain  $f(a + b) = 4(a + b) - 3$ .

14.  $f(x) = -3x + 4$

- a.**  $f(-1) = -3(-1) + 4 = 3 + 4 = 7$      **b.**  $f(0) = -3(0) + 4 = 0 + 4 = 4$   
**c.**  $f(1) = -3(1) + 4 = -3 + 4 = 1$      **d.** Substitute  $y$  for  $x$  to obtain  $f(y) = -3y + 4$   
**e.** Substitute  $(a + b)$  for  $x$  to obtain  $f(a + b) = -3(a + b) + 4$ .

15.  $f(x) = x^2 + 2x + 3$

- a.**  $f(0) = (0)^2 + 2(0) + 3 = 0 + 0 + 3 = 3$      **b.**  $f(1) = 1^2 + 2(1) + 3 = 1 + 2 + 3 = 6$

## Solutions Section 1.1

- c.  $f(-1) = (-1)^2 + 2(-1) + 3 = 1 - 2 + 3 = 2$       d.  $f(-3) = (-3)^2 + 2(-3) + 3 = 9 - 6 + 3 = 6$   
 e. Substitute  $a$  for  $x$  to obtain  $f(a) = a^2 + 2a + 3$ .      f. Substitute  $(x + h)$  for  $x$  to obtain  
 $f(x + h) = (x + h)^2 + 2(x + h) + 3$ .

16.  $g(x) = 2x^2 - x + 1$

- a.  $g(0) = 2(0)^2 - 0 + 1 = 0 - 0 + 1 = 1$       b.  $g(-1) = 2(-1)^2 - (-1) + 1 = 2 + 1 + 1 = 4$   
 c. Substitute  $r$  for  $x$  to obtain  $g(r) = 2r^2 - r + 1$ .  
 d. Substitute  $(x + h)$  for  $x$  to obtain  $g(x + h) = 2(x + h)^2 - (x + h) + 1$ .

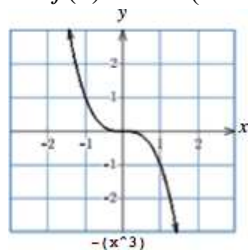
17.  $g(s) = s^2 + \frac{1}{s}$

- a.  $g(1) = 1^2 + \frac{1}{1} = 1 + 1 = 2$       b.  $g(-1) = (-1)^2 + \frac{1}{-1} = 1 - 1 = 0$   
 c.  $g(4) = 4^2 + \frac{1}{4} = 16 + \frac{1}{4} = \frac{65}{4}$  or 16.25      d. Substitute  $x$  for  $s$  to obtain  $g(x) = x^2 + \frac{1}{x}$   
 e. Substitute  $(s + h)$  for  $s$  to obtain  $g(s + h) = (s + h)^2 + \frac{1}{s + h}$   
 f.  $g(s + h) - g(s) = \text{Answer to part (e)} - \text{Original function} = \left((s + h)^2 + \frac{1}{s + h}\right) - \left(s^2 + \frac{1}{s}\right)$

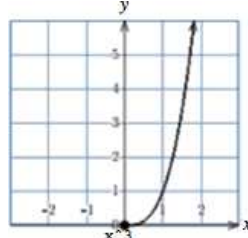
18.  $h(r) = \frac{1}{r + 4}$

- a.  $h(0) = \frac{1}{0 + 4} = \frac{1}{4}$       b.  $h(-3) = \frac{1}{(-3) + 4} = \frac{1}{1} = 1$   
 c.  $h(-5) = \frac{1}{(-5) + 4} = \frac{1}{-1} = -1$       d. Substitute  $x^2$  for  $r$  to obtain  $h(x^2) = \frac{1}{x^2 + 4}$ .  
 e. Substitute  $(x^2 + 1)$  for  $r$  to obtain  $h(x^2 + 1) = \frac{1}{(x^2 + 1) + 4} = \frac{1}{x^2 + 5}$ .  
 f.  $h(x^2) + 1 = \text{Answer to part (d)} + 1 = \frac{1}{x^2 + 4} + 1$

19.  $f(x) = -x^3$  (domain  $(-\infty, \infty)$ )      Technology formula:  $-(x^3)$



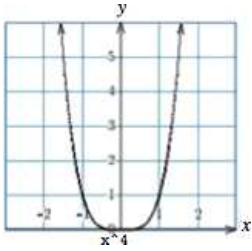
20.  $f(x) = x^3$  (domain  $[0, \infty)$ )      Technology formula:  $x^3$



## Solutions Section 1.1

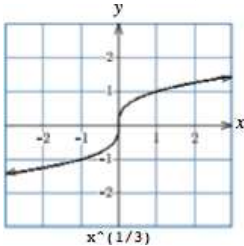
21.  $f(x) = x^4$  (domain  $(-\infty, \infty)$ )

Technology formula:  $x^4$



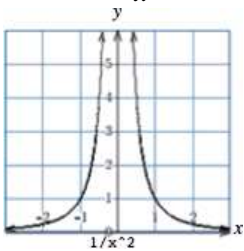
22.  $f(x) = \sqrt[3]{x}$  (domain  $(-\infty, \infty)$ )

Technology formula:  $x^{(1/3)}$



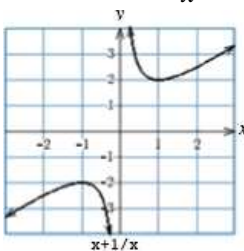
23.  $f(x) = \frac{1}{x^2}$  ( $x \neq 0$ )

Technology formula:  $1/(x^2)$



24.  $f(x) = x + \frac{1}{x}$  ( $x \neq 0$ )

Technology formula:  $x+1/x$



25. a.  $f(x) = x$  ( $-1 \leq x \leq 1$ )

Since the graph of  $f(x) = x$  is a diagonal  $45^\circ$  line through the origin inclining up from left to right, the correct graph is (A).

b.  $f(x) = -x$  ( $-1 \leq x \leq 1$ )

Since the graph of  $f(x) = -x$  is a diagonal  $45^\circ$  line through the origin inclining down from left to right, the correct graph is (D).

c.  $f(x) = \sqrt{x}$  ( $0 < x < 4$ )

Since the graph of  $f(x) = \sqrt{x}$  is the top half of a sideways parabola, the correct graph is (E).

d.  $f(x) = x + \frac{1}{x} - 2$  ( $0 < x < 4$ )

If we plot a few points like  $x = 1/2, 1, 2$ , and  $3$ , we find that the correct graph is (F).

e.  $f(x) = |x|$  ( $-1 \leq x \leq 1$ )

Since the graph of  $f(x) = |x|$  is a "V"-shape with its vertex at the origin, the correct graph is (C).

f.  $f(x) = x - 1$  ( $-1 \leq x \leq 1$ )

## Solutions Section 1.1

Since the graph of  $f(x) = x - 1$  is a straight line through  $(0, -1)$  and  $(1, 0)$ , the correct graph is (B).

**26. a.**  $f(x) = -x + 3$  ( $0 < x \leq 3$ )

Since the graph of  $f(x) = -x + 3$  is a straight line inclining down from left to right, the correct graph must be (D).

**b.**  $f(x) = 2 - |x|$  ( $-2 < x \leq 2$ )

Since  $f(x) = 2 - |x|$  is obtained from the graph of  $y = |x|$  by flipping it vertically (the minus sign in front of  $|x|$ ) and then moving it 2 units vertically up (adding 2 to all the values), the correct graph is (F).

**c.**  $f(x) = \sqrt{x+2}$  ( $-2 < x \leq 2$ )

The graph of  $f(x) = \sqrt{x+2}$  is similar to that of  $y = \sqrt{x}$ , which is half a parabola on its side, and the correct graph is (A).

**d.**  $f(x) = -x^2 + 2$  ( $-2 < x \leq 2$ )

The graph of  $f(x) = -x^2 + 2$  is a parabola opening down, so the correct graph is (C).

**e.**  $f(x) = \frac{1}{x} - 1$

The graph of  $f(x) = \frac{1}{x} - 1$  ( $0 < x \leq 3$ ) is part of a hyperbola, and the correct graph is (E).

**f.**  $f(x) = x^2 - 1$  ( $-2 < x \leq 2$ )

The graph of  $f(x) = x^2 - 1$  is a parabola opening up, so the correct graph is (B).

**27.** Technology formula:  $0.1 \cdot x^2 - 4 \cdot x + 5$

Table of values:

| $x$    | 0 | 1   | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     | 10  |
|--------|---|-----|------|------|------|-------|-------|-------|-------|-------|-----|
| $f(x)$ | 5 | 1.1 | -2.6 | -6.1 | -9.4 | -12.5 | -15.4 | -18.1 | -20.6 | -22.9 | -25 |

**28.** Technology formula:  $0.4 \cdot x^2 - 6 \cdot x - 0.1$

Table of values:

| $x$    | -5   | -4   | -3   | -2   | -1  | 0    | 1    | 2     | 3     | 4     | 5     |
|--------|------|------|------|------|-----|------|------|-------|-------|-------|-------|
| $g(x)$ | 39.9 | 30.3 | 21.5 | 13.5 | 6.3 | -0.1 | -5.7 | -10.5 | -14.5 | -17.7 | -20.1 |

**29.** Technology formula:  $(x^2 - 1) / (x^2 + 1)$

Table of values:

| $x$    | 0.5     | 1.5    | 2.5    | 3.5    | 4.5    | 5.5    | 6.5    | 7.5    | 8.5    | 9.5    | 10.5   |
|--------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $h(x)$ | -0.6000 | 0.3846 | 0.7241 | 0.8491 | 0.9059 | 0.9360 | 0.9538 | 0.9651 | 0.9727 | 0.9781 | 0.9820 |

**30.** Technology formula:  $(2 \cdot x^2 + 1) / (2 \cdot x^2 - 1)$

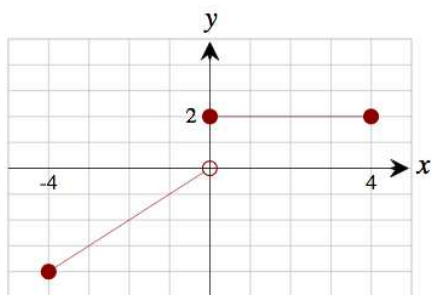
Table of values:

| $x$    | -1     | 0       | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |
|--------|--------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $r(x)$ | 3.0000 | -1.0000 | 3.0000 | 1.2857 | 1.1176 | 1.0645 | 1.0408 | 1.0282 | 1.0206 | 1.0157 | 1.0124 |

**31.**  $f(x) = \begin{cases} x & \text{if } -4 \leq x < 0 \\ 2 & \text{if } 0 \leq x \leq 4 \end{cases}$

Technology formula:  $x \cdot (x \leq 0) + 2 \cdot (x > 0)$  (For a graphing calculator, use  $\geq$  instead of  $>$ .)

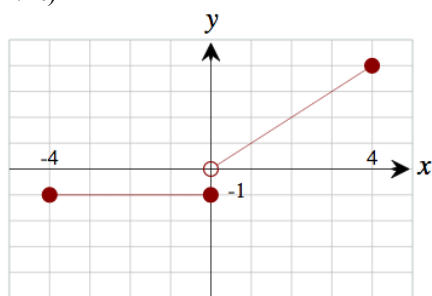
## Solutions Section 1.1



- a.  $f(-1) = -1$ . We used the first formula, since  $-1$  is in  $[-4, 0)$ .
- b.  $f(0) = 2$ . We used the second formula, since  $0$  is in  $[0, 4]$ .
- c.  $f(1) = 2$ . We used the second formula, since  $1$  is in  $[0, 4]$ .

32. 
$$f(x) = \begin{cases} -1 & \text{if } -4 \leq x \leq 0 \\ x & \text{if } 0 < x \leq 4 \end{cases}$$

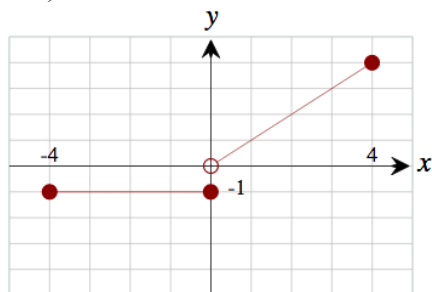
Technology formula:  $(-1) * (x \leq 0) + x * (x > 0)$  (For a graphing calculator, use  $\leq$  instead of  $<=$ .)



- a.  $f(-1) = -1$ . We used the first formula, since  $-1$  is in  $[-4, 0]$ .
- b.  $f(0) = -1$ . We used the first formula, since  $0$  is in  $[-4, 0]$ .
- c.  $f(1) = 1$ . We used the second formula, since  $1$  is in  $(0, 4]$ .

33. 
$$f(x) = \begin{cases} x^2 & \text{if } -2 < x \leq 0 \\ \frac{1}{x} & \text{if } 0 < x \leq 4 \end{cases}$$

Technology formula:  $(x^2) * (x \leq 0) + (1/x) * (0 < x)$  (For a graphing calculator, use  $\leq$  instead of  $<=$ .)



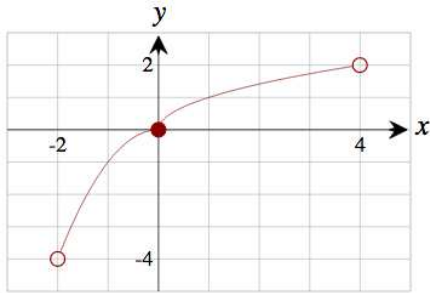
- a.  $f(-1) = 1^2 = 1$ . We used the first formula, since  $-1$  is in  $(-2, 0]$ .
- b.  $f(0) = 0^2 = 0$ . We used the first formula, since  $0$  is in  $(-2, 0]$ .
- c.  $f(1) = 1/1 = 1$ . We used the second formula, since  $1$  is in  $(0, 4]$ .

