

***Plates and Shells Theory and Analysis 4th edition
Ugural Solutions Manual***

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CHAPTER 2

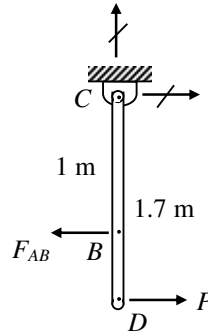
SOLUTION (2.1)

Free Body: CD

$$\sum M_C = 0: F_{AB} = 1.7P$$

and

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}}$$

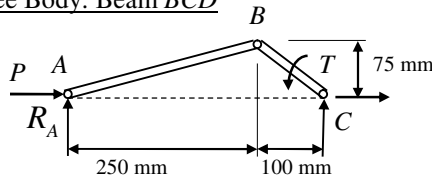


Substitute the numerical values:

$$50(10^6) = \frac{1.7P}{500(10^{-6})}, \quad P = 14.71 \text{ kN}$$

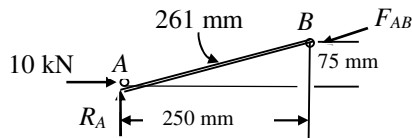
SOLUTION (2.2)

(a) Free Body: Beam BCD



$$\sum M_C = 0: -R_A(0.35) + T = 0, \quad T = 0.35R_A \quad (a)$$

Free Body: Rod AB



$$AB = \sqrt{250^2 + 75^2} = 261 \text{ mm}$$

$$\sum M_B = 0: 10(0.075) - R_A(0.25) = 0 \quad R_A = 3 \text{ kN}$$

Equation (a):

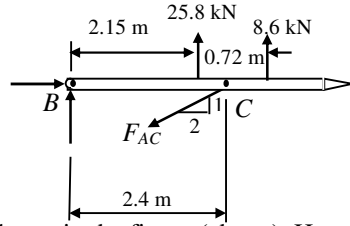
$$T = 0.35(3) = 1.05 \text{ kN} \cdot \text{m}$$

(b)

$$\sum F_x = 0: 10 - \frac{250}{261} F_{AB} = 0 \quad F_{AB} = 10.44 \text{ kN}$$

Thus,

$$\sigma_{AB} = \frac{10.44(10^3)}{0.5} = 20.88 \text{ kN}$$

SOLUTION (2.3)

Load resultants are shown in the figure (above). Hence

$$\sum M_B = 0: 25.8(2.15) + 8.6(2.15 + 0.72) - F_{AC} \frac{1}{\sqrt{5}}(2.4) = 0$$

or

$$F_{AC} = 74.68 \text{ kN}$$

Thus,

$$A_{AC} = \frac{74.68(10^3)}{80 \times 10^6} = 933.5(10^{-6}) \text{ m}^2 = 933.5 \text{ mm}^2 = 908.4 \text{ mm}^2 \quad \blacktriangleleft$$

SOLUTION (2.4)

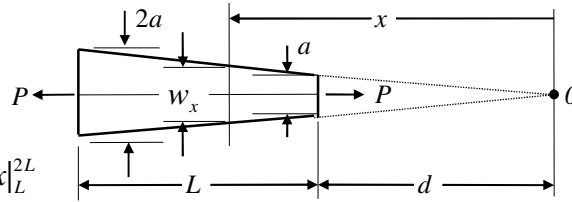
From geometry $\frac{d+L}{2a} = \frac{d}{a}; \quad d=L$

$$\frac{a}{L} = \frac{w_x}{x}; \quad w_x = \frac{a}{L} x$$

and $A_x = w_x t = \frac{at}{L} x$

Thus,

$$\begin{aligned} \delta &= \int_L^{2L} \frac{P dx}{A_x E} = \frac{PL}{atE} \int_L^{2L} \frac{dx}{x} = \frac{PL}{atE} \ln x \Big|_L^{2L} \\ &= \frac{PL}{atE} \ln 2 \end{aligned} \quad \blacktriangleleft$$



SOLUTION (2.5)

$$J = \frac{\pi}{2} (75^4 - 65^4) = 21.661 \times 10^{-6} \text{ m}^4$$

$$c = 0.075 \text{ m} \quad \theta = 40^\circ$$

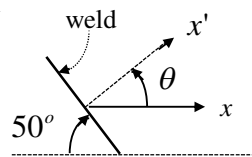
$$\tau_{xy} = \frac{T r}{J}, \quad \sigma_x = \sigma_y = 0$$

Using Eq. (1.11a);

$$\sigma_x' = 0 + 0 + \tau_{xy} \sin 2\theta$$

or

$$200(10^6) = \frac{T(0.075)(0.985)}{21.661(10^{-6})}; \quad T = 58.65 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$



SOLUTION (2.6)

$$J_b = \frac{\pi}{2} (0.03^4 - 0.02^4) = 102.102(10^{-8}) \text{ m}^4$$

$$\text{Statics: } T_s + T_b = 1 \text{ kN} \cdot \text{m} \quad (a)$$

Geometry:

$$\phi_c = \frac{T_b b}{\frac{\pi}{2} (0.03^4 - 0.02^4) (42 \times 10^9)} = \frac{T_s b}{\frac{\pi}{2} (0.02^4) (80 \times 10^9)}$$

or

$$T_b = 2.1328 T_s \quad (b)$$

(CONT.)

