

***Two Phase Flow Boiling and Condensation In
Conventional and Miniature Systems 2nd edition
Ghiaasiaan Solutions Manual***

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Problem 2.1

We have

$$\frac{dP}{dx} = \mu \frac{\partial^2 U}{\partial y^2} \quad (1)$$

Also

$$\frac{dP}{dy} = -\rho_L g \quad (2)$$

Assuming incompressible flow and steady-state, mass conservation requires that:

$$\int_0^H U(y) dy = 0 \quad (3)$$

The velocity boundary conditions are:

$$U = 0 \quad \text{at } y = 0 \quad (4)$$

$$\mu \frac{\partial U}{\partial y} = \frac{d\delta}{dx} \quad \text{at } y = H \quad (5)$$

Eq. (1) can now be integrated, with boundary conditions (4) & (5), to get:

$$\mu U = \left(\frac{d\delta}{dx} - H \frac{\partial P}{\partial x} \right) y + \frac{1}{2} \frac{\partial P}{\partial x} y^2 \quad (6)$$

Given that at any location $P = P_\infty + \rho_L g (H - y)$,
then

$$\frac{\partial P}{\partial x} = \rho_L g \frac{dH}{dx} \quad (7)$$

We can now substitute for U in Eq. (3) from Eq. (6), and perform the integration to get

$$\frac{\partial \sigma}{\partial x} = \frac{2}{3} H \frac{\partial P}{\partial x} \quad (8)$$

Elimination of $\frac{\partial P}{\partial x}$ between Eqs. (7) and (8) then gives

$$\frac{d\sigma}{dx} = \frac{2}{3} \rho_L g H \frac{dH}{dx} \quad (9)$$

Substitution in Eq. (6), from (7) and (9) eventually leads to the desired result.

Reference:

R. F. Probstein, Physicochemical Hydrodynamics, Butterworth, 1989.

Problem 2.3

Referring to Fig 1 below, at an arbitrary point Q we must have

$$r_1 + r_2 = \frac{\sigma}{P_B - P_\infty} \quad (1)$$

where r_1 & r_2 are the principle radii of curvature. Taking r_1 as the radius in the plane of the page, we can write

$$r_1 = \left[\frac{(1 + y'^2)^{3/2}}{|y''|} \right]_Q \quad (2)$$

where $y' = dy/dx$ on the bubble surface.

The radius of curvature in the plane perpendicular to the page and passing through the point Q is r_2 as shown in the figure (note that the bubble is axis-symmetric), and it can be easily shown from geometry that

$$r_2 = \left\{ |x| \left[1 + \frac{1}{y'^2} \right]^{1/2} \right\} \quad (3)$$

Combining Eqs. (1) ~ (3), the equation

defining the shape of the bubble's profile is derived as

$$\frac{(1+y'^2)^{\frac{3}{2}}}{|y''|} + |x| \left[1 + \frac{1}{y'^2} \right]^{1/2} = \frac{\sigma}{P_B - P_\infty} \quad (4)$$

To find $P_B - P_\infty$, we note that it is a constant, and must apply to point D as well. At point D, we can assume the bubble's surface is spherical with a radius of curvature $r_{c0}/\sin\theta_0$. Then

$$P_B - P_\infty = \frac{2\sigma_w}{r_{c0}/\sin\theta_0} \quad (5)$$

The right side of Eq. (4) then becomes

$$\frac{\sigma r_{c0}/\sin\theta_0}{2\sigma_w} \quad (6)$$

Substitution of Eq. (6) into Eq. (4) then gives a second order ordinary differential equation. The (boundary (initial) conditions for the equation will be:

$$\text{At } x = r_{c0}$$

$$y = 0$$

$$\frac{dy}{dx} \approx y' = \tan \theta_0$$

Now, non-dimensionalization using the definitions of X and Y , will give the desired dimensionless second-order ordinary differential equation. The initial conditions will then be:

$$\text{at } X = 0$$

$$Y = 0$$

$$\frac{dY}{dX} = \tan \theta_0$$

2.3-4

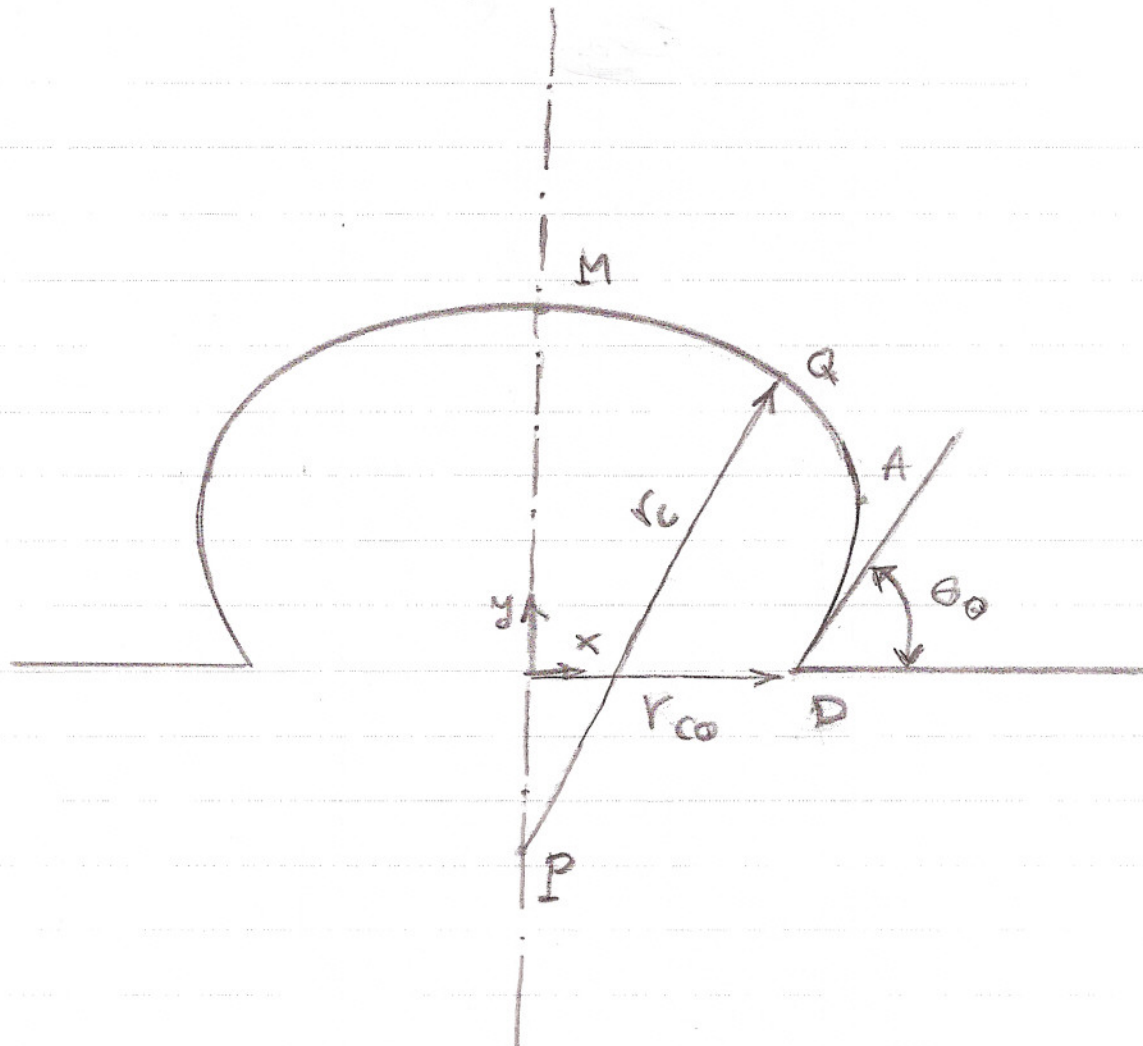


Figure 1

(Problem 2.3)

Problem 2.4

a)

$$X_G = 3.774 \times 10^{-4} \quad (\text{CO}_2 \text{ mole fraction in air})$$

$$C_{\text{He}, 290} = 1.28 \times 10^8 \text{ Pa} \quad (C_{\text{He}} \text{ at } 290 \text{ K})$$

$$C_{\text{He}, 300} = 1.71 \times 10^8 \text{ Pa} \quad (C_{\text{He}} \text{ at } 300 \text{ K})$$