## Solutions Section 1.1

$$34. f(x) = \begin{cases} -x^2 & \text{if } -2 < x \leq 0 \\ \sqrt{x} & \text{if } 0 < x < 4 \end{cases}$$

Technology formula: Excel:  $(-1 * x^2) * (x \leq 0) + \text{SQRT}(\text{ABS}(x)) * (x > 0)$

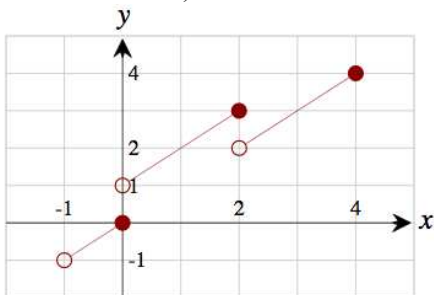
TI-83/84 Plus:  $(-1 * x^2) * (x \leq 0) + \sqrt{x} * (x > 0)$



- a.  $f(-1) = -(-1)^2 = -1$ . We used the first formula, since  $-1$  is in  $(-2, 0]$ .
- b.  $f(0) = -0^2 = 0$ . We used the first formula, since  $0$  is in  $(-2, 0]$ .
- c.  $f(1) = \sqrt{1} = 1$ . We used the second formula, since  $1$  is in  $(0, 4)$ .

$$35. f(x) = \begin{cases} x & \text{if } -1 < x \leq 0 \\ x + 1 & \text{if } 0 < x \leq 2 \\ x & \text{if } 2 < x \leq 4 \end{cases}$$

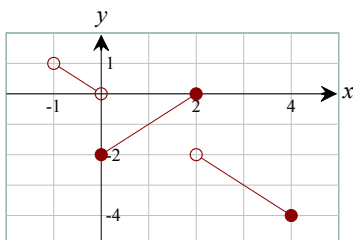
Technology formula:  $x * (x \leq 0) + (x + 1) * (0 < x) * (x \leq 2) + x * (2 < x)$  (For a graphing calculator, use  $\leq$  instead of  $<=$ .)



- a.  $f(0) = 0$ . We used the first formula, since  $0$  is in  $(-1, 0]$ .
- b.  $f(1) = 1 + 1 = 2$ . We used the second formula, since  $1$  is in  $(0, 2]$ .
- c.  $f(2) = 2 + 1 = 3$ . We used the second formula, since  $2$  is in  $(0, 2]$ .
- d.  $f(3) = 3$ . We used the third formula, since  $3$  is in  $(2, 4]$ .

$$36. f(x) = \begin{cases} -x & \text{if } -1 < x < 0 \\ x - 2 & \text{if } 0 \leq x \leq 2 \\ -x & \text{if } 2 < x \leq 4 \end{cases}$$

Technology formula:  $x * (x < 0) + (x - 2) * (0 \leq x) * (x \leq 2) + (-x) * (2 < x)$  (For a graphing calculator, use  $\leq$  instead of  $<=$ .)



## Solutions Section 1.1

- a.  $f(0) = 0 - 2 = -2$ . We used the second formula, since 0 is in  $[0, 2]$ .  
 b.  $f(1) = 1 - 2 = -1$ . We used the second formula, since 1 is in  $[0, 2]$ .  
 c.  $f(2) = 2 - 2 = 0$ . We used the second formula, since 2 is in  $[0, 2]$ .  
 d.  $f(3) = -3$ . We used the third formula, since 3 is in  $(2, 4]$ .

**37.**  $f(x) = x^2$

a.  $f(x + h) = (x + h)^2$  Therefore,

$$\begin{aligned} f(x + h) - f(x) &= (x + h)^2 - x^2 \\ &= x^2 + 2xh + h^2 - x^2 \\ &= 2xh + h^2 = h(2x + h) \end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x + h) - f(x)}{h} = \frac{h(2x + h)}{h} = 2x + h$$

**38.**  $f(x) = 3x - 1$

a.  $f(x + h) = 3(x + h) - 1 = 3x + 3h - 1$  Therefore,

$$\begin{aligned} f(x + h) - f(x) &= 3x + 3h - 1 - (3x - 1) \\ &= 3x + 3h - 1 - 3x + 1 = 3h \end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x + h) - f(x)}{h} = \frac{3h}{h} = 3$$

**39.**  $f(x) = 2 - x^2$

a.  $f(x + h) = 2 - (x + h)^2$  Therefore,

$$\begin{aligned} f(x + h) - f(x) &= 2 - (x + h)^2 - (2 - x^2) \\ &= 2 - x^2 - 2xh - h^2 - 2 + x^2 \\ &= -2xh - h^2 = -h(2x + h) \end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x + h) - f(x)}{h} = \frac{-h(2x + h)}{h} = -(2x + h)$$

**40.**  $f(x) = x^2 + x$

a.  $f(x + h) = (x + h)^2 + (x + h)$  Therefore,

$$\begin{aligned} f(x + h) - f(x) &= (x + h)^2 + (x + h) - (x^2 + x) \\ &= x^2 + 2xh + h^2 + x + h - x^2 - x \\ &= 2xh + h^2 + h = h(2x + h + 1) \end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x + h) - f(x)}{h} = \frac{h(2x + h + 1)}{h} = 2x + h + 1$$

**41.** From the table,

a.  $p(2) = 0.67$ ; Pemex produced 0.67 billion barrels of crude oil in 2017 ( $t = 2$ ).

$p(3) = 0.61$ ; Pemex produced 0.61 billion barrels of crude oil in 2018 ( $t = 3$ ).

$p(6) = 0.62$ ; Pemex produced 0.62 billion barrels of crude oil in 2021 ( $t = 6$ ).

## Solutions Section 1.1

**b.**  $p(6) - p(3) = 0.62 - 0.61 = 0.01$ ; Annual crude oil production by Pemex increased by 0.01 billion barrels from 2018 ( $t = 3$ ) to 2021 ( $t = 6$ ).

**42.** From the table,

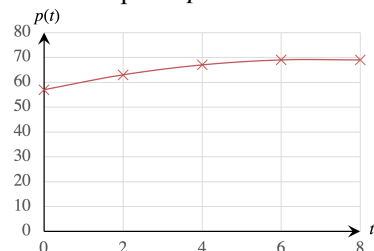
**a.**  $s(0) = 0.69$ ; Pemex produced 0.69 billion barrels of offshore crude oil in 2015 ( $t = 0$ ).

$s(2) = 0.56$ ; Pemex produced 0.56 billion barrels of offshore crude oil in 2017 ( $t = 2$ ).

$s(4) = 0.51$ ; Pemex produced 0.51 billion barrels of offshore crude oil in 2019 ( $t = 4$ ).

**b.**  $s(4) - s(0) = 0.51 - 0.69 = -0.18$ ; Annual offshore crude oil production by Pemex decreased by 0.18 billion barrels from 2015 ( $t = 0$ ) to 2019 ( $t = 4$ ).

**43. a.** Graph of  $p$ :



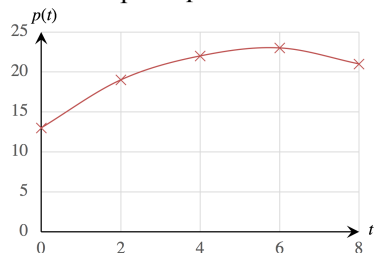
The graph suggests a curve, so we exclude the linear models (A) and (C). Model (B) gives a parabola that curves up whereas the curve suggested by the graph curves down, leaving us with Model (D). Also, model (D) gives almost the exact values shown in the chart. (Use the technology formula  $-0.25x^2 + 3.5x + 57$ .)

**b.** Using Model (D),  $p(5) = -0.25(5)^2 + 3.5(5) + 57 = 68.25$

Interpretation:  $t = 5$  represents 5 years since the start of 2013, or the start of 2018. Thus, we interpret the answer as follows: Approximately 68.25% of U.S. adults used Facebook at the start of 2018

**c.** The graph becomes less steep as time increases, indicating decelerating Facebook membership over the period 2013–2021.

**44. a.** Graph of  $p$ :



The graph suggests a curve, so we exclude the linear model (B). Model (D) gives a parabola that curves up whereas the curve suggested by the graph curves down, leaving us with Models (A) and (C). Model (A) gives values that round to the values shown in the chart whereas Model (C) is way off. (Use the technology formula  $-0.32x^2 + 3.6x + 13$ .)

**b.** Using Model (A),  $p(7.5) = -0.32(7.5)^2 + 3.6(7.5) + 13 = 22$

Interpretation:  $t = 7.5$  represents 7.5 years since the start of 2013, or midway through 2020. Thus, we interpret the answer as follows: Approximately 22% of U.S. adults used Twitter midway through 2020.

**c.** The graph becomes less steep as time increases, indicating decelerating Twitter membership over the period 2013–2021.

**45.** From the graph,  $f(11) \approx 800$ . Because  $f$  is the number of thousands of housing starts in year 11, we interpret the result as follows: Approximately 800,000 homes were started in 2016.

Similarly,  $f(15) \approx 1,000$ . Approximately 1,000,000 homes were started in 2020.

Also, we estimate  $f(2.5) \approx 800$ . Because  $t = 2.5$  is midway between 2007 and 2008, we interpret the result as follows: approximately 800,000 homes were started in the year beginning midway through 2007.

## Solutions Section 1.1

**46.** From the graph,  $f(3) \approx 600$ . Because  $f$  is the number of thousands of housing starts in year 3, we interpret the result as follows: Approximately 600,000 homes were started in 2008.  
 $f(12) \approx 900$ : Approximately 900,000 homes were started in 2016.  
 $f(14.5) \approx 950$ : Approximately 950,000 homes were started in the year beginning midway through 2019.

**47.**  $f(14 - 10) = f(4) \approx 400$ . Interpretation: Approximately 400,000 homes were started in 2009 ( $t = 4$ ).  
 $f(14) - f(10) \approx 900 - 700 = 200$ . Interpretation:  $f(14) - f(10)$  is the change in the number of housing starts (in thousands) from 2015 to 2019; there were approximately 200,000 more housing starts in 2019 than in 2015.

**48.**  $f(15 - 1) = f(14) \approx 900$ . Interpretation: Approximately 900,000 homes were started in 2019 ( $t = 14$ ).  
 $f(15) - f(1) \approx 1,000 - 1,500 = -500$ . Interpretation:  $f(15) - f(1)$  is the change in the number of housing starts (in thousands) from 2006 to 2020; there were approximately 500,000 fewer housing starts in 2020 than in 2006.

**49.**  $f(t + 5) - f(t)$  measures the change from year  $t$  to the year five years later. It is greatest when the line segment from year  $t$  to year  $t + 5$  is steepest upward-sloping. From the graph, or by computing differences of values estimated from the graph, this occurs when  $t = 6$  or 7:

$$f(11) - f(6) = 800 - 400 = 400; f(12) - f(7) = 900 - 500 = 400.$$

Interpretation: The greatest five-year increase in the number of housing starts occurred in 2011–2016 and again in 2012–2017.

**50.**  $f(t) - f(t - 1)$  measures the change from year  $t - 1$  to the following year. It is least when the line segment from year  $t - 1$  to year  $t$  is steepest downward-sloping. From the graph, this occurs when  $t = 2$ , for a change of  $f(2) - f(1) = 1,000 - 1,500 = -500$ .  
Interpretation: The greatest annual decrease in the number of housing starts occurred in 2006–2007.

**51. a.** From the graph,  $b(3) \approx 50$  and  $b(5) \approx 35$ . The (50-day average) price of Bitcoin was approximately \$50,000 on June 1, 2021 ( $t = 3$ ) and \$35,000 on August 1, 2021 ( $t = 5$ ).

$b(7) - b(5) \approx 50 - 35 = 15$ . The Bitcoin price increased by around \$15,000 from August 1 to October 1, 2021.

**b.** Increasing most rapidly when the graph is steepest upward from left to right on the interval  $[1, 3]$ , which occurs at  $t = 1$  (integer answer required). Thus, during the period April 1–June 1, 2021, the price of Bitcoin was increasing most rapidly around April 1.

**c.** Decreasing most rapidly when the graph is steepest down from left to right, which occurs at  $t = 3$  (integer answer required). Thus, during the period April 1–June 1, 2021, the price of Bitcoin was decreasing most rapidly around June 1.

**52. a.** From the graph,  $e(2) \approx 2.5$  and  $e(4) \approx 2.5$ . The price of Ethereum was approximately \$2,500 on May 1, 2021 ( $t = 2$ ) and again on July 1, 2021 ( $t = 4$ ).

$e(3) - e(1) \approx 2 - 3 = -1$ . The price of Ethereum decreased by around \$1,000 from April 1 to June 1, 2021.

**b.** Increasing most rapidly when the graph is steepest upward from left to right on the interval  $[1, 5]$ , which occurs at  $t = 4$  (integer answer required). Thus, during the period April 1–August 1, 2021, the price of Ethereum was increasing most rapidly around July 1.

**c.** Decreasing most rapidly when the graph is steepest down from left to right, which occurs at  $t = 2$  (integer answer required). Thus, during the period April 1–August 1, 2021, the price of Ethereum was decreasing most rapidly around May 1.