(2.6 CONT.)

From Eqs (a) and (b):

$$T_b = 680.8 \text{ N} \cdot \text{m} \quad T_s = 319.2 \text{ N} \cdot \text{m}$$

It is required that, $\phi_A = \phi_C + \phi_{AC}$:

$$0.01 = \frac{1}{J_b E_b} [680.8b + 1000(0.5 - b)] = \frac{500 - 319.2b}{102.102(10^{-8})(42 \times 10^9)}$$

Solving, $b = 0.223 \text{ m} = 223 \text{ mm}$ ◀

SOLUTION (2.7)

State of pure shear; $\sigma_1 = -\sigma_2 = \tau$

$$\varepsilon_{\max} = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{\tau}{E}(1 + \nu)$$

Thus,

$$\tau = \frac{E\varepsilon_{\max}}{1 + \nu} = \frac{120(10^3)(1900)}{1 + 0.33} = 171.4 \text{ MPa}$$

We have

$$J = \frac{T_r}{\tau} = \frac{150(0.015)}{171.4(10^6)} = 13.127(10^9) \text{ m}^4$$

Hence

$$J = \frac{\pi}{32}(30^4 - d^4) = 13.172(10^3) \text{ mm}^4$$

Solving, $d = 28.68 \text{ mm}$ ◀

SOLUTION (2.8)

$$(a) \quad \tau_{\max} = \frac{3V}{2A} = \frac{3}{2} \frac{pL/2}{bh} = \frac{3}{4} \frac{pL}{bh} \quad (a)$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(pL^2/8)(h/2)}{bh^3/12} = \frac{3}{4} \frac{pL^2}{bh^2} \quad (b)$$

Thus,

$$\tau_{\max} / \sigma_{\max} = h/L \quad (c)$$

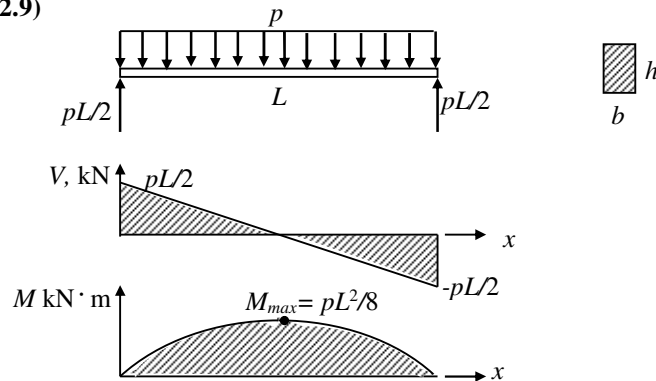
Equation (c):

$$L = h \frac{\sigma_{all}}{\tau_{all}} = 0.15 \left(\frac{9}{1.5} \right) = 0.9 \text{ m} \quad \blacktriangleleft$$

(b) Equation (a):

$$p_{all} = \frac{4}{3} \frac{bh}{L} \tau_{all} = \frac{4}{3} \frac{0.05 \times 0.15}{0.9} (1.5 \times 10^6) = 16.67 \text{ kN/m} \quad \blacktriangleleft$$

SOLUTION (2.9)



(CONT.)

(2.9 CONT.)

$$\tau_{\max} = \frac{3V}{2A} = \frac{3}{2} \frac{pL/2}{bh} = \frac{3}{4} \frac{pL}{bh} \quad (a)$$

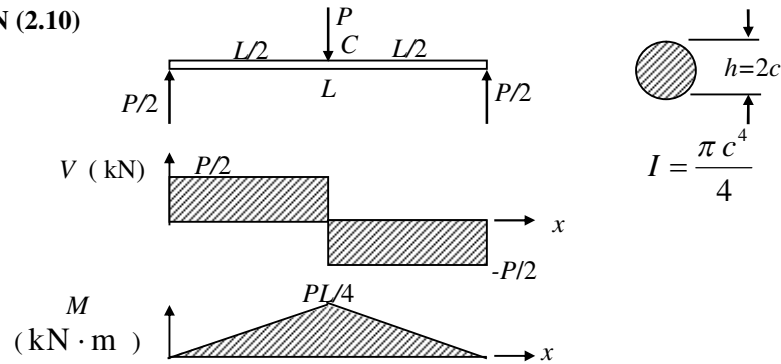
$$\sigma_{\max} = \frac{Mc}{I} = \frac{pL^2/8(h/2)}{bh^3/12} = \frac{3}{4} \frac{pL^2}{bh^2} \quad (b)$$

Thus,

$$\tau_{\max} / \sigma_{\max} = h/L \quad (c) \quad \blacktriangleleft$$

For example, if $L = 10h$, the above ratio is $1/10$.

SOLUTION (2.10)



From Table B.6:

$$\tau_{\max} = \frac{4V}{3A} = \frac{4}{3} \frac{P/2}{\pi c^2} = \frac{2}{3} \frac{P}{\pi c^2}$$

Also

$$\sigma_{\max} = \frac{Mc}{I} = \frac{4M}{\pi c^3} = \frac{PL}{\pi c^3}$$

Thus, $\tau_{\max} / \sigma_{\max} = 2c/3L = h/3L \quad \blacktriangleleft$

For example, if $L = 10h$, the above quotient is $1/30$.