Apply Henry's law to the interphase

$$P_0 = 101330 \text{ Pa}$$

$$X_{L, 290} = P_0 X_G / C_{\text{He}, 290} = 2.988 \times 10^{-7}$$

$$X_{L, 300} = P_0 X_G / C_{\text{He}, 300} = 2.236 \times 10^{-7}$$

$$m_{L, 290} = X_{L, 290} \frac{M_{\text{CO}_2}}{M_{\text{H}_2\text{O}}} = 7.30 \times 10^{-7}$$

$$m_{L, 300} = X_{L, 300} \frac{M_{\text{CO}_2}}{M_{\text{H}_2\text{O}}} = 5.47 \times 10^{-7}$$

b) Interpolation gives, at 287.5°C

$$C_{\text{He}} = 1.17 \times 10^8 \text{ Pa}$$

$$X_L = P_0 \frac{X_G}{C_{\text{He}}} = 3.264 \times 10^{-7}$$

$$m_L = X_L M_{\text{CO}_2} / M_{\text{H}_2\text{O}} = 7.984 \times 10^{-7}$$

$$c) \quad V_W = 1.35 \times 10^{18} \text{ m}^3$$

$$m_{\text{mass}, W} = \rho_L V_W = 1.35 \times 10^{21} \text{ kg} \quad (\text{Water mass})$$

2.4-2

$$X_{G,1} = 3.756 \times 10^{-4}$$

$$X_{L,1} = P_0 X_{G,1} / C_{H_2O} = 3.248 \times 10^{-7}$$

$$m_{L,1} = X_{L,1} M_{CO_2} / M_{H_2O} = 7.94 \times 10^{-7}$$

$$mass_{CO_2,1} = V_w m_{L,1} = 1.072 \times 10^{15}$$

$$\Delta mass_{CO_2} = mass_{CO_2} - mass_{CO_2,1} = 5.137 \times 10^{12} \text{ kg}$$

This is change in the mass of dissolved CO_2 , if the oceans remain at equilibrium with the atmosphere.

$$Ratio = \frac{\Delta mass_{CO_2}}{7.25 \times 10^{12}} = 0.7086$$

*{Example 2.5}**{Based on assumptions, $T_s=25\text{ C}$, and concentration of CO_2 at s surface is zero.}* $\text{MM}_v=18.$ $\text{MM}_{\text{air}}=29.$ $\text{MM}_{\text{CO}_2}=44$ $D_d=0.55\text{e-}3$ $U_d=2.$ $T_G=25.+273.$ $T_L=25.+273.$ $P_G=1.0133\text{e}5$ $\phi=0.6$ $P_{\text{sat}}=P_{\text{sat}}(\text{Water}, T=T_G)$ $P_v=\phi*P_{\text{sat}}$ $X_v=P_v/P_G$ $m_v=X_v*\text{MM}_v/(X_v*\text{MM}_v+(1.-X_v)*\text{MM}_{\text{air}})$ $X_{\text{vs}}=P_{\text{sat}}/P_G$ $m_{\text{vs}}=X_{\text{vs}}*\text{MM}_v/(X_{\text{vs}}*\text{MM}_v+(1.-X_{\text{vs}})*\text{MM}_{\text{air}})$ $\mu_G=\text{viscosity}(\text{AIR}, T=T_G)$ $\rho_G=\text{density}(\text{AIR}, T=T_G, P=P_G)$ $k_G=\text{conductivity}(\text{AIR}, T=T_G)$ $\nu_G=\mu_G/\rho_G$ $C_{\text{PG}}=\text{cp}(\text{STEAM}, T=T_G, x=1)$ $D_{12}=(1.97\text{e-}5)*(101330./P_G)*((T_G/256.)^{(1.685)})$ $\text{Sc}_G=\nu_G/D_{12}$ $\text{Pr}_G=\text{prandtl}(\text{AIR}, T=T_G)$ $\text{Sh}_G=2.$ $\text{Sh}_G=K_{\text{massG}}*D_d/(\rho_G*D_{12})$ $\text{Nus}_G=2.$ $\text{Nus}_G=H_G*D_d/k_G$ *{Part a: First neglect the blowing effect}* $m_{\text{dd_va}}=K_{\text{massG}}*(m_{\text{vs}}-m_v)$ *{Now include blowing effect}* $B_{\text{mG}}=(m_v-m_{\text{vs}})/(m_{\text{vs}}-1.)$ $m_{\text{dd_va}}=K_{\text{massG}}*\ln(1.+B_{\text{mg}})$ *{Part b: Now we have heat transfer as well. However, by fixing T_s at 50 C, evaporation rate will also be fixed, and will be decoupled from HT}* $T_{\text{sb}}=273+50$ $P_{\text{vsb}}=P_{\text{sat}}(\text{water}, T=T_{\text{sb}})$ $X_{\text{vsb}}=P_{\text{vsb}}/P_G$ $m_{\text{vsb}}=X_{\text{vsb}}*\text{MM}_v/(X_{\text{vsb}}*\text{MM}_v+(1.-X_{\text{vsb}})*\text{MM}_{\text{air}})$ $B_{\text{mGb}}=(m_v-m_{\text{vsb}})/(m_{\text{vsb}}-1.)$ $m_{\text{dd_vb}}=K_{\text{massG}}*\ln(1.+B_{\text{mgb}})$ *{c: This time we deal with CO_2 mass transfer rate. Let us solve for case (a)}* $m_{\text{CO}_2\text{s}}=0.0$ $X_{\text{CO}_2\text{G}}=500.\text{e-}6$ $m_{\text{CO}_2\text{G}}=X_{\text{CO}_2\text{G}}*\text{MM}_{\text{CO}_2}/(X_{\text{CO}_2\text{G}}*\text{MM}_{\text{CO}_2}+X_v*\text{MM}_v+(1.-X_v-X_{\text{CO}_2\text{G}})*\text{MM}_{\text{air}})$ $D_{\text{aCO}_2}=0.16\text{e-}4$ $\text{Sh}_G=K_{\text{massCO}_2}*D_d/(\rho_G*D_{\text{aCO}_2})$ *{First without the blowing effect:}* $m_{\text{ddCO}_2}=K_{\text{massCO}_2}*(m_{\text{CO}_2\text{s}}-m_{\text{CO}_2\text{G}})$ *{Now with blowing}* $m_{\text{ddtot}}=m_{\text{dd_va}}+m_{\text{ddCO}_2\text{b}}$ $B_{\text{mGCO}_2}=(m_{\text{CO}_2\text{G}}-m_{\text{CO}_2\text{s}})/(m_{\text{CO}_2\text{s}}-(m_{\text{ddCO}_2\text{b}}/m_{\text{ddtot}}))$ $m_{\text{ddCO}_2\text{b}}=m_{\text{ddtot}}*m_{\text{CO}_2\text{s}}+K_{\text{massCO}_2}*(\ln(1.+B_{\text{mGCO}_2})/B_{\text{mGCO}_2})*(m_{\text{CO}_2\text{s}}-m_{\text{CO}_2\text{G}})$ $\text{MM}_v = 18$ $\text{MM}_{\text{air}} = 29$

$$MM_{CO_2} = 44$$

$$D_d = 0.00055$$

$$U_d = 2$$

$$T_G = 25 + 273$$

$$T_L = 25 + 273$$

$$P_G = 101330$$

$$\phi = 0.6$$

$$P_{sat} = P_{sat} \left[\text{'Water'}, T = T_G \right]$$

$$P_v = \phi \cdot P_{sat}$$

$$X_v = \frac{P_v}{P_G}$$

$$m_v = X_v \cdot \left[\frac{MM_v}{X_v \cdot MM_v + (1 - X_v) \cdot MM_{air}} \right]$$

$$X_{vs} = \frac{P_{sat}}{P_G}$$

$$m_{vs} = X_{vs} \cdot \left[\frac{MM_v}{X_{vs} \cdot MM_v + (1 - X_{vs}) \cdot MM_{air}} \right]$$

$$\mu_G = \mathbf{Visc} \left[\text{'Air'}, T = T_G \right]$$

$$\rho_G = \rho \left[\text{'Air'}, T = T_G, P = P_G \right]$$

$$k_G = \mathbf{k} \left[\text{'Air'}, T = T_G \right]$$

$$v_G = \frac{\mu_G}{\rho_G}$$

$$C_{PG} = \mathbf{Cp} \left[\text{'Steam'}, T = T_G, x = 1 \right]$$

$$D_{12} = 0.0000197 \cdot \frac{101330}{P_G} \cdot \left[\frac{T_G}{256} \right]^{1.685}$$

