Applied Calculus 7th edition Berresford Solutions Manual

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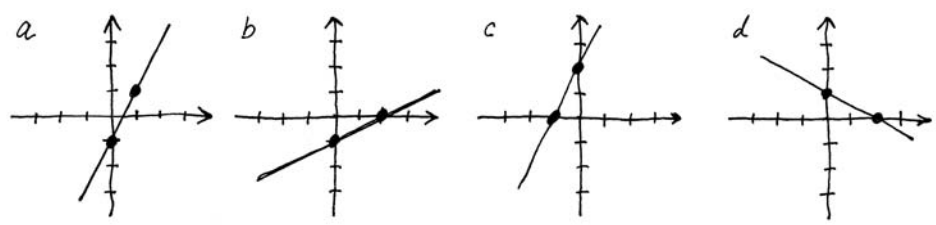
DIAGNOSTIC TEST

Are you ready to study calculus?

Algebra is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some algebra.

If you are comfortable with the algebra covered in the following problems, you are ready to begin your study of calculus. If not, turn to the *Algebra Appendix* beginning on page A.xxx and review the *Complete Solutions* to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.

Problems	**Answers**
1. True or False? $\frac{1}{2} < -3$	False
2. Express $\{x \mid -4 < x \leq 5\}$ in interval notation.	$(-4, 5]$
3. What is the slope of the line through the points $(6, -7)$ and $(9, 8)$?	5
4. On the line $y = 3x + 4$, what value of Δy corresponds to $\Delta x = 2$?	6
5. Which sketch shows the graph of the line $y = 2x - 1$?	a



6. True or False? $\left(\dfrac{\sqrt{x}}{y}\right)^{-2} = \dfrac{y^2}{x}$	True
7. Find the zeros of the function $f(x) = 9x^2 - 6x - 1$	$x = \frac{1 \pm \sqrt{2}}{3}$
8. Expand and simplify $x(8 - x) - (3x + 7)$.	$-x^2 + 5x - 7$
9. What is the domain of $f(x) = \dfrac{x^2 - 3x + 2}{x^3 + x^2 - 6x}$?	$\{x \mid x \neq -3, x \neq 0, x \neq 2\}$
10. For $f(x) = x^2 - 5x$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.	$2x - 5 + h$

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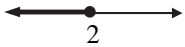
Chapter 1: Functions

EXERCISES 1.1

1. $\{x \mid 0 \le x < 6\}$

2. $\{x \mid -3 < x \le 5\}$

3. $\{x \mid x \le 2\}$



4. $\{x \mid x \ge 7\}$

5. **a.** Since $\Delta x = 3$ and $m = 5$, then Δy, the change in y, is

$$\Delta y = 3 \bullet m = 3 \bullet 5 = 15$$

b. Since $\Delta x = -2$ and $m = 5$, then Δy, the change in y, is

$$\Delta y = -2 \bullet m = -2 \bullet 5 = -10$$

6. **a.** Since $\Delta x = 5$ and $m = -2$, then Δy, the change in y, is

$$\Delta y = 5 \bullet m = 5 \bullet (-2) = -10$$

b. Since $\Delta x = -4$ and $m = -2$, then Δy, the change in y, is

$$\Delta y = -4 \bullet m = -4 \bullet (-2) = 8$$

7. For (2, 3) and (4, –1), the slope is

$$\frac{-1-3}{4-2} = \frac{-4}{2} = -2$$

8. For (3, –1) and (5, 7), the slope is

$$\frac{7-(-1)}{5-3} = \frac{7+1}{2} = \frac{8}{2} = 4$$

9. For (–4, 0) and (2, 2), the slope is

$$\frac{2-0}{2-(-4)} = \frac{2}{2+4} = \frac{2}{6} = \frac{1}{3}$$

10. For (–1, 4) and (5, 1), the slope is

$$\frac{1-4}{5-(-1)} = \frac{-3}{5+1} = \frac{-3}{6} = -\frac{1}{2}$$

11. For (0, –1) and (4, –1), the slope is

$$\frac{-1-(-1)}{4-0} = \frac{-1+1}{4} = \frac{0}{4} = 0$$

12. For $\left(-2, \frac{1}{2}\right)$ and $\left(5, \frac{1}{2}\right)$, the slope is

$$\frac{\frac{1}{2}-\frac{1}{2}}{5-(-2)} = \frac{0}{5+2} = \frac{0}{7} = 0$$

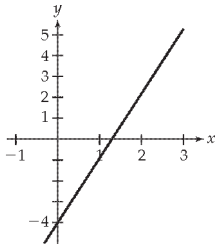
13. For (2, –1) and (2, 5), the slope is

$$\frac{5-(-1)}{2-2} = \frac{5+1}{0} \qquad \text{undefined}$$

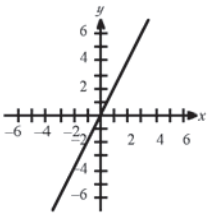
14. For (6, –4) and (6, –3), the slope is

$$\frac{-3-(-4)}{6-6} = \frac{-3+4}{0} \qquad \text{undefined}$$

15. Since $y = 3x - 4$ is in slope-intercept form, $m = 3$ and the y-intercept is (0, –4). Using the slope $m = 3$, we see that the point 1 unit to the right and 3 units up is also on the line.



16. Since $y = 2x$ is in slope-intercept form, $m = 2$ and the y-intercept is (0, 0). Using $m = 2$, we see that the point 1 to the right and 2 units up is also on the line.



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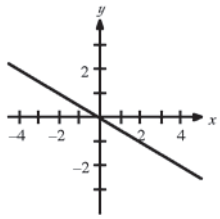
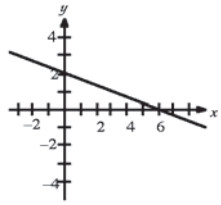
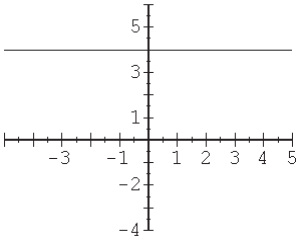
17. Since $y = -\frac{1}{2}x$ is in slope-intercept form, $m = -\frac{1}{2}$ and the y-intercept is (0, 0). Using $m = -\frac{1}{2}$, we see that the point 2 units to the right and 1 unit down is also on the line.



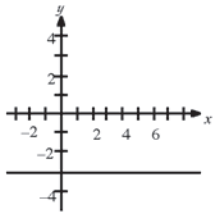
18. Since $y = -\frac{1}{3}x + 2$ is in slope-intercept form, $m = -\frac{1}{3}$ and the y-intercept is (0, 2). Using $m = -\frac{1}{3}$, we see that the point 3 units to the right and 1 unit down is also on the line.



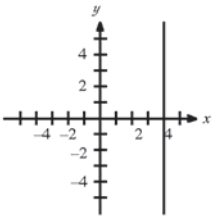
19. The equation $y = 4$ is the equation of the horizontal line through all points with y-coordinate 4. Thus, $m = 0$ and the y-intercept is (0, 4).



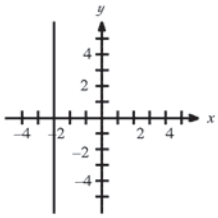
20. The equation $y = -3$ is the equation of the horizontal line through all points with y-coordinate –3. Thus, $m = 0$ and the y-intercept is (0, –3).



21. The equation $x = 4$ is the equation of the vertical line through all points with x-coordinate 4. Thus, m is not defined and there is no y-intercept.



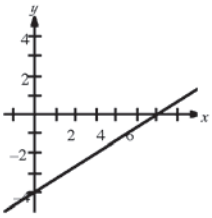
22. The equation $x = -3$ is the equation of the vertical line through all points with x-coordinate –3. Thus, m is not defined and there is no y-intercept.



23. First, solve for y:

$$2x - 3y = 12$$
$$-3y = -2x + 12$$
$$y = \frac{2}{3}x - 4$$

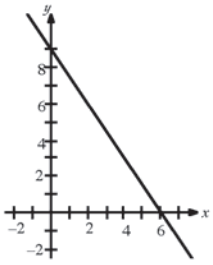
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –4).



24. First, solve for y:

$$3x + 2y = 18$$
$$2y = -3x + 18$$
$$y = -\frac{3}{2}x + 9$$

Therefore, $m = -\frac{3}{2}$ and the y-intercept is (0, 9).



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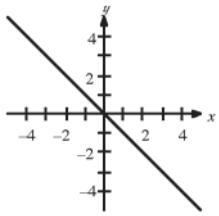
25. First, solve for y:

$$x + y = 0$$
$$y = -x$$

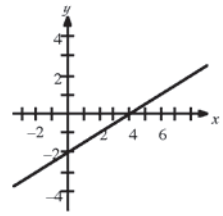
Therefore, $m = -1$ and the y-intercept is (0, 0).



26. First, solve for y:

$$x = 2y + 4$$
$$-2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$

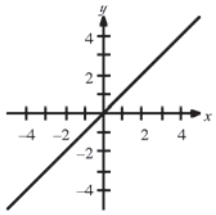
Therefore, $m = \frac{1}{2}$ and the y-intercept is (0, –2).



27. First, solve for y:

$$x - y = 0$$
$$-y = -x$$
$$y = x$$

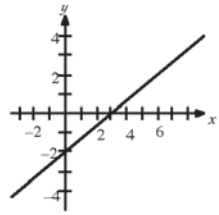
Therefore, $m = 1$ and the y-intercept is (0, 0).



28. First, put the equation in slope-intercept form:

$$y = \frac{2}{3}(x - 3)$$
$$y = \frac{2}{3}x - 2$$

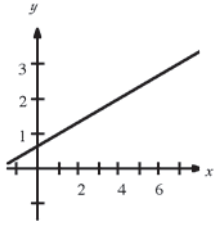
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –2).



29. First, put the equation in slope-intercept form:

$$y = \frac{x+2}{3}$$
$$y = \frac{1}{3}x + \frac{2}{3}$$

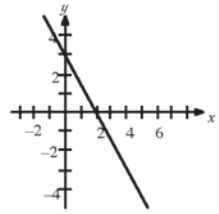
Therefore, $m = \frac{1}{3}$ and the y-intercept is $\left(0, \frac{2}{3}\right)$.



30. First, solve for y:

$$\frac{x}{2} + \frac{y}{3} = 1$$
$$\frac{y}{3} = -\frac{1}{2}x + 1$$
$$y = -\frac{3}{2}x + 3$$

Therefore, $m = -\frac{3}{2}$ and the y-intercept is (0, 3).



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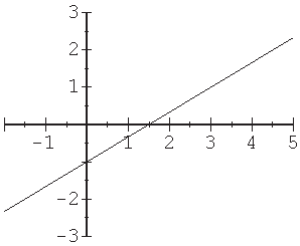
31. First, solve for y:

$$\frac{2x}{3} - y = 1$$
$$-y = -\frac{2}{3}x + 1$$
$$y = \frac{2}{3}x - 1$$

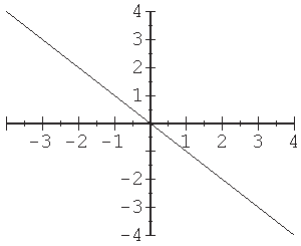
Therefore, $m = \frac{2}{3}$ and the y-intercept is (0, –1).



32. First, solve for y:

$$\frac{x+1}{2} + \frac{y+1}{2} = 1$$
$$x + 1 + y + 1 = 2$$
$$x + y + 2 = 2$$
$$y = -x$$

Therefore, $m = -1$ and the y-intercept is (0, 0).



33. $y = -2.25x + 3$

34. $y = \frac{2}{3}x - 8$

35. $y - (-2) = 5[x - (-1)]$

$$y + 2 = 5x + 5$$
$$y = 5x + 3$$

36. $y - 3 = -1(x - 4)$

$$y - 3 = -x + 4$$
$$y = -x + 7$$

37. $y = -4$

38. $y = \frac{3}{4}$

39. $x = 1.5$

40. $x = \frac{1}{2}$

41. First, find the slope.

$$m = \frac{-1-3}{7-5} = \frac{-4}{2} = -2$$

Then use the point-slope formula with this slope and the point (5, 3).

$$y - 3 = -2(x - 5)$$
$$y - 3 = -2x + 10$$
$$y = -2x + 13$$

42. First, find the slope.

$$m = \frac{0-(-1)}{6-3} = \frac{1}{3}$$

Then use the point-slope formula with this slope and the point (6, 0).

$$y - 0 = \frac{1}{3}(x - 6)$$
$$y = \frac{1}{3}x - 2$$

43. First, find the slope.

$$m = \frac{-1-(-1)}{5-1} = \frac{-1+1}{4} = 0$$

Then use the point-slope formula with this slope and the point (1,–1).

$$y - (-1) = 0(x - 1)$$
$$y + 1 = 0$$
$$y = -1$$

44. First, find the slope.

$$m = \frac{-4-0}{2-2} = \frac{-4}{0} \quad \text{undefined}$$

Since the slope of the line is undefined, the line is a vertical line. Because the x-coordinates of the points are 2, the equation is $x = 2$.

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45. **a.** First find the slope of the line $4y-3x=5$. Write the equation in slope-intercept form.

$$y=\frac{3}{4}x+\frac{5}{4}$$

The slope of the parallel line is $m=\frac{3}{4}$.

Next, use the point-slope form with the point (12, 2):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-2&=\frac{3}{4}(x-12)\\ y&=\frac{3}{4}x-7 \end{aligned}$$

b. The slope of the line perpendicular to $y=\frac{3}{4}x+\frac{5}{4}$. is $m=-\frac{4}{3}$.

Next, use the point-slope form with the point (12, 2):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-2&=-\frac{4}{3}(x-12)\\ y&=-\frac{4}{3}x+18 \end{aligned}$$

46. **a.** First find the slope of the line $x+3y=7$. Write the equation in slope-intercept form.

$$y=-\frac{1}{3}x+\frac{7}{3}.$$

The slope of the parallel line is $m=-\frac{1}{3}$.

Next, use the point-slope form with the point (–6, 5):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-5&=-\frac{1}{3}(x+6)\\ y&=-\frac{1}{3}x+3 \end{aligned}$$

b. The slope of the line perpendicular to $y=-\frac{1}{3}x+\frac{7}{3}$. is $m=3$.

Next, use the point-slope form with the point (–6, 5):

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-5&=3(x+6)\\ y&=3x+23 \end{aligned}$$

47. The y-intercept of the line is (0, 1), and $\Delta y=-2$ for $\Delta x=1$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{-2}{1}=-2$. Now, use the slope-intercept form of the line: $y=-2x+1$.