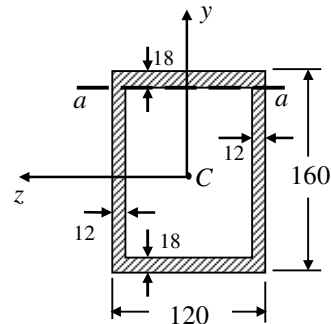
SOLUTION (2.11)

$$\begin{aligned} I &= \frac{1}{12}(120)(160)^3 - \frac{1}{12}(96)(124)^3 \\ &= 25.7(10^6) \text{ mm}^4 \end{aligned}$$

(a) Maximum shear stress (at N.A.):

$$\begin{aligned} Q &= (120)(80)(40) - (96)(62)(31) \\ &= 199.5(10^3) \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \tau_{\max} &= \frac{VQ}{Ib} = \frac{250(10^3)(199.5 \times 10^{-6})}{25.7(10^{-6})(0.24)} \\ &= 8.09 \text{ MPa} \end{aligned}$$



(CONT.)

(2.11 CONT.)

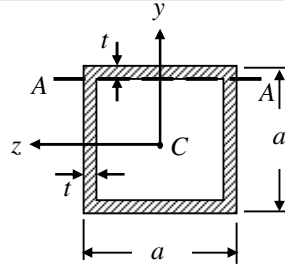
(b) Minimum shear stress (at section $a-a$):

$$Q = (120)(18)(80 - 9) = 153.36(10^3) \text{ mm}^3$$

$$\tau_{\min} = \frac{VQ}{Ib} = \frac{250(10^3)(153.36 \times 10^{-6})}{25.7(10^{-6})(0.24)} = 6.22 \text{ MPa}$$

SOLUTION (2.12)

$$\begin{aligned} I_z &= \frac{a^4}{12} - \frac{1}{12}(a - 2t)^4 \\ &= \frac{200^4}{12} - \frac{1}{12}(170)^4 = 63.73(10^6) \text{ mm}^4 \end{aligned}$$



(a) Maximum shear stress (at N.A.).

$$\begin{aligned} Q &= a\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) - (a - 2t)\left(\frac{a}{2} - t\right)\left(\frac{a}{2} - t\right)\left(\frac{1}{2}\right) \\ &= (200)(100)(50) - (170)(85)(85)\left(\frac{1}{2}\right) \\ &= 385,875 \text{ mm}^3 \end{aligned}$$

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{120(10^3)(385,875)}{(63.73)(10^6)(2 \times 15)} = 24.22 \text{ MPa}$$

(b) Minimum shear stress (at section A-A).

$$Q = at\left(\frac{a}{2} - \frac{t}{2}\right) = (200)(15)(92.5) = 88,425 \text{ mm}^3$$

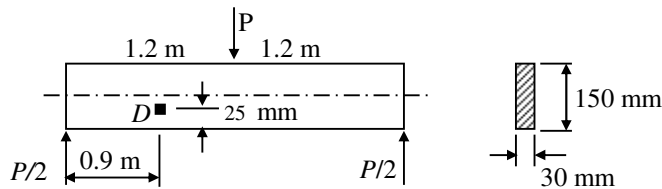
$$\tau_{\max} = \frac{VQ}{Ib} = \frac{120(10^3)(88,425)}{(63.73)(10^6)(2 \times 15)} = 5.55 \text{ MPa}$$

SOLUTION (2.13)

We have

$$I = \frac{1}{12}(30)(150)^3 = 8.44(10^{-6}) \text{ m}^4$$

$$Q_D = (0.03)(0.05)(0.0625) = 93.75(10^{-6}) \text{ m}^3$$



(CONT.)

(2.13 CONT.)

$$M_D = 0.45P \quad V_D = 0.5P$$

$$\sigma_D = \frac{Mc}{I} = \frac{0.45P(0.025)}{8.44 \times 10^{-6}} = 1333P$$

$$\tau_D = \frac{VQ}{Ib} = \frac{0.5P(93.75 \times 10^{-6})}{8.44 \times 10^{-6}(0.03)} = 185.13P$$

Equation (1.13):

$$\begin{aligned} (\sigma_1)_D &= 15 \times 10^6 = \frac{1333P}{2} + P \sqrt{\left(\frac{1333}{2}\right)^2 + (185.13)^2} \\ &= 666.5P + 691.7P = 1358.2P \end{aligned}$$

or

$$P_{all} = 11.04 \text{ kN}$$

SOLUTION (2.14)

$$EIw^{IV} = p = \frac{p_o}{L^2}(L^2 - x^2), \quad EIw''' = \frac{p_o}{L^2}(L^2x - \frac{x^3}{3}) + c_1$$