**53.**  $r(x) = 0.4x^2 + x + 26.5 \quad (0 \leq x \leq 5)$

**a.** The domain of  $r$  is  $[0, 5]$ .

## Solutions Section 1.1

$$r(0) = 0.4(0)^2 + 0 + 26.5 = 26.5;$$

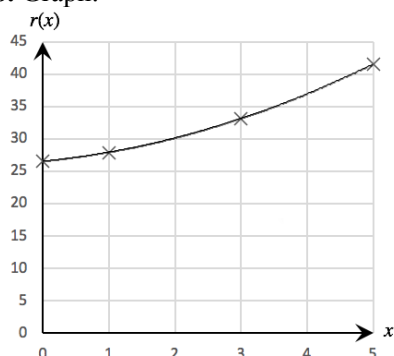
$$r(1) = 0.4(1)^2 + 1 + 26.5 = 27.9$$

$$r(3) = 0.4(3)^2 + 3 + 26.5 = 33.1$$

$$r(5) = 0.4(5)^2 + 5 + 26.5 = 41.5$$

Food delivery app revenues in the U.S. were projected to be \$26.5 billion in 2020, \$27.9 billion in 2021, \$33.1 billion in 2023, and \$41.5 billion in 2025.

b. Graph:



(As the curve is concave up the projected revenue was accelerating.)

54.  $r(x) = -0.2x^2 + 3x + 18 \quad (0 \leq x \leq 5)$

a. The domain of  $r$  is  $[0, 5]$ .

$$r(0) = -0.2(0)^2 + 3(0) + 18 = 18;$$

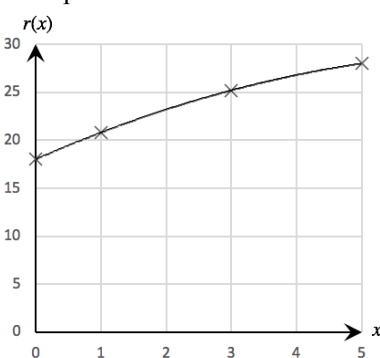
$$r(1) = -0.2(1)^2 + 3(1) + 18 = 20.8$$

$$r(3) = -0.2(3)^2 + 3(3) + 18 = 25.2$$

$$r(5) = -0.2(5)^2 + 3(5) + 18 = 28$$

Food delivery app revenues in Europe were projected to be \$18 billion in 2020, \$20.8 billion in 2021, \$25.2 billion in 2023, and \$28 billion in 2025.

b. Graph



(As the graph is concave down the projected revenue was decelerating).

55. a. The model is valid for the range 1958 ( $t = 0$ ) through 1966 ( $t = 8$ ). Thus, an appropriate domain is  $[0, 8]$ .  $t \geq 0$  is not an appropriate domain because it would predict federal funding of NASA beyond 1966, whereas the model is based only on data up to 1966.

b.  $p(t) = \frac{4.5}{1.07^{(t-8)^2}}$

$$\Rightarrow p(5) = \frac{4.5}{1.07^{(5-8)^2}} \approx 2.4 \quad \text{Technology formula: } 4.5 / (1.07^{((t-8)^2)})$$

$t = 5$  represents  $1958 + 5 = 1963$ , and therefore we interpret the result as follows: In 1963, 2.4% of the U.S. federal budget was allocated to NASA.

c.  $p(t)$  is increasing most rapidly when the graph is steepest upward-sloping from left to right, and, among the given values of  $t$ , this occurs when  $t = 5$ . Thus, the percentage of the budget allocated to NASA was

## Solutions Section 1.1

increasing most rapidly in 1963.

**56. a.**  $[1, 55]$ ;  $[0, 55]$  is not an appropriate domain because  $p$  is undefined at 0.

**b.**  $p(t) = 0.03 + \frac{5}{t^{0.6}}$

$\Rightarrow p(40) = 0.03 + \frac{5}{40^{0.6}} \approx 0.58$       Technology formula:  $0.03+5/t^{0.6}$

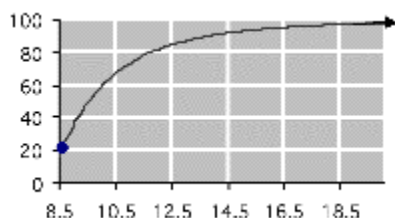
$t = 40$  represents  $1965 + 40 = 2005$ , and therefore we interpret the result as follows: In 2005, 0.58% of the US federal budget was allocated to NASA.

**c.** If we evaluate  $p(t)$  for  $t = 100, 1,000, 100,000, 1,000,000 \dots$ , we find values of  $p(t)$  decreasing toward 0.03. Thus, in the (very) long term, the percentage of the budget allocated to NASA is predicted to approach 0.03%

**57. a.**  $p(t) = 100\left(1 - \frac{12,200}{t^{4.48}}\right)$       ( $t \geq 8.5$ )

Technology formula:  $100*(1-12200/t^{4.48})$

**b.** Graph:



**c.** Table of values:

| $t$    | 9    | 10   | 11   | 12   | 13   | 14   | 15   | 16   | 17   | 18   | 19   | 20   |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|
| $p(t)$ | 35.2 | 59.6 | 73.6 | 82.2 | 87.5 | 91.1 | 93.4 | 95.1 | 96.3 | 97.1 | 97.7 | 98.2 |

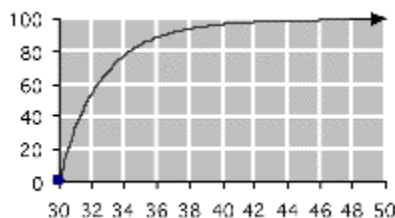
**d.** From the table,  $p(12) = 82.2$ , so that 82.2% of children are able to speak in at least single words by the age of 12 months.

**e.** We seek the first value of  $t$  such that  $p(t)$  is at least 90. Since  $t = 14$  has this property ( $p(14) = 91.1$ ) we conclude that, at 14 months, 90% or more children are able to speak in at least single words.

**58. a.**  $p(t) = 100\left(1 - \frac{5.27 \times 10^{17}}{t^{12}}\right)$       ( $t \geq 30$ )

Technology formula:  $100*(1-5.27*10^{17}/t^{12})$

**b.** Graph:



**c.** Table of values:

| $t$    | 30  | 31   | 32   | 33   | 34   | 35   | 36   | 37   | 38   | 39   | 40   |
|--------|-----|------|------|------|------|------|------|------|------|------|------|
| $p(t)$ | 0.8 | 33.1 | 54.3 | 68.4 | 77.9 | 84.4 | 88.9 | 92.0 | 94.2 | 95.7 | 96.9 |

**d.** From the table,  $p(36) = 88.9$ , so that 88.9% of children are able to speak in sentences of five or more words by the age of 36 months.

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e. We seek the first value of  $t$  such that  $p(t)$  is at least 75. Since  $t = 34$  has this property ( $p(34) = 77.9$ ) we conclude that, at 34 months, 75% or more children are able to speak in sentences of five or more words.

$$59. v(t) = \begin{cases} 8(1.22)^t & \text{if } 0 \leq t < 16 \\ 400t - 6,200 & \text{if } 16 \leq t < 25 \\ 0.45(t - 25)^3 + 3,800 & \text{if } 25 \leq t \leq 40 \end{cases}$$

a.  $v(10) = 8(1.22)^{10} \approx 58$ . We used the first formula, since 10 is in  $[0, 16)$ .

$v(16) = 400(16) - 6,200 = 200$ . We used the second formula, since 16 is in  $[16, 25)$ .

$v(40) = 0.45(40 - 25)^3 + 3,800 \approx 5,319$ . We used the third formula, since 40 is in  $[25, 40]$ .

Interpretation: Processor speeds were about 58 MHz in 1990, 200 MHz in 1996, and 5,319 MHz in 2020.

b. Technology formula (using  $x$  as the independent variable):

$$(8 * (1.22)^x) * (x < 16) + (400 * x - 6200) * (x \geq 16) * (x < 25) + (0.45 * (x - 25)^3 + 3800) * (x \geq 25)$$

(For a graphing calculator, use  $\leq$  instead of  $<=$ .)

c. Using the above technology formula (for instance, on the Function Evaluator and Grapher on the Web site) we obtain the graph and table of values.

Graph:

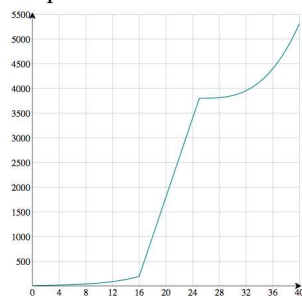


Table of values:

| $t$    | 0 | 4  | 8  | 12 | 16  | 20    | 24    | 28    | 32    | 36    | 40    |
|--------|---|----|----|----|-----|-------|-------|-------|-------|-------|-------|
| $v(t)$ | 8 | 18 | 39 | 87 | 200 | 1,800 | 3,400 | 3,800 | 4,000 | 4,400 | 5,300 |

d. From either the graph or the table, we see that the speed reached 3,000 MHz between  $t = 20$  and  $t = 24$ . We can obtain a more precise answer algebraically by using the formula for the corresponding portion of the graph:

$$3,000 = 400t - 6,200$$

giving

$$t = \frac{9,200}{400} = 23$$

Since  $t$  is time since 1980,  $t = 23$  corresponds to 2003.

$$60. v(t) = \begin{cases} 0.12t^2 + 0.04t + 0.2 & \text{if } 0 \leq t < 12 \\ 1.1(1.22)^t & \text{if } 12 \leq t < 26 \\ 400t - 10,200 & \text{if } 26 \leq t \leq 30 \end{cases}$$

a.  $v(2) = 0.12(2)^2 + 0.04(2) + 0.2 = 0.76$ . We used the first formula, since 2 is in  $[0, 12)$ .

$v(12) = 1.1(1.22)^{12} \approx 12$ . We used the second formula, since 12 is in  $[12, 26)$ .

$v(28) = 400(28) - 10,200 = 1,000$ . We used the third formula, since 28 is in  $[26, 30]$ .

Interpretation: Processor speeds were about 0.76 MHz in 1972, 12 MHz in 1982, and 1,000 MHz in 1998.

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**b.** Technology formula (using  $x$  as the independent variable):

$$(0.12x^2 + 0.04x + 0.2) * (x < 12) + (1.1 * (1.22)^x) * (x \geq 12) * (x < 26) + (400x - 10200) * (x \geq 26)$$

(For a graphing calculator, use  $\leq$  instead of  $<=$ .)

**c.** Using the above technology formula (for instance, on the Function Evaluator and Grapher on the Web site) we obtain the graph and table of values.

Graph:

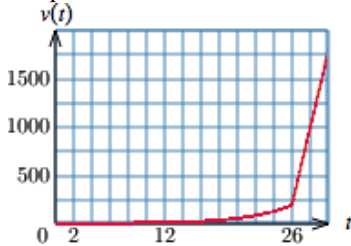


Table of values:

| $t$    | 0    | 2    | 4   | 6   | 8   | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24  | 26  | 28    | 30    |
|--------|------|------|-----|-----|-----|----|----|----|----|----|----|----|-----|-----|-------|-------|
| $v(t)$ | 0.20 | 0.76 | 2.3 | 4.8 | 8.2 | 13 | 12 | 18 | 26 | 39 | 59 | 87 | 130 | 200 | 1,000 | 1,800 |

**d.** From either the graph or the table, we see that the speed reached 500 MHz around  $t = 27$ . We can obtain a more precise answer algebraically by using the formula for the corresponding portion of the graph:

$$500 = 400t - 10,200$$

giving

$$t = \frac{10,700}{400} = 26.75 \approx 27 \text{ to the nearest year}$$

Since  $t$  is time since 1970,  $t = 27$  corresponds to 1997.

**61. a.** Each row of the table gives us a formula with a condition:

*First row in words:* 10% of the amount over \$0 if your income is over \$0 and not over \$9,950.

*Translation to formula:*

$$0.10x \quad \text{if } 0 < x \leq 9,950.$$

*Second row in words:* \$995.00 + 12% of the amount over \$9,950 if your income is over \$9,950 and not over \$40,525.

*Translation to formula:*

$$995.00 + 0.12(x - 9,950) \quad \text{if } 9,950 < x \leq 40,525.$$

Continuing in this way leads to the following piecewise-defined function:

$$T(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 9,950 \\ 995.00 + 0.12(x - 9,950) & \text{if } 9,950 < x \leq 40,525 \\ 4,664 + 0.22(x - 40,525) & \text{if } 40,525 < x \leq 86,375 \\ 14,751 + 0.24(x - 86,375) & \text{if } 86,375 < x \leq 164,925 \\ 33,603 + 0.32(x - 164,925) & \text{if } 164,925 < x \leq 209,425 \\ 47,843 + 0.35(x - 209,425) & \text{if } 209,425 < x \leq 523,600 \\ 157,804.25 + 0.37(x - 523,600) & \text{if } 523,600 < x \end{cases}$$

## Solutions Section 1.1

**b.** A taxable income of \$45,000 falls in the bracket  $40,525 < x \leq 86,375$  and so we use the formula  $4,664 + 0.22(x - 40,525)$  :

$$4,664 + 0.22(45,000 - 40,525) = 4,664 + 0.22(4,475) = \$5,648.50$$

**62. a.** Each row of the table gives us a formula with a condition:

*First row in words:* 10% of the amount over \$0 if your income is over \$0 and not over \$8,700.