48. The y-intercept of the line is (0, –2), and $\Delta y=3$ for $\Delta x=1$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{3}{1}=3$. Now, use the slope-intercept form of the line: $y=3x-2$

49. The y-intercept is (0, –2), and $\Delta y=3$ for $\Delta x=2$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{3}{2}$. Now, use the slope-intercept form of the line: $y=\frac{3}{2}x-2$

50. The y-intercept is (0, 1), and $\Delta y=-2$ for $\Delta x=3$. Thus, $m=\frac{\Delta y}{\Delta x}=\frac{-2}{3}=-\frac{2}{3}$. Now, use the slope-intercept form of the line: $y=-\frac{2}{3}x+1$

51. First, consider the line through the points (0, 5) and (5, 0). The slope of this line is $m=\frac{0-5}{5-0}=\frac{-5}{5}=-1$. Since (0, 5) is the y-intercept of this line, use the slope-intercept form of the line: $y=-1x+5$ or $y=-x+5$.
Now consider the line through the points (5, 0) and (0, –5). The slope of this line is $m=\frac{-5-0}{0-5}=\frac{-5}{-5}=1$. Since (0,–5) is the y-intercept of the line, use the slope-intercept form of the line: $y=1x-5$ or $y=x-5$
Next, consider the line through the points (0, –5) and (–5, 0). The slope of this line is $m=\frac{0-(-5)}{-5-0}=\frac{5}{-5}=-1$.
Since (0, –5) is the y-intercept, use the slope-intercept form of the line: $y=-1x-5$ or $y=-x-5$
Finally, consider the line through the points (–5, 0) and (0, 5). The slope of this line is $m=\frac{5-0}{0-(-5)}=\frac{5}{5}=1$. Since (0, 5) is the y-intercept, use the slope-intercept form of the line: $y=1x+5$ or $y=x+5$

52. The equation of the vertical line through (5, 0) is $x=5$.
The equation of the vertical line through (–5, 0) is $x=-5$.
The equation of the horizontal line through (0, 5) is $y=5$.
The equation of the horizontal line through (0, –5) is $y=-5$.

53. If the point (x_1, y_1) is the y-intercept $(0, b)$, then substituting into the point-slope form of the line gives

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-b&=m(x-0)\\ y-b&=mx\\ y&=mx+b \end{aligned}$$

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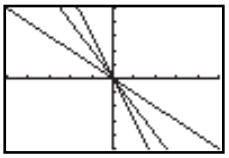
54. To find the x-intercept, substitute $y = 0$ into the equation and solve for x:

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{x}{a}+\frac{0}{b}=1$$
$$\frac{x}{a}=1$$
$$x=a$$ Thus, $(a, 0)$ is the x-intercept.

To find the y-intercept, substitute $x = 0$ into the equation and solve for y:

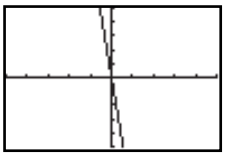
$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{0}{a}+\frac{y}{b}=1$$
$$\frac{y}{b}=1$$
$$y=b$$ Thus, $(0, b)$ is the y-intercept.

55. **a.**



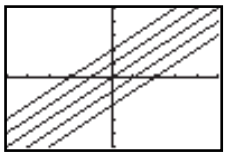
on [–5, 5] by [–5, 5]

b.



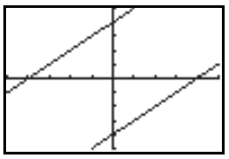
on [–5, 5] by [–5, 5]

56. **a.**



on [–5, 5] by [–5, 5]

b.



on [–5, 5] by [–5, 5]

57. Low demand: [0, 8);
average demand: [8, 20);
high demand: [20, 40);
critical demand: [40, ∞)

58. A: [90, 100]; B: [80,90); C: [70, 80);
D: [60, 70); F: [0, 60)

59. **a.** The value of x corresponding to the year 2020 is $x = 2020 - 1900 = 120$. Substituting $x = 120$ into the equation for the regression line gives

$$y=-0.356x+257.44$$
$$y=-0.356(120)+257.44=214.72 \text{ seconds}$$

Since 3 minutes = 180 seconds, 214.72 = 3 minutes 34.72 seconds. Thus, the world record in the year 2020 will be 3 minutes 34.72 seconds.

b. To find the year when the record will be 3 minutes 30 seconds, first convert 3 minutes 30 seconds to seconds: 3 minutes 30 seconds = 3 minutes • $\frac{60 \text{ sec}}{1 \text{ min}}$ + 30 seconds = 210 seconds.

Now substitute $y = 210$ seconds into the equation for the regression line and solve for x.

$$y=-0.356x+257.44$$
$$210=-0.356x+257.44$$
$$0.356x=257.44-210$$
$$0.356x=47.44$$
$$x=\frac{47.44}{0.356}\approx 133.26$$

Since x represents the number of years after 1900, the year corresponding to this value of x is $1900 + 133.26 = 2033.26 \approx 2033$. The world record will be 3 minutes 30 seconds in 2033.

60. For $x = 720$:

$$y=-0.356x+257.44$$
$$=-0.356(720)+257.44$$
$$=-256.32+257.44=1.12 \text{ seconds}$$

These are both unreasonable times for running 1 mile.

For $x = 722$:

$$y=-0.356x+257.44$$
$$=-0.356(722)+257.44$$
$$=-257.744+257.44=0.408 \text{ second}$$

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61. **a.** To find the linear equation, first find the slope of the line containing these points.

$$m = \frac{146-70}{3-1} = \frac{76}{2} = 38$$

Next, use the point-slope form with the point (1, 70):

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 70 &= 38(x-1) \\ y &= 38x + 32 \end{aligned}$$

b. Sales are increasing by 38 million units per year.

c. The sales at the end of 2020 is $y = 38(10) + 32 = 412$ million units.

62. **a.** First, find the slope of the line containing the points.

$$m = \frac{42.8-38.6}{4-1} = \frac{4.2}{3} = 1.4$$

Next, use the point-slope form with the point (1, 38.6):

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 38.6 &= 1.4(x-1) \\ y &= 1.4x + 37.2 \end{aligned}$$

b. PCPI increases by about \$1400 (or \$1.4 thousand) each year.

c. The value of x corresponding to 2020 is $x = 2020 - 2008 = 12$. Substitute 12 into the equation:
$y = 1.4(12) + 37.2 = \$54$ thousand or \$54,000

63. **a.** First, find the slope of the line containing the points.

$$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

Next, use the point-slope form with the point (0, 32):

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 32 &= \frac{9}{5}(x-0) \\ y &= \frac{9}{5}x + 32 \end{aligned}$$

b. Substitute 20 into the equation.

$$y = \frac{9}{5}x + 32$$
$$y = \frac{9}{5}(20) + 32 = 36 + 32 = 68^\circ \text{F}$$

64. **a.** First, find the slope of the line containing the points.

$$m = \frac{89.8-74.8}{4-0} = \frac{15}{4} = 3.75$$

Next, use the point-slope form with the point (0, 74.8):

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 74.8 &= 3.75(x-0) \\ y &= 3.75x + 74.8 \end{aligned}$$

b. Since 2021 is 12 years after 2009, substitute 11 into the equation.
$y = 3.75x + 74.8$
$y = 3.75(12) + 74.8 = 119.8$ thousand dollars or \$119,800

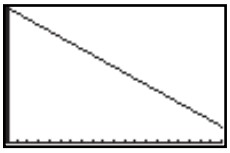
65. **a.** Price = \$50,000; useful lifetime = 20 years; scrap value = \$6000

$$\begin{aligned} V &= 50{,}000 - \left(\frac{50{,}000-6000}{20}\right)t \quad 0 \le t \le 20 \\ &= 50{,}000 - 2200t \quad 0 \le t \le 20 \end{aligned}$$

b. Substitute $t = 5$ into the equation.

$$\begin{aligned} V &= 50{,}000 - 2200t \\ &= 50{,}000 - 2200(5) \\ &= 50{,}000 - 11{,}000 = \$39{,}000 \end{aligned}$$

c.



on [0, 20] by [0, 50,000]

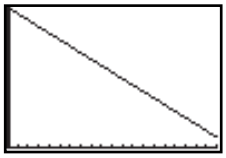
66. **a.** Price = \$800,000; useful lifetime = 20 yrs; scrap value = \$60,000

$$\begin{aligned} V &= 800{,}000 - \left(\frac{800{,}000-60{,}000}{20}\right)t \\ &\quad 0 \le t \le 20 \\ &= 800{,}000 - 37{,}000t \quad 0 \le t \le 20 \end{aligned}$$

b. Substitute $t = 10$ into the equation.

$$\begin{aligned} V &= 800{,}000 - 37{,}000t \\ &= 800{,}000 - 37{,}000(10) \\ &= 800{,}000 - 370{,}000 = \$430{,}000 \end{aligned}$$

c.



on [0, 20] by [0, 800,000]

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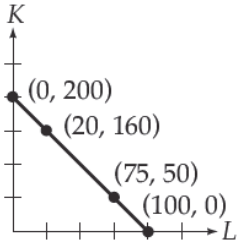
67. **a.** Substitute $w = 10$, $r = 5$, $C = 1000$ into the equation.

$10L + 5K = 1000$



b. Substitute each pair into the equation.

For (100, 0), $10(100) + 5(0) = 1000$

For (75, 50), $10(75) + 5(50) = 1000$

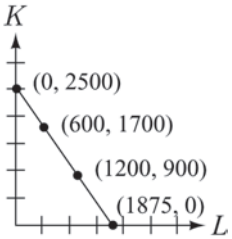
For (20, 160), $10(20) + 5(160) = 1000$

For (0, 200), $10(0) + 5(200) = 1000$

Every pair gives 1000.

68. **a.** Substitute $w = 8$, $r = 6$, $C = 15{,}000$ into the equation.

$8L + 6K = 15{,}000$



b. Substitute each pair into the equation.

For (1875, 0), $8(1875) + 6(0) = 15{,}000$

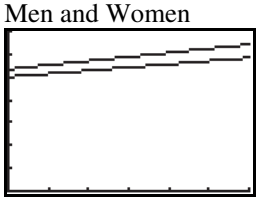
For (1200, 900), $8(1200) + 6(900) = 15{,}000$

For (600, 1700), $8(600) + 6(1700) = 15{,}000$

For (0, 2500), $8(0) + 6(2500) = 15{,}000$

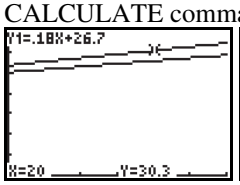
Every pair gives 15,000.

69. **a.** Median Marriage Age for Men and Women

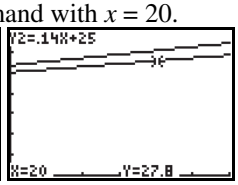


on [0, 30] by [0, 35]

b. The x-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screens are a result of the CALCULATE command with $x = 20$.



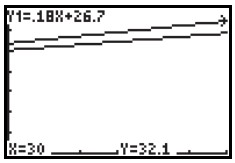
Median Age at Marriage for Men in 2020



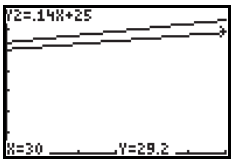
Median Age at Marriage for Women in 2020.

So, the median marriage age for men in 2020 will be 30.3 years and for women it will be 27.8 years.

c. The x-value corresponding to the year 2030 is $x = 2030 - 2000 = 30$. The following screens are a result of the CALCULATE command with $x = 30$.



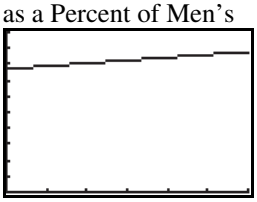
Median Age at Marriage for Men in 2030



Median Age at Marriage for Women in 2030.

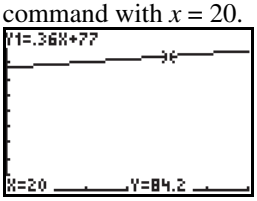
So, the median marriage age for men in 2030 will be 32.1 years and for women it will be 29.2 years.

70. **a.** Women's Annual Earnings as a Percent of Men's



on [0, 30] by [0, 100]

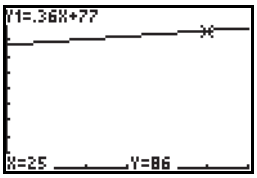
b. The x-value corresponding to the year 2020 is $x = 2020 - 2000 = 20$. The following screen is a result of the CALCULATE command with $x = 20$.



Women's Annual Earnings as a Percent of Men's

So, in the year 2020 women's wages will be about 84.2% of men's wages.

c. The x-value corresponding to the year 2025 is $x = 2025 - 2000 = 25$. The following screen is a result of the CALCULATE command with $x = 25$.



Women's Annual Earnings as a Percent of Men's

So in the year 2025 women's wages will be about 86% of men's wages.

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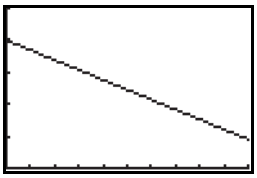
71. **a.**



on [0, 100] by [0, 50]

b. To find the probability that a person with a family income of \$40,000 is a smoker, substitute 40 into the equation
$y = -0.31x + 40$
$y = -0.31(40) + 40 = 27.6$ or 28%.

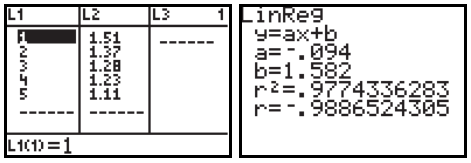
c. The probability that a person with a family income of \$70,000 is a smoker is
$y = -0.31(70) + 40 = 18.3$ or 18%.

72. **a.** To find the reported "happiness" of a person with an income of \$25,000, substitute 25 into the equation
$y = 0.065x - 0.613$
$y = 0.065(25) - 0.613 = 1.0.$

b. The reported "happiness" of a person with an income of \$35,000 is
$y = 0.065(35) - 0.613 = 1.7.$

c. The reported "happiness" of a person with an income of \$45,000 is
$y = 0.065(45) - 0.613 = 2.3.$

73. **a.**

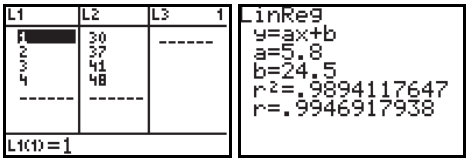


$y = -0.094x + 1.582$

b. Cigarette consumption is declining by about 94 cigarettes (from 0.094 thousand, so about 5 packs) per person per year.

c. $y = -0.094(13) + 1.582 = 0.36$ thousand (360 cigarettes)

74. **a.**

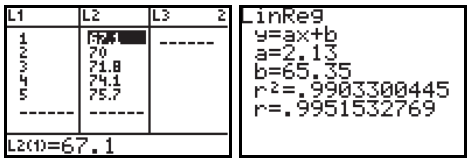


$y = 5.8x + 24.5$

b. Each year the usage increases by about 5.8 percentage points.

c. $y = 5.8(11) + 24.5 = 88.3\%$

75. **a.**

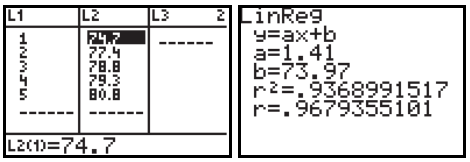


$y = 2.13x + 65.35$

b. The male life expectancy is increasing by 2.13 years per decade, which is 0.213 years (or about 2.6 months per year).

c. $y = 2.13(6.5) + 65.35 \approx 79.2$ years

76. **a.**

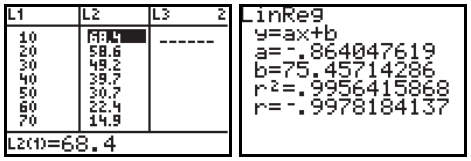


$y = 1.41x + 73.97$

b. The female life expectancy is increasing by 1.41 years per decade, which is 0.141 years (or about 1.7 months per year).

c. $y = 1.41(6.5) + 73.97 \approx 83.1$ years

77. **a.**



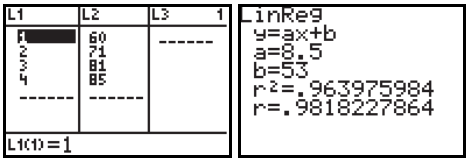
$y = -0.864x + 75.46$

b. Future longevity decreases by 0.864 (or about 10.44 months) per year.

c. $y = -0.864(25) + 75.46 \approx 53.9$ years

d. It would not make sense to use the regression line to predict future longevity at age 90 because the line predicts –2.3 years of life remaining.