$$Sc_G = \frac{v_G}{D_{12}}$$

$$Pr_G = \mathbf{Pr} \left[\text{'Air'}, T = T_G \right]$$

$$Sh_G = 2$$

$$Sh_G = K_{massG} \cdot \frac{D_d}{\rho_G \cdot D_{12}}$$

$$\text{Nus}_G = 2$$

$$\text{Nus}_G = \text{H}_G \cdot \frac{D_d}{k_G}$$

$$m_{dd,va1} = K_{\text{mass}G} \cdot [m_{vs} - m_v]$$

$$B_{mG} = \frac{m_v - m_{vs}}{m_{vs} - 1}$$

$$m_{dd,va} = K_{\text{mass}G} \cdot \ln [1 + B_{mG}]$$

$$T_{sb} = 273 + 50$$

$$P_{vsb} = P_{\text{sat}} [\text{'Water'}, T = T_{sb}]$$

$$X_{vsb} = \frac{P_{vsb}}{P_G}$$

$$m_{vsb} = X_{vsb} \cdot \left[\frac{MM_v}{X_{vsb} \cdot MM_v + (1 - X_{vsb}) \cdot MM_{\text{air}}} \right]$$

$$B_{mGb} = \frac{m_v - m_{vsb}}{m_{vsb} - 1}$$

$$m_{dd,vb} = K_{\text{mass}G} \cdot \ln [1 + B_{mGb}]$$

$$m_{\text{CO}2s} = 0$$

$$X_{\text{CO}2G} = 0.0005$$

$$m_{\text{CO}2G} = X_{\text{CO}2G} \cdot \left[\frac{MM_{\text{CO}2}}{X_{\text{CO}2G} \cdot MM_{\text{CO}2} + X_v \cdot MM_v + (1 - X_v - X_{\text{CO}2G}) \cdot MM_{\text{air}}} \right]$$

$$D_{a\text{CO}2} = 0.000016$$

$$\text{Sh}_G = K_{\text{massCO}2} \cdot \frac{D_d}{\rho_G \cdot D_{a\text{CO}2}}$$

$$m_{dd\text{CO}2} = K_{\text{massCO}2} \cdot [m_{\text{CO}2s} - m_{\text{CO}2G}]$$

$$m_{\text{ddtot}} = m_{dd,va} + m_{dd\text{CO}2b}$$

$$B_{m\text{GCO}2} = \frac{m_{\text{CO}2G} - m_{\text{CO}2s}}{m_{\text{CO}2s} - \frac{m_{dd\text{CO}2b}}{m_{\text{ddtot}}}}$$

$$m_{dd\text{CO}2b} = m_{\text{ddtot}} \cdot m_{\text{CO}2s} + K_{\text{massCO}2} \cdot \frac{\ln [1 + B_{m\text{GCO}2}]}{B_{m\text{GCO}2}} \cdot [m_{\text{CO}2s} - m_{\text{CO}2G}]$$

SOLUTION

Unit Settings: [J]/[K]/[Pa]/[kg]/[degrees]

$$B_{mG} = 0.007998$$

$$B_{mGb} = 0.07276$$

$$B_{m\text{GCO}2} = 0.01198$$

CPG = 1887	D ₁₂ = 0.00002545	D _{aCO2} = 0.000016
D _d = 0.00055	H _G = 92.72	k _G = 0.0255
K _{massCO2} = 0.06893	K _{massG} = 0.1096	MM _{air} = 29
MM _{CO2} = 44	MM _v = 18	μ _G = 0.00001848
m _{CO2G} = 0.0007638	m _{CO2s} = 0	m _{ddCO2} = -0.00005265
m _{ddCO2b} = -0.00005233	m _{ddtot} = 0.000821	m _{dd,va} = 0.0008733
m _{dd,va1} = 0.0008597	m _{dd,vb} = 0.007699	m _v = 0.01163
m _{vs} = 0.01947	m _{vsvb} = 0.07866	Nus _G = 2
v _G = 0.0000156	φ = 0.6	Pr _G = 0.7281
P _G = 101330	P _{sat} = 3141	P _v = 1884
P _{vsvb} = 12253	ρ _G = 1.185	Sc _G = 0.613
Sh _G = 2	T _G = 298	T _L = 298
T _{sb} = 323	U _d = 2	X _{CO2G} = 0.0005
X _v = 0.0186	X _{vs} = 0.031	X _{vsvb} = 0.1209

6 potential unit problems were detected.

Problem 2.5

The assumptions mentioned regarding the droplet are met with $T_s = 25^\circ\text{C}$ and $m_{\text{CO}_2, s} = 0$.

Define

① $\rightarrow$ vapor

② $\rightarrow$ air

③ $\rightarrow$ CO₂

$$a) \quad T_L = T_G = T_s = 25^\circ\text{C} = 298\text{ K}$$

$$\phi = 0.6$$

$$P_1 = \phi P_{\text{sat}} = (0.6)(3141\text{ Pa}) = 1884\text{ Pa}$$

$$P_0 = 101330\text{ Pa}$$

$$X_{1,G} = P_1 / P_0 = 0.0186$$

$$m_{1,G} = \frac{X_1 M_1}{X_1 M_1 + (1 - X_1) M_2} = 0.01163$$

$$X_{1,s} = P_{\text{sat}}|_{T_s} / P = 3141 / 101330 = 0.031$$

$$m_{1,s} = \frac{X_{1,s} M_1}{X_{1,s} M_1 + (1 - X_{1,s}) M_2} = 0.0195$$

From Appendix E

$$\begin{aligned} D_{1,2} &= (1.97 \times 10^{-5} \text{ m}^2/\text{s}) \left(\frac{P_0}{P_0} \right) \left(\frac{T_G}{296} \right)^{1.685} \\ &= 2.545 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

$$Sh_G = \frac{K_G D}{\rho_G D_{12}} = 2$$

As an approximation use

$$\rho_a = \rho_{air} = 1.185 \text{ kg/m}^3$$

$$\text{Also, } D = 5.5 \times 10^{-4} \text{ m}$$

$$\Rightarrow K_G = 0.118513 \text{ kg/m}^2\text{s}$$

Without consideration for the blowing effect, we will have

$$\dot{m}_1'' = K_G (m_{1,s} - m_{1,g}) = 0.00088 \text{ kg/m}^2\text{s}$$

With the blowing effect included, then

$$B_{m,g} = \frac{m_{1,g} - m_{1,s}}{m_{1,s} - 1} \approx 0.0086$$

$$\dot{m}_1'' = K_G \ln(1 + B_{m,g}) = 0.000873 \text{ kg/m}^2\text{s}$$

$$b) \quad T_{s,b} = 323 \text{ K}$$

$$P_{1,sb} = P_{sat} \Big|_{T_{s,b}} = 1.225 \times 10^4 \text{ Pa}$$

$$X_{1,sb} = P_{1,sb} / P_0 = 0.1209$$

$$\Rightarrow m_{1,sb} = 0.0787$$

$$B_{m,g,b} = \frac{m_{1,g} - m_{1,sb}}{m_{1,sb} - 1} = 0.07276$$

$$\dot{m}_{1,b}'' = K_G \ln(1 + B_{m,g,b}) = 0.0077 \text{ kg/m}^2\text{s}$$

c) We will do this for Part a only.

$$m_{3,s} = 0$$

$$X_{3,g} = 500 \times 10^{-6}$$

$$m_{3,g} = \frac{X_{3,g} M_3}{X_{3,g} M_3 + X_{1,g} M_1 + (1 - X_{1,g} - X_{3,g}) M_2} = 7.64 \times 10^{-4}$$

For diffusion coefficient, let us assume diffusivity of CO₂ with respect to the mixture is equal to D_{13} for simplicity.

$$D_{13} = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Sh}_{G,\text{CO}_2} = \frac{K_{G,\text{CO}_2} D}{P_G \phi_{13}} \Rightarrow K_{G,\text{CO}_2} = 0.0689 \text{ kg/m}^2\text{s}$$

$$m_{3,s} = 0$$

$$B_{mG} = \frac{m_{3,g} - m_{3,s}}{m_{3,s} - \frac{m''_{\text{CO}_2}}{m''_{\text{tot}}}} \quad (1)$$

$$m''_3 = m''_{\text{tot}} m_{3,s} + K_{G,\text{CO}_2} \frac{\ln(1 + B_{mG})}{B_{mG}}$$

$$\rightarrow (m_{3,s} - m_{3,g}) \quad (2)$$

$$m''_{\text{tot}} = m''_3 + m''_1 \quad (3)$$

Solution of (1) ~ (3) gives

$$m''_3 = -0.0000523 \text{ kg/m}^2\text{s}.$$

Problem 2.6

The threshold diameter, according to Suo and Griffith (1964) is

$$D = 0.3 \left[\frac{\sigma}{g \Delta \rho} \right]^{1/2}$$

Define

- ① air-water at room temperature
- ② saturated water-steam at 10 MPa
- ③ saturated R-134a mixture at 1.02 MPa