*Translation to formula:*

$$0.10x \quad \text{if } 0 < x \leq 8,700.$$

*Second row in words:* \$870.00 + 15% of the amount over \$8,700 if your income is over \$8,700 and not over \$35,350.

*Translation to formula:*

$$870.00 + 0.15(x - 8,700) \quad \text{if } 8,700 < x \leq 35,350.$$

Continuing in this way leads to the following piecewise-defined function:

$$T(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 8,700 \\ 870.00 + 0.15(x - 8,700) & \text{if } 8,700 < x \leq 35,350 \\ 4,867.50 + 0.22(x - 35,350) & \text{if } 35,350 < x \leq 85,650 \\ 17,442.50 + 0.28(x - 85,650) & \text{if } 85,650 < x \leq 178,650 \\ 43,482.50 + 0.33(x - 178,650) & \text{if } 178,650 < x \leq 388,350 \\ 112,683.50 + 0.35(x - 388,350) & \text{if } 388,350 < x \end{cases}$$

**b.** A taxable income of \$45,000 falls in the bracket  $35,350 < x \leq 85,650$  and so we use the formula  $4,867.50 + 0.22(x - 35,350)$  :

$$4,867.50 + 0.22(45,000 - 35,350) = 4,867.50 + 0.22(9,650) = \$9,990.50.$$

**63.** The dependent variable is a function of the independent variable. Here, the market price of gold  $m$  is a function of time  $t$ . Thus, the independent variable is  $t$  and the dependent variable is  $m$ .

**64.** The dependent variable is a function of the independent variable. Here, the weekly profit  $P$  is a function of the selling price  $s$ . Thus, the independent variable is  $s$  and the dependent variable is  $P$ .

**65.** To obtain the function notation, write the dependent variable as a function of the independent variable.

Thus  $y = 4x^2 - 2$  can be written as

$$f(x) = 4x^2 - 2 \text{ or } y(x) = 4x^2 - 2$$

**66.** To obtain the equation notation, introduce a dependent variable instead of the function notation. Thus

$C(t) = -0.34t^2 + 0.1t$  can be written as

$$c = -0.34t^2 + 0.1t \text{ or } y = -0.34t^2 + 0.1t$$

**67.** False. A graph usually gives infinitely many values of the function while a numerical table will give only a finite number of values.

**68.** True. An algebraically specified function  $f$  is specified by algebraic formulas for  $f(x)$ . Given such formulas, we can construct the graph of  $f$  by plotting the points  $(x, f(x))$  for values of  $x$  in the domain of  $f$ .

**69.** False. In a numerically specified function, only certain values of the function are specified so we cannot know its value on every real number in  $[0, 10]$ , whereas an algebraically specified function would give values for every real number in  $[0, 10]$ .

## Solutions Section 1.1

**70.** False. A graphically specified function is specified by a graph. However, we cannot always expect to find an algebraic formula whose graph is exactly the graph that is given.

**71.** Functions with infinitely many points in their domain (such as  $f(x) = x^2$ ) cannot be specified numerically. So, the assertion is false.

**72.** A numerical model supplies only the values of a function at specific values of the independent variable, whereas an algebraic model supplies the value of a function at every point in its domain. Thus, an algebraic model supplies more information.

**73.** (Answers may vary.) Take  $f(x) = x^2$ . Then

a.  $f(3 + 2) = f(5) = 5^2 = 25$ , whereas  $f(3) + f(2) = 3^2 + 2^2 = 9 + 4 = 13$ . Thus,  $f(3 + 2) \neq f(3) + f(2)$ .

b.  $f(3 - 2) = f(1) = 1^2 = 1$ , whereas  $f(3) - f(2) = 3^2 - 2^2 = 9 - 4 = 5$ . Thus,  $f(3 - 2) \neq f(3) - f(2)$ .

**74.** (Answers may vary.) Take  $f(x) = x + 1$ . Then

a.  $f(3 \times 2) = f(6) = 6 + 1 = 7$ , whereas  $3f(2) = 3(2 + 1) = 3 \times 3 = 9$ . Thus,  $f(3 \times 2) \neq 3f(2)$ .

b.  $f(3 \times 2) = f(6) = 6 + 1 = 7$ , whereas  $f(3)f(2) = (3 + 1)(2 + 1) = 4 \times 3 = 12$ . Thus,  $f(3 \times 2) \neq f(3)f(2)$ .

**75.** As the text reminds us: to evaluate  $f$  of a quantity (such as  $x + h$ ) replace  $x$  everywhere by the *whole quantity*  $x + h$  :

$$f(x) = x^2 - 1$$

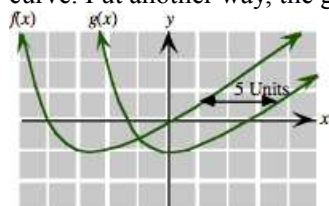
$$f(x + h) = (x + h)^2 - 1.$$

**76.** Knowing  $f(x)$  for two values of  $x$  does not convey any information about  $f(x)$  at any other value of  $x$ . Interpolation is only a way of *estimating*  $f(x)$  at values of  $x$  not given.

**77.** If two functions are specified by the same formula  $f(x)$ , say, their graphs must follow the same curve  $y = f(x)$ . However, it is the domain of the function that specifies what portion of the curve appears on the graph. Thus, if the functions have different domains, their graphs will be different portions of the curve  $y = f(x)$ .

**78.** If we plot points of the graphs  $y = f(x)$  and  $y = g(x)$ , we see that, since  $g(x) = f(x) + 10$ , we must add 10 to the  $y$ -coordinate of each point in the graph of  $f$  to get a point on the graph of  $g$ . Thus, the graph of  $g$  is 10 units higher up than the graph of  $f$ .

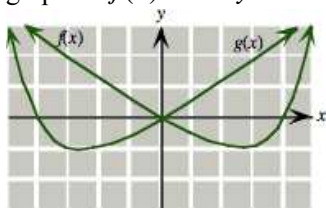
**79.** Suppose we already have the graph of  $f$  and want to construct the graph of  $g$ . We can plot a point of the graph of  $g$  as follows: Choose a value for  $x$  ( $x = 7$ , say) and then "look back" 5 units to read off  $f(x - 5)$  ( $f(2)$  in this instance). This value gives the  $y$ -coordinate we want. In other words, points on the graph of  $g$  are obtained by "looking back 5 units" to the graph of  $f$  and then copying that portion of the curve. Put another way, the graph of  $g$  is the same as the graph of  $f$ , but shifted 5 units to the right:



**80.** Suppose we already have the graph of  $f$  and want to construct the graph of  $g$ . We can plot a point of

### Solutions Section 1.1

the graph of  $g$  as follows: Choose a value for  $x$  ( $x = 7$ , say) and then look on the other side of the  $y$ -axis to read off  $f(-x)$  ( $f(-7)$  in this instance). This value gives the  $y$ -coordinate we want. In other words, points on the graph of  $g$  are obtained by "looking back" to the graph of  $f$  on the opposite side of the  $y$ -axis and then copying that portion of the curve. Put another way, the graph of  $g(x)$  is the mirror image of the graph of  $f(x)$  in the  $y$ -axis:



## Section 1.2

1.  $f(x) = x^2 + 1$  with domain  $(-\infty, \infty)$

$g(x) = x - 1$  with domain  $(-\infty, \infty)$

a.  $s(x) = f(x) + g(x) = (x^2 + 1) + (x - 1) = x^2 + x$

b. Since both functions are defined for every real number  $x$ , the domain of  $s$  is the set of all real numbers:  $(-\infty, \infty)$ .

c.  $s(-3) = (-3)^2 + (-3) = 9 - 3 = 6$

2.  $f(x) = x^2 + 1$  with domain  $(-\infty, \infty)$

$g(x) = x - 1$  with domain  $(-\infty, \infty)$

a.  $d(x) = g(x) - f(x) = (x - 1) - (x^2 + 1) = -x^2 + x - 2$

b. Since both functions are defined for every real number  $x$ , the domain of  $d$  is the set of all real numbers:  $(-\infty, \infty)$ .

c.  $d(-1) = -(-1)^2 + (-1) - 2 = -4$

3.  $g(x) = x - 1$  with domain  $(-\infty, \infty)$

$u(x) = \sqrt{x + 10}$  with domain  $[-10, \infty)$

a.  $p(x) = g(x)u(x) = (x - 1)\sqrt{x + 10}$

b. The domain of  $p$  consists of all real numbers  $x$  simultaneously in the domains of  $g$  and  $u$ ; that is,  $[-10, \infty)$ .

c.  $p(-6) = (-6 - 1)\sqrt{-6 + 10} = (-7)(2) = -14$

4.  $h(x) = x + 4$  with domain  $[10, \infty)$

$v(x) = \sqrt{10 - x}$  with domain  $[-\infty, 10]$

a.  $p(x) = h(x)v(x) = (x + 4)\sqrt{10 - x}$

b. The domain of  $p$  consists of all real numbers  $x$  simultaneously in the domains of  $h$  and  $v$ ; that is, the single point  $x = 10$ .

c. As 1 is not in the domain of  $p$ ,  $p(1)$  is not defined.

5.  $g(x) = x - 1$  with domain  $(-\infty, \infty)$

$v(x) = \sqrt{10 - x}$  with domain  $[-\infty, 10]$

a.  $q(x) = \frac{v(x)}{g(x)} = \frac{\sqrt{10 - x}}{x - 1}$

b. The domain of  $q$  consists of all real numbers  $x$  simultaneously in the domains of  $v$  and  $g$  such that  $g(x) \neq 0$ . Since

$$g(x) = 0 \text{ when } x - 1 = 0, \text{ or } x = 1$$

we exclude  $x = 1$  from the domain of the quotient. Thus, the domain consists of all  $x$  in  $[0, 10]$  excluding  $x = 1$  (since  $g(1) = 0$ ), or  $0 \leq x \leq 10; x \neq 1$ .

c. As 1 is not in the domain of  $q$ ,  $q(1)$  is not defined.

6.  $g(x) = x - 1$  with domain  $(-\infty, \infty)$

$v(x) = \sqrt{10 - x}$  with domain  $[-\infty, 10]$

a.  $q(x) = \frac{g(x)}{v(x)} = \frac{x - 1}{\sqrt{10 - x}}$

b. The domain of  $q$  consists of all real numbers  $x$  simultaneously in the domains of  $v$  and  $g$  such that  $v(x) \neq 0$ . Since

$$v(x) = 0 \text{ when } \sqrt{10 - x} = 0, \text{ or } x = 10$$

we exclude  $x = 10$  from the domain of the quotient. Thus, the domain consists of all  $x$  in  $[0, 10]$  excluding  $x = 10$ ; that is,  $[0, 10)$ .

## Solutions Section 1.2

c. 1 is in the domain of  $q$ , and  $q(1) = \frac{1-1}{\sqrt{10-1}} = 0$

7.  $f(x) = x^2 + 1$  with domain  $(-\infty, \infty)$

a.  $m(x) = 5f(x) = 5(x^2 + 1)$

b. The domain of  $m$  is the same as the domain of  $f : (-\infty, \infty)$ .

c.  $m(1) = 5f(1) = 5(1^2 + 1) = 10$

8.  $u(x) = \sqrt{x+10}$  with domain  $[-10, \infty)$

a.  $m(x) = 3u(x) = 3\sqrt{x+10}$

b. The domain of  $m$  is the same as the domain of  $u : [-10, \infty)$

c.  $m(-1) = 3u(-1) = 3\sqrt{-1+10} = 9$

9. Number of music files = Starting number + New files =  $200 + 10 \times$  Number of days

So,  $N(t) = 200 + 10t$  ( $N$  = number of music files,  $t$  = time in days)

10. Free space left = Current amount – Decrease =  $50 - 5 \times$  Number of months

So,  $S(t) = 50 - 5t$  ( $S$  = space on your HD,  $t$  = time in months)

11. The number of hours you study,  $h(n)$ , equals 4 on Sunday through Thursday and equals 0 on the remaining days. Since Sunday corresponds to  $n = 1$  and Thursday to  $n = 5$ , we get

$$\begin{cases} 4 & \text{if } 1 \leq n \leq 5 \\ 0 & \text{if } n > 5 \end{cases}$$

12. The number of hours you watch movies,  $h(n)$ , equals 5 on Saturday ( $n = 7$ ) and Sunday ( $n = 1$ ) and equals 2 on the remaining days.

$$\begin{cases} 5 & \text{if } n = 1 \text{ or } n = 7 \\ 2 & \text{otherwise} \end{cases}$$

13. For a linear cost function,  $C(x) = mx + b$ . Here,  $m$  = marginal cost = \$1,500 per piano,  $b$  = fixed cost = \$1,000.

Thus, the daily cost function is

$$C(x) = 1,500x + 1,000.$$

a. The cost of manufacturing 3 pianos is

$$C(3) = 1,500(3) + 1,000 = 4,500 + 1,000 = \$5,500.$$

b. The cost of manufacturing each additional piano (such as the third one or the 11th one) is the marginal cost,  $m = \$1,500$ .

c. Same answer as (b).