78. **a.**



$y = 8.5x + 53$

b. Seat belt use increases by 8.5% each 5 years (or about 1.7% per year).

c. $y = 8.5(5.4) + 53 = 98.9\%$

d. It would not make sense to use the regression line to predict seat belt use in 2025 ($x = 7$) because the line predicts 112.5%.

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79. False: Infinity is not a number.

80. True: All negative numbers must be less than zero, and all positive numbers are more than zero. Therefore, all negative numbers are less than all positive numbers.

81. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ for any two points (x_1, y_1) and (x_1, y_1) on the line *or* the slope is the amount that the line rises when x increases by 1.

82. "Slope" is the answer to the first blank. The second blank would be describing it as negative, because the slope of a line slanting downward as you go to the right is a "fall" over "run".

83. False: The slope of a vertical line is undefined.

84. False: The slope of a vertical line is undefined, so a vertical line does not have a slope.

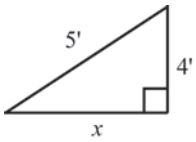
85. True: The slope is $\dfrac{-a}{b}$ and the y-intercept is $\dfrac{c}{b}$.

86. True: $x = c$ will always be a vertical line because the x values do not change.

87. False: It should be $\dfrac{y_2 - y_1}{x_2 - x_1}$.

88. False. A vertical line has no slope, so there is no m for $y = mx + b$.

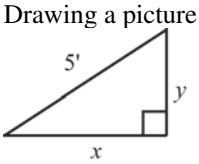
89. Drawing a picture of a right triangle.



$$\begin{aligned} x^2 + 4^2 &= 5^2 \\ x^2 + 16 &= 25 \\ x^2 &= 9 \\ x &= 3 \end{aligned}$$

The slope is $m = \dfrac{4}{3}$ or $-\dfrac{4}{3}$ if the ladder slopes downward.

90. Drawing a picture of a right triangle.



$$x^2 + y^2 = 5^2$$
$$\frac{y}{x} = 0.75 \Rightarrow y = 0.75x$$

$$\begin{aligned} x^2 + (0.75x)^2 &= 5^2 \\ x^2 + 0.5625x^2 &= 25 \\ 1.5625x^2 &= 25 \\ x^2 &= 16 \\ x &= 4 \\ y &= 0.75(4) = 3 \end{aligned}$$

The upper end is 3 feet high.

91. To find the x-intercept, substitute $y = 0$ into the equation and solve for x:

$$\begin{aligned} y &= mx + b \\ 0 &= mx + b \\ -mx &= b \\ x &= -\frac{b}{m} \end{aligned}$$

If $m \neq 0$, then a single x-intercept exists. So $a = -\dfrac{b}{m}$. Thus, the x-intercept is $\left(-\dfrac{b}{m}, 0\right)$.

92. **i.** To obtain the slope-intercept form of a line, solve the equation for y:

$$\begin{aligned} ax + by &= c \\ by &= -ax + c \\ y &= -\frac{a}{b}x + \frac{c}{b} \end{aligned}$$

ii. Substitute 0 for b and solve for x:

$$\begin{aligned} ax + by &= c \\ ax + 0 \cdot y &= c \\ ax &= c \\ x &= \frac{c}{a} \end{aligned}$$

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93. Consider $R > 1$ and $0 < x < K$

$x < K$ means that $K - x > 0$ and $0<\frac{x}{K}<1$.

Since $K - x > 0$, then

$$K-x+Rx>Rx$$
$$K+(R-1)x>Rx$$
$$K\left[1+\left(\frac{R-1}{K}\right)x\right]>Rx$$

Therefore, $K>\dfrac{Rx}{1+\left(\frac{R-1}{K}\right)x}=y$

Additionally, since $0<\frac{x}{K}<1$,

$$1+(R-1)\left(\frac{x}{K}\right)<1+(R-1)\cdot 1$$

So, $y=\dfrac{Rx}{1+\left(\frac{R-1}{K}\right)x}>\dfrac{Rx}{1+(R-1)}=\dfrac{Rx}{R}=x$

We have $x < y < K$.

94. $x > K$ means that $K - x < 0$ and $\frac{x}{K}>1$.

Since $K - x < 0$, then

$$K-x+Rx<Rx$$
$$K+(R-1)x<Rx$$
$$K\left[1+\left(\frac{R-1}{K}\right)x\right]<Rx$$

Therefore, $K<\dfrac{Rx}{1+\left(\frac{R-1}{K}\right)x}=y$

Additionally, since $\frac{x}{K}>1$,

$$1+(R-1)\left(\frac{x}{K}\right)>1+(R-1)\cdot 1$$

So, $y=\dfrac{Rx}{1+\left(\frac{R-1}{K}\right)x}<\dfrac{Rx}{1+(R-1)}=\dfrac{Rx}{R}=x$

We have $K < y < x$.

EXERCISES 1.2

1. $\left(2^2\cdot 2\right)^2=\left(2^2\cdot 2^1\right)^2=\left(2^3\right)^2=2^6=64$

2. $\left(5^2\cdot 4\right)^2=\left(5^2\cdot 2^2\right)^2=\left(10^2\right)^2=10^4=10{,}000$

3. $2^{-4}=\dfrac{1}{2^4}=\dfrac{1}{16}$

4. $3^{-3}=\dfrac{1}{3^3}=\dfrac{1}{27}$

5. $\left(\frac{1}{2}\right)^{-3}=\left(2^{-1}\right)^{-3}=2^3=8$

6. $\left(\frac{1}{3}\right)^{-2}=\left(3^{-1}\right)^{-2}=3^2=9$

7. $\left(\frac{5}{8}\right)^{-1}=\frac{8}{5}$

8. $\left(\frac{3}{4}\right)^{-1}=\frac{4}{3}$

9. $4^{-2}\cdot 2^{-1}=\left(2^2\right)^{-2}\cdot 2^{-1}$

$$=2^{-4}\cdot 2^{-1}=2^{-5}=\frac{1}{2^5}=\frac{1}{32}$$

10. $3^{-2}\cdot 9^{-1}=3^{-2}\cdot\left(3^2\right)^{-1}$

$$=3^{-2}\cdot 3^{-2}=3^{-4}=\frac{1}{3^4}=\frac{1}{81}$$

11. $\left(\frac{3}{2}\right)^{-3}=\left(\frac{2}{3}\right)^3=\frac{2^3}{3^3}=\frac{8}{27}$

12. $\left(\frac{2}{3}\right)^{-3}=\left(\frac{3}{2}\right)^3=\frac{3^3}{2^3}=\frac{27}{8}$

13. $\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}=\left(\frac{3}{1}\right)^2-\left(\frac{2}{1}\right)^3=\frac{3^2}{1^2}-\frac{2^3}{1^3}$

$$=9-8=1$$

14. $\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-2}=\left(\frac{3}{1}\right)^2-\left(\frac{2}{1}\right)^2=\frac{3^2}{1^2}-\frac{2^2}{1^2}$

$$=9-4=5$$

15. $\left[\left(\frac{2}{3}\right)^{-2}\right]^{-1}=\left[\left(\frac{3}{2}\right)^2\right]^{-1}=\left(\frac{3}{2}\right)^{-2}=\left(\frac{2}{3}\right)^2=\frac{2^2}{3^2}=\frac{4}{9}$

16. $\left[\left(\frac{2}{5}\right)^{-2}\right]^{-1}=\left[\left(\frac{5}{2}\right)^2\right]^{-1}=\left(\frac{5}{2}\right)^{-2}=\left(\frac{2}{5}\right)^2=\frac{2^2}{5^2}=\frac{4}{25}$

17. $25^{1/2}=\sqrt{25}=5$

18. $36^{1/2}=\sqrt{36}=6$

19. $25^{3/2}=\left(\sqrt{25}\right)^3=5^3=125$

20. $16^{3/2}=\left(\sqrt{16}\right)^3=4^3=64$

21. $16^{3/4}=\left(\sqrt[4]{64}\right)^3=2^3=8$

22. $27^{2/3}=\left(\sqrt[3]{27}\right)^2=3^2=9$

23. $(-8)^{2/3}=\left(\sqrt[3]{-8}\right)^2=(-2)^2=4$

24. $(-27)^{2/3}=\left(\sqrt[3]{-27}\right)^2=(-3)^2=9$

25. $(-8)^{5/3}=\left(\sqrt[3]{-8}\right)^5=(-2)^5=-32$

26. $(-27)^{5/3}=\left(\sqrt[3]{-27}\right)^5=(-3)^5=-243$

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27. $\left(\frac{25}{36}\right)^{3/2}=\left(\sqrt{\frac{25}{36}}\right)^{3}=\left(\frac{5}{6}\right)^{3}=\frac{5^3}{6^3}=\frac{125}{216}$

28. $\left(\frac{16}{25}\right)^{3/2}=\left(\sqrt{\frac{16}{25}}\right)^{3}=\left(\frac{4}{5}\right)^{3}=\frac{4^3}{5^3}=\frac{64}{125}$

29. $\left(\frac{27}{125}\right)^{2/3}=\left(\sqrt[3]{\frac{27}{125}}\right)^{2}=\left(\frac{3}{5}\right)^{2}=\frac{9}{25}$

30. $\left(\frac{125}{8}\right)^{2/3}=\left(\sqrt[3]{\frac{125}{8}}\right)^{2}=\left(\frac{5}{2}\right)^{2}=\frac{25}{4}$

31. $\left(\frac{1}{32}\right)^{2/5}=\left(\sqrt[5]{\frac{1}{32}}\right)^{2}=\left(\frac{1}{2}\right)^{2}=\frac{1}{4}$

32. $\left(\frac{1}{32}\right)^{3/5}=\left(\sqrt[5]{\frac{1}{32}}\right)^{3}=\left(\frac{1}{2}\right)^{3}=\frac{1}{8}$

33. $4^{-1/2}=\frac{1}{4^{1/2}}=\frac{1}{\sqrt{4}}=\frac{1}{2}$

34. $9^{-1/2}=\frac{1}{9^{1/2}}=\frac{1}{\sqrt{9}}=\frac{1}{3}$

35. $4^{-3/2}=\frac{1}{4^{3/2}}=\frac{1}{\left(\sqrt{4}\right)^{3}}=\frac{1}{2^3}=\frac{1}{8}$

36. $9^{-3/2}=\frac{1}{9^{3/2}}=\frac{1}{\left(\sqrt{9}\right)^{3}}=\frac{1}{3^3}=\frac{1}{27}$

37. $8^{-2/3}=\frac{1}{8^{2/3}}=\frac{1}{\left(\sqrt[3]{8}\right)^{2}}=\frac{1}{2^2}=\frac{1}{4}$

38. $16^{-3/4}=\frac{1}{16^{3/4}}=\frac{1}{\left(\sqrt[4]{16}\right)^{3}}=\frac{1}{2^3}=\frac{1}{8}$

39. $(-8)^{-1/3}=\frac{1}{(-8)^{1/3}}=\frac{1}{\left(\sqrt[3]{-8}\right)}=\frac{1}{-2}=-\frac{1}{2}$

40. $(-27)^{-1/3}=\frac{1}{(-27)^{1/3}}=\frac{1}{\left(\sqrt[3]{-27}\right)}=\frac{1}{-3}=-\frac{1}{3}$

41. $(-8)^{-2/3}=\frac{1}{(-8)^{2/3}}=\frac{1}{\left(\sqrt[3]{(-8)^2}\right)}=\frac{1}{(-2)^2}=\frac{1}{4}$

42. $(-27)^{-2/3}=\frac{1}{(-27)^{2/3}}=\frac{1}{\left(\sqrt[3]{(-27)^2}\right)}=\frac{1}{(-3)^2}=\frac{1}{9}$

43. $\left(\frac{25}{16}\right)^{-1/2}=\left(\frac{16}{25}\right)^{1/2}=\left(\sqrt{\frac{16}{25}}\right)=\frac{4}{5}$

44. $\left(\frac{16}{9}\right)^{-1/2}=\left(\frac{9}{16}\right)^{1/2}=\left(\sqrt{\frac{9}{16}}\right)=\frac{3}{4}$

45. $\left(\frac{25}{16}\right)^{-3/2}=\left(\frac{16}{25}\right)^{3/2}=\left(\sqrt{\frac{16}{25}}\right)^{3}=\left(\frac{4}{5}\right)^{3}=\frac{64}{125}$

46. $\left(\frac{16}{9}\right)^{-3/2}=\left(\frac{9}{16}\right)^{3/2}=\left(\sqrt{\frac{9}{16}}\right)^{3}=\left(\frac{3}{4}\right)^{3}=\frac{27}{64}$

47. $\left(-\frac{1}{27}\right)^{-5/3}=(-27)^{5/3}=\left(\sqrt[3]{-27}\right)^{5}=(-3)^5=-243$

48. $\left(-\frac{1}{8}\right)^{-5/3}=(-8)^{5/3}=\left(\sqrt[3]{-8}\right)^{5}=(-2)^5=-32$

49. $7^{0.39}\approx 2.14$

50. $5^{0.47}\approx 2.13$

51. $8^{2.7}\approx 274.37$

52. $5^{3.9}\approx 532.09$

53. $\left[(0.1)^{0.1}\right]^{0.1}\approx 0.977$

54. $\left(1+\frac{1}{1000}\right)^{1000}\approx 2.717$

55. $\left(1-\frac{1}{1000}\right)^{-1000}\approx 2.720$

56. $\left(1+10^{-6}\right)^{10^6}\approx 2.718$

57. $\frac{4}{x^5}=4x^{-5}$

58. $\frac{6}{2x^3}=\frac{3}{x^3}=3x^{-3}$

59. $\frac{4}{\sqrt[3]{8x^3}}=\frac{4}{2x^{4/3}}=2x^{-4/3}$

60. $\frac{6}{\sqrt{4x^3}}=\frac{6}{2x^{3/2}}=3x^{-3/2}$

61. $\frac{24}{\left(2\sqrt{x}\right)^3}=\frac{24}{8x^{3/2}}=3x^{-3/2}$

62. $\frac{18}{\left(3\sqrt[3]{x}\right)^2}=\frac{18}{9x^{2/3}}=2x^{-2/3}$

63. $\sqrt{\frac{9}{x^4}}=\frac{3}{x^2}=3x^{-2}$

64. $\sqrt[3]{\frac{8}{x^6}}=\frac{2}{x^2}=2x^{-2}$

65. $\frac{5x^2}{\sqrt{x}}=\frac{5x^2}{x^{1/2}}=5x^{2-1/2}=5x^{3/2}$

66. $\frac{3\sqrt{x}}{x}=\frac{3x^{1/2}}{x}=3x^{1/2-1}=3x^{-1/2}$

67. $\frac{12\sqrt[3]{x^2}}{3x^2}=\frac{12x^{2/3}}{3x^2}=\frac{12}{3}x^{2/3-2}=4x^{-4/3}$

68. $\frac{10\sqrt{x}}{2\sqrt[3]{x}}=\frac{10x^{1/2}}{2x^{1/3}}=\frac{10}{2}x^{1/2-1/3}=5x^{1/6}$

69. $\frac{\sqrt{36x}}{2x}=\frac{6\sqrt{x}}{2x}=\frac{6x^{1/2}}{2x}=\frac{6}{2}x^{1/2-1}=3x^{-1/2}$