Boundary Condition:

$$w'''(L) = 0; \quad c_1 = -\frac{2}{3}p_oL$$

$$EIw'' = \frac{p_o}{L^2}(L^2\frac{x^2}{2} - \frac{x^4}{12}) + c_1x + c_2$$

Boundary Conditions:

$$EIw''(L) = 0; \quad c_2 = \frac{p_oL^2}{4}$$

$$EIw'' = \frac{p_o}{12L^2}(3L^4 - 8L^3x + 6L^2x^2 - x^4)$$

$$EIw' = \frac{p_o}{12L^2}(3L^4x - 8L^3\frac{x^2}{2} + 6L^2\frac{x^3}{3} - \frac{x^5}{5}) + c_3$$

Boundary Condition:

$$EIw'(0) = 0; \quad c_3 = 0.$$

$$EIw = \frac{p_o}{12L^2}(3L^4\frac{x^2}{2} - 4L^3\frac{x^3}{3} + 2L^2\frac{x^4}{4} - \frac{x^6}{30}) + c_4$$

Boundary Condition:

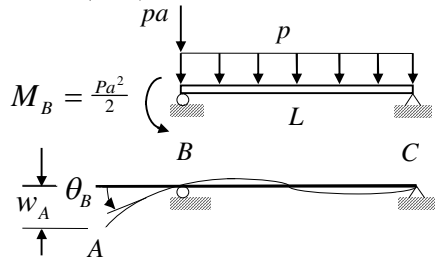
$$w(0) = 0; \quad c_4 = 0$$

$$\text{Thus, } w = \frac{p_o x^2}{360EI L^2}(45L^4 - 40L^3x + 15L^2x^2 - x^4)$$

At $x=L$;

$$w_B = \frac{19p_oL^4}{360EI} \quad \theta_B = w_B' = \frac{p_oL^3}{15EI}$$

SOLUTION (2.15)



Refer to Table B.7 (Case 5 and 7):

$$\theta_B = \frac{-pL^3}{24EI} + \frac{M_B L}{3EI} = \frac{pL(4a^2 + L^2)}{24EI}$$

Deflection w_1 of A due to only θ_B :

$$w_1 = \theta_B a = \frac{paL(4a^2 + L^2)}{24EI}$$

(CONT.)

(2.15 CONT.)

Table B.7 (Case 2 with $b=0$), cantilever:

$$w_2 = \frac{pL^3}{24EI}(4L - L) = \frac{pL^4}{8EI}$$

Total deflection

$$w_A = w_1 + w_2 = \frac{pa}{24EI}(3L^4 + 4a^2L - L^3) \quad \blacktriangleleft$$

SOLUTION (2.16)

$$EIw^{IV} = p_0 \sin \frac{\pi x}{L}; \quad EIw''' = -p_0 \left(\frac{L}{\pi}\right) \cos \frac{\pi x}{L} + c_1$$

$$EIw'' = -p_0 \left(\frac{L}{\pi}\right)^2 \sin \frac{\pi x}{L} + c_1 x + c_2$$

Boundary Conditions:

$$w''(0) = 0, \quad c_2 = 0; \quad w''(L) = 0, \quad c_1 = 0$$

$$EIw' = p_0 \left(\frac{L}{\pi}\right)^3 \cos \frac{\pi x}{L} + c_3$$

$$EIw = p_0 \left(\frac{L}{\pi}\right)^4 \sin \frac{\pi x}{L} + c_3 x + c_4$$

Boundary Conditions:

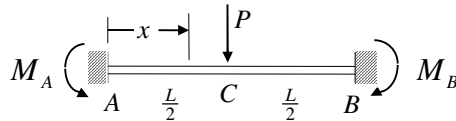
$$w(0) = 0, \quad c_4 = 0; \quad w(L) = 0, \quad c_3 = 0$$

Thus

$$w = \frac{p_0 L^4}{\pi^4 EI} \sin \frac{\pi x}{L} \quad \blacktriangleleft$$

$$\text{Slope at } x=0: \quad \theta_A = w'_1(0) = \frac{p_0 L^3}{\pi^3 EI} \curvearrowright = -\theta_B \quad \blacktriangleleft$$

SOLUTION (2.17)



Symmetry

$$M_A = -M_B$$

$$R_A = R_B = \frac{P}{2} \uparrow \quad \blacktriangleleft$$

Segment AC

$$EIw^{IV} = 0, \quad EIw''' = c_1, \quad EIw'' = c_1 x + c_2$$

$$EIw' = \frac{1}{2} c_1 x^2 + c_2 x + c_3$$

$$EIw = \frac{1}{6} c_1 x^3 + \frac{1}{2} c_2 x^2 + c_3 x + c_4 \quad (a)$$

We have

$$EIw'''(0) = c_1 = -V = -\frac{P}{2}$$

$$EIw''(0) = c_2 = -(-M_A): \quad c_2 = M_A$$

$$w'(0) = 0: \quad c_3 = 0, \quad w'\left(\frac{L}{2}\right) = 0: \quad M_A = -M_B = \frac{PL}{8} \curvearrowright$$

$$w(0) = 0: \quad c_4 = 0$$

Equation (a) is thus

$$w = \frac{Px^2}{48EI}(3L - 4x) \quad \blacktriangleleft$$

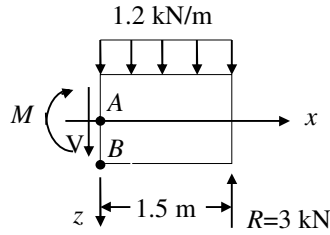
SOLUTION (2.18)