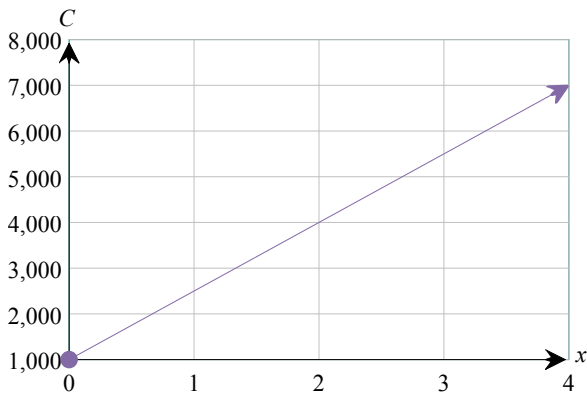
d. Variable cost = part of the cost function that depends on  $x = \$1,500x$

Fixed cost = constant summand of the cost function = \$1,000

Marginal cost = slope of the cost function = \$1,500 per piano

## Solutions Section 1.2

e. Graph:



**14.** For a linear cost function,  $C(x) = mx + b$ . Here,  $m$  = marginal cost = \$88 per tuxedo,  $b$  = fixed cost = \$20.

Thus, the cost function is

$$C(x) = 88x + 20.$$

a. The cost of renting 2 tuxes is

$$C(2) = 88(2) + 20 = \$196$$

b. The cost of each additional tux is the marginal cost  $m = \$88$ .

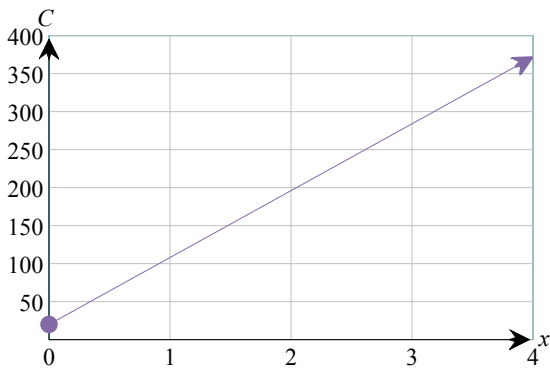
c. Same answer as (b).

d. Variable cost = part of the cost function that depends on  $x = \$88x$

Fixed cost = constant summand of the cost function = \$20

Marginal cost = slope of the cost function = \$88 per tuxedo

e. Graph:



**15. a.** For a linear cost function,  $C(x) = mx + b$ . Here,  $m$  = marginal cost = \$0.40 per copy,  $b$  = fixed cost = \$70.

Thus, the cost function is  $C(x) = 0.4x + 70$ .

The revenue function is  $R(x) = 0.50x$ . ( $x$  copies at 50¢ per copy)

The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.5x - (0.4x + 70) \\ &= 0.5x - 0.4x - 70 \\ &= 0.1x - 70 \end{aligned}$$

b.  $P(500) = 0.1(500) - 70 = 50 - 70 = -20$

Since  $P$  is negative, this represents a loss of \$20.

c. For breakeven,  $P(x) = 0$  :

## Solutions Section 1.2

$$\begin{aligned}0.1x - 70 &= 0 \\0.1x &= 70 \\x &= \frac{70}{0.1} = 700 \text{ copies}\end{aligned}$$

**16. a.** For a linear cost function,  $C(x) = mx + b$ . Here,  $m$  = marginal cost = \$0.15 per serving,  $b$  = fixed cost = \$350.

Thus, the cost function is  $C(x) = 0.15x + 350$ .

The revenue function is  $R(x) = 0.50x$ .

The profit function is

$$\begin{aligned}P(x) &= R(x) - C(x) \\&= 0.50x - (0.15x + 350) \\&= 0.35x - 350\end{aligned}$$

**b.** For break-even,  $P(x) = 0$  :

$$\begin{aligned}0.35x - 350 &= 0 \\0.35x &= 350 \\x &= 1,000 \text{ servings}\end{aligned}$$

**c.**  $P(1,500) = 0.35(1,500) - 350 = 525 - 350 = \$175$ , representing a profit of \$175.

**17.** The revenue per jersey is \$100. Therefore, Revenue  $R(x) = \$100x$ .

Profit = Revenue - Cost

$$\begin{aligned}P(x) &= R(x) - C(x) \\&= 100x - (2,000 + 10x + 0.2x^2) \\&= -2,000 + 90x - 0.2x^2\end{aligned}$$

To break even,  $P(x) = 0$ , so  $-2,000 + 90x - 0.2x^2 = 0$ .

This is a quadratic equation with  $a = -0.2$ ,  $b = 90$ ,  $c = -2,000$  and solution

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-90 \pm \sqrt{(90)^2 - 4(-2,000)(-0.2)}}{2(-0.2)} \approx 23.44 \text{ or } 426.56 \text{ jerseys.}\end{aligned}$$

Since the second value is outside the domain, we use the first:  $x = 23.44$  jerseys. To make a profit,  $x$  should be larger than this value: at least 24 jerseys.

**18.** The revenue per pair is \$120. Therefore, Revenue  $R(x) = \$120x$ .

Profit = Revenue - Cost

$$\begin{aligned}P(x) &= R(x) - C(x) \\&= 120x - (3,000 + 8x + 0.1x^2) \\&= -3,000 + 112 - 0.1x^2\end{aligned}$$

To break even,  $P(x) = 0$ , so  $-3,000 + 112 - 0.1x^2 = 0$ .

This is a quadratic equation with  $a = -0.1$ ,  $b = 112$ ,  $c = -3,000$  and solution

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-112 \pm \sqrt{(112)^2 - 4(-3,000)(-0.1)}}{2(-0.1)} \approx 27.46 \text{ or } 1092.54 \text{ jerseys.}\end{aligned}$$

## Solutions Section 1.2

Since the second value is outside the domain, we use the first:  $x = 27.46$  jerseys. To make a profit,  $x$  should be larger than this value: at least 28 pairs of cleats.

**19.** The revenue from one thousand square feet ( $x = 1$ ) is \$0.1 million. Therefore, Revenue  $R(x) = \$0.1x$ .

Profit = Revenue – Cost

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.1x - (1.7 + 0.12x - 0.0001x^2) \\ &= -1.7 - 0.02x + 0.0001x^2 \end{aligned}$$

To break even,  $P(x) = 0$ , so  $-1.7 - 0.02x + 0.0001x^2 = 0$ .

This is a quadratic equation with  $a = 0.0001$ ,  $b = -0.02$ ,  $c = -1.7$  and solution\

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{0.02 \pm \sqrt{(-0.02)^2 - 4(0.0001)(-1.7)}}{2(0.0001)} = \frac{0.02 \pm 0.03286}{0.0002} \approx 264 \text{ thousand square feet} \end{aligned}$$

**20.** The revenue from one thousand square feet ( $x = 1$ ) is \$0.2 million. Therefore, Revenue  $R(x) = \$0.2x$ .

Profit = Revenue – Cost

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.2x - (1.7 + 0.14x - 0.0001x^2) \\ &= -1.7 + 0.06x + 0.0001x^2 \end{aligned}$$

To break even,  $P(x) = 0$ , so  $-1.7 + 0.06x + 0.0001x^2 = 0$ .

This is a quadratic equation with  $a = 0.0001$ ,  $b = 0.06$ ,  $c = -1.7$  and solution\

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0.06 \pm \sqrt{(0.06)^2 - 4(0.0001)(-1.7)}}{2(0.0001)} = \frac{-0.06 \pm 0.0654}{0.0002} \approx 27 \text{ thousand square feet} \end{aligned}$$

**21.** The hourly profit function is given by

Profit = Revenue – Cost

$$P(x) = R(x) - C(x)$$

(Hourly) cost function: This is a fixed cost of \$10,200 only:

$$C(x) = 10,200$$

(Hourly) revenue function: This is a variable of \$200 per passenger cost only:

$$R(x) = 200x$$

Thus, the profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 200x - 10,200$$

For the domain of  $P(x)$ , the number of passengers  $x$  cannot exceed the capacity: 381. Also,  $x$  cannot be negative. Thus, the domain is given by  $0 \leq x \leq 381$ , or  $[0, 381]$ .

For breakeven,  $P(x) = 0$

$$200x - 10,200 = 0$$

$$200x = 10,200, \text{ or } x = \frac{10,200}{200} = 51$$

If  $x$  is larger than this, then the profit function is positive, and so there should be at least 52 passengers;  $x \geq 52$ , for a profit.

## Solutions Section 1.2

22. The hourly profit function is given by

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P(x) = R(x) - C(x)$$

(Hourly) cost function: This is a fixed cost of \$6,500 only:

$$C(x) = 6,500$$

(Hourly) revenue function: This is a variable of \$100 per passenger cost only:

$$R(x) = 200x$$

Thus, the profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 200x - 6,500$$

For the domain of  $P(x)$ , the number of passengers  $x$  cannot exceed the capacity: 150. Also,  $x$  cannot be negative. Thus, the domain is given by  $0 \leq x \leq 150$ , or  $[0, 150]$ .

For breakeven,  $P(x) = 0$

$$200x - 5,600 = 0$$

$$200x = 5,600, \text{ or } x = \frac{5,600}{200} = 28$$

If  $x$  is larger than this, then the profit function is positive, and so there should be at least 29 passengers;  $x \geq 29$ , for a profit.

23. To compute the break-even point, we use the profit function: Profit = Revenue - Cost

$$P(x) = R(x) - C(x)$$

$$R(x) = 2x \quad \$2 \text{ per unit}$$

$$C(x) = \text{Variable Cost} + \text{Fixed Cost}$$

$$= 40\% \text{ of Revenue} + 6,000 = 0.4(2x) + 6,000 = 0.8x + 6,000$$

Thus,  $P(x) = R(x) - C(x)$

$$P(x) = 2x - (0.8x + 6,000) = 1.2x - 6,000$$

For breakeven,  $P(x) = 0$

$$1.2x - 6,000 = 0$$

$$1.2x = 6,000, \text{ so } x = \frac{6,000}{1.2x} = 5,000$$

Therefore, 5,000 units should be made to break even.

24. To compute the break-even point, we use the profit function: Profit = Revenue - Cost

$$P(x) = R(x) - C(x)$$

$$R(x) = 5x \quad \$5 \text{ per unit}$$

$$C(x) = \text{Variable Cost} + \text{Fixed Cost}$$

$$= 30\% \text{ of Revenue} + 7,000 = 0.3(5x) + 7,000 = 1.5x + 7,000$$

Thus,  $P(x) = R(x) - C(x)$

$$P(x) = 5x - (1.5x + 7,000) = 3.5x - 7,000$$

For breakeven,  $P(x) = 0$

$$3.5x = 7,000, \text{ so } x = 2,000 \text{ units.}$$

25. To compute the break-even point, we use the revenue and cost functions:

$$R(x) = \text{Selling price} \times \text{Number of units} = SPx$$

$$C(x) = \text{Variable Cost} + \text{Fixed Cost} = VCx + FC$$

(Note that "variable cost per unit" is marginal cost.) For breakeven

$$R(x) = C(x)$$

$$SPx = VCx + FC$$

Solve for  $x$  :

$$SPx - VCx = FC$$

$$x(SP - VC) = FC, \text{ so } x = \frac{FC}{SP - VC}.$$

## Solutions Section 1.2

**26.** To compute the break-even point, we use the revenue and cost functions:

$$R(x) = SPx; C(x) = VCx + FC$$

At breakeven

$$R(BE) = C(BE)$$

$$SP(BE) = VC(BE) + FC$$

$$\text{Thus, } FC = SP(BE) - VC(BE) = BE(SP - VC).$$

**27.** Take  $x$  to be the number of grams of perfume he buys and sells. The profit function is given by Profit = Revenue – Cost: that is,  $P(x) = R(x) - C(x)$

Cost function  $C(x)$  :

$$\text{Fixed costs: } 20,000$$

$$\text{Cheap perfume @ \$20 per g: } 20x$$

$$\text{Transportation @ \$30 per 100 g: } 0.3x$$

Thus the cost function is

$$C(x) = 20x + 0.3x + 20,000 = 20.3x + 20,000$$

Revenue function  $R(x)$

$$R(x) = 600x \quad \text{\$600 per gram}$$

Thus, the profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 600x - (20.3x + 20,000) = 579.7x - 20,000, \text{ with domain } x \geq 0.$$

For breakeven,  $P(x) = 0$

$$579.7x - 20,000 = 0$$

$$579.7x = 20,000, \text{ so } x = \frac{20,000}{579.7} \approx 34.50$$

Thus, he should buy and sell 34.50 grams of perfume per day to break even.

**28.** Take  $x$  to be the number of grams of perfume he buys and sells. The profit function is given by

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Cost function: } C(x) = 400x + 30x = 430x$$

$$\text{Revenue: } R(x) = 420x$$

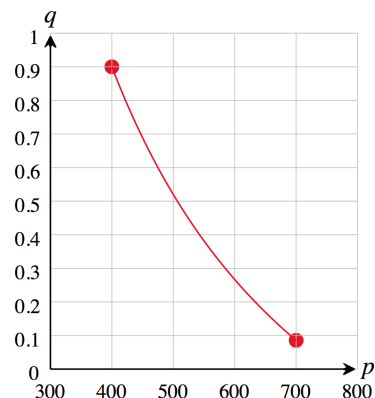
Thus, the profit function is  $P(x) = R(x) - C(x) = 420x - 430x = -10x$ , with domain  $x \geq 0$ .

For breakeven,  $-10x = 0$ , so  $x = 0$  grams per day; he should shut down the operation.

**29. a.** To graph the demand function we use technology with the formula

$$760/x - 1$$

and with xMin = 400 and xMax = 700. Graph:



$$\text{b. } q(400) = \frac{760}{400} - 1 = 0.9, \quad q(500) = \frac{760}{500} - 1 = 0.52$$

So, the change in demand is  $0.52 - 0.9 = -0.38$  billion units, which means that demand decreases by about 380 million units sold per year.