	ρ_L (kg/m ³)	ρ_G (kg/m ³)	σ (N/m)	$0.3 \sqrt{\frac{\sigma}{g \Delta \rho}}$ (m)
①	997	1.185	0.072	8.15×10^{-4}
②	688.6	55.48	0.012	4.19×10^{-4}
③	1,146	50.23	0.0061	2.26×10^{-4}

{P 2.7}

$$P_{inf}=101330.$$

$$T_{sat_inf}=T_{sat}(\text{Water}, P=P_{inf})$$

$$T_{cr_w}=647.15$$

$$\sigma=0.238*(1-T_{sat_inf}/T_{cr_w})^{1.25}*(1.-0.639*(1.-T_{sat_inf}/T_{cr_w}))$$

$$\rho_f=\text{density}(\text{water}, P=P_{inf}, x=0.0)$$

$$\rho_g=\text{density}(\text{water}, P=P_{inf}, x=1.)$$

$$h_f=\text{enthalpy}(\text{Water}, P=P_{inf}, x=0.)$$

$$h_g=\text{enthalpy}(\text{Water}, P=P_{inf}, x=1.)$$

$$h_{fg}=h_g-h_f$$

{R=1.e-6}

$$\text{DelT}_{sup}=2.*\sigma*T_{sat_inf}/(R*h_{fg}*\rho_g)$$

$$\text{DelT}_{subc}=2.*\sigma*T_{sat_inf}/(R*h_{fg}*\rho_f)$$

$$P_{inf} = 101330$$

$$T_{sat,inf} = T_{sat} [\text{water} , P = P_{inf}]$$

$$T_{cr,w} = 647.15$$

$$\sigma = 0.238 \cdot \left[1 - \frac{T_{sat,inf}}{T_{cr,w}} \right]^{1.25} \cdot \left[1 - 0.639 \cdot \left(1 - \frac{T_{sat,inf}}{T_{cr,w}} \right) \right]$$

$$\rho_f = \rho [\text{water} , P = P_{inf} , x = 0]$$

$$\rho_g = \rho [\text{water} , P = P_{inf} , x = 1]$$

$$h_f = h [\text{water} , P = P_{inf} , x = 0]$$

$$h_g = h [\text{water} , P = P_{inf} , x = 1]$$

$$h_{fg} = h_g - h_f$$

$$\text{DelT}_{sup} = 2 \cdot \sigma \cdot \frac{T_{sat,inf}}{R \cdot h_{fg} \cdot \rho_g}$$

$$\text{DelT}_{subc} = 2 \cdot \sigma \cdot \frac{T_{sat,inf}}{R \cdot h_{fg} \cdot \rho_f}$$

SOLUTION

Unit Settings: SI K Pa J mass deg

(Table 1, Run 14)

$$\text{DelT}_{subc} = 0.0004092$$

$$h_f = 419073$$

$$h_g = 2.676\text{E}+06$$

$$R = 0.00005$$

$$\rho_g = 0.5975$$

$$T_{cr,w} = 647.2$$

$$\text{DelT}_{sup} = 0.6563$$

$$h_{fg} = 2.257\text{E}+06$$

$$P_{inf} = 101330$$

$$\rho_f = 958.4$$

$$\sigma = 0.05929$$

$$T_{sat,inf} = 373.2$$

5 potential unit problems were detected.

Parametric Table: Table 1

	R	DelT _{sup}
Run 1	1.000E-06	32.82
Run 2	0.000002	16.41
Run 3	0.000002	16.41
Run 4	0.000003	10.94
Run 5	0.000004	8.204
Run 6	0.000005	6.563
Run 7	0.000006	5.469
Run 8	0.000007	4.688
Run 9	0.000008	4.102
Run 10	0.000009	3.646
Run 11	0.00001	3.282
Run 12	0.00002	1.641
Run 13	0.00004	0.8204
Run 14	0.00005	0.6563

Problem 2.8

We have

$$\lambda = 100 \text{ m}$$

$$J_0 = 0.5 \text{ m}$$

$$k = \frac{2\pi}{\lambda} = 0.0628 \text{ m}^{-1}$$

$$\sigma = 0.071 \text{ N/m}$$

$$\rho_L \approx 1000 \text{ kg/m}^3$$

From Eq. (2.105)

$$\omega = \left[\sigma k^3 / \rho_L + gk \right]^{1/2} = 0.785 \text{ Rad/s}$$

Eq. (2.104) gives

$$A = J_0 \frac{\omega}{k} = 6.248 \text{ m}^2/\text{s}$$

The velocity in z direction is $\partial \phi / \partial z$,

and from Eq. (2.98)

$$V_{z, \max} \Big|_{z=0} = Ak = 0.3924 \text{ m/s}$$

At location $-z^*$ (i.e., at a depth z^*), again from Eq. (2.98)

$$V_{z, \max} \Big|_{z=-z^*} = Ake^{-kz^*}$$

We thus get

$$V_{z, \max} \Big|_{z=-10 \text{ m}} = 1.78 \times 10^{-5} \text{ m/s}$$

$$\rho_L \left(\frac{\partial \phi'_L}{\partial t} + U_L \frac{\partial \phi'_L}{\partial x} + U_L \frac{\partial \phi'_L}{\partial y} - g \zeta \right)$$

$$\rho_G \left(\frac{\partial \phi'_G}{\partial t} + U_G \frac{\partial \phi'_G}{\partial x} + U_G \frac{\partial \phi'_G}{\partial y} - g \zeta \right) = \sigma \left(-\frac{\partial^2 \zeta}{\partial x^2} - \frac{\partial^2 \zeta}{\partial y^2} \right) \quad (1)$$

$$\frac{\partial \zeta}{\partial t} + U_L \frac{\partial \zeta}{\partial x} + U_L \frac{\partial \zeta}{\partial y} \approx \frac{\partial \phi'_L}{\partial z} \Big|_{z=0} \quad (2)$$

$$\frac{\partial \zeta}{\partial t} + U_G \frac{\partial \zeta}{\partial x} + U_G \frac{\partial \zeta}{\partial y} \approx \frac{\partial \phi'_G}{\partial z} \Big|_{z=0} \quad (3)$$

$$\zeta = \zeta_0 \exp [i(\omega t - k_x x - k_y y)] \quad (4)$$

Employing (4) into (2)

$$-b k = \zeta_0 [i\omega + U_L (-ik_x) + U_L (-ik_y)] = i\zeta_0 [\omega - U_L k_x - U_L k_y]$$

Employing (4) into (3)

$$b' k = \zeta_0 [i\omega - U_G ik_x - iU_G k_y] = i\zeta_0 [\omega - U_G k_x - U_G k_y]$$

$$\rho_L [i\omega b - ik_x U_L b - ik_y U_L b - g \zeta_0]$$

$$- \rho_G [i\omega b' - ik_x U_G b' - ik_y U_G b' - g \zeta_0] = \sigma \zeta_0 (k_x^2 + k_y^2)$$

$$\checkmark \nabla^2 \phi_L = 0$$

$$(-ik_x)^2 + (-ik_y)^2 + (-k)^2 = -k_x^2 - k_y^2 + k^2 = 0$$

$$\Rightarrow k^2 = k_x^2 + k_y^2 \quad \checkmark$$

$$P_L \left\{ \frac{\zeta_0 w [w - U_L (k_x + k_y)]}{-k^2} - U_L (k_x + k_y) \frac{\zeta_0 [w - U_L (k_x + k_y)]}{k^2} \right. \\ \left. - g \frac{\zeta_0}{k} \right\} - P_G \left\{ \frac{-\zeta_0 w [w - U_G (k_x + k_y)]}{k^2} + U_G (k_x + k_y) \frac{\zeta_0 [w - U_G (k_x + k_y)]}{k^2} \right.$$