We have $250/2 = 125$ MPa . Equation (2.30b) gives the limiting value of pressure for the tangential stress as

$$p = \frac{\sigma_{all} t}{r} = \frac{125 \times 10^6 (0.005)}{0.2} = 3.125 \text{ MPa}$$

Note that, the axial stress formula, Eq. (2.38a) requires

$$p = 2 \frac{\sigma_{all} t}{r} = 6.25 \text{ MPa}$$

SOLUTION (2.19)

$$V = 3 - 1.5(1.2) = 1.2 \text{ kN}$$

$$M = 3(1.5) - \frac{1}{2}(1.2)(1.5)^2 = 3.15 \text{ kN} \cdot \text{m}$$

Point A

$$\sigma = \frac{M_c}{I} = 0, \quad \sigma_\theta = \frac{pr}{t} = \frac{4(10^4)(0.5)}{0.005} = 4 \text{ MPa}$$

$$\sigma_x = \sigma_a = 2 \text{ MPa}$$

Table B-4;

$$\tau = \frac{VQ}{Ib} = \frac{12000(\pi r t)(2r/\pi)}{\pi^3 t(2t)} = 153 \text{ kPa}$$

Thus $\tau_{\max} = [(\frac{2-4}{2})^2 + (0.153)^2]^{1/2} = 1.012 \text{ MPa}$

$$\theta_s = \frac{1}{2} \tan^{-1} \frac{2-4}{2(0.153)} = -40.65^\circ$$

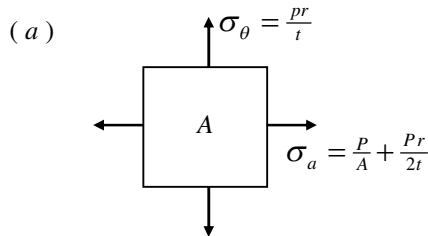
Point B:

$$\sigma = \frac{M_c}{I} = \frac{3150(0.5)}{\pi(0.5)^3(0.005)} = 802.1 \text{ kPa}, \quad \tau = \frac{VQ}{Ib} = 0$$

$$\sigma_1 = \sigma_\theta = 4 \text{ MPa}, \quad \sigma_2 = 2 + 0.802 = 2.802 \text{ MPa}$$

and

$$\tau_{\max} = \frac{1}{2}(4 - 2.802) = 599 \text{ kPa}, \quad \theta_s = 45^\circ$$

SOLUTION (2.20)

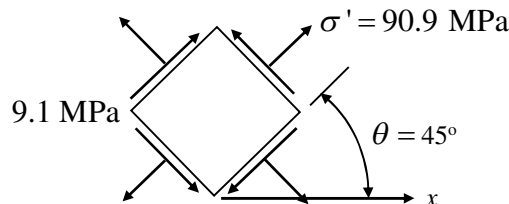
$$A = 2\pi r t$$

$$= 2\pi(250)(10) = 15,708 \text{ mm}^2$$

$$\sigma_\theta = \frac{4(10^6)(0.25)}{0.01} = 100 \text{ MPa}$$

$$\sigma_x = \frac{500(10^3)}{15,708(10^{-6})} + 50 = 81.8 \text{ MPa}$$

(b) $\tau_{\max} = \frac{1}{2}[100 - 81.8] = 9.1 \text{ MPa}$

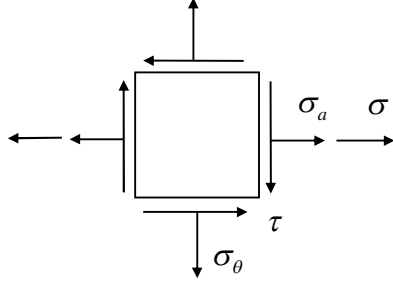


$$\sigma' = \frac{1}{2}(\sigma_\theta + \sigma_x) = 90.9 \text{ MPa}$$

SOLUTION (2.21)

At a point on circumference, we have

$$\sigma_a = \frac{pr}{2t} = \frac{4(100)}{2(8)} = 25 \text{ MPa}, \quad \sigma_\theta = 50 \text{ MPa}$$



$$\sigma = \frac{p}{2\pi r t} = \frac{50(10^3)}{2\pi(100)(8)10^{-6}} = 9.947 \text{ MPa}$$

$$\tau = \frac{Tr}{J} = \frac{-30(10^3)(0.1)}{2\pi(0.1)^3(0.008)} = -59.68 \text{ MPa}$$

Thus

$$\sigma_{1,2} = \frac{34.95+50}{2} \pm \left[\left(\frac{34.95-50}{2} \right)^2 + 59.68^2 \right]^{1/2}$$

$$= 42.48 \pm 60.12$$

or

$$\sigma_1 = 102.6 \text{ MPa}, \quad \sigma_2 = -17.64 \text{ MPa}$$

$$(a) \quad |\sigma_1| \leq \sigma_u; \quad |102.6| < 240 \quad \therefore \text{no failure} \quad \blacktriangleleft$$

$$(b) \quad \frac{\sigma_1}{\sigma_u} = \frac{\sigma_2}{\sigma_{uc}} = 1; \quad \frac{102.6}{240} - \frac{-17.64}{600} = 1$$

or

$$0.428 + 0.029 < 1 \quad \therefore \text{no failure} \quad \blacktriangleleft$$