70. $\frac{\sqrt[3]{8x^2}}{4x}=\frac{2\sqrt[3]{x^2}}{4x}=\frac{2x^{2/3}}{4x}=\frac{2}{4}x^{2/3-1}=\frac{1}{2}x^{-1/3}$

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71. $\left(x^3 \cdot x^2\right)^2 = \left(x^5\right)^2 = x^{10}$

72. $\left(x^4 \cdot x^3\right)^2 = \left(x^7\right)^2 = x^{14}$

73. $\left[z^2\left(z \cdot z^2\right)^2 z\right]^3 = \left[z^2\left(z^3\right)^2 z\right]^3 = \left(z^2 \cdot z^6 \cdot z\right)^3$
$= \left(z^9\right)^3 = z^{27}$

74. $\left[z\left(z^3 \cdot z\right)^2 z^2\right]^2 = \left[z\left(z^4\right)^2 z^2\right]^2 = \left(z \cdot z^8 \cdot z^2\right)^2$
$= \left(z^{11}\right)^2 = z^{22}$

75. $\left[\left(x^2\right)^2\right]^2 = \left(x^4\right)^2 = x^8$

76. $\left[\left(x^3\right)^3\right]^3 = \left(x^9\right)^3 = x^{27}$

77. $(3x^2y^5z)^3 = 3^3x^{2\cdot 3}y^{5\cdot 3}z^3 = 27x^6y^{15}z^3$

78. $(2x^4yz^6)^4 = 2^4x^{4\cdot 4}y^4z^{6\cdot 4} = 16x^{16}y^4z^{24}$

79. $\dfrac{(ww^2)^3}{w^3w} = \dfrac{(w^3)^3}{w^4} = \dfrac{w^9}{w^4} = w^5$

80. $\dfrac{(ww^3)^2}{w^3w^2} = \dfrac{(w^4)^2}{w^5} = \dfrac{w^8}{w^5} = w^3$

81. $\dfrac{\left(5xy^4\right)^2}{25x^3y^3} = \dfrac{25x^2y^8}{25x^3y^3} = \dfrac{y^5}{x}$

82. $\dfrac{\left(4x^3y\right)^2}{8x^2y^3} = \dfrac{16x^6y^2}{8x^2y^3} = \dfrac{2x^4}{y}$

83. $\dfrac{\left(9xy^3z\right)^2}{3\left(xyz\right)^2} = \dfrac{81x^2y^6z^2}{3x^2y^2z^2} = 27y^4$

84. $\dfrac{\left(5x^2y^3z\right)^2}{5\left(xyz\right)^2} = \dfrac{25x^4y^6z^2}{5x^2y^2z^2} = 5x^2y^4$

85. $\dfrac{\left(2u^2vw^3\right)^2}{4\left(uw^2\right)^2} = \dfrac{4u^4v^2w^6}{4u^2w^4} = u^2v^2w^2$

86. $\dfrac{\left(u^3vw^2\right)^2}{9\left(u^2w\right)^2} = \dfrac{u^6v^2w^4}{9u^4w^2} = \dfrac{u^2v^2w^2}{9}$

87. Average body thickness
$= 0.4(\text{hip-to-shoulder length})^{3/2}$
$= 0.4(16)^{3/2}$
$= 0.4\left(\sqrt{16}\right)^3$
≈ 25.6 ft

88. Average body thickness
$= 0.4(\text{hip-to-shoulder length})^{3/2}$
$= 0.4(14)^{3/2}$
$= 0.4\left(\sqrt{14}\right)^3$
≈ 21.0 ft

89. $C' = x^{0.6}C$
$= 4^{0.6}C \approx 2.3C$
To quadruple the capacity costs about 2.3 times as much.

90. $C' = x^{0.6}C$
$= 3^{0.6}C \approx 1.9C$
To triple the capacity costs about 1.9 times as much

91. **a.** Given the unemployment rate of 2 percent, the inflation rate is
$y = 9.638(2)^{-1.394} - 0.900$
≈ 2.77 percent.

b. Given the unemployment rate of 5 percent, the inflation rate is
$y = 9.638(5)^{-1.394} - 0.900$
≈ 0.12 percent.

92. **a.** Given the unemployment rate of 3 percent, the inflation rate is
$y = 45.4(3)^{-1.54} - 1$
≈ 7.36 percent.

b. Given the unemployment rate of 8 percent, the inflation rate is
$y = 45.4(8)^{-1.54} - 1$
≈ 0.85 percent.

93. $(\text{Heart rate}) = 250(\text{weight})^{-1/4}$
$= 250(16)^{-1/4}$
$= 125$ beats per minute

94. $(\text{Heart rate}) = 250(\text{weight})^{-1/4}$
$= 250(625)^{-1/4}$
$= 50$ beats per minute

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95. (Time to build the 50th Boeing 707)
$=150(50)^{-0.322}$
≈ 42.6 thousand work-hours

It took approximately 42,600 work-hours to build the 50th Boeing 707.

96. (Time to build the 250th Boeing 707)
$=150(250)^{-0.322}$
≈ 25.3 thousand work-hours

It took approximately 25,300 work-hours to build the 250th Boeing 707.

97. Increase in energy $= 32^{B-A}$
$= 32^{7.8-6.7}$
$= 32^{1.1} \approx 45$

The 1906 San Francisco earthquake had about 45 times more energy released than the 1994 Northridge earthquake.

98. Increase in energy $= 32^{B-A}$
$= 32^{9.0-7.7}$
$= 32^{1.3} \approx 91$

The 2011 Japan earthquake had about 91 times more energy released than the 2011 India earthquake.

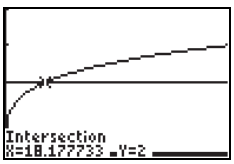
99. $K = 3000(225)^{-1/2} = 200$

100. $K = 4000(125)^{-2/3} = 160$

101. $S = \frac{60}{11}x^{0.5}$
$= \frac{60}{11}(3281)^{0.5} \approx 312$ mph

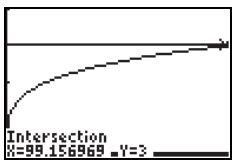
102. $S = \frac{60}{11}x^{0.5}$
$= \frac{60}{11}(1650)^{0.5} \approx 222$ mph

103.



on [0, 100] by [0,4]
$x \approx 18.2$. Therefore, the land area must be increased by a factor of more than 18 to double the number of species.

104.

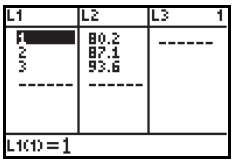


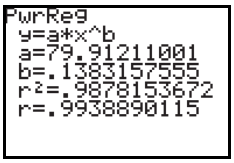
on [0, 100] by [0,4]
$x \approx 99$. Therefore, the land area must be increased by almost 100 times to triple the number of species.

105. $y = 9.4x^{0.37}$
$y = 9.4(150)^{0.37} = 60$ miles per hour
The speed of a car that left 150-foot skid marks was 60 miles per hour.

106. $y = 9.4(350)^{0.37} = 82$ miles per hour
The speed of a car that left 350-foot skid marks was 82 miles per hour.

107. **a.**

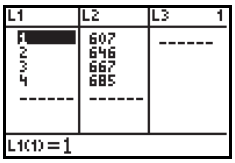


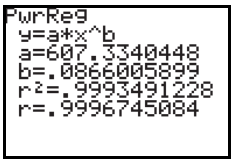


$y = 79.9x^{0.138}$ (rounded)

b. For year 2020, $x = 10$.
$y = 79.9(10)^{0.138} \approx \$110$ billion

108. **a.**

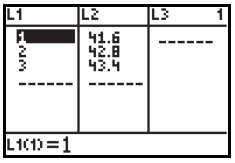


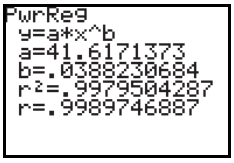


$y = 607x^{0.0866}$ (rounded)

b. For year 2020, $x = 12$
$y = 607(12)^{0.0866} \approx \$753$

109. **a.**

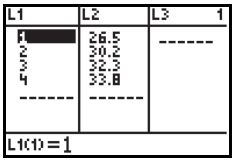


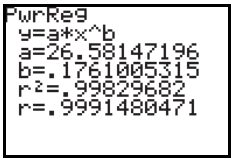


$y = 41.6x^{0.0388}$ (rounded)

b. For year 2020, $x = 11$.
$y = 41.6(11)^{0.0388} \approx \$45.7$ million

110. **a.**





$y = 26.6x^{0.176}$ (rounded)

b. For year 2020, $x = 12$.
$y = 26.6(12)^{0.176} \approx \$41.2$ billion

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111. 3, since $\sqrt{9}$ means the principal square foot. (To get ± 3 you would have to write $\pm\sqrt{9}$.)

112. False: $2^2 \cdot 2^3 = 4 \cdot 8 = 32$, while $2^{2\cdot 3} = 2^6 = 64$. (The correct statement is $x^m \cdot x^n = x^{m+n}$.

113. False: $\frac{2^6}{2^6} = \frac{64}{4} = 16$, while $2^{6/2} = 2^3 = 8$. (The correct statement is $\frac{x^m}{x^n} = x^{m-n}$.)

114. False: $(2^3)^2 = 8^2 = 64$, while $2^{3^2} = 2^9 = 512$. (The correct statement is $(x^m)^n = x^{m\cdot n}$.

115. $x^{1/2} = \sqrt{x}$, so x must be nonnegative for the expression to be defined.

116. $x^{1/3} = \sqrt[3]{x}$, so all values of x. For example, $8^{1/3} = 2$ and $(-8)^{1/3} = -2$.

117. $x^{-1} = \frac{1}{x}$, so all values of x except 0, because you cannot divide by 0.

118. If the exponent $\frac{m}{n}$ is not fully reduced, it will indicate an even root of a negative number, which is not defined in the real number set.

EXERCISES 1.3

1. Yes **2.** No **3.** No **4.** Yes

5. No **6.** Yes **7.** No **8.** Yes

9. Domain = $\{x \mid x \le 0 \text{ or } x \ge 1\}$
Range = $\{y \mid y \ge -1\}$

10. Domain = $\{x \mid x \le -1 \text{ or } x \ge 0\}$
Range = $\{y \mid y \le 1\}$

11. **a.** $f(x) = \sqrt{x-1}$
$f(10) = \sqrt{10-1} = \sqrt{9} = 3$

b. Domain = $\{x \mid x \ge 1\}$ since $f(x) = \sqrt{x-1}$ is defined for all values of $x \ge 1$.

c. Range = $\{y \mid y \ge 0\}$

12. **a.** $f(x) = \sqrt{x-4}$
$f(40) = \sqrt{40-4} = \sqrt{36} = 6$

b. Domain = $\{x \mid x \ge 4\}$ since $f(x) = \sqrt{x-4}$ is defined for all values of $x \ge 4$.

c. Range = $\{y \mid y \ge 0\}$

13. **a.** $h(z) = \frac{1}{z+4}$
$h(-5) = \frac{1}{-5+4} = -1$

b. Domain $=\{z \mid z \ne -4\}$ since $h(z) = \frac{1}{z+4}$ is defined for all values of z except $z = -4$.

c. Range = $\{y \mid y \ne 0\}$

14. **a.** $h(z) = \frac{1}{z+7}$
$h(-8) = \frac{1}{-8+7} = -1$

b. Domain = $\{z \mid z \ne -7\}$ since $h(z) = \frac{1}{z+7}$ is defined for all values of z except $z = -7$.

c. Range = $\{y \mid y \ne 0\}$

15. **a.** $h(x) = x^{1/4}$
$h(81) = 81^{1/4} = \sqrt[4]{81} = 3$

b. Domain = $\{x \mid x \ge 0\}$ since $h(x) = x^{1/4}$ is defined only for nonnegative values of x.

c. Range = $\{y \mid y \ge 0\}$

16. **a.** $h(x) = x^{1/6}$
$h(81) = 64^{1/4} = \sqrt[6]{64} = 2$

b. Domain = $\{x \mid x \ge 0\}$ since $h(x) = x^{1/6}$ is defined for nonnegative values of x.

c. Range = $\{y \mid y \ge 0\}$

17. **a.** $f(x) = x^{2/3}$
$f(-8) = (-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \ge 0\}$

18. **a.** $f(x) = x^{4/5}$
$f(-32) = (-32)^{4/5} = \left(\sqrt[5]{-32}\right)^4 = (-2)^4 = 16$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \ge 0\}$

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19. **a.** $f(x)=\sqrt{4-x^2}$

$f(0)=\sqrt{4-0^2}=\sqrt{4}=2$

b. $f(x)=\sqrt{4-x^2}$ is defined for values of x such that $4-x^2\geq 0$. Thus,

$4-x^2\geq 0$

$-x^2\geq -4$

$x^2\leq 4$

$-2\leq x\leq 2$

Domain = $\{x \mid -2\leq x\leq 2\}$

c. Range = $\{y \mid 0\leq y\leq 2\}$

20. **a.** $f(x)=\dfrac{1}{\sqrt{x}}$

$f(4)=\dfrac{1}{\sqrt{4}}=\dfrac{1}{2}$

b. Domain = $\{x \mid x>0\}$ since $f(x)=\dfrac{1}{\sqrt{x}}$ is defined only for positive values of x.

c. Range = $\{y \mid y>0\}$

21. **a.** $f(x)=\sqrt{-x}$

$f(-25)=\sqrt{-(-25)}=\sqrt{25}=5$

b. $f(x)=\sqrt{-x}$ is defined only for values of x such that $-x\geq 0$. Thus $x\leq 0$.

Domain = $\{x \mid x\leq 0\}$

c. Range = $\{y \mid y\geq 0\}$

22. **a.** $f(x)=-\sqrt{-x}$

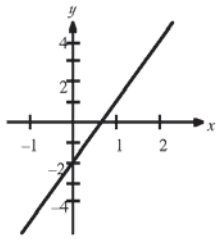
$f(-100)=-\sqrt{-(-100)}=-\sqrt{100}=-10$

b. $f(x)=-\sqrt{-x}$ is defined only for values of x such that $-x\geq 0$. Thus $x\leq 0$.

Domain = $\{x \mid x\leq 0\}$

c. Range = $\{y \mid y\leq 0\}$

23.



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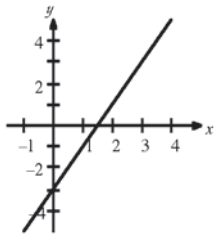

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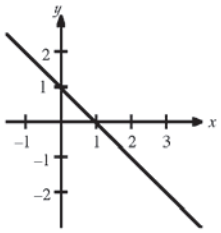

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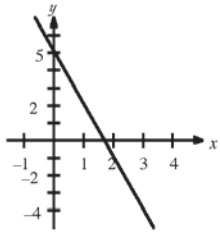
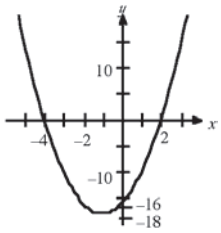


27.



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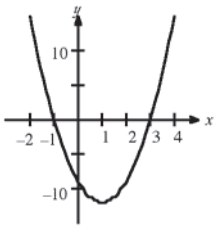

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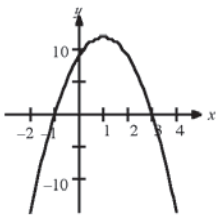
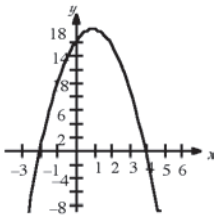
29.



30.



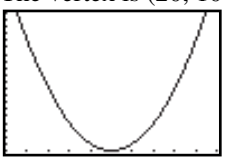
31. **a.** $x = \frac{-b}{2a} = \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

To find the y-coordinate, evaluate f at $x = 20$.