$$\left. - g \frac{\zeta_0}{k} \right\} = \sigma \zeta_0 k$$

$$(P_L + P_G) \left(\frac{w}{k} \right)^2 - \frac{2U_L (k_x + k_y) P_L \left(\frac{w}{k} \right) - 2P_G U_G (k_x + k_y) \left(\frac{w}{k} \right)}{k}$$

$$+ \frac{P_L U_L^2 (k_x + k_y)^2}{k^2} + \frac{P_G U_G^2 (k_x + k_y)^2}{k^2} - \frac{(P_L g - P_G g + \sigma k)}{k} = 0$$

$$\Rightarrow (P_L + P_G) \left(\frac{w}{k} \right)^2 - \frac{2(U_L P_L + U_G P_G) (k_x + k_y) \left(\frac{w}{k} \right)}{k} + \frac{(P_L U_L^2 + P_G U_G^2) (k_x + k_y)^2}{k^2} \\ - g \frac{(P_L - P_G)}{k} - \sigma k = 0$$

$$\frac{w}{k} = \frac{(U_L P_L + U_G P_G) (k_x + k_y)}{(P_L + P_G) k} + \frac{1}{2(P_L + P_G)} \left\{ 4 \frac{(U_L P_L + U_G P_G)^2 (k_x + k_y)^2}{k^2} \right.$$

$$\left. - 4 \left[\frac{(P_L U_L^2 + P_G U_G^2) (k_x + k_y)^2}{k^2} - g \frac{(P_L - P_G)}{k} - \sigma k \right] (P_L + P_G) \right\}$$

$$\frac{w}{k} = \frac{(U_L \rho_L + U_G \rho_G)(k_x + k_y)}{(\rho_L + \rho_G) k} + \frac{1}{(\rho_L + \rho_G)} \left\{ (k_x^2 + k_y^2) (2 \rho_L \rho_G U_L U_G - \rho_L \rho_G U_G^2 - \rho_L \rho_G U_L^2) - k g (\rho_L^2 - \rho_G^2) + \sigma k^3 (\rho_L + \rho_G) \right\}^{\frac{1}{2}}$$

critical happens when

$$\frac{g}{k} \left(\frac{\rho_L - \rho_G}{\rho_L + \rho_G} \right) + \frac{(k_x + k_y)^2}{k^2} \frac{\rho_L \rho_G}{(\rho_L + \rho_G)^2} (U_L - U_G)^2 - \frac{\sigma k}{\rho_L + \rho_G} = 0$$

Taylor Instability happens when $U_L = U_G = 0$

$$\frac{g}{k} (\rho_L - \rho_G) - \sigma k = 0$$

$$k_c = \sqrt{\frac{g \Delta \rho}{\sigma}} = \frac{1}{L}$$

$$\lambda_c = \frac{2\pi}{k_c} = 2\pi L \quad \lambda_{c2} = \sqrt{2} \lambda_c = 2\pi \sqrt{2} L$$

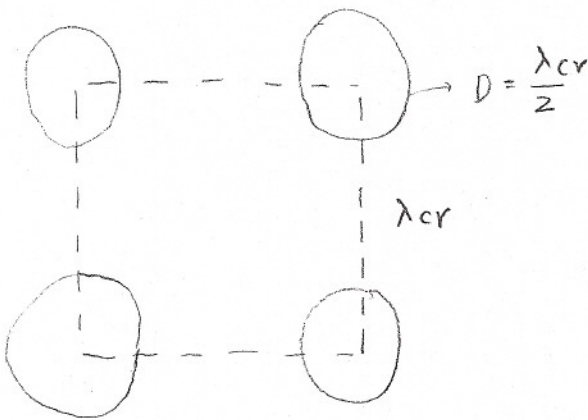
To find the fastest growing wave length

$$\frac{dw}{dk} = 0, \quad U_L = U_G = 0$$

$$w = \left(g \left(\frac{\rho_L - \rho_G}{\rho_L + \rho_G} \right) k - \frac{\sigma k^3}{\rho_L + \rho_G} \right)^{\frac{1}{2}}$$

$$\frac{dw}{dk} = \frac{g \left(\frac{\rho_L - \rho_G}{\rho_L + \rho_G} \right) - 3 \frac{\sigma}{\rho_L + \rho_G} k^2}{\sqrt{g k \left(\frac{\rho_L - \rho_G}{\rho_L + \rho_G} \right) - \frac{\sigma k^3}{\rho_L + \rho_G}}} = 0 \Rightarrow k = \sqrt{\frac{g \Delta \rho}{3\sigma}} = \frac{1}{\sqrt{3} L}$$

$$\lambda_{d2} = \sqrt{2} \lambda_d = 2\pi \sqrt{6} L$$



$$\lambda_H = \lambda_c = 2\pi R_j, \quad U_{G,cr} = \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_H}}$$

mass flow rate

$$\dot{m} = \rho_G A_G U_G$$

$$= \rho_G \frac{\pi}{4} \left(\frac{\lambda_{cr}}{2}\right)^2 \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_H}}$$

$$= \rho_G \frac{\pi}{4} \frac{1}{4} \lambda_c^2 \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_c}}$$

$$= \frac{1}{16} \rho_G \pi \lambda_c^2 \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_c}}$$

$$= \frac{1}{16} \rho_G \pi 4\pi^2 R_j^2 \sqrt{\frac{\sigma}{\rho_G R_j}}$$

$$= \frac{1}{4} \rho_G \pi^3 R_j^2 \sqrt{\frac{\sigma}{\rho_G R_j}}$$

$$\lambda_H = 2\pi \left(\frac{\lambda_c}{2}\right) = \pi \lambda_c$$

OK

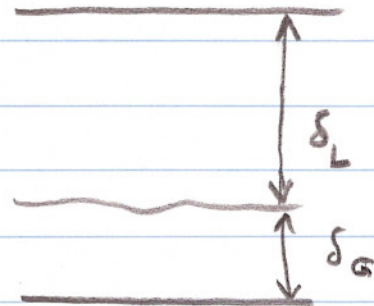
mass flow rate per unit surface area:

$$\frac{\rho_G \frac{1}{16} \pi \cdot \lambda_c^2 \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_c}}}{\lambda_c^2} = \frac{1}{16} \rho_G \pi \sqrt{\frac{2\pi\sigma}{\rho_G \lambda_c}} = \frac{1}{16} \sqrt{\frac{2\pi^3 \sigma \rho_G}{2\pi R_j}}$$

Numbers can now be plugged in

$$= \frac{1}{16} \sqrt{\frac{\pi \sigma \rho_G}{2 R_j}}$$

Problem 2.11 and 2.12



Starting from Eqs. (2.117) and (2.118) we must express ϕ'_L and ϕ'_G such that

$$\frac{d\phi'_G}{dz} = 0 \quad \text{at } z = -\delta_G$$

$$\frac{d\phi'_L}{dz} = 0 \quad \text{at } z = +\delta_L$$

Thus, instead of Eq. (2.117) we

$$\phi'_L = b \cosh[+k(z - \delta_L)] e^{i(\omega t - kx)}$$

and instead of Eq. (2.118) we

$$\phi'_G = b' \cosh[k(z + \delta_G)] e^{i(\omega t - kx)}$$

The result of the analysis will be similar to Eq. (2.124) except that P_L and P_G are

2.11.2

replaced with ρ_L' and ρ_G' where

$$\rho_L' = \rho_L / \tanh(k\delta_L)$$

$$\rho_G' = \rho_G / \tanh(k\delta_G)$$

The neutral and the fastest growing wavelengths are found to be, when

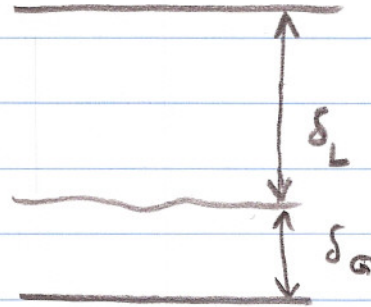
$$U_L \rightarrow 0 \text{ and } U_G \rightarrow 0:$$

$$\lambda_{cr}' = 2\pi \lambda_L'$$

$$\lambda_d = 2\pi\sqrt{3} \lambda_L'$$

$$\lambda_L'' = \sqrt{\frac{\sigma}{g(\rho_L' - \rho_G')}}}$$

Problem 2.11



Starting from Eqs. (2.117) and (2.118) we must express ϕ'_L and ϕ'_G such that

$$\frac{d\phi'_G}{dz} = 0 \quad \text{at } z = -\delta_G$$

$$\frac{d\phi'_L}{dz} = 0 \quad \text{at } z = +\delta_L$$

Thus, instead of Eq. (2.117) we

$$\phi'_L = b \cosh[-k(z - \delta_L)] e^{i(\omega t - kx)}$$

and instead of Eq. (2.118) we

$$\phi'_G = b' \cosh[k(z + \delta_G)] e^{i(\omega t - kx)}$$

The result of the analysis will be similar to Eq. (2.129) except that P_L and P_G are