SOLUTION (2.22)

$$\sigma_{1,2} = \frac{-100-50}{2} \pm \left[\left(\frac{-100+50}{2} \right)^2 + 30^2 \right]^{1/2}$$

or

$$\sigma_2 = -36 \text{ MPa}, \quad \sigma_3 = -114 \text{ MPa}, \quad \sigma_1 = 60 \text{ MPa}$$

$$(a) \quad n = \left| \frac{\sigma_u}{\sigma_1} \right| = \left| \frac{150}{60} \right| = 2.5$$

$$\text{or} \quad n = \left| \frac{150}{114} \right| = 1.32 \quad \blacktriangleleft$$

$$(b) \quad \frac{60}{150} - \frac{-114}{600} = \frac{1}{n}; \quad 0.4 + 0.19 = \frac{1}{n}$$

$$\text{Solving} \quad n = 1.7 \quad \blacktriangleleft$$

SOLUTION (2.23)

We have $\sigma_{all} = 250/2.2 = 113.6 \text{ MPa}$. From Eq. (2.38b) we find that the limiting value of pressure

$$p = \frac{\sigma_{all} t}{r} = \frac{113.6(10^6)(0.0036)}{0.9} = 454.4 \text{ kPa}$$

for circumferential stress. The axial stress is thus

$$p = 2 \frac{\sigma_{all} t}{r} = 908.8 \text{ kPa}$$

Comment: The gage pressure may not exceed 454.4 kPa.

SOLUTION (2.24)

The tangential, axial, and radial stresses are:

$$\sigma_{\theta} = \frac{pr}{t} = 25p \quad \sigma_a = \frac{pr}{2t} = 12.5p \quad \sigma_z = 0$$

(a) Using Equation 2.42a, we have:

$$25p = \frac{1}{1.8}(260)(10^6), \quad p = 5.78 \text{ MPa}$$

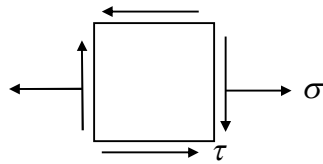
(b) From Equation:

$$p(25^2 - 25 \times 12.5 + 12.5^2)^{1/2} = \frac{260}{1.8}, \quad p = 6.67 \text{ MPa}$$

Comment: The allowable value of the maximum pressure is limited to 5.78 MPa.

SOLUTION (2.25)

At a point on the surface of the shaft, we have



$$J = \frac{\pi}{32}(75)^4 = 3.106(10^6) \text{ mm}^4$$

$$A = \frac{\pi}{4}(75)^2 = 4.418(10^3) \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{4.418(10^{-3})} = 9.054 \text{ MPa}$$

$$\tau = \frac{Tr}{J} = \frac{6(10^3)(0.0375)}{3.106(10^{-6})} = 72.44 \text{ MPa}$$

Thus

$$\sigma_{1,2} = \frac{9.054}{2} \pm [(\frac{9.054}{2})^2 + 72.44^2]^{1/2} = 4.527 \pm 72.581$$

$$\sigma_1 = 77.11 \text{ MPa}, \quad \sigma_2 = -68.05 \text{ MPa}$$

$$(a) \quad \frac{250}{n} = [(77.11)^2 - (77.11)(-68.05) + (-68.05)^2]^{1/2}$$

◀

or $n = 1.99$

$$(b) \quad |\sigma_1 - \sigma_2| = \frac{\sigma_{yp}}{n}; \quad |77.11 + 68.05| = \frac{250}{n}$$

◀

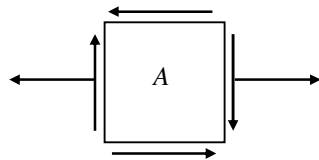
or $n = 1.72$

SOLUTION (2.26)

At the fixed end A (see Fig. P2.26):

$$T = 1.2R \quad M_z = 1.6R \quad V_y = -R$$

The effect of V_y may be neglected. Thus, at a point A on the top of the bar at the fixed end:



$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1.6R)}{\pi(0.06)^3} = 7.545R$$

$$\tau = -\frac{16T}{\pi d^3} = -\frac{16(1.2R)}{\pi(0.06)^3} = -2.829R$$

(CONT.)

(2.26 CONT.)