$f(20) = (20)^2 - 40(20) + 500 = 100$

The vertex is (20, 100).

b.



on [15, 25] by [100, 120]

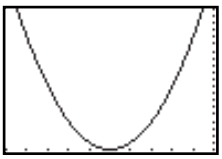
32. **a.** $x = \frac{-b}{2a} = \frac{-40}{2(1)} = \frac{-40}{2} = -20$

To find the y-coordinate, evaluate f at $x = -20$.

$f(-20) = (-20)^2 + 40(-20) + 500 = 100$

The vertex is (–20, 100).

b.



on [–25, –15] by [100, 120]

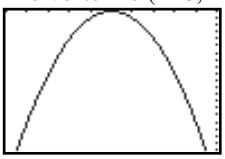
33. **a.** $x = \frac{-b}{2a} = \frac{-(-80)}{2(-1)} = \frac{80}{-2} = -40$

To find the y-coordinate, evaluate f at $x = -40$.

$f(-40) = -(-40)^2 - 80(-40) - 1800$
$= -200$

The vertex is (–40, –200).

b.



on [–45, –35] by [–220, –200]

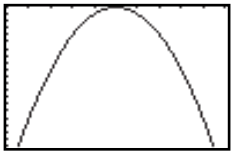
34. **a.** $x = \frac{-b}{2a} = \frac{-80}{2(-1)} = \frac{-80}{-2} = 40$

To find the y-coordinate, evaluate f at $x = 40$.

$f(-40) = -(40)^2 + 80(40) - 1800$
$= -200$

The vertex is (40, –200).

b.



on [35, 45] by [–220, –200]

35. $x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

Equals 0 at $x = 7$ Equals 0 at $x = -1$

$x = 7, \quad x = -1$

36. $x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

Equals 0 at $x = 5$ Equals 0 at $x = -4$

$x = 5, \quad x = -4$

37. $x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

Equals 0 at $x = -5$ Equals 0 at $x = 3$

$x = -5, \quad x = 3$

38. $x^2 - 3x = 54$

$x^2 - 3x - 54 = 0$

$(x-9)(x+6) = 0$

Equals 0 at $x = 9$ Equals 0 at $x = -6$

$x = 9, \quad x = -6$

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39.
$$2x^2 + 40 = 18x$$
$$2x^2 - 18x + 40 = 0$$
$$x^2 - 9x + 20 = 0$$
$$(x-4)(x-5) = 0$$
Equals 0 at $x = 4$ Equals 0 at $x = 5$
$x = 4, \quad x = 5$

40.
$$3x^2 + 18 = 15x$$
$$3x^2 - 15x + 18 = 0$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
Equals 0 at $x = 3$ Equals 0 at $x = 2$
$x = 3, \quad x = 2$

41.
$$5x^2 - 50x = 0$$
$$x^2 - 10x = 0$$
$$x(x-10) = 0$$
Equals 0 at $x=0$ Equals 0 at $x=10$
$x=0, \quad x=10$

42.
$$3x^2 - 36x = 0$$
$$x^2 - 12x = 0$$
$$x(x-12) = 0$$
Equals 0 at $x=0$ Equals 0 at $x=12$
$x=0, \quad x=12$

43.
$$2x^2 - 50 = 0$$
$$x^2 - 25 = 0$$
$$(x-5)(x+5) = 0$$
Equals 0 at $x = 5$ Equals 0 at $x = -5$
$x = 5, \quad x = -5$

44.
$$3x^2 - 27 = 0$$
$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$
Equals 0 at $x = 3$ Equals 0 at $x = -3$
$x = 3, \quad x = -3$

45.
$$4x^2+24x+40=4$$
$$4x^2+24x+36=0$$
$$x^2+6x+9=0$$
$$(x+3)^2=0$$
Equals 0 at $x=-3$

46.
$$3x^2-6x+9=6$$
$$3x^2-6x+3=0$$
$$x^2-2x+1=0$$
$$(x-1)^2=0$$
Equals 0 at $x=1$

47.
$$-4x^2 + 12x = 8$$
$$-4x^2 + 12x - 8 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
Equals 0 at $x = 2$ Equals 0 at $x = 1$
$x = 2, \quad x = 1$

48.
$$-3x^2 + 6x = -24$$
$$-3x^2 + 6x + 24 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
Equals 0 at $x = 4$ Equals 0 at $x = -2$
$x = 4, \quad x = -2$

49.
$$2x^2 - 12x + 20 = 0$$
$$x^2 - 6x + 10 = 0$$
Use the quadratic formula with $a = 1$, $b = -6$, and $c = 10$.
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{36-40}}{2}$$
$$= \frac{6 \pm \sqrt{-4}}{2} \quad \text{Undefined}$$
$2x^2 - 12x + 20 = 0$ has no real solutions.

50.
$$2x^2 - 8x + 10 = 0$$
$$x^2 - 4x + 5 = 0$$
Use the quadratic formula with $a = 1$, $b = -4$, and $c = 5$.
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16-20}}{2}$$
$$= \frac{4 \pm \sqrt{-4}}{2} \quad \text{Undefined}$$
$2x^2 - 8x + 10 = 0$ has no real solutions.

51.
$$3x^2 + 12 = 0$$
$$x^2 + 4 = 0$$
$$x^2 = -4$$
$$x = \pm\sqrt{-4} \quad \text{Undefined}$$
$3x^2 + 12 = 0$ has no real solutions.

52.
$$5x^2 + 20 = 0$$
$$x^2 + 4 = 0$$
$$x^2 = -4$$
$$x = \pm\sqrt{-4} \quad \text{Undefined}$$
$5x^2 + 20 = 0$ has no real solutions.

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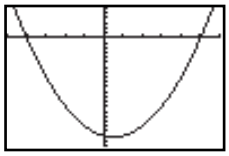
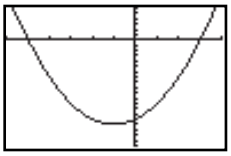
53.



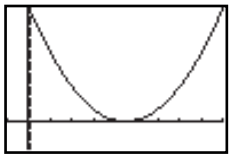
on [–5, 6] by [–22, 6]
$x = -4$, $x = 5$

54.



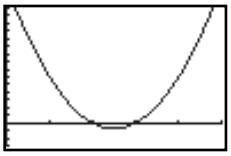
on [–6, 4] by [–20, 6]
$x = -5$, $x = 3$

55.



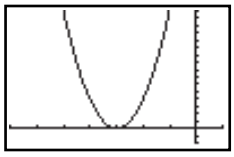
on [–1, 9] by [–10, 40]
$x = 4$, $x = 5$

56.



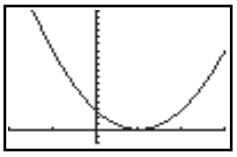
on [0, 5] by [–3, 15]
$x = 2$, $x = 3$

57.



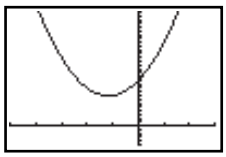
on [–7, 1] by [–2, 16]
$x = -3$

58.



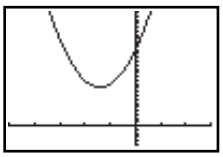
on [–2, 3] by [–2, 18]
$x = 1$

59.



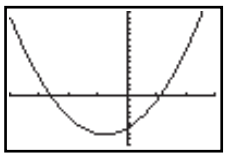
on [–5, 3] by [–5, 30]
No real solutions

60.



on [–5, 3] by [–5, 30]
No real solutions

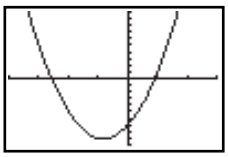
61.



on [–4, 3] by [–9, 15]

$x = -2.64$, $x = 1.14$

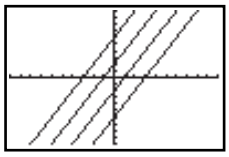
62.



on [–4, 3] by [–10, 10]

$x = -2.57$, $x = 0.91$

63.

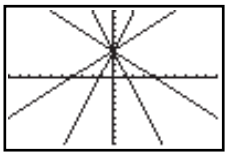


on [–10, 10] by [–10, 10]

a. Their slopes are all 2, but they have different y-intercepts.

b. The line 2 units below the line of the equation $y = 2x - 6$ must have y-intercept –8. Thus, the equation of this line is $y = 2x - 8$.

64.



on [–10, 10] by [–10, 10]

a. The lines have the same y-intercept, but their slopes are different.

b. $y = \frac{1}{2}x + 4$

65. Let x = the number of board feet of wood. Then $C(x) = 4x + 20$

66. Let x = the number of bicycles. Then $C(x) = 55x + 900$

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67. Let x = the number of hours of overtime. Then $P(x) = 15x + 500$

68. Let x = the total week's sales. Then $P(x) = 0.02x + 300$

69. **a.**
$$p(d) = 0.45d + 15$$
$$p(6) = 0.45(6) + 15$$
$$= 17.7 \text{ pounds per square inch}$$

b.
$$p(d) = 0.45d + 15$$
$$p(35,000) = 0.45(35,000) + 15$$
$$= 15,765 \text{ pounds per square inch}$$

70.
$$B(h) = -1.8h + 212$$
$$98.6 = -1.8h + 212$$
$$1.8h = 113.4$$
$$h = 63 \text{ thousand feet above sea level}$$

71.
$$D(v) = 0.055v^2 + 1.1v$$
$$D(40) = 0.055(40)^2 + 1.1(40) = 132 \text{ ft}$$

72.
$$D(v) = 0.55v^2 + 1.1v$$
$$D(60) = 0.55(60)^2 + 1.1(60) = 264 \text{ ft}$$

73. **a.**
$$N(t) = 200 + 50t^2$$
$$N(2) = 200 + 50(2)^2$$
$$= 400 \text{ cells}$$

b.
$$N(t) = 200 + 50t^2$$
$$N(10) = 200 + 50(10)^2$$
$$= 5200 \text{ cells}$$

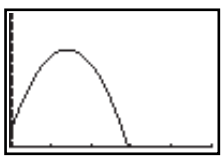
74. **a.**
$$T(h) = 0.5\sqrt{h}$$
$$T(4) = 0.5\sqrt{4} = 1 \text{ second}$$
$$T(8) = 0.5\sqrt{8} \approx 1.4 \text{ seconds}$$

b.
$$T(h) = 0.5\sqrt{h} \qquad T(h) = 0.5\sqrt{h}$$
$$2 = 0.5\sqrt{h} \qquad 3 = 0.5\sqrt{h}$$
$$4 = \sqrt{h} \qquad 6 = \sqrt{h}$$
$$h = 16 \text{ ft} \qquad h = 36 \text{ ft}$$

75.
$$v(x) = \frac{60}{11}\sqrt{x}$$
$$v(1776) = \frac{60}{11}\sqrt{1776} \approx 230 \text{ mph}$$

76.
$$s(d) = 3.86\sqrt{d}$$
$$s(15,000) = 3.86\sqrt{15,000} \approx 473 \text{ mph}$$

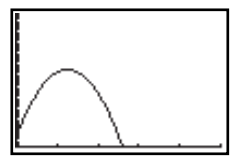
77.



on [0, 5] by [0, 50]

The object hits the ground in about 2.92 seconds

78.



on [0, 5] by [0, 50]

The object hits the ground in about 2.6 seconds.

79. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.
$$C(x) = R(x)$$
$$180x + 16,000 = -2x^2 + 660x$$
$$2x^2 - 480x + 16,000 = 0$$
Use the quadratic formula with $a = 2$, $b = -480$ and $c = 16,000$.
$$x = \frac{480 \pm \sqrt{(-480)^2 - 4(2)(16,000)}}{2(2)}$$
$$x = \frac{480 \pm \sqrt{102,400}}{4} = \frac{480 \pm 320}{4}$$
$$x = \frac{800}{4} \text{ or } \frac{160}{4}$$
$$x = 200 \text{ or } 40$$
The company will break even when it makes either 40 devices or 200 devices.

b. To find the number of devices that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.
$$P(x) = (-2x^2 + 660x) - (180x + 16,000)$$
$$= -2x^2 + 480x - 16,000$$
Since this is a parabola that opens downward, the maximum profit is found at the vertex.
$$x = \frac{-480}{2(-2)} = \frac{-480}{-4} = 120$$
Thus, profit is maximized when 120 devices are produced per week. The maximum profit is found by evaluating $P(120)$.
$$P(120) = -2(120)^2 + 480(120) - 16,000$$
$$= \$12,800$$
Therefore, the maximum profit is \$12,800.

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80. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$C(x)=R(x)$$
$$420x+72,000=-3x^2+1800x$$
$$3x^2-1380x+72,000=0$$

Use the quadratic formula with $a = 3$, $b = -1380$ and $c = 72,000$.

$$x=\frac{1380\pm\sqrt{(-1380)^2-4(3)(72,000)}}{2(3)}$$
$$x=\frac{1380\pm\sqrt{1,040,400}}{6}=\frac{1380\pm1020}{6}$$
$$x=\frac{2400}{6} \text{ or } \frac{360}{6}$$
$$x=400 \text{ or } 60$$

The store will break even when it sells either 60 bicycles or 400 bicycles.

b. To find the number of bicycles that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x)=(-3x^2+1800x)-(420x+72,000)$$
$$=-3x^2+1380x-72,000$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x=\frac{-1380}{2(-3)}=\frac{-1380}{-6}=230$$

Thus, profit is maximized when 230 bicycles are sold per month. The maximum profit is found by evaluating $P(230)$.

$$P(230)=-3(230)^2+1380(230)-72,000$$
$$=\$86,700$$

Therefore, the maximum profit is \$86,700.

81. **a.** To find the break-even points, set $C(x)$ equal to $R(x)$ and solve the resulting equation.

$$C(x)=R(x)$$
$$100x+3200=-2x^2+300x$$
$$2x^2-200x+3200=0$$

Use the quadratic formula with $a = 2$, $b = -200$ and $c = 3200$.

$$x=\frac{200\pm\sqrt{(-1020)^2-4(2)(3200)}}{2(2)}$$
$$x=\frac{200\pm\sqrt{14,400}}{4}=\frac{200\pm120}{4}$$
$$x=\frac{320}{4} \text{ or } \frac{80}{4}$$
$$x=80 \text{ or } 20$$

The store will break even when it sells either 20 exercise machines or 80 exercise machines.

b. To find the number of exercise machines that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x)=(-2x^2+300x)-(100x+3200)$$
$$=-2x^2+200x-3200$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x=\frac{-200}{2(-2)}=\frac{-200}{-4}=50$$

Thus, profit is maximized when 50 exercise machines are sold per day. The maximum profit is found by evaluating $P(50)$.

$$P(50)=-2(50)^2+200(50)-3200$$
$$=\$1800$$

Therefore, the maximum profit is \$1800.

82. Since this is a parabola that opens downward, the monthly price that maximizes visits is found at the vertex.

$$x=\frac{-0.56}{2(-0.004)}=\$70$$

83. $(w+a)(v+b)=c$

$$v+b=\frac{c}{w+a}$$
$$v=\frac{c}{w+a}-b$$

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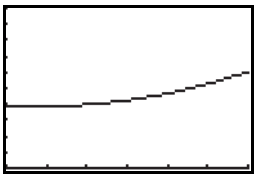
84. **a.** $f(1)=-0.077(1)^2-0.057(1)+1$
$=-0.077-0.057+1$
$=0.866$
So a 65-year-old person has an 86.6% chance of living another decade.

b. $f(2)=-0.077(2)^2-0.057(2)+1$
$=-0.308-0.114+1$
$=0.578$
So a 65-year-old person has a 57.8% chance of living two more decades.

c. $f(3)=-0.077(3)^2-0.057(3)+1$
$=-0.693-0.171+1$
$=0.136$
So a 65-year-old person has a 13.6% chance of living three more decades.