2.11.2

replaced with p'_L and p'_G where

$$p'_L = p_L / \tanh(k\delta_L)$$

$$p'_G = p_G / \tanh(k\delta_G)$$

Solution

$$\nabla^2 \phi = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) = 0 \quad (a)$$

$$\frac{\partial \phi}{\partial r} = \dot{R} \quad \text{at } r = R$$

$$\frac{\partial \phi}{\partial r} \rightarrow 0 \quad \text{at } r \rightarrow \infty$$

The solution for these equations is

$$\phi = R \dot{R} \ln r$$

Therefore

$$\frac{\partial \phi}{\partial t} = \ln r [\dot{R}^2 + R \ddot{R}]$$

$$U = u_r = \frac{\partial \phi}{\partial r} = \frac{R \dot{R}}{r}$$

Insert in Euler's equation:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} U^2 + \frac{P}{\rho} = f(t)$$

$$\Rightarrow [\dot{R}^2 + R \ddot{R}] \ln r + \frac{1}{2} \left(\frac{R \dot{R}}{r} \right)^2 + \frac{P}{\rho} = f(t) \quad (b)$$

Equation (b) must be applicable to a point far away, where

$$[\dot{R}^2 + R \ddot{R}] \ln R_\infty + \frac{P_\infty}{\rho} = f(t) \quad (c)$$

At the surface of the cylinder:

$$[\dot{R}^2 + R \ddot{R}] \ln R + \frac{1}{2} \dot{R}^2 + \frac{P_L}{\rho} = f(t) \quad (d)$$

Subtracting (c) from (d) gives

$$[\dot{R}^2 + R \ddot{R}] \ln \frac{R}{R_\infty} + \frac{1}{2} \dot{R}^2 + \frac{P_L - P_\infty}{\rho} = 0$$

Since $\lim_{R \rightarrow \infty} \ln(R/R_\infty) \rightarrow -\infty$,

the following equations must both apply in order for the above expression to make sense:

$$R + R \ddot{R} = 0$$

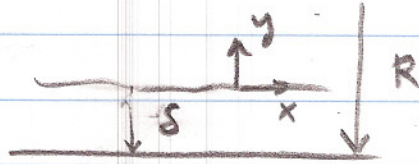
$$\frac{1}{2} \dot{R}^2 + \frac{P_L - P_\infty}{\rho} = 0$$

These imply:

$$R \ddot{R} - \frac{2(P_L - P_\infty)}{\rho} = 0$$

2.14 - 1

Problem 2.14



Assume

1. $\delta/R \ll 1$; therefore liquid can be infinitely thick
2. Both phases are incompressible & inviscid
3. No gravitational effect
4. Axis-symmetric conditions

In view of assumption 1, the waves can be treated as 2-D, in (x, y) coordinates.

The solution proceeds as:

$$\phi_L = U_L x + \phi'_L \quad (1)$$

$$\phi_G = U_G x + \phi'_G \quad (2)$$

Integration of Euler's equation for the two phases (similar to Eq. (2.119)) gives:

$$P_L \left(\frac{\partial \phi'_L}{\partial t} + U_L \frac{\partial \phi'_L}{\partial x} \right) = -P_L + P_{0L} \quad (3)$$

$$P_G \left(\frac{\partial \phi'_G}{\partial t} + U_G \frac{\partial \phi'_G}{\partial x} \right) = -P_G + P_{0G} \quad (4)$$

At equilibrium we would have

$$P_{0L} - P_{0G} = \frac{\sigma}{R-h} \quad (5)$$

Also

Therefore

$$P_L - P_G = \frac{\sigma}{R-h-\delta} + \sigma \frac{\partial^2 \zeta}{\partial x^2} \quad (6)$$

Now assume

$$\phi'_L = A_L e^{-ky} e^{i(\omega t - kx)} \quad (7)$$

$$\phi'_G = A_G \cosh [k(y+\delta)] e^{i(\omega t - kx)} \quad (8)$$

These equations satisfy the following

$$\nabla^2 \phi'_L = \nabla^2 \phi'_G = 0 \quad (9)$$

$$\nabla \phi'_L \big|_{y \rightarrow \infty} = 0 \quad (10)$$

$$\frac{d\phi'_G}{dy} \big|_{y=-\delta} = 0 \quad (11)$$

Assume a disturbance of the form

$$\zeta = \zeta_0 e^{i(\omega t - kx)} \quad (12)$$

We now must satisfy

$$v_L|_{y=\zeta} \approx \left. \frac{\partial \phi_L}{\partial y} \right|_{y=0} = \frac{\partial \zeta}{\partial t} + U_L \frac{\partial \zeta}{\partial x} \quad (13)$$

$$v_G|_{y=\zeta} \approx \left. \frac{\partial \phi_G}{\partial y} \right|_{y=0} = \frac{\partial \zeta}{\partial t} + U_G \frac{\partial \zeta}{\partial x} \quad (14)$$

$$v_L|_{y=\zeta} = v_G|_{y=\zeta} \quad (15)$$

⇒ We also note that

$$-(P_L - P_G) + (P_{0L} + P_{0G}) = \frac{\sigma}{R-\zeta} - \frac{\sigma}{(R-\zeta)-\zeta} - \sigma \frac{\partial^2 \zeta}{\partial x^2} \quad (16-a)$$

$$\frac{\sigma}{(R-\zeta)-\zeta} \approx \frac{1}{R-\zeta} \sigma \left[1 - \frac{\zeta}{R-\zeta} \right] \quad (16-b)$$

Eqs. (13), (14), (15), then lead to, respectively

$$k A_L = -i \zeta_0 (\omega - U_L k) \quad (17)$$

$$k A_G \sinh(k\delta) = i \zeta_0 (\omega - U_G k) \quad (18)$$

$$\begin{aligned} p_L [i\omega A_L + U_L A_L (-ik)] - p_G [i\omega A_G \cosh(k\delta) \\ + U_G A_G \cosh(k\delta) (-ik)] = \sigma \zeta_0 k^2 - \frac{\sigma \zeta_0^2}{(R-\zeta)^2} \end{aligned} \quad (19)$$

$$(17) \Rightarrow A_L = -i \frac{J_0(\omega - U_L k)}{k} \quad (20)$$

$$(18) \Rightarrow A_G = i \frac{J_0(\omega - U_G k)}{k \sinh(k\delta)} \quad (21)$$

substitution from Eqs. (20) and (21) in (19) gives the desired result.

Problem 2.16

Assume;

1. steady - state
2. $\delta/R \ll 1$, so that a planar, 2-D stability analysis at the interphase is justified
3. $M_G \ll M_L$, so that the gas can be treated as inviscid
4. Axis-symmetric flow
5. Negligible gravitational effect

2.16.2

The conservation equations for the liquid phase can then be represented as:

$$\frac{\partial u_L}{\partial x} + \frac{\partial v_L}{\partial y} = 0 \quad (5)$$

$$\rho_L \frac{\partial u_L}{\partial t} = -\frac{\partial P_L}{\partial x} + \mu_L \left(\frac{\partial^2 u_L}{\partial x^2} + \frac{\partial^2 u_L}{\partial y^2} \right) \quad (6)$$

$$\rho_L \frac{\partial v_L}{\partial t} = -\frac{\partial P_L}{\partial y} + \mu_L \left(\frac{\partial^2 v_L}{\partial x^2} + \frac{\partial^2 v_L}{\partial y^2} \right) \quad (7)$$

The gas phase conservation equations are similar, except that they do not include viscosity.