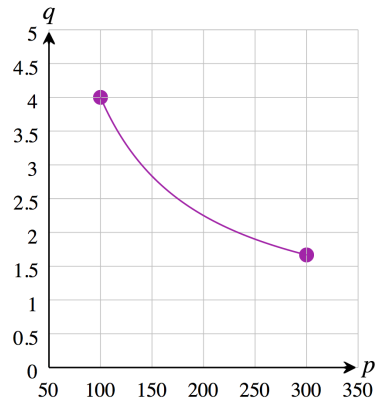
## Solutions Section 1.2

c. The value of  $q$  on the graph decreases by smaller and smaller amounts as we move to the right on the graph, indicating that the demand decreases at a smaller and smaller rate (Choice (D)).

30. a. To graph the demand function we use technology with the formula

$$350/x + 0.5$$

and with  $x_{\min} = 100$  and  $x_{\max} = 300$ . Graph:



b.  $q(100) = \frac{350}{100} + 0.5 = 4$ ,  $q(200) = \frac{350}{200} + 0.5 = 2.25$

So, the change in demand is  $2.25 - 4 = -1.75$  billion units, which means that demand decreases by about 1.75 billion units sold per year.

c. The value of  $q$  on the graph increases by larger and larger amounts as we move to the left on the graph, indicating that the demand increases at a greater and greater rate (Choice (A)).

31. The price at which there is neither a shortage nor surplus is the equilibrium price, which occurs when demand = supply:

$$-3p + 700 = 2p - 500$$

$$5p = 1200$$

$$p = \$240 \text{ per skateboard}$$

32. The price at which there is neither a shortage nor surplus is the equilibrium price, which occurs when demand = supply:

$$-5p + 50 = 3p - 30$$

$$8p = 80$$

$$p = \$10 \text{ per skateboard}$$

33. a. The equilibrium price occurs when demand = supply:

$$-p + 57 = 4p - 136$$

$$5p = 193$$

$$p = 38.6, \text{ or } \$38,600 \text{ per vehicle.}$$

b. Since \$38,400 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

$$\text{Demand: } q = -38.4 + 57 = 18.6 \text{ million vehicles}$$

$$\text{Supply: } q = 4(38.4) - 136 = 17.6 \text{ million vehicles}$$

$$\text{Shortage} = \text{Demand} - \text{Supply} = 18.6 - 17.6 = 1 \text{ million vehicles}$$

34. a. The equilibrium price occurs when demand = supply:

$$-0.15p + 23 = 3.85p - 131$$

$$4p = 154$$

$$p = 38.5, \text{ or } \$38,500 \text{ per vehicle.}$$

b. Since \$38,600 is above the equilibrium price, there would be a surplus at that price. To calculate it, compute demand and supply:

$$\text{Demand: } q = -0.15(38.6) + 23 = 17.21 \text{ million vehicles}$$

## Solutions Section 1.2

Supply:  $q = 3.85(38.6) - 131 = 17.61$  million vehicles

Surplus = Supply - Demand =  $17.61 - 17.21 = 0.4$  million vehicles

**35. a.** For equilibrium, Demand = Supply:

$$\frac{760}{p} - 1 = 0.00304p - 1$$

$$\frac{760}{p} = 0.00304p$$

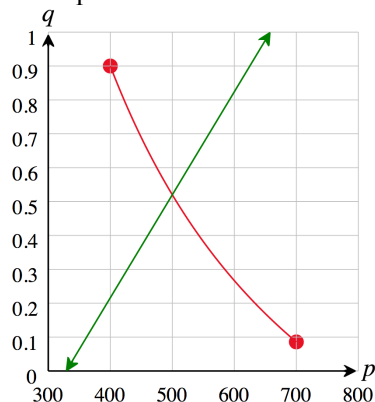
Cross-multiply:

$$0.00304p^2 = 760 \Rightarrow p^2 = \frac{760}{0.00304} = 250,000$$

So  $p = \sqrt{250,000} = \$500$ .

Thus, the equilibrium price is \$500, and the equilibrium demand (or supply) is  $760/500 - 1 = 0.52$  billion phones

**b.** Graph:



Technology formulas:

Demand:  $y = 760/x - 1$

Supply:  $y = 0.00304x - 1$

The graphs cross at (500, 0.52) confirming the calculation in part (a).

**c.** Since \$400 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

$$\text{Demand: } q = \frac{760}{400} - 1 = 0.9 \text{ billion phones}$$

$$\text{Supply: } 0.00304(400) - 1 = 0.216 \text{ billion phones}$$

$$\text{Shortage} = \text{Demand} - \text{Supply} \approx 0.9 - 0.216 = 0.684 \text{ billion phones, or 684 million phones.}$$

**36. a.** For equilibrium, Demand = Supply:

$$\frac{350}{p} + 0.5 = 0.0056p + 0.5$$

$$\frac{350}{p} = 0.0056p$$

Cross-multiply:

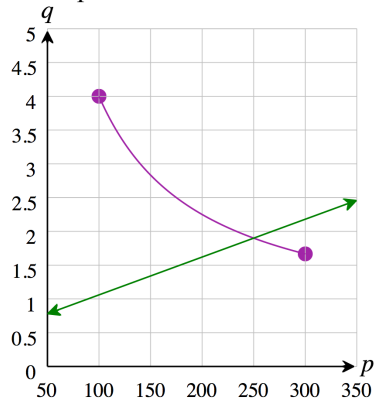
$$0.0056p^2 = 350 \Rightarrow p^2 = \frac{350}{0.0056} = 62,500$$

So  $p = \sqrt{62,500} = \$250$ .

Thus, the equilibrium price is \$250, and the equilibrium demand (or supply) is  $350/250 + 0.5 = 1.9$  billion phones

## Solutions Section 1.2

b. Graph:



Technology formulas:

Demand:  $y = 350/x + 0.5$

Supply:  $y = 0.0056x + 0.5$

The graphs cross at (250, 1.9) confirming the calculation in part (a).

c. Since \$200 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

$$\text{Demand: } q = \frac{350}{200} + 0.5 = 2.25 \text{ billion phones}$$

$$\text{Supply: } 0.0056(200) + 0.5 = 1.62 \text{ billion phones}$$

$$\text{Shortage} = \text{Demand} - \text{Supply} \approx 2.25 - 1.62 = 0.63 \text{ billion phones, or 630 million phones.}$$

37.  $C(q) = 2,000 + 100q^2$

a.  $C(10) = 2,000 + 100(10)^2 = 2,000 + 10,000 = \$12,000$

b.  $N = C - S$ , so

$$N(q) = C(q) - S(q) = 2,000 + 100q^2 - 500q$$

This is the cost of removing  $q$  lb of PCPs per day after the subsidy is taken into account.

c.  $N(20) = 2,000 + 100(20)^2 - 500(20) = 2,000 + 40,000 - 10,000 = \$32,000$

38.  $C(q) = 1,000 + 100\sqrt{q}$

a.  $C(100) = 1,000 + 100\sqrt{100} = 1,000 + 100(10) = \$2,000$

b.  $N = C - S$ , so

$$N(q) = C(q) - S(q) = 1,000 + 100\sqrt{q} - 200q$$

This is the cost for dental coverage to the company if it has  $q$  employees, after the subsidy is taken into account.

c.  $N(100) = 1,000 + 100\sqrt{100} - 200(100) = 1000 + 100(10) - 20,000 = -\$18,000$

The company makes \$18,000 from the government for dental coverage if it employs 100 people.

39. The technology formulas are:

(A)  $-0.2 \cdot t^2 + t + 16$

(B)  $0.2 \cdot t^2 + t + 16$

(C)  $t + 16$

The following table shows the values predicted by the three models:

## Solutions Section 1.2

|        |    |      |      |      |      |
|--------|----|------|------|------|------|
| $t$    | 0  | 2    | 4    | 6    | 7    |
| $S(t)$ | 16 | 18   | 22   | 28   | 30   |
| (A)    | 16 | 17.2 | 16.8 | 14.8 | 13.2 |
| (B)    | 16 | 18.8 | 23.2 | 29.2 | 32.8 |
| (C)    | 16 | 18   | 20   | 22   | 23   |

As shown in the table, the values predicted by model (B) are much closer to the observed values  $S(t)$  than those predicted by the other models.

b. Since 1998 corresponds to  $t = 8$ ,

$$S(t) = 0.2t^2 + t + 16$$

$$S(8) = 0.2(8)^2 + 8 + 16 = 36.8$$

So the spending on corrections in 1998 was predicted to be approximately \$37 billion.

40. The technology formulas are:

(A)  $16+2*t$

(B)  $16+t+0.5*t^2$

(C)  $16+t-0.5*t^2$

The following table shows the values predicted by the three models:

|        |    |    |    |    |      |
|--------|----|----|----|----|------|
| $t$    | 0  | 2  | 4  | 6  | 7    |
| $S(t)$ | 16 | 18 | 22 | 28 | 30   |
| (A)    | 16 | 20 | 24 | 28 | 30   |
| (B)    | 16 | 20 | 28 | 40 | 47.5 |
| (C)    | 16 | 16 | 12 | 4  | -1.5 |

As shown in the table, the values predicted by model (A) are much closer to the observed values  $S(t)$  than those predicted by the other models.

b. Since 1998 corresponds to  $t = 8$ ,

$$S(t) = 16 + 2t$$

$$S(8) = 16 + 2(8) = 32$$

So the spending on corrections in 1998 was predicted to be \$32 billion.

41. The technology formulas are:

(A)  $0.005*x+20.75$

(B)  $0.01*x+20+25/x$

(C)  $0.0005*x^2-0.07*x+23.25$

(The stars are optional for some technologies like graphing calculators and the Evaluator and Grapher App)

Here is result from the Evaluator and Grapher App:

**Functions box** (Cartesian mode):

| Enter function of $x$ |                             |
|-----------------------|-----------------------------|
| $y_1(x) =$            | $0.005x + 20.75$            |
| $y_2(x) =$            | $0.01x + 20 + 25/x$         |
| $y_3(x) =$            | $0.0005x^2 - 0.07x + 23.25$ |

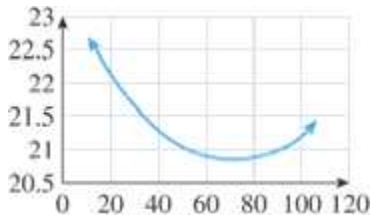
**Evaluator box** (Set to 2 decimal places):

| $x$ -Values | $y_1$ -Values | $y_2$ -Values | $y_3$ -Values |
|-------------|---------------|---------------|---------------|
| 5           | 20.78         | 25.05         | 22.91         |
| 25          | 20.88         | 21.25         | 21.81         |
| 40          | 20.95         | 21.03         | 21.25         |
| 100         | 21.25         | 21.25         | 21.25         |
| 125         | 21.38         | 21.45         | 22.31         |

Model (C) fits the data perfectly to two decimal places—more closely than any of the other models.

## Solutions Section 1.2

b. Graph of model (C):



$$0.0005x^2 - 0.07x + 23.25$$

The lowest point on the graph occurs at  $x = 70$  with a  $y$ -coordinate of 20.8. Thus, the lowest cost per shirt is \$20.80, which the team can obtain by buying 70 shirts.

42. The technology formulas are:

(A)  $0.05x + 20.75$

(B)  $0.1x + 20 + 25/x$

(C)  $0.0008x^2 - 0.07x + 23.25$

(The stars are optional for some technologies like graphing calculators and the Evaluator and Grapher App)

Here is result from the Evaluator and Grapher App:

**Functions box** (Cartesian mode):

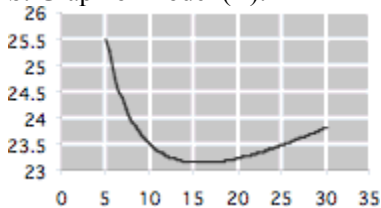
| Enter function of $x$ |                             |
|-----------------------|-----------------------------|
| $y_1(x) =$            | $0.05x + 20.75$             |
| $y_2(x) =$            | $0.1x + 20 + 25/x$          |
| $y_3(x) =$            | $0.0008x^2 - 0.07x + 23.25$ |

**Evaluator box** (Set to 2 decimal places):

| $x$ -Values | $y_1$ -Values | $y_2$ -Values | $y_3$ -Values |
|-------------|---------------|---------------|---------------|
| 5           | 21            | 25.5          | 22.92         |
| 25          | 22            | 23.5          | 22            |
| 40          | 22.75         | 24.63         | 21.73         |
| 100         | 25.75         | 30.25         | 24.25         |
| 125         | 27            | 32.7          | 27            |

Model (B) fits the data perfectly to two decimal places—more closely than any of the other models.

b. Graph of model (B):



$$0.1x + 20 + 25/x$$

The lowest point on the graph occurs at  $x = 16$  with a  $y$ -coordinate of 23.1625. Thus, the lowest cost per hat is \$23.16, which the team can obtain by buying 16 hats.