85. **a.**

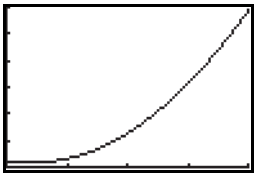


On [10, 16] by [0, 100].

b. $y=0.831x^2-18.1x+137.3$
$y=0.831(12)^2-18.1(12)+137.3$
$y=39.764$
The probability that a high school graduate smoker will quit is 40%.

c. $y=0.831x^2-18.1x+137.3$
$y=0.831(16)^2-18.1(16)+137.3$
$y=60.436$
The probability that a college graduate smoker will quit is 60%.

86. **a.**



On [0, 20] by [0, 300].

b. 2015 – 1995 = 20
$y=0.9x^2-3.9x+12.4$
$y=0.9(20)^2-3.9(20)+12.4$
$y=294.4$
So the global wind power generating capacity in the year 2015 is about 294 thousand megawatts.

c. 2020 – 1995 = 25
$y=0.9x^2-3.9x+12.4$
$y=0.9(25)^2-3.9(25)+12.4$
$y=477.4$
So the global wind power generating capacity in the year 2020 is about 477 thousand megawatts.

87. **a.** $(100-x)x=100x-x^2$ or $-x^2+100$
$f(x)=100x-x^2$ or $f(x)=-x^2+100x$

b. Since this function represents a parabola opening downward (because $a=-1$), it is maximized at its vertex, which is found using the vertex formula, $x=\frac{-b}{2a}$, with $a=-1$ and $b=100$.

$x=\frac{-100}{-2}=50$
She should charge \$50 to maximize her revenue.

88. **a.** The upper limit is
$f(x)=0.7(220-x)=154-0.7x$
The lower limit is
$f(x)=0.5(220-x)=110-0.5x$

b. The lower cardio limit for a 20-year old is
$g(20)=110-0.5(20)=100$ bpm
The upper cardio limit for a 20-year old is
$f(20)=154-0.7(20)=140$ bpm
The lower cardio limit for a 60-year old is
$g(60)=110-0.5(60)=80$ bpm
The upper cardio limit for a 60-year old is
$f(60)=154-0.7(60)=112$ bpm

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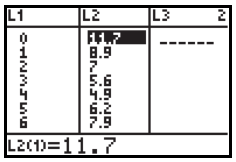
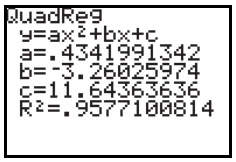
89. **a.**



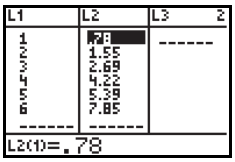


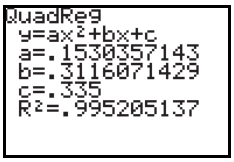
$y = 0.434x^2 - 3.26x + 11.6$

b. For year 2016, $x = 8.6$.

$y = 0.434(8.6)^2 - 3.26(8.6) + 11.6 \approx 15.7\%$

90. **a.**





$y = 0.153x^2 + 0.312x + 0.335$

b. For year 2020, $x = 7$.

$y = 0.153(7)^2 + 0.312(7) + 0.335 \approx \$10$

For year 2030, $x = 8$.

$y = 0.153(8)^2 + 0.312(8) + 0.335 \approx \$12.60$

91. A function can have more than one x-intercept. Many parabolas cross the x-axis twice. A function cannot have more than one y-intercept because that would violate the vertical line test.

92. $f(4) = 9$, (since the two given values show that x increasing by 1 means y increases by 2.).

93. Because the function is linear and 5 is halfway between 4 and 6, $f(5) = 9$ (halfway between 7 and 11).

94. The units of $f(x)$ is widgets and the units of x are blivets, so the units of the slope would be widgets per blivet.

95. m is blargs per prendle and $\frac{\Delta y}{\Delta x}$, so x is in prendles and y is in blargs.

96. If a is negative, then it will have a vertex that is its highest value. If a is positive, then the equation will have a vertex that is its lowest value.

97. No, that would violate the vertical line test. Note: A *parabola* is a geometric shape and so *may* open sideways, but a quadratic function, being a *function*, must pass the vertical line test.

98. Either by the symmetry of parabolas, or, better, by taking the average of the two x-intercepts: the $\pm$ part of the quadratic formula will cancel out, leaving just $\frac{-b}{2a}$.

EXERCISES 1.4

1. Domain = $\{x \mid x < -4 \text{ or } x > 0\}$
Range = $\{y \mid y < -2 \text{ or } y > 0\}$

2. Domain = $\{x \mid x \le 0 \text{ or } x > 3\}$
Range = $\{y \mid y \le -2 \text{ or } y > 2\}$

3. **a.**

$$f(x) = \frac{1}{x+4}$$
$$f(-3) = \frac{1}{-3+4} = 1$$

b. Domain = $\{x \mid x \ne -4\}$
c. Range = $\{y \mid y \ne 0\}$

4. **a.**

$$f(x) = \frac{1}{(x-1)^2}$$
$$f(-1) = \frac{1}{(-1-1)^2} = \frac{1}{4}$$

b. Domain = $\{x \mid x \ne 1\}$
c. Range = $\{y \mid y > 0\}$

5. **a.**

$$f(x) = \frac{x^2}{x-1}$$
$$f(-1) = \frac{(-1)^2}{-1-1} = -\frac{1}{2}$$

b. Domain = $\{x \mid x \ne 1\}$
c. Range = $\{y \mid y \le 0 \text{ or } y \ge 4\}$

6. **a.**

$$f(x) = \frac{x^2}{x+2}$$
$$f(2) = \frac{2^2}{2+2} = 1$$

b. Domain = $\{x \mid x \ne 2\}$
c. Range = $\{y \mid y \le -8 \text{ or } y \ge 0\}$

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7. **a.** $f(x)=\dfrac{12}{x(x+4)}$

$f(2)=\dfrac{12}{2(2+4)}=1$

b. Domain = $\{x \mid x \neq 0, x \neq -4\}$

c. Range = $\{y \mid y \leq -3 \text{ or } y > 0\}$

8. **a.** $f(x)=\dfrac{16}{x(x-4)}$

$f(-4)=\dfrac{16}{-4(-4-4)}=\dfrac{1}{2}$

b. Domain = $\{x \mid x \neq 0, x \neq 4\}$

c. Range = $\{y \mid y \leq -4 \text{ or } y > 0\}$

9. **a.** $g(x)=|x+2|$

$g(-5)=|(-5)+2|=|-3|=3$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \geq 0\}$

10. **a.** $g(x)=|x|+2$

$g(-5)=|-5|+2=5+2=7$

b. Domain = $\mathbb{R}$

c. Range = $\{y \mid y \geq 2\}$

11.
$$x^5+2x^4-3x^3=0$$
$$x^3\left(x^2+2x-3\right)=0$$
$$x^3(x+3)(x-1)=0$$
Equals 0 at $x=0$ Equals 0 at $x=-3$ Equals 0 at $x=1$

$x=0, x=-3,$ and $x=1$

12.
$$x^6-x^5-6x^4=0$$
$$x^4\left(x^2-x-6\right)=0$$
$$x^4(x-3)(x+2)=0$$
Equals 0 at $x=0$ Equals 0 at $x=3$ Equals 0 at $x=-2$

$x=0, x=3,$ and $x=-2$

13.
$$5x^3-20x=0$$
$$5x\left(x^2-4\right)=0$$
$$5x(x-2)(x+2)=0$$
Equals 0 at $x=0$ Equals 0 at $x=2$ Equals 0 at $x=-2$

$x=0, x=2,$ and $x=-2$

14.
$$2x^5-50x^3=0$$
$$2x^3\left(x^2-25\right)=0$$
$$2x^3(x-5)(x+5)=0$$
Equals 0 at $x=0$ Equals 0 at $x=5$ Equals 0 at $x=-5$

$x=0, x=5,$ and $x=-5$

15.
$$2x^3+18x=12x^2$$
$$2x^3-12x^2+18x=0$$
$$2x\left(x^2-6x+9\right)=0$$
$$2x(x-3)^3=0$$
Equals 0 at $x=0$ Equals 0 at $x=3$

$x=0$ and $x=3$

16.
$$3x^3+12x^2=12x^3$$
$$3x^4-12x^3+12x^2=0$$
$$3x^2\left(x^2-4x+4\right)=0$$
$$3x^2(x-2)^2=0$$
Equals 0 at $x=0$ Equals 0 at $x=2$

$x=0$ and $x=2$

17.
$$6x^5=30x^4$$
$$6x^5-30x^4=0$$
$$6x^4(x-5)=0$$
Equals 0 at $x=0$ Equals 0 at $x=5$

$x=0$ and $x=5$

18.
$$5x^4=20x^3$$
$$5x^4-20x^3=0$$
$$5x^3(x-4)=0$$
Equals 0 at $x=0$ Equals 0 at $x=4$

$x=0$ and $x=4$

19.
$$3x^{5/2}-6x^{3/2}=9x^{1/2}$$
$$3x^{5/2}-6x^{3/2}-9x^{1/2}=0$$
$$3x^{1/2}\left(x^2-2x-3\right)=0$$
$$3x^{1/2}(x-3)(x+1)=0$$
Equals 0 at $x=0$ Equals 0 at $x=3$ Equals 0 at $\cancel{x=-1}$

$x=0,$ $x=3$ and $\cancel{x=-1}$

Valid solutions are $x=0$ and $x=3$.

20.
$$2x^{7/2}+8x^{5/2}=24x^{3/2}$$
$$2x^{7/2}+8x^{5/2}-24x^{3/2}=0$$
$$2x^{3/2}\left(x^2+4x-12\right)=0$$
$$2x^{3/2}(x+6)(x-2)=0$$
Equals 0 at $x=0$ Equals 0 at $\cancel{x=-6}$ Equals 0 at $x=2$

$x=0,$ $\cancel{x=-6}$ and $x=2$

Valid solutions are $x=0$ and $x=2$.

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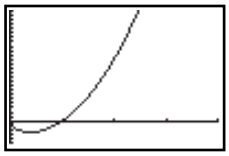
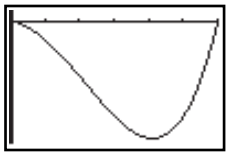
21.



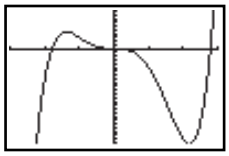
on [0, 4] by [–5,25]
$x = 0$ and $x = 1$

22.



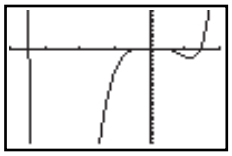
on [0, 6] by [–300, 25]
$x = 0$ and $x = 6$

23.



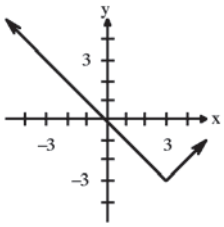
on [–3, 3] by [–25, 10]
$x \approx -1.79$, $x = 0$, and $x \approx 2.79$

24.



on [–4,2] by [–25, 10]
$x \approx -3.45$, $x = 0$, and $x \approx 1.45$

25.



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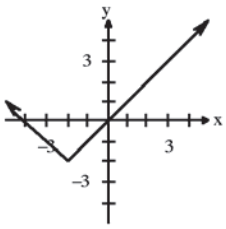
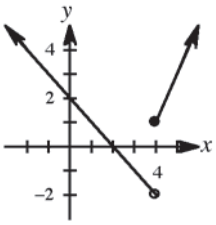


27.



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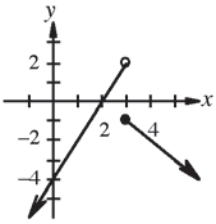

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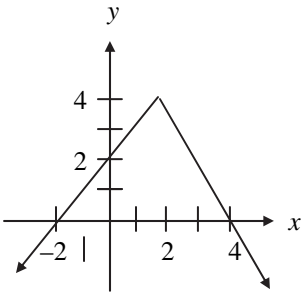

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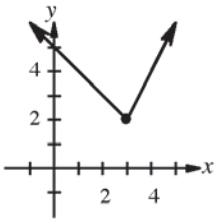
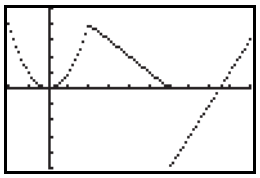
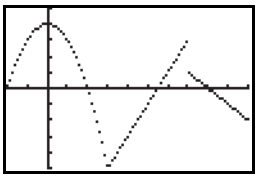


31.



on [–2, 10] by [–5, 5]

32.



on [–2, 10] by [–5, 5]

33. Polynomial
34. Piecewise linear function
35. Piecewise linear function
36. Polynomial
37. Polynomial
38. Piecewise linear function
39. Rational function
40. Polynomial

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41. Piecewise linear function

42. Rational function

43. Polynomial

44. None of these

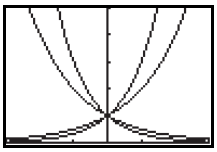
45. None of these

46. Polynomial

47. **a.** y_4

b. y_1

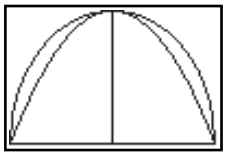
c.



on [–3, 3] by [0, 5]

d. (0, 1) because $a^0 = 1$ for any constant $a \neq 0$.

48.



on [–1, 1] by [0, 1]
The parabola is inside and the semicircle is outside.