Then

$$\sigma_{1,2} = \frac{7.545R}{2} \pm \sqrt{\left(\frac{7.545R}{2}\right)^2 + (-2.829R)^2}$$
$$\sigma_1 = 8.487 \times 10^4 R \quad \sigma_2 = -0.943 \times 10^4 R \quad \tau_{\max} = 4.715 \times 10^4 R$$

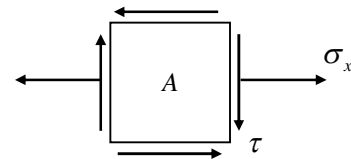
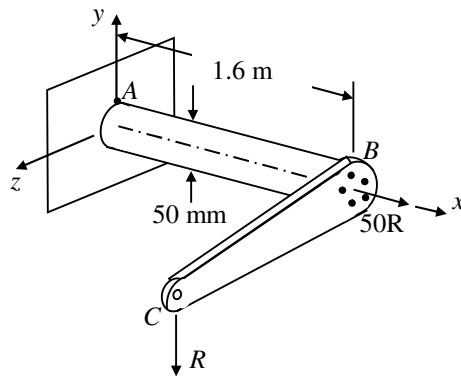
$$(a) \quad \frac{130 \times 10^6}{1.9} = 4.715 \times 10^4 R, \quad R = 1.45 \text{ kN}$$

$$(b) \quad \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = (\sigma_{yp} / n_s)^2$$
$$[8.487^2 - 8.487(-0.943) + (-0.943)^2]^{1/2} (10^4) R = 240 \times 10^6 / 1.9$$

or

$$R = 1.404 \text{ kN}$$

SOLUTION (2.27)



At the fixed end:

$$T = 1.2R \quad M_z = 1.6R$$
$$P_x = 50R \quad V_y = -R$$

The effect of V_y may be neglected. Therefore, at point A:

$$\sigma_x' = \frac{P_x}{A} = \frac{50R}{\frac{\pi}{4}(0.06)^2} = 1.768 \times 10^4 R$$
$$\sigma_x'' = \frac{32}{\pi d^3} = \frac{32(1.6R)}{\pi(0.06)^3} = 7.545 \times 10^4 R$$
$$\tau = -\frac{16T}{\pi d^3} = -\frac{16(1.2R)}{\pi(0.06)^3} = -2.829 \times 10^4 R$$
$$\sigma_{1,2} = 10^4 R \left[\frac{9.313}{2} \pm \sqrt{\left(\frac{9.313}{2}\right)^2 + (-2.829)^2} \right]$$
$$\sigma_1 = 10.106 \times 10^4 R \quad \sigma_2 = -0.793 \times 10^4 R \quad \tau_{\max} = 5.449 \times 10^4 R$$

$$(a) \quad \frac{130 \times 10^6}{1.9} = 5.449 \times 10^4 R, \quad R = 1.26 \text{ kN}$$

(CONT.)

(2.27 CONT.)

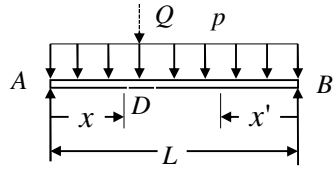
$$(b) [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} = \frac{\sigma_{yp}}{2}$$

$$10^4 R[(10.106)^2 - (10.106)(-0.793) + (-0.793)^2]^{1/2} = 240 \times 10^6 / 1.9$$

Solving,

$$R = 1.2 \text{ kN}$$

SOLUTION (2.28)



$$\frac{\partial M_{AD}}{\partial Q} = \frac{bx}{L}, \quad \frac{\partial M_{BD}}{\partial Q} = \frac{ax'}{L}$$

Applying Eq. (2.57):

$$w_D = \frac{p}{2EI} \left[\int_0^a (Lx - x^2)(bx)dx + \int_0^b (Lx' - x'^2)(ax')dx' \right]$$

Integrating, we have

$$w_D = \frac{pab}{24EIL} [4L(a^2 + b^2) - 3(a^3 + b^3)]$$

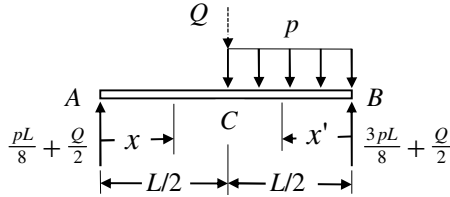
Using equations of statics;

$$R_A = \frac{pL}{2} + \frac{Qb}{L}, \quad R_B = \frac{pL}{2} + \frac{Qa}{L}$$

Then

$$M_{AD} = R_A x + \frac{px^2}{2}, \quad M_{BD} = R_B x' + \frac{px'^2}{2};$$

SOLUTION (2.29)



Segment AC

$$M_1 = \left(\frac{pL}{8} + \frac{Q}{2} \right) x, \quad \frac{\partial M_1}{\partial Q} = \frac{x}{2}$$

Segment BC

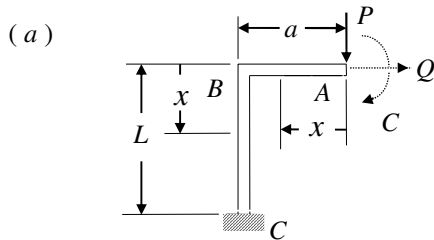
$$M_2 = \left(\frac{3pL}{8} + \frac{Q}{2} \right) x' - \frac{px'^2}{2}, \quad \frac{\partial M_2}{\partial Q} = \frac{x'}{2}$$