43. Here are the technology formulas as entered in the online Function Evaluator and Grapher at the Web Site:

(A)  $0.075 \cdot 0.83^x$

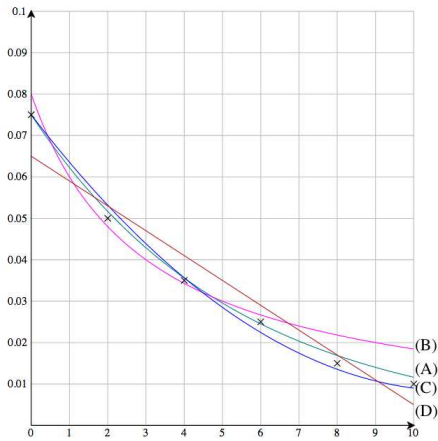
(B)  $0.24 / (x+3)$

(C)  $0.00054x^2 - 0.012x + 0.075$

(D)  $-0.006x + 0.065$

The following graph shows all four curves together with the plotted points (entered as shown in the margin technology note with Example 5).

## Solutions Section 1.2



As shown in the graph, the values predicted by models (A) and (C) are much closer to the observed values than those predicted by the other models.

**b.** Since 2030 corresponds to  $t = 20$ ,

$$\text{Model (A): } c(20) = 0.075 * 0.83^{20} \approx \$0.00181$$

$$\text{Model (C): } c(20) = 0.00054(20^2) - 0.012(20) + 0.075 \approx \$0.0510$$

So model (A) gives the lower price: approximately \$0.0018 per GB.

**44.** Here are the technology formulas as entered in the online Function Evaluator and Grapher at the Web Site:

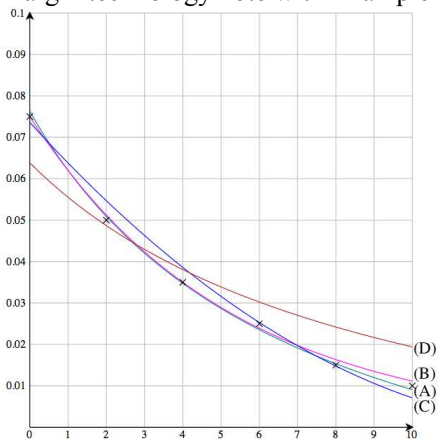
(A)  $-0.038 + 0.8 / (x + 7)$

(B)  $0.075 * 1.21^{-x}$

(C)  $0.00034 * (x - 14.5)^2$

(D)  $-0.022 + 0.8 / (x + 9.32)$

The following graph shows all three curves together with the plotted points (entered as shown in the margin technology note with Example 5).



As shown in the graph, the values predicted by models (A), (B), and (C) are much closer to the observed values than those predicted by (D).

**b.** Since 2030 corresponds to  $t = 20$ ,

$$\text{Model (A): } c(20) = -0.038 + \frac{0.8}{20 + 7} \approx -\$0.084 \text{ (unreasonable as prices cannot be negative)}$$

$$\text{Model (B): } c(20) = 0.075(1.21^{-20}) \approx \$0.0017 \text{ (reasonable given the current trends)}$$

Model (C):  $c(20) = 0.00034(20 - 14.5)^2 \approx \$0.010$  (unreasonable that the price would be the same again in 10 years)

**45.** A plot of the given points gives a straight line (Option (A)). Options (B) and (C) give curves, so (A) is the best choice.

## Solutions Section 1.2

**46.** Plotting the three data points suggests a concave down curve, suggesting that a linear model may not be the best fit. An exponential model  $p(t) = Ab^t$  would result in a concave up curve, regardless of whether  $b$  is larger than 1 or less than 1. This leaves a quadratic model as the only possible choice. In fact, a quadratic can always be found that passes through any three points not on the same straight line with different  $x$ -coordinates. Therefore, a quadratic model would give an exact fit.

**47.** A plot of the given data suggests a concave-down curve that becomes steeper downward as the price  $p$  increases, suggesting Model (D). Model (A) would predict increasing demand with increasing price, Model (B) would correspond to a descending curve that becomes less steep as  $p$  increases (a concave up curve), and Model (C) would give a concave-up parabola.

**48.** Model (B) is the best choice; Model (A) would predict increasing demand with increasing price, Model (D) would correspond to a concave down parabola, and Model (C), would predict demand that, rather than flattening out as the price increases, would begin to climb again.

**49.** Apply the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

with  $P = \$1,000$ ,  $r = 6/100 = 0.06$ ,  $n = 4$ ,  $t = 4$ .

$$\begin{aligned} A(4) &= 1,000\left(1 + \frac{0.06}{4}\right)^{4 \times 4} \\ &= 1,000(1.015)^{16} \approx \$1,268.99 \end{aligned}$$

**50.** Apply the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

with  $P = \$10,000$ ,  $r = 2/100 = 0.02$ ,  $n = 4$ ,  $t = 5$

$$\begin{aligned} A(5) &= 1,000\left(1 + \frac{0.02}{4}\right)^{4 \times 5} \\ &= 1,000(1.005)^{20} \approx \$11,048.96 \end{aligned}$$

**51.** Apply the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

with  $P = 5,000$ ,  $r = 0.15/100 = 0.0015$ , and  $n = 12$ . We get the model

$$A(t) = 5,000(1 + 0.0015/12)^{12t}$$

In June 2028 ( $t = 7$ ), the deposit would be worth  $5,000(1 + 0.0015/12)^{12(7)} \approx \$5,053$ .

**52.** Apply the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

with  $P = 4,000$ ,  $r = 0.0120$ , and  $n = 365$ . We get the model

$$A(t) = 4,000(1 + 0.0120/365)^{365t}$$

In March 2029 ( $t = 8$ ), the deposit would be worth  $4,000(1 + 0.0120/365)^{365(8)} \approx \$4,403$ .

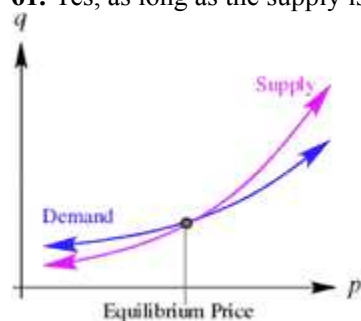
**53.**  $P(0) = 200$ ,  $P(1) = 230$ ,  $P(2) = 260$ , ... and so on. Thus, the population is increasing by 30 per year.

**54.**  $B(0) = 5000$ ,  $B(1) = 4800$ ,  $B(2) = 4600$ , ... and so on. Thus, the balance is decreasing by \$200 per day.

## Solutions Section 1.2

- 55.** Curve fitting. The model is based on fitting a curve to a given set of observed data.
- 56.** Analytical. The model is obtained by analyzing the situation being modeled.
- 57.** The given model is  $c(t) = 4 - 0.2t$ . This tells us that  $c$  is \$4 at time  $t = 0$  (January) and is decreasing by \$0.20 per month. So, the cost of downloading a movie was \$4 in January and is decreasing by 20¢ per month.
- 58.** The given model is  $c(t) = 4 - 0.2t$  and therefore passes through the points  $(t, c) = (0, 4)$  and  $(1, 3.8)$ . So, the cost of downloading a movie was \$4 in January and \$3.80 in February.
- 59.** In a linear cost function, the variable cost is  $x$  times the marginal cost.
- 60.** In a linear cost function, the marginal cost is the additional (or incremental) cost per item.

- 61.** Yes, as long as the supply is going up at a faster rate, as illustrated by the following graph:



- 62.** There would be a shortage at any given price. Therefore, consumers would be willing to pay more for a scarce commodity and sellers would naturally oblige by charging more, resulting in an upward spiral of prices for the commodity.



- 63.** Extrapolate both models and choose the one that gives the most reasonable predictions.
- 64.** No; as long as  $a$  is negative, the value of  $s(t)$  for large  $t$  will be negative, making the model unreasonable for large value of  $t$ .
- 65.** The value of  $f - g$  at  $x$  is  $f(x) - g(x)$ . Since  $f(x) \geq g(x)$  for every  $x$ , it follows that  $f(x) - g(x) \geq 0$  for every  $x$ .
- 66.** The value of  $\frac{f}{g}$  at  $x$  is  $\frac{f(x)}{g(x)}$ . Since  $f(x) > g(x) > 0$  for every  $x$ , it follows that  $\frac{f(x)}{g(x)} > 1$  for every  $x$ .
- 67.** Since the values of  $\frac{f}{g}$  at  $x$  are ratios  $\frac{f(x)}{g(x)}$ , it follows that the units of measurement of  $\frac{f}{g}$  are units of  $f$  per unit of  $g$ ; that is, books per person.
- 68.** Write  $f(x) = mx + b$  and  $g(x) = nx + c$ . Then  

$$f(x) - g(x) = mx + b - (nx + c) = (m - n)x + (b - c),$$
also a linear function.

## Section 1.3

1.

|   |    |   |   |
|---|----|---|---|
| x | -1 | 0 | 1 |
| y | 5  | 8 |   |

We calculate the slope  $m$  first. The first two points shown give changes in  $x$  and  $y$  of

$$\Delta x = 0 - (-1) = 1$$

$$\Delta y = 8 - 5 = 3 \text{ This gives a slope of}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3.$$

Now look at the second and third points: The change in  $x$  is again

$$\Delta x = 1 - 0 = 1$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = 3(1) = 3$$

This means that the missing value of  $y$  is

$$8 + \Delta y = 8 + 3 = 11.$$

2.

|   |    |    |   |
|---|----|----|---|
| x | -1 | 0  | 1 |
| y | -1 | -3 |   |

We calculate the slope  $m$  first. The first two points shown give changes in  $x$  and  $y$  of

$$\Delta x = 0 - (-1) = 1$$

$$\Delta y = -3 - (-1) = -2$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2.$$

Now look at the second and third points: The change in  $x$  is again

$$\Delta x = 1 - 0 = 1$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = -2(1) = -2$$

This means that the missing value of  $y$  is

$$-3 + \Delta y = -3 + (-2) = -5.$$

3.

|   |    |    |   |
|---|----|----|---|
| x | 2  | 3  | 5 |
| y | -1 | -2 |   |

We calculate the slope  $m$  first. The first two points shown give changes in  $x$  and  $y$  of

$$\Delta x = 3 - 2 = 1$$

$$\Delta y = -2 - (-1) = -1$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1.$$

Now look at the second and third points: The change in  $x$  is

$$\Delta x = 5 - 3 = 2$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = (-1)(2) = -2$$

This means that the missing value of  $y$  is

$$-2 + \Delta y = -2 + (-2) = -4.$$

## Solutions Section 1.3

4.

|     |    |    |   |
|-----|----|----|---|
| $x$ | 2  | 4  | 5 |
| $y$ | -1 | -2 |   |

We calculate the slope  $m$  first. The first two points shown give changes in  $x$  and  $y$  of

$$\Delta x = 4 - 2 = 2$$

$$\Delta y = -2 - (-1) = -1$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{2} = -\frac{1}{2}.$$

Now look at the second and third points: The change in  $x$  is

$$\Delta x = 5 - 4 = 1$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(-\frac{1}{2}\right)(1) = -\frac{1}{2}$$

This means that the missing value of  $y$  is

$$-2 + \Delta y = -2 + \left(-\frac{1}{2}\right) = -\frac{5}{2} \text{ or } -2.5.$$

5.

|     |    |   |    |
|-----|----|---|----|
| $x$ | -2 | 0 | 2  |
| $y$ | 4  |   | 10 |

We calculate the slope  $m$  first. The first and third points shown give changes in  $x$  and  $y$  of

$$\Delta x = 2 - (-2) = 4$$

$$\Delta y = 10 - 4 = 6$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{6}{4} = \frac{3}{2}.$$

Now look at the first and second points: The change in  $x$  is

$$\Delta x = 0 - (-2) = 2$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(\frac{3}{2}\right)(2) = 3$$

This means that the missing value of  $y$  is

$$4 + \Delta y = 4 + 3 = 7.$$

6.

|     |    |   |    |
|-----|----|---|----|
| $x$ | 0  | 3 | 6  |
| $y$ | -1 |   | -5 |

We calculate the slope  $m$  first. The first and third points shown give changes in  $x$  and  $y$  of

$$\Delta x = 6 - 0 = 6$$

$$\Delta y = -5 - (-1) = -4$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-4}{6} = -\frac{2}{3}.$$

Now look at the first and second points: The change in  $x$  is

$$\Delta x = 3 - 0 = 3$$

and so  $\Delta y$  must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(-\frac{2}{3}\right)(3) = -2$$

### Solutions Section 1.3

This means that the missing value of  $y$  is

$$-1 + \Delta y = -1 + (-2) = -3$$

7. From the table,  $b = f(0) = -2$ .

The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{0 - (-2)} = \frac{-1}{2} = -\frac{1}{2}.$$

Thus, the linear equation is

$$f(x) = mx + b = -\frac{1}{2}x - 2, \text{ or } f(x) = -\frac{x}{2} - 2.$$

8. From the table,  $b = f(0) = 3$ .

The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{(-3) - (-6)} = \frac{1}{3}$$

Thus, the linear equation is

$$f(x) = mx + b = \frac{1}{3}x + 3, \text{ or } f(x) = \frac{x}{3} + 3.$$

9. The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{-3 - (-4)} = \frac{-1}{1} = -1.$$

To obtain  $f(0) = b$ , use the formula for  $b$ :

$$f(0) = b = y_1 - mx_1 = -1 - (-1)(-4) = -5 \quad \text{Using the point } (x_1, y_1) = (-4, -1)$$

This gives

$$f(x) = mx + b = -x - 5.$$

10. The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4}{2 - 1} = \frac{2}{1} = 2$$

To obtain  $f(0) = b$ , use the formula for  $b$ :

$$f(0) = b = y_1 - mx_1 = 4 - (2)(1) = 2 \quad \text{Using the point } (x_1, y_1) = (1, 4)$$

This gives

$$f(x) = mx + b = 2x + 2.$$

11. In the table,  $x$  increases in steps of 1 and  $f$  increases in steps of 4, showing that  $f$  is linear with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{4}{1} = 4$$

and intercept

$$b = f(0) = 6$$

giving

$$f(x) = mx + b = 4x + 6.$$

The function  $g$  does not increase in equal steps, so  $g$  is not linear.