49. **a.** $f(g(x)) = [g(x)]^5 = (7x-1)^5$

b. $g(f(x)) = 7[f(x)] - 1$
$= 7(x^5) - 1$
$= 7x^5 - 1$

50. **a.** $f(g(x)) = [g(x)]^8 = (2x+5)^8$

b. $g(f(x)) = 2[f(x)] + 5 = 2x^8 + 5$

51. **a.** $f(g(x)) = \dfrac{1}{g(x)} = \dfrac{1}{x^2+1}$

b. $g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{x^2} + 1$

52. **a.** $f(g(x)) = \sqrt{g(x)} = \sqrt{x^3 - 1}$

b. $g(f(x)) = [f(x)]^3 - 1$
$= \left(\sqrt{x}\right)^3 - 1 = x^{3/2} - 1$

53. **a.** $f(g(x)) = [g(x)]^3 - [g(x)]^2$
$= (\sqrt{x} - 1)^3 - (\sqrt{x} - 1)^2$

b. $g(f(x)) = \sqrt{f(x)} - 1 = \sqrt{x^3 - x^2} - 1$

54. **a.** $f(g(x)) = g(x) - \sqrt{g(x)}$
$= x^2 + 1 - \sqrt{x^2 + 1}$

b. $g(f(x)) = [f(x)]^2 + 1 = \left(x - \sqrt{x}\right)^2 + 1$

55. **a.** $f(g(x)) = \dfrac{[g(x)]^3 - 1}{[g(x)]^3 + 1} = \dfrac{(x^2 - x)^3 - 1}{(x^2 - x)^3 + 1}$

b. $g(f(x)) = [f(x)]^2 - f(x)$
$= \left(\dfrac{x^3 - 1}{x^3 + 1}\right)^2 - \dfrac{x^3 - 1}{x^3 + 1}$

56. **a.** $f(g(x)) = \dfrac{[g(x)]^4 + 1}{[g(x)]^4 - 1} = \dfrac{(x^3 + x)^4 + 1}{(x^3 + x)^4 - 1}$

b. $g(f(x)) = [f(x)]^3 + f(x)$
$= \left(\dfrac{x^4 + 1}{x^4 - 1}\right)^3 + \dfrac{x^4 + 1}{x^4 - 1}$

57. **a.** $f(g(x)) = 2[g(x)] - 6$
$= 2\left(\dfrac{x}{2} + 3\right) - 6$
$= x + 6 - 6$

b. $g(f(x)) = \dfrac{f(x)}{2} + 3$
$= \dfrac{2x - 6}{2} + 3$
$= x - 3 + 3$
$= x$

58. **a.** $f(g(x)) = [g(x)]^3 + 1$
$= \left(\sqrt[3]{x - 1}\right)^3 + 1$
$= x - 1 + 1$
$= x$

b. $g(f(x)) = \sqrt[3]{(f(x)) - 1}$
$= \sqrt[3]{x^3 + 1 - 1}$
$= \sqrt[3]{x^3}$
$= x$

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59. $f(x+h)=5(x+h)^2=5(x^2+2hx+h^2)$
$=5x^2+10hx+5h^2$

60. $f(x+h)=3(x+h)^2=3(x^2+2hx+h^2)$
$=3x^2+6hx+3h^2$

61. $f(x+h)=2(x+h)^2-5(x+h)+1$
$=2x^2+4xh+2h^2-5x-5h+1$

62. $f(x+h)=3(x+h)^2-5(x+h)+2$
$=3x^2+6hx+3h^2-5x-5h+2$

63. $f(x)=5x^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{5(x+h)^2-5x^2}{h}=\frac{5(x^2+2xh+h^2)-5x^2}{h}=\frac{5x^2+10xh+5h^2-5x^2}{h}=\frac{10xh+5h^2}{h}=\frac{h(10x+5h)}{h}=10x+5h \text{ or } 5(2x+h)$$

64. $f(x)=3x^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-3x^2}{h}=\frac{3(x^2+2xh+h^2)-3x^2}{h}=\frac{3x^2+6xh+3h^2-3x^2}{h}=\frac{6xh+3h^2}{h}=\frac{h(6x+3h)}{h}=6x+3h \text{ or } 3(2x+h)$$

65. $f(x)=2x^2-5x+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)^2-5(x+h)+1-(2x^2-5x+1)}{h}=\frac{2(x^2+2xh+h^2)-5(x+h)+1-(2x^2-5x+1)}{h}=\frac{2x^2+4xh+2h^2-5x-5h+1-2x^2+5x-1}{h}=\frac{4xh+2h^2-5h}{h}=\frac{h(4x+2h-5)}{h}=4x+2h-5$$

66. $f(x)=3x^2-5x+2$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-5(x+h)+2-(3x^2-5x+2)}{h}=\frac{3(x^2+2xh+h^2)-5(x+h)+2-(3x^2-5x+2)}{h}=\frac{3x^2+6xh+3h^2-5x-5h+2-3x^2+5x-2}{h}=\frac{6xh+3h^2-5h}{h}=\frac{h(6x+3h-5)}{h}=6x+3h-5$$

67. $f(x)=7x^2-3x+2$

$$\frac{f(x+h)-f(x)}{h}=\frac{7(x+h)^2-3(x+h)+2-(7x^2-3x+2)}{h}=\frac{7(x^2+2xh+h^2)-3(x+h)+2-(7x^2-3x+2)}{h}=\frac{7x^2+14xh+7h^2-3x-3h+2-7x^2+3x-2}{h}=\frac{14xh+7h^2-3h}{h}=\frac{h(14x+7h-3)}{h}=14x+7h-3$$

68. $f(x)=4x^2-5x+3$

$$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)^2-5(x+h)+3-(4x^2-5x+3)}{h}=\frac{4(x^2+2xh+h^2)-5(x+h)+3-(4x^2-5x+3)}{h}=\frac{4x^2+8xh+4h^2-5x-5h+3-4x^2+5x-3}{h}=\frac{8xh+4h^2-5h}{h}=\frac{h(8x+4h-5)}{h}=8x+4h-5$$

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69. $f(x)=x^3$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^3-x^3}{h}\\ &= \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}\\ &= \frac{3x^2h+3xh^2+h^3}{h}\\ &= \frac{h(3x^2+3xh+h^2)}{h}\\ &= 3x^2+3xh+h^2\end{aligned}$$

70. $f(x)=x^4$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^4-x^4}{h}\\ &= \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}\\ &= \frac{4x^3h+6x^2h^2+4xh^3+h^4}{h}\\ &= \frac{h(4x^3+6x^2h+4xh^2+h^3)}{h}\\ &= 4x^3+6x^2h+4xh^2+h^3\end{aligned}$$

71. $f(x)=\frac{2}{x}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{2}{x+h}-\frac{2}{x}}{h}\\ &= \frac{\frac{2}{x+h}-\frac{2}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}\\ &= \frac{2x-2(x+h)}{hx(x+h)}\\ &= \frac{2x-2x-2h}{hx(x+h)}\\ &= \frac{-2h}{hx(x+h)}\\ &= \frac{-2}{x(x+h)}\end{aligned}$$

72. $f(x)=\frac{3}{x}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{3}{x+h}-\frac{3}{x}}{h}\\ &= \frac{\frac{3}{x+h}-\frac{3}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}\\ &= \frac{3x-3(x+h)}{hx(x+h)}\\ &= \frac{3x-3x-3h}{hx(x+h)}\\ &= \frac{-3h}{hx(x+h)}\\ &= \frac{-3}{x(x+h)}\end{aligned}$$

73. $f(x)=\frac{1}{x^2}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\ &= \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\cdot\frac{x^2(x+h)^2}{x^2(x+h)^2}\\ &= \frac{x^2-(x+h)^2}{hx^2(x+h)^2}\\ &= \frac{x^2-x^2-2xh-h^2}{hx^2(x+h)^2}\\ &= \frac{-2xh-h^2}{hx^2(x+h)^2}\\ &= \frac{h(-2x-h)}{hx^2(x+h)^2}\\ &= \frac{-2x-h}{x^2(x+h)^2} \text{ or } \frac{-2x-h}{x^2(x^2+2xh+h^2)}\\ &\text{ or } \frac{-2x-h}{x^4+2x^3h+x^2h^2}\end{aligned}$$

74. $f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

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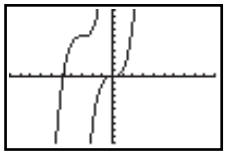
75. **a.** 2.70481
b. 2.71815
c. 2.71828
d. Yes, 2.71828

76.

x	$\left(1+\frac{1}{x}\right)^x$
100	2.70481
10,000	2.71815
1,000,000	2.71828
10,000,000	2.71828

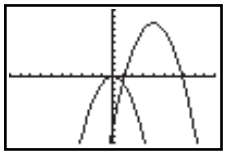
Yes, 2.71828

77. The graph of $y=(x+3)^3+6$ is the same shape as the graph of $y=x^3$ but it is shifted left 3 units and up 6 units.
Check:



on [–10, 10] by [–10, 10]

78. The graph of $y=-(x-4)^2+8$ is the same shape as the graph of $y=-x^2$ but it is shifted right 4 units and up 8 units.
Check:



on [–10, 10] by [–10, 10]

79. $P(x)=522(1.0053)^x$
$P(50)=522(1.0053)^{50}\approx 680$ million people in 1750

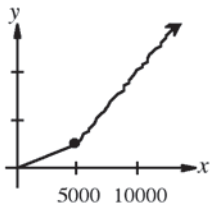
80. $P(100)=522(1.0053)^{100}$
≈ 886 million people in 1800

81. **a.** For $x = 3000$, use $f(x) = 0.10x$.
$f(x)=0.10x$
$f(3000)=0.10(3000)=\$300$

b. For $x = 5000$,
use $f(x) = 0.10x$.
$f(x)=0.10x$
$f(5000)=0.10(5000)=\$500$

c. For $x = 10{,}000$,
use $f(x) = 500 + 0.30(x - 5000)$.
$f(x)=500+0.30(x-5000)$
$f(10{,}000)=500+0.30(10{,}000-5000)$
$=500+0.30(5000)$
$=\$2000$

d.

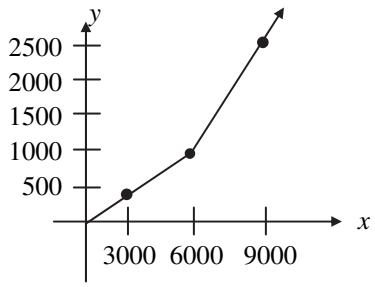


82. **a.** For $x = 3000$, use $f(x) = 0.15x$.
$f(x)=0.15x$
$f(3000)=0.15(3000)=\$450$

b. For $x = 6000$,
use $f(x) = 0.15x$.
$f(x)=0.15x$
$f(6000)=0.15(6000)=\$900$

c. For $x = 10{,}000$,
use $f(x) = 900 + 0.40(x - 6000)$.
$f(x)=900+0.40(x-6000)$
$f(10{,}000)=900+0.40(10{,}000-6000)$
$=900+0.40(4000)$
$=\$2500$

d.



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83. **a.** For $x=\frac{2}{3}$, use $f(x)=10.5x$.

$$f(x)=10.5x$$
$$f\left(\frac{2}{3}\right)=10.5\left(\frac{2}{3}\right)$$
$$=7 \text{ years}$$

For $x=\frac{4}{3}$, use $f(x)=10.5x$.

$$f(x)=10.5x$$
$$f\left(\frac{4}{3}\right)=10.5\left(\frac{4}{3}\right)$$
$$=14 \text{ years}$$

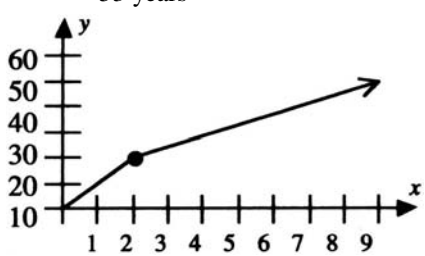
For $x=4$, use $f(x)=21+4(x-2)$.

$$f(x)=21+4(x-2)$$
$$f(4)=21+4(4-2)$$
$$=21+4(2)$$
$$=29 \text{ years}$$

For $x=10$, use $f(x)=21+4(x-2)$.

$$f(x)=21+4(x-2)$$
$$f(10)=21+4(10-2)$$
$$=21+4(8)$$
$$=53 \text{ years}$$

b.



84. **a.** For $x=\frac{2}{3}$, use $f(x)=15x$.

$$f(x)=15x$$
$$f\left(\frac{2}{3}\right)=15\left(\frac{2}{3}\right)$$
$$=10 \text{ years}$$

For $x=\frac{4}{3}$, use $f(x)=15+9(x-1)$.

$$f(x)=15+9(x-1)$$
$$f\left(\frac{4}{3}\right)=15+9\left(\frac{4}{3}-1\right)$$
$$=18 \text{ years}$$

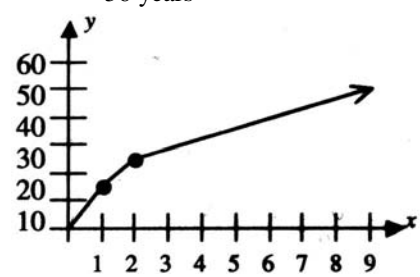
For $x=4$, use $f(x)=15+9(x-1)$.

$$f(x)=15+9(x-1)$$
$$f(4)=15+9(4-1)$$
$$=15+9(3)$$
$$=32 \text{ years}$$

For $x=10$, use $f(x)=15+9(x-1)$.

$$f(x)=15+9(x-1)$$
$$f(10)=15+9(10-1)$$
$$=15+9(8)$$
$$=56 \text{ years}$$

b.



85. Substitute $K=24\cdot L^{-1}$ into $3L+8K=48$.

$$3L+8\left(24\cdot L^{-1}\right)=48$$
$$3L+\frac{192}{L}=48$$
$$3L^2+192=48L$$
$$3L^2-48L+192=0$$
$$3\left(L^2-16L+64\right)=0$$
$$3(L-8)^2=0$$

So, $L=8$.

And $K=24\cdot 8^{-1}=\frac{24}{8}=3$

The intersection point is (8, 3).

86. Substitute $K=180\cdot L^{-1}$ into $5L+4K=120$.

$$5L+4\left(180\cdot L^{-1}\right)=120$$
$$5L+\frac{720}{L}=120$$
$$5L^2+720=120L$$
$$5L^2-120L+720=0$$
$$5\left(L^2-24L+144\right)=0$$
$$5(L-12)^2=0$$

So, $L=12$.

And $K=180\cdot 12^{-1}=\frac{180}{12}=15$

The intersection point is (12, 15).

87. First find the composition $R(v(t))$.

$$R\left(v(t)\right)=2\left[v(t)\right]^{0.3}$$
$$=2(60+3t)^{0.3}$$

Then find $R(v(10))$.

$$R\left(v(10)\right)=2\left(60+3(10)\right)^{0.3}=2(90)^{0.3}$$
$$\approx \$7.714 \text{ million}$$

88. We must find the composition $R(p(t))$.

$$R\left(p(t)\right)=3\left[p(t)\right]^{0.25}$$
$$=3\left(55+4t\right)^{0.25}$$

$$R(5)=3[55+4(5)]^{0.25}$$
$$=3(75)^{0.25}\approx \$8.8 \text{ million}$$

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89. **a.** $f(x) = 4^{10} = 1{,}048{,}576$ cells
≈ 1 million cells

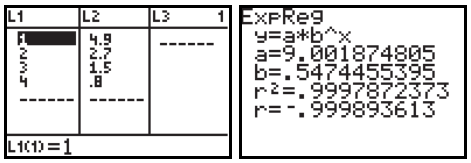
b. $f(15) = 4^{15} = 1{,}073{,}741{,}824$ cells
No, the mouse will not survive beyond day 15.

90. The value 2020 – 2012 = 8 corresponds to the year 2020. Substitute 8 for x.

$f(8) = 226(1.11)^8 \approx \$521$ billion

There will be about \$521 billion of e-commerce in the year 2020.

91. **a.**



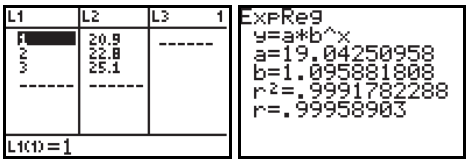
$y = 9 \cdot 0.547^x$ (rounded)

b. For year 2020, $x = 6$.

$y = 9 \cdot 0.547^6 \approx 0.24$ weeks

or less than 2 days

92. **a.**



$y = 19.04 \cdot 1.096^x$ (rounded)

b. For year 2020, $x = 5$.

$y = 19.04 \cdot 1.096^5 \approx 30.1$ million

93. One will have "missing points" at the excluded x-values.

94. $x^{\sqrt{2}} + 1$ is not a polynomial because the exponent is not a non-negative integer.

95. A slope of 1 is a tax of 100%. That means, all dollars taxed are paid as the tax.

96.
$$\begin{aligned} f\left(f(x)\right) &= a\left(f(x)\right) \\ &= a(ax) \\ &= a^2 x \end{aligned}$$

97.
$$\begin{aligned} f\left(f(x)\right) &= \left(f(x)\right) + a \\ &= (x+a) + a \\ &= x + 2a \end{aligned}$$

98. $f(x) + 10$ is translated up by 10 units.