Periodic motion of incompressible fluid surfaces obeying the Navier-Stokes equations can be represented using a potential and a stream function. Each velocity component of the liquid is therefore assumed to consist of an inviscid term and a perturbation representing the effect of viscosity [17].

$$u_L = u_L^0 + U_L \quad (8)$$

$$v_L = v_L^0 + V_L \quad (9)$$

$$P = P^0 \quad (10)$$

where parameters with superscript 0 represent ideal fluid conditions, and U and V represent the velocities in x and y directions due to viscosity, respectively. The gas phase is assumed to be inviscid; the gas velocity components therefore do not include terms representing viscosity. The above ideal-fluid velocities are represented by a potential function of the form:

$$\Phi_L = A_L e^{-ky + \omega t} \cos kx \quad (11)$$

$$\Phi_G = A_G \cosh[k(y + \delta)] e^{\omega t} \cos kx \quad (12)$$

where for both phases:

$$(u^o, v^o) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \Phi \quad (13)$$

These potential functions fully represent the irrotational, ideal liquid and gas fields. The liquid velocity terms that account for the viscosity-induced rotational motion can be represented by a stream function, ψ_L , that must satisfy [17]:

$$\frac{\partial \psi_L}{\partial t} = \nu_L \nabla^2 \psi_L \quad (14)$$

where:

$$(U_L, V_L) = \left(-\frac{\partial}{\partial y}, \frac{\partial}{\partial x} \right) \psi_L \quad (15)$$

Equation (14) and all the necessary boundary conditions can be satisfied using:

$$\psi_L = B_L e^{-my + \omega t} \sin kx \quad (16)$$

Note that the above equation approximates the liquid core as infinitely-deep. Using Eq. (14), it can be shown that:

$$m^2 = k^2 + \frac{\omega}{\nu_L} \quad (17)$$

The above potential and stream functions are now utilized in a linear stability analysis, using the following boundary conditions at the interphase:

$$v_L = v_G \quad (18)$$

$$-P_L + 2\mu_L \left(\frac{\partial v_L}{\partial y} \right) = -P_G - \sigma \frac{\partial^2 \eta}{\partial x^2} - \Delta P_\sigma \quad (19)$$

$$\mu_L \left(\frac{\partial u_L}{\partial y} + \frac{\partial v_L}{\partial x} \right) = 0 \quad (20)$$

where:

$$\eta = \frac{k}{\omega} (B_L - A_L) e^{\omega t} \cos kx \quad (21)$$

$$P_L - P_G = -\rho_L \left. \frac{\partial \Phi_L}{\partial t} \right|_{y=0} + \rho_G \left. \frac{\partial \Phi_G}{\partial t} \right|_{y=0} \quad (22)$$

The term ΔP_σ in Eqn. (19), which is de-stabilizing, is the surface tension force caused by the curvature of the channel, and can be represented as:

$$\Delta P_\sigma = \frac{\sigma}{R-\delta} - \frac{\sigma}{R-\delta-\eta} \approx \frac{\sigma}{R-\delta} \left(1 + \frac{\eta}{R-\delta} \right) \quad (23)$$

Equations (19-21) following substitution for velocities using the aforementioned potential and stream functions, lead to:

$$-A_L + B_L - A_G \sinh(kh) = 0 \quad (24)$$

$$2k^2 A_L - (m^2 + k^2) B_L = 0 \quad (25)$$

$$\begin{aligned} & \left[2\mu_L k^2 + \frac{k^3 \sigma}{\omega} - \frac{\sigma}{(R-\delta)^2} \frac{k}{\omega} + \rho_L \omega \right] A_L + \\ & \left[-2\mu_L km - \frac{k^3 \sigma}{\omega} + \frac{\sigma}{(R-\delta)^2} \frac{k}{\omega} \right] B_L - [\rho_G \alpha \cosh(kh)] A_G = 0 \end{aligned} \quad (26)$$

A non-trivial solution is possible if the determinant of the coefficient matrix of Eqns. (24-26) vanishes, and that leads to:

$$\left[\rho^* \coth(R^*KH) + 1 \right] \Omega^2 - \frac{K}{R^{*2}(1-H)^2} + K^3 + 4K^4 \left[1 - \left(1 + \frac{\Omega}{K^2} \right)^{1/2} \right] + 4\Omega K^2 = 0 \quad (27)$$

where $\rho^* = \rho_G / \rho_L$; $H = \delta / R$; $R^* = R \frac{\sigma}{\rho_L v_L^2}$; $K = k \left(\frac{\sigma}{\rho_L v_L^2} \right)^{-1}$; and $\Omega = \omega \left(\frac{\rho_L^2 v_L^3}{\sigma^2} \right)$. The

interface is evidently unstable when $\Omega > 0$, and the neutral condition ($\Omega = 0$) occurs when

$$(KR^*)_{cr} = \frac{1}{1-H} \quad (28)$$

Disturbances with wavelengths longer than λ_{cr} thus cause instability, where:

$$\lambda_{cr} = 2\pi(R - \delta) \quad (29)$$

The neutral wavelength thus is similar to the predictions of the Kelvin-Helmholtz stability; and does not depend on liquid viscosity, in agreement with the Taylor stability analysis for film boiling of viscous liquids. The fastest growing wavelength, Ω_d , occurs when $d\Omega/dK = 0$, whereby:

$$\left(\frac{\rho^* R^* H}{\sinh^2(R^*KH)} \right) \Omega_d^2 - 4K \left[2 + \left(1 + \frac{\Omega_d}{K_d^2} \right)^{-1/2} \right] \Omega_d - 16K_d^3 \left[1 - \left(1 + \frac{\Omega_d}{K_d^2} \right)^{1/2} \right] - 3K_d^2 = 0 \quad (30)$$

Problem 2.17

We have

$$P_L = 1.0133 \times 10^5 \text{ Pa}$$

$$T_L = 298$$

$$\sigma = 0.071 \text{ N/m}$$

$$\rho_L \approx 1000 \text{ kg/m}^3$$

$$\gamma = C_{pG} / C_{vG} = 1.4$$

$$\rho_G = 1.18 \text{ kg/m}^3$$

The volumetric and second mode oscillation frequencies are found from

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{3\gamma P_L}{\rho_L R_0^2}}$$

$$f_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi} \left\{ \frac{24\sigma}{R_0^3 [3\rho_G + 2\rho_L]} \right\}^{1/2}$$

The results are as follows

R_0 (m)	f_0 (Hz)	f_2 (Hz)
1.0×10^{-5}	3.28×10^5	1.47×10^5
1.0×10^{-4}	3.28×10^4	4,641
5×10^{-4}	6,567	415

```

{P 2.17}
R_01=10.e-6
R_02=100.e-6
R_03=500.e-6
gamma=1.4
P_L=101330.
T_L=298.
rho_L=1000.
sigma=0.071
omega_01=sqrt(3.*gamma*P_L/(rho_L*R_01^2))
f_01=omega_01/(2.*pi)
omega_02=sqrt(3.*gamma*P_L/(rho_L*R_02^2))
f_02=omega_02/(2.*pi)
omega_03=sqrt(3.*gamma*P_L/(rho_L*R_03^2))
f_03=omega_03/(2.*pi)
rho_a=density(air, P=P_L, T=T_L)
omega_21=sqrt((24.*sigma)/(R_01^3*(3.*rho_a+2.*rho_L)))
omega_22=sqrt((24.*sigma)/(R_02^3*(3.*rho_a+2.*rho_L)))
omega_23=sqrt((24.*sigma)/(R_03^3*(3.*rho_a+2.*rho_L)))
f_21=omega_21/(2.*pi)
f_22=omega_22/(2.*pi)
f_23=omega_23/(2.*pi)

```

$$R_{01} = 0.00001$$

$$R_{02} = 0.0001$$

$$R_{03} = 0.0005$$

$$\gamma = 1.4$$

$$P_L = 101330$$

$$T_L = 298$$

$$\rho_L = 1000$$

$$\sigma = 0.071$$

$$\omega_{01} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{01}^2}}$$

$$f_{01} = \frac{\omega_{01}}{2 \cdot \pi}$$

$$\omega_{02} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{02}^2}}$$

$$f_{02} = \frac{\omega_{02}}{2 \cdot \pi}$$

$$\omega_{03} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{03}^2}}$$

$$f_{03} = \frac{\omega_{03}}{2 \cdot \pi}$$

$$\rho_a = \rho[\text{'Air'} , P = P_L , T = T_L]$$

$$\omega_{21} = \sqrt{\frac{24 \cdot \sigma}{R_{01}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

$$\omega_{22} = \sqrt{\frac{24 \cdot \sigma}{R_{02}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

$$\omega_{23} = \sqrt{\frac{24 \cdot \sigma}{R_{03}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

$$f_{21} = \frac{\omega_{21}}{2 \cdot \pi}$$

$$f_{22} = \frac{\omega_{22}}{2 \cdot \pi}$$

$$f_{23} = \frac{\omega_{23}}{2 \cdot \pi}$$

SOLUTION

Unit Settings: [J]/[K]/[Pa]/[kg]/[degrees]