Let $Q=0$, Thus, Eq. (2.57):

$$EIw_C = \int_0^{L/2} \frac{pLx}{8} \frac{x}{2} dx + \int_0^{L/2} \left(\frac{3pL}{8} x' - \frac{px'^2}{2} \right) \frac{x'}{2} dx' = \frac{5}{16} \frac{pL^4}{48}$$

or $w_C = \frac{5}{768} \frac{pL^4}{EI} \downarrow$

SOLUTION (2.30)



We have

$$M_{AB} = Px \quad M_{BC} = Pa$$

$$\delta_v = \frac{1}{EI} \int M_i \frac{\partial M_i}{\partial P} dx$$

$$= \frac{1}{EI} \left[\int_0^a (Px)(x)dx + \int_0^L (Pa)(a)dy \right]$$

$$= \frac{1}{EI} \left(\frac{a^3}{3} + a^2 L \right) = \frac{Pa^2}{3EI} (a + 3L)$$

(CONT.)

(2.30 CONT.)

(b) Add Q at A. Hence,

$$\delta_H = \frac{1}{EI} \left[\int_0^a M_{AB} \frac{\partial M_{AB}}{\partial Q} dx + \int_0^L M_{BC} \frac{\partial M_{BC}}{\partial Q} dx \right],$$

where $M_{AB} = Px$, $M_{BC} = Pa + Qx$

Let $Q=0$, Then

$$\delta_H = 0 + \int_0^L (Pa)(x)dx = \frac{PaL^2}{2EI}$$

(c) Add C at A. We have

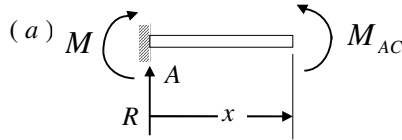
$$\theta = \frac{1}{EI} \int M_i \frac{\partial M_i}{\partial C} dx \quad \text{where } M_{AB} = Px + C, \quad M_{BC} = Pa + C$$

$$\theta_A = \frac{1}{EI} \left[\int_0^a (Px + C)dx + \int_0^L (Pa + C)dx \right]$$

For $C=0$:

$$\theta_A = \frac{P}{EI} \left(\frac{a^2}{2} + aL \right) = \frac{Pa}{2EI} (a + 2L)$$

SOLUTION (2.31)



$$M_{AC} = Rx + M, \quad M_{BC} = Rx + M - P(x - \frac{L}{2})$$

$$\theta = \frac{1}{EI} \int M_i \frac{\partial M_i}{\partial M} dx, \quad w = \frac{1}{EI} \int M_i \frac{\partial M_i}{\partial R} dx$$

We have

$$\begin{aligned} \theta_A &= \frac{1}{EI} \left\{ \int_0^{L/2} (Rx + M)dx + \int_{L/2}^L [Rx + M - P(x - \frac{L}{2})]dx \right\} \\ w_A &= \frac{1}{EI} \left\{ \int_0^{L/2} (Rx + M)x dx + \int_{L/2}^L [Rx + M - P(x - \frac{L}{2})]x dx \right\} \end{aligned} \quad (a)$$

Boundary conditions are $\theta(0) = 0$ and $w(0) = 0$. Thus, after integrating Eqs.(a):

$$\theta_A = \frac{RL^2}{2EI} + \frac{ML}{EI} - \frac{PL^2}{8EI} = 0$$

$$w_A = \frac{PL^3}{3EI} + \frac{ML^2}{2EI} - \frac{5PL^3}{48EI} = 0$$

From which

$$RL^2 + 2ML = \frac{PL^2}{4}$$

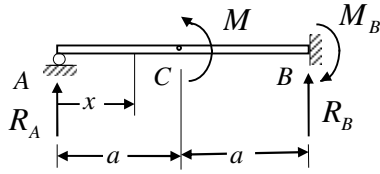
$$RL^3 + \frac{3}{2} ML^2 = \frac{5}{16} PL^3$$

Solving

$$M = -\frac{PL}{8} = \frac{PL}{8} \quad R = \frac{P}{2} \uparrow$$

$$\begin{aligned} (c) \quad w_C &= \frac{1}{EI} \left[\int_0^{L/2} (Rx + M)x dx \right] = \frac{1}{EI} \left[\int_0^{L/2} \left(\frac{P}{2} x - \frac{PL}{8} \right) x dx \right] \\ &= \frac{PL^3}{192EI} = w_{\max} \downarrow \end{aligned}$$

SOLUTION (2.32)



$$M_{AC} = R_A x, \quad M_{BC} = R_A x - M$$

$$w_A = \frac{1}{EI} \int M_i \frac{\partial M_i}{\partial R_A} dx$$

$$w_A = \frac{1}{EI} \left[\int_0^a (R_A x)(x) dx + \int_a^{2a} (R_A x - M)(x) dx \right]$$

$$= \frac{8}{3} \frac{R_A a^3}{EI} - \frac{3}{2} \frac{M a^2}{EI} = 0$$

$$\text{from which } R_A = \frac{9}{16} \frac{M}{a} \uparrow$$

Statics:

$$\sum F_y = 0: \quad R_B = -R_A \downarrow$$

$$\sum M_B = 0: \quad M_B - M + R_A(2a) = 0$$

or

$$M_B = -\frac{M}{8} = \frac{M}{8} \curvearrowright$$