12. In the table,  $x$  increases in steps of 10 and  $g$  increases in steps of 5, showing that  $g$  is linear with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{5}{10} = \frac{1}{2}$$

and intercept

$$b = g(0) = -4$$

giving

$$g(x) = mx + b = \frac{1}{2}x - 4.$$

### Solutions Section 1.3

The function  $f$  does not increase in equal steps, so  $f$  is not linear.

**13.** In the first three points listed in the table,  $x$  increases in steps of 3, but  $f$  does not increase in equal steps, whereas  $g$  increases in steps of 6. Thus, based on the first three points, only  $g$  could possibly be linear, with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{6}{3} = 2$$

and intercept

$$b = g(0) = -1$$

giving

$$g(x) = mx + b = 2x - 1.$$

We can now check that the remaining points in the table fit the formula  $g(x) = 2x - 1$ , showing that  $g$  is indeed linear.

**14.** In the first and last pairs of points listed in the table,  $x$  increases in steps of 3, but  $f$  does not increase in equal steps, whereas  $g$  increases in steps of 9. Thus, based on those points, only  $g$  could possibly be linear, with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{9}{3} = 3$$

and intercept

$$b = g(0) = -1$$

giving

$$g(x) = mx + b = 3x - 1.$$

We can now check that the remaining points in the table fit the formula  $g(x) = 3x - 1$ , showing that  $g$  is indeed linear.

**15.** Slope = coefficient of  $x = -\frac{3}{2}$

**16.** Slope = coefficient of  $x = \frac{2}{3}$

**17.** Slope = coefficient of  $x = \frac{1}{6}$

**18.** Write the equation as  $y = -\frac{2x}{3} + \frac{1}{3}$

Slope = coefficient of  $x = -\frac{2}{3}$

**19.** If we solve for  $x$  we find that the given equation represents the vertical line  $x = -1/3$ , and so its slope is infinite (undefined).

**20.**  $8x - 2y = 1$ . Solving for  $y$  :

$$2y = 8x - 1$$

$$y = 4x - \frac{1}{2}$$

Slope = coefficient of  $x = 4$

**21.**  $3y + 1 = 0$ . Solving for  $y$  :

$$3y = -1$$

$$y = -\frac{1}{3}$$

## Solutions Section 1.3

Slope = coefficient of  $x = 0$

**22.** If we solve for  $x$  we find that the given equation represents the vertical line  $x = -3/2$ , and so its slope is infinite (undefined).

**23.**  $4x + 3y = 7$ . Solve for  $y$  :

$$3y = -4x + 7$$

$$y = -\frac{4}{3}x + \frac{7}{3}$$

Slope = coefficient of  $x = -\frac{4}{3}$

**24.**  $2y + 3 = 0$ . Solve for  $y$  :

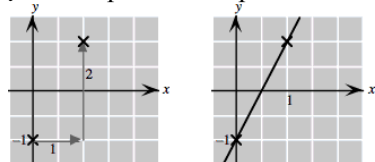
$$2y = -3$$

$$y = -\frac{3}{2}$$

Slope = coefficient of  $x = 0$

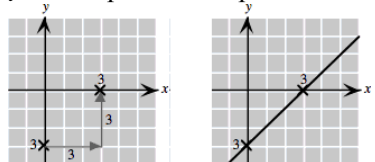
**25.**  $y = 2x - 1$

$y$ -intercept =  $-1$ , slope =  $2$

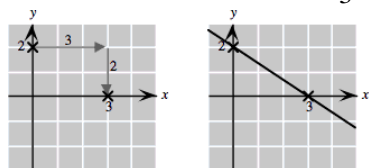


**26.**  $y = x - 3$

$y$ -intercept =  $-3$ , slope =  $1$

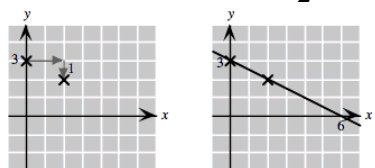


**27.**  $y$ -intercept =  $2$ , slope =  $-\frac{2}{3}$



**28.**  $y = -\frac{1}{2}x + 3$

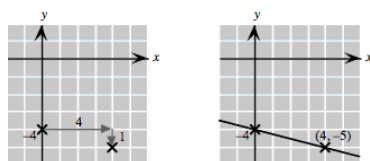
$y$ -intercept =  $3$ , slope =  $-\frac{1}{2}$



**29.**  $y + \frac{1}{4}x = -4$ . Solve for  $y$  to obtain  $y = -\frac{1}{4}x - 4$

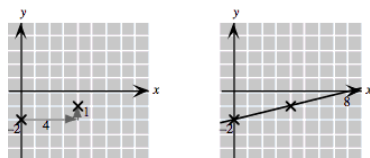
## Solutions Section 1.3

y-intercept =  $-4$ , slope =  $-\frac{1}{4}$



**30.**  $y - \frac{1}{4}x = -2$ . Solve for  $y$  to obtain  $y = \frac{1}{4}x - 2$

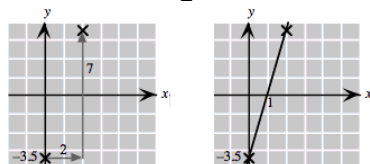
y-intercept =  $-2$ , slope =  $\frac{1}{4}$



**31.**  $7x - 2y = 7$ . Solve for  $y$  :

$$-2y = -7x + 7, \text{ so } y = \frac{7}{2}x - \frac{7}{2}$$

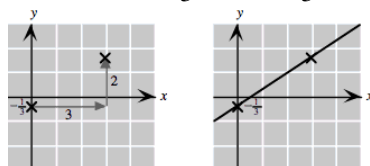
y-intercept =  $-\frac{7}{2} = -3.5$ , slope =  $\frac{7}{2} = 3.5$



**32.**  $2x - 3y = 1$ . Solve for  $y$  :

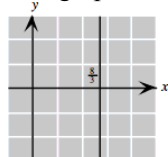
$$-3y = -2x + 1, \text{ so } y = \frac{2}{3}x - \frac{1}{3}$$

y-intercept =  $-\frac{1}{3}$ , slope =  $\frac{2}{3}$



**33.**  $3x = 8$ . Solve for  $x$  to obtain  $x = \frac{8}{3}$ .

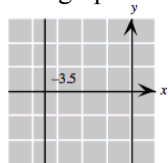
The graph is a vertical line:



### Solutions Section 1.3

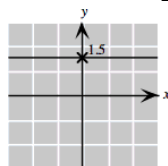
34.  $2x = -7$ . Solve for  $x$  to obtain  $x = -\frac{7}{2} = -3.5$ .

The graph is a vertical line:



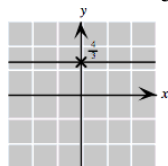
35.  $6y = 9$ . Solve for  $y$  to obtain  $y = \frac{9}{6} = \frac{3}{2} = 1.5$

$y$ -intercept  $= \frac{3}{2} = 1.5$ , slope  $= 0$ . The graph is a horizontal line:



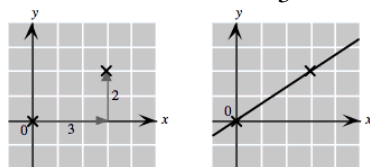
36.  $3y = 4$ . Solve for  $y$  to obtain  $y = \frac{4}{3}$

$y$ -intercept  $= \frac{4}{3}$ , slope  $= 0$ . The graph is a horizontal line:



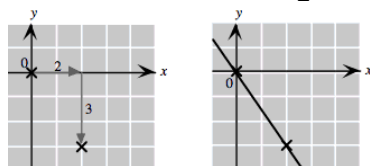
37.  $2x = 3y$ . Solve for  $y$  to obtain  $y = \frac{2}{3}x$

$y$ -intercept  $= 0$ , slope  $= \frac{2}{3}$



38.  $3x = -2y$ . Solve for  $y$  to obtain  $y = -\frac{3}{2}x$

$y$ -intercept  $= 0$ , slope  $= -\frac{3}{2}$



39.  $(0, 0)$  and  $(1, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{1 - 0} = 2$$

40.  $(0, 0)$  and  $(-1, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-1 - 0} = -2$$

### Solutions Section 1.3

41.  $(-1, -2)$  and  $(0, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-2)}{0 - (-1)} = 2$$

42.  $(2, 1)$  and  $(0, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{0 - 2} = \frac{1}{2}$$

43.  $(4, 3)$  and  $(5, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{5 - 4} = \frac{-2}{1} = -2$$

44.  $(4, 3)$  and  $(4, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 4} \text{ Undefined}$$

45.  $(1, -1)$  and  $(1, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{1 - 1} \text{ Undefined}$$

46.  $(-2, 2)$  and  $(-1, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{-1 - (-2)} = \frac{-3}{1} = -3$$

47.  $(2, 3.5)$  and  $(4, 6.5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.5 - 3.5}{4 - 2} = \frac{3}{2} = 1.5$$

48.  $(10, -3.5)$  and  $(0, -1.5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1.5 - (-3.5)}{0 - 10} = \frac{2}{-10} = -0.2$$

49.  $(300, 20.2)$  and  $(400, 11.2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11.2 - 20.2}{400 - 300} = \frac{-9}{100} = -0.09$$

50.  $(1, -20.2)$  and  $(2, 3.2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.2 - (-20.2)}{2 - 1} = \frac{23.4}{1} = 23.4$$

51.  $(0, 1)$  and  $(-\frac{1}{2}, \frac{3}{4})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3/4 - 1}{-1/2 - 0} = \frac{-1/4}{-1/2} = \frac{2}{4} = \frac{1}{2}$$

52.  $(\frac{1}{2}, 1)$  and  $(-\frac{1}{2}, \frac{3}{4})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3/4 - 1}{-1/2 - 1/2} = \frac{-1/4}{-1} = \frac{1}{4}$$

53.  $(a, b)$  and  $(c, d)$  ( $a \neq c$ )

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - b}{c - a}$$

54.  $(a, b)$  and  $(c, b)$  ( $a \neq c$ )

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b - b}{c - a} = 0$$

55.  $(a, b)$  and  $(a, d)$  ( $b \neq d$ )

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - b}{a - a} \text{ Undefined}$$

56.  $(a, b)$  and  $(-a, -b)$  ( $a \neq 0$ )

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-b - b}{-a - a} = \frac{-2b}{-2a} = \frac{b}{a}$$

57.  $(-a, b)$  and  $(a, -b)$  ( $a \neq 0$ )

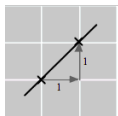
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-b - b}{a - (-a)} = \frac{-2b}{2a} = -\frac{b}{a}$$

58.  $(a, b)$  and  $(b, a)$  ( $a \neq b$ )

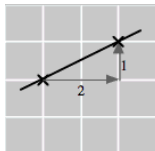
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - b}{b - a} = -1$$

## Solutions Section 1.3

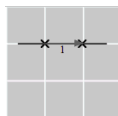
59. a.  $m = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$



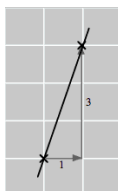
b.  $m = \frac{\Delta y}{\Delta x} = \frac{1}{2}$



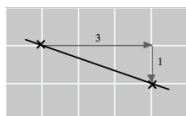
c.  $m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = 0$



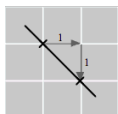
d.  $m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$



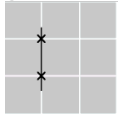
e.  $m = \frac{\Delta y}{\Delta x} = \frac{-1}{3} = -\frac{1}{3}$



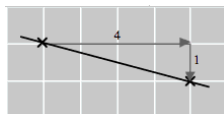
f.  $m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1$



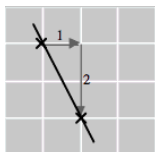
g. Vertical line; undefined slope



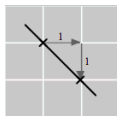
h.  $m = \frac{\Delta y}{\Delta x} = \frac{-1}{4} = -\frac{1}{4}$



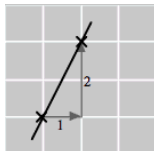
i.  $m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$



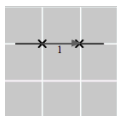
60. a.  $m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1$



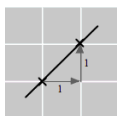
b.  $m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$



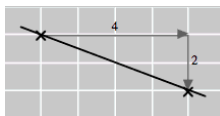
c.  $m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = 0$



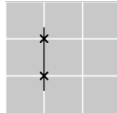
d.  $m = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$



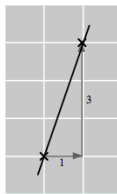
e.  $m = \frac{\Delta y}{\Delta x} = \frac{-2}{4} = -\frac{1}{2}$



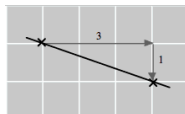
f. Vertical line; undefined slope



g.  $m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$



h.  $m = \frac{\Delta y}{\Delta x} = \frac{-1}{3} = -\frac{1}{3}$



i.  $m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$