$$f_{01} = 328332$$

$$f_{21} = 146776$$

$$\gamma = 1.4$$

$$\omega_{03} = 41259$$

$$\omega_{23} = 2608$$

$$\rho_L = 1000$$

$$R_{03} = 0.0005$$

$$f_{02} = 32833$$

$$f_{22} = 4641$$

$$\omega_{01} = 2.063E+06$$

$$\omega_{21} = 922219$$

$$P_L = 101330$$

$$R_{01} = 0.00001$$

$$\sigma = 0.071$$

$$f_{03} = 6567$$

$$f_{23} = 415.1$$

$$\omega_{02} = 206297$$

$$\omega_{22} = 29163$$

$$\rho_a = 1.185$$

$$R_{02} = 0.0001$$

$$T_L = 298$$

{P 2.17}

$$R_{01}=10.e-6$$

$$R_{02}=100.e-6$$

$$R_{03}=500.e-6$$

$$\gamma=1.4$$

$$P_L=101330.$$

$$T_L=298.$$

$$\rho_L=1000.$$

$$\sigma=0.071$$

$$\omega_{01}=\sqrt{3 \cdot \gamma \cdot P_L / (\rho_L \cdot R_{01}^2)}$$

$$f_{01}=\omega_{01} / (2 \cdot \pi)$$

$$\omega_{02}=\sqrt{3 \cdot \gamma \cdot P_L / (\rho_L \cdot R_{02}^2)}$$

$$f_{02}=\omega_{02} / (2 \cdot \pi)$$

$$\omega_{03}=\sqrt{3 \cdot \gamma \cdot P_L / (\rho_L \cdot R_{03}^2)}$$

$$f_{03}=\omega_{03} / (2 \cdot \pi)$$

$$\rho_a=\text{density}(\text{air}, P=P_L, T=T_L)$$

$$\omega_{21}=\sqrt{(24 \cdot \sigma) / (R_{01}^3 \cdot (3 \cdot \rho_a + 2 \cdot \rho_L))}$$

$$\omega_{22}=\sqrt{(24 \cdot \sigma) / (R_{02}^3 \cdot (3 \cdot \rho_a + 2 \cdot \rho_L))}$$

$$\omega_{23}=\sqrt{(24 \cdot \sigma) / (R_{03}^3 \cdot (3 \cdot \rho_a + 2 \cdot \rho_L))}$$

$$f_{21}=\omega_{21} / (2 \cdot \pi)$$

$$f_{22}=\omega_{22} / (2 \cdot \pi)$$

$$f_{23}=\omega_{23} / (2 \cdot \pi)$$

$$R_{01} = 0.00001$$

$$R_{02} = 0.0001$$

$$R_{03} = 0.0005$$

$$\gamma = 1.4$$

$$P_L = 101330$$

$$T_L = 298$$

$$\rho_L = 1000$$

$$\sigma = 0.071$$

$$\omega_{01} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{01}^2}}$$

$$f_{01} = \frac{\omega_{01}}{2 \cdot \pi}$$

$$\omega_{02} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{02}^2}}$$

$$f_{02} = \frac{\omega_{02}}{2 \cdot \pi}$$

$$\omega_{03} = \sqrt{3 \cdot \gamma \cdot \frac{P_L}{\rho_L \cdot R_{03}^2}}$$

$$f_{03} = \frac{\omega_{03}}{2 \cdot \pi}$$

$$\rho_a = \rho [\text{'Air'} , P = P_L , T = T_L]$$

$$\omega_{21} = \sqrt{\frac{24 \cdot \sigma}{R_{01}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

$$\omega_{22} = \sqrt{\frac{24 \cdot \sigma}{R_{02}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

$$\omega_{23} = \sqrt{\frac{24 \cdot \sigma}{R_{03}^3 \cdot [3 \cdot \rho_a + 2 \cdot \rho_L]}}$$

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SOLUTION

Unit Settings: [J]/[K]/[Pa]/[kg]/[degrees]

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$$\omega_{02} = 206297$$

$$\omega_{22} = 29163$$

$$\rho_a = 1.185$$

$$R_{02} = 0.0001$$

$$T_L = 298$$

1 potential unit problem was detected.

Problem 2.7. For a microbubble surrounded and at thermal and mechanical equilibrium with liquid, prove that the liquid must be superheated according to:

$$P_{sat} - P_L \approx \frac{2\sigma}{R} \frac{\rho_L + \rho_v}{\rho_L}$$

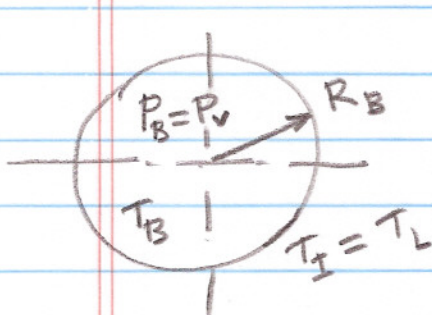
where P_L is the pressure in the liquid phase and P_{sat} is the saturation pressure corresponding to the temperature of the bubble and its surroundings.

Also, for a micro droplet surrounded and at thermal and mechanical equilibrium with its vapor, prove that the vapor must be supercooled according to

$$P_{sat} - P_v \approx -\frac{2\sigma}{R} \frac{\rho_v}{\rho_L}$$

where P_v is the pressure in the vapor phase and P_{sat} is the saturation pressure corresponding to the temperature of the bubble and its surroundings.

Problem 2.7

 P_L, T_L $T_B = T_L$ (from thermal equilibrium)

From Eq. (2.50)

$$P_B = P_{\text{sat}}(T_L) \exp\left(-\frac{2\sigma}{P_L \frac{R_u}{M} T_L R}\right) \quad (1)$$

Since $\frac{2\sigma}{P_L \frac{R_u}{M} T_L R} \ll 1$, then

$$P_B \approx P_{\text{sat}}|_{T_L} \left[1 - \frac{2\sigma}{P_L \frac{R_u}{M} T_L R}\right]$$

$$P_{\text{sat}}|_{T_L} - P_B \approx P_{\text{sat}}|_{T_L} \frac{2\sigma}{P_L \frac{R_u}{M} T_L R} \approx \frac{2\sigma P_g}{P_L R} \quad (2)$$

Mechanical equilibrium requires that

$$P_B - P_\infty = \frac{2\sigma}{R} \quad (3)$$

$$(2) \& (3) \Rightarrow P_{\text{sat}}|_{T_L} - P_\infty = \frac{2\sigma(P_L - P_g)}{P_L R} \approx \frac{2\sigma}{R} \quad (4)$$

Now, from Clapeyron's

relation:

$$\frac{dP}{dT} = \frac{h_{fg}}{T_{sat} v_{fg}}$$

$$\Rightarrow \Delta T \approx \Delta P \frac{T_{sat} v_{fg}}{h_{fg}} \quad (5)$$

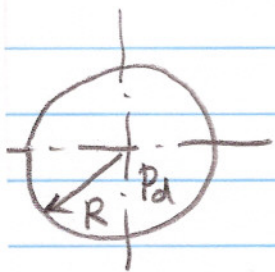
(4) & (5) $\Rightarrow$

$$T_L - T_{sat}|_{P_{\infty}} = \frac{2\sigma T_{sat} v_{fg}}{R h_{fg}} \quad (6)$$

This expression represents the liquid superheat needed for the bubble with radius R to be at equilibrium with its surrounding liquid.

The attached Table 1 shows some numbers.

Now, for a droplet



$$P_{\infty} = P_v$$

$$T_{\infty} = T_d = T_v$$

$$\ln \frac{P_v}{P_{sat}|_{T_v}} = + \frac{2\sigma}{\rho_L \frac{R_u}{M} T_v R} \quad (7)$$

$$\Rightarrow P_v \approx P_{sat}|_{T_v} \left[1 + \frac{2\sigma}{\rho_L \frac{R_u}{M} T_v R} \right] \approx P_{sat}|_{T_v} + \frac{2\sigma P_v}{\rho_L R} \quad (8)$$

$$P_v - P_{sat}|_{T_v} = \frac{2\sigma P_v}{\rho_L R}$$

$$\Rightarrow T_{sat}|_{P_v} - T_v \approx \frac{2\sigma T_v}{\rho_L R h_{fg}} \quad (9)$$

Thus, for a microdroplet with radius R to be stable in pure vapor, the vapor must be supercooled by the above amount.

The attached Table 2 shows some numerical results.