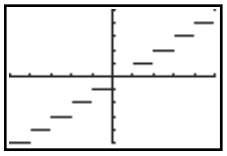
99. $f(x+10)$ is shifted to the left by 10 units.

100. $f(x+10) + 10$ is shifted up 10 units and left 10 units.

101. False: $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$, not $x^2 + h^2$.

102. True:
$$\begin{aligned} f(x+h) &= m(x+h) + b \\ &= mx + b + mh \\ &= f(x) + mh \end{aligned}$$

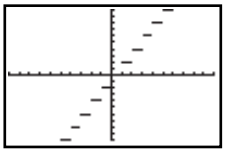
103. **a.**



on [–5, 5] by [–5, 5]
Note that each line segment in this graph includes its left end-point, but excludes its right endpoint. So it should be drawn like •——○.

b. Domain = $\mathbb{R}$; range = the set of integers

104. **a.**



on [–10, 10] by [–10, 10]

b. Domain = $\mathbb{R}$; range = the set of even integers.

105. **a.**
$$\begin{aligned} f\left(g(x)\right) &= a\left[g(x)\right] + b = a(cx+d) + b \\ &= acx + ad + b \end{aligned}$$

b. Yes

106. **a.** $f\left(g(x)\right) = \left[g(x)\right]^2 = \left(x^2\right)^2 = x^4$

b. Yes, because the composition of two polynomials involves raising integral powers to integral powers.

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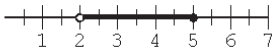
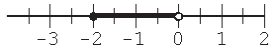
REVIEW EXERCISES AND CHAPTER TEST FOR CHAPTER 1

1. $\{x \mid 2 < x \le 5\}$



2. $\{x \mid -2 \le x < 0\}$



3. $\{x \mid x \ge 100\}$

4. $\{x \mid x \le 6\}$

5. Hurricane: $[74, \infty)$; storm: $[55, 74)$; gale: $[38, 55)$; small craft warning: $[21,38)$

6. **a.** $(0, \infty)$
b. $(-\infty, 0)$
c. $[0, \infty)$
d. $(-\infty, 0]$

7.
$$\begin{aligned} y-(-3) &= 2(x-1) \\ y+3 &= 2x-2 \\ y &= 2x-5 \end{aligned}$$

8.
$$\begin{aligned} y-6 &= -3[x-(-1)] \\ y-6 &= -3x-3 \\ y &= -3x+3 \end{aligned}$$

9. Since the vertical line passes through the point with x-coordinate 2, the equation of the line is $x = 2$.

10. Since the horizontal line passes through the point with y-coordinate 3, the equation of the line is
$y = 3$.

11. First, calculate the slope from the two points.
$$m = \frac{-3-3}{2-(-1)} = \frac{-6}{3} = -2$$
Now use the point-slope formula with this slope and the point $(-1, 3)$.
$$\begin{aligned} y-3 &= -2[x-(-1)] \\ y-3 &= -2x-2 \\ y &= -2x+1 \end{aligned}$$

12. First find the slope of the line $x+2y=8$.
Write the equation in slope-intercept form.
$$y = -\frac{1}{2}x+4.$$
The slope of the perpendicular line is $m = 2$.
Next, use the point-slope form with the point $(6, -1)$:
$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y+1 &= 2(x-6) \\ y &= 2x-13 \end{aligned}$$

13. Since the y-intercept is $(0, -1)$, $b = -1$. To find the slope, use the slope formula with the points $(0, -1)$ and $(1, 1)$.
$$m = \frac{1-(-1)}{1-0} = 2$$
Thus the equation of the line is $y = 2x - 1$.

14. Since the y-intercept is $(0, 1)$, $b = 1$. To find the slope, use the slope formula with the points $(0, 1)$ and $(2, 0)$.
$$m = \frac{0-1}{2-0} = -\frac{1}{2}$$
The equation of the line is $y = -\frac{1}{2}x+1$

15. **a.** Use the straight-line depreciation formula with price = 25,000, useful lifetime = 8, and scrap value = 1000.
$$\begin{aligned} \text{Value} &= \text{price} - \left(\frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\right)t \\ &= 25{,}000 - \left(\frac{25{,}000-1000}{8}\right)t \\ &= 25{,}000 - \left(\frac{24{,}000}{8}\right)t \\ &= 25{,}000 - 3000t \end{aligned}$$
b.
$$\begin{aligned} \text{Value after 4 years} &= 25{,}000 - 3000(4) \\ &= 25{,}000 - 12{,}000 \\ &= \$13{,}000 \end{aligned}$$

16. **a.** Use the straight-line depreciation formula with price = 78,000, useful lifetime = 15, and scrap value = 3000.
$$\begin{aligned} \text{Value} &= \text{price} - \left(\frac{\text{price} - \text{scrap value}}{\text{useful lifetime}}\right)t \\ &= 78{,}000 - \left(\frac{78{,}000-3000}{15}\right)t \\ &= 78{,}000 - \left(\frac{75{,}000}{15}\right)t \\ &= 78{,}000 - 5000t \end{aligned}$$
b.
$$\begin{aligned} \text{Value after 8 years} &= 78{,}000 - 5000(8) \\ &= 78{,}000 - 40{,}000 \\ &= \$38{,}000 \end{aligned}$$

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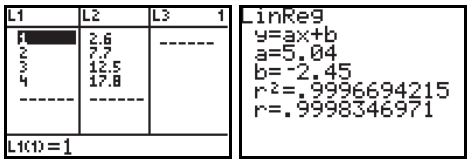
17. **a.**



$y = 5.04x - 2.45$

b. The number is increasing by about 5000 3D screens per year.

c. For the year 2020, $x = 12$

$y = 5.04(12) - 2.45 = 58.03$

or about 58 thousand

18. $\left(\frac{6}{1}\right)^2 = \frac{6^2}{1^2} = 36$

19. $\left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$

20. $64^{1/2} = \sqrt{64} = 8$

21. $1000^{1/3} = \sqrt[3]{1000} = 10$

22. $(81)^{-3/4} = \left(\frac{1}{81}\right)^{3/4} = \left(\sqrt[4]{\frac{1}{81}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

23. $100^{-3/2} = \left(\frac{1}{100}\right)^{3/2} = \left(\sqrt{\frac{1}{100}}\right)^3 = \left(\frac{1}{10}\right)^3 = \frac{1}{1000}$

24. $\left(-\frac{8}{27}\right)^{-2/3} = \left(-\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{-\frac{27}{8}}\right)^2$

$= \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

25. $\left(\frac{9}{16}\right)^{-3/2} = \left(\frac{16}{9}\right)^{3/2} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

26. 13.97

27. 112.32

28. **a.** $y = 0.86x^{0.47}$

$y = 0.86(4000)^{0.47} = 42.4$

The weight for the top cold-blooded meat-eating animals in Hawaii is 42.4 lbs.

b. $y = 0.86x^{0.47}$

$y = 0.86(9{,}400{,}000)^{0.47} = 1628.8$

The weight for the top cold-blooded meat-eating animals in North America is 1628.8 lbs.

29. **a.** $y = 1.7x^{0.52}$

$y = 1.7(4000)^{0.52} = 126.9$

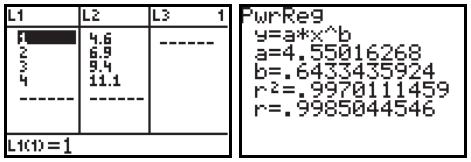
The weight for the top warm-blooded plant-eating animals in Hawaii is 126.9 lbs.

b. $y = 1.7x^{0.52}$

$y = 1.7(9{,}400{,}000)^{0.52} = 7185.8$

The weight for the top warm-blooded plant-eating animals in Hawaii is 7185.8 lbs.

30. **a.**



$y = 4.55x^{0.643}$ (rounded)

b. For year 2020, $x = 12$.

$y = 4.55(12)^{0.643} \approx \$22.5$ billion

31. **a.** $f(11) = \sqrt{11-7} = \sqrt{4} = 2$

b. Domain = $\{x \mid x \geq 7\}$ because $\sqrt{x-7}$ is defined only for all values of $x \geq 7$.

c. Range = $\{y \mid y \geq 0\}$

32. **a.** $g(-1) = \frac{1}{-1+3} = \frac{1}{2}$

b. Domain = $\{t \mid t \neq -3\}$

c. Range = $\{y \mid y \neq 0\}$

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33. **a.** $h(16) = 16^{-3/4} = \left(\frac{1}{16}\right)^{3/4} = \left(\sqrt[4]{\frac{1}{16}}\right)^3$

$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

b. Domain = $\{w \mid w > 0\}$ because the fourth root is defined only for nonnegative numbers and division by 0 is not defined.

c. Range = $\{y \mid y > 0\}$

34. **a.** $w(8) = 8^{-4/3} = \left(\frac{1}{8}\right)^{4/3} = \left(\sqrt[3]{\frac{1}{8}}\right)^4$

$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

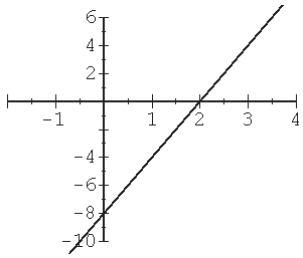
b. Domain = $\{z \mid z \neq 0\}$ because division by 0 is not defined.

c. Range = $\{y \mid y > 0\}$

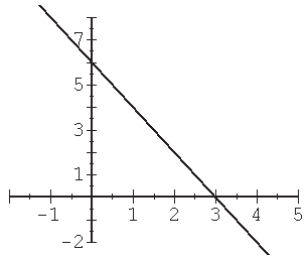
35. Yes

36. No

37.



38.



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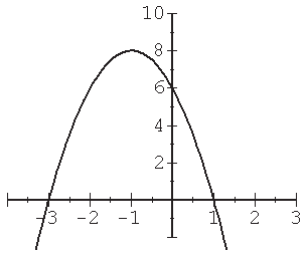
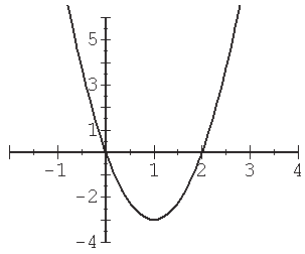


40.



41. **a.**

$$3x^2 + 9x = 0$$
$$3x(x+3) = 0$$

Equals 0 at $x = 0$ Equals 0 at $x = -3$

$x = 0$ and $x = -3$

b. Use the quadratic formula with $a = 3$, $b = 9$, and $c = 0$

$$\frac{-9 \pm \sqrt{9^2 - 4(3)(0)}}{2(3)} = \frac{-9 \pm \sqrt{81}}{6}$$
$$= \frac{-9 \pm 9}{6}$$
$$= 0, -3$$

$x = 0$ and $x = -3$

42. **a.**

$$2x^2 - 8x - 10 = 0$$
$$2(x^2 - 4x - 5) = 0$$
$$(x-5)(x+1) = 0$$

Equals 0 at $x = 5$ Equals 0 at $x = -1$

$x = 5$ and $x = -1$

b. Use the quadratic formula with $a = 2$, $b = -8$, and $c = -10$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{64 + 80}}{4}$$
$$= \frac{8 \pm 12}{4}$$
$$= 5, -1$$

$x = 5$ and $x = -1$

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43. **a.**

$$3x^2+3x+5=11$$
$$3x^2+3x-6=0$$
$$3(x^2+x-2)=0$$
$$(x+2)(x-1)=0$$

Equals 0 at $x=-2$ Equals 0 at $x=1$

$x=-2$ and $x=1$

b.

$$\frac{-3\pm\sqrt{3^2-4(3)(-6)}}{2(3)}=\frac{-3\pm\sqrt{9+72}}{6}$$
$$=\frac{-3\pm\sqrt{81}}{6}$$
$$=\frac{-3\pm 9}{6}$$
$$=-2,1$$

$x=-2$ and $x=1$

44. **a.**

$$4x^2-2=2$$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$

$x=1$ and $x=-1$

b.

$$\frac{-0\pm\sqrt{0^2-4(4)(-4)}}{2(4)}=\frac{\pm\sqrt{64}}{8}$$
$$=\frac{\pm 8}{8}$$
$$=\pm 1$$

$x=1$ and $x=-1$

45. **a.** Use the vertex formula with $a=1$ and $b=-10$.

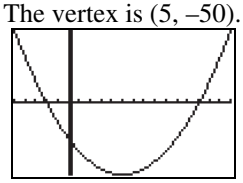
$$x=\frac{-b}{2a}=\frac{-(-10)}{2(1)}=\frac{10}{2}=5$$

To find y, evaluate $f(5)$.

$$f(5)=(5)^2-10(5)-25=-50$$

The vertex is $(5, -50)$.

b.



on $[-5, 15]$ by $[-50, 50]$

46. **a.** Use the vertex formula with $a=1$ and $b=14$.

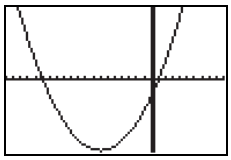
$$x=\frac{-b}{2a}=\frac{-14}{2(1)}=-7$$

To find y, evaluate $f(-7)$.

$$f(-7)=(-7)^2+14(-7)-15=-64$$

The vertex is $(-7, -64)$

b.



on $[-20, 10]$ by $[-65, 65]$

47. Let x = number of miles per day.
$C(x)=0.12x+45$

48. Use the interest formula with $P=10{,}000$ and $r=0.08$.
$I(t)=10{,}000(0.08)t=800t$

49. Let x = the altitude in feet.

$$T(x)=70-\frac{x}{300}$$

50. Let t = the number of years after 2010.

$$C(t)=0.45t+20.3$$
$$25=0.45t+20.3$$

$t\approx 10.4$ years after 2010; in the year 2020

51. **a.** To find the break even points, solve the equation $C(x)=R(x)$ for x.

$$C(x)=R(x)$$
$$80x+1950=-2x^2+240x$$
$$2x^2-160x+1950=0$$
$$x^2-80x+975=0$$
$$(x-65)(x-15)=0$$

Equals 0 at $x=65$ Equals 0 at $x=15$

$x=65$ and $x=15$

The store breaks even at 15 receivers and at 65 receivers.

52. **a.** To find the break even points, solve the equation $C(x)=R(x)$ for x.

$$C(x)=R(x)$$
$$220x+202{,}500=-3x^2+2020x$$
$$3x^2-1800x+202{,}500=0$$
$$x^2-600x+67{,}500=0$$
$$(x-450)(x-150)=0$$

Equals 0 at $x=450$ Equals 0 at $x=150$

$x=450$ and $x=150$

The outlet breaks even at 150 units and 450 units.

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51. **b.** To find the number of receivers that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x) = \left(-2x^2 + 240x\right) - (80x + 1950)$$
$$= -2x^2 + 160x - 1950$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

$$x = \frac{-b}{2a} = \frac{-160}{2(-2)} = \frac{-160}{-4} = 40$$

Thus, profit is maximized when 40 receivers are installed per week. The maximum profit is found by evaluating $P(40)$.

$$P(40) = -2(40)^2 + 160(40) - 1950$$
$$= \$1250$$

Therefore, the maximum profit is \$1250.

52. **b.** To find the number of units that maximizes profit, first find the profit function, $P(x) = R(x) - C(x)$.

$$P(x) = \left(-3x^2 + 2020x\right) - \left(220x + 202{,}500\right)$$
$$= -3x^2 + 1800x - 202{,}500$$

Since this is a parabola that opens downward, the maximum profit is found at the vertex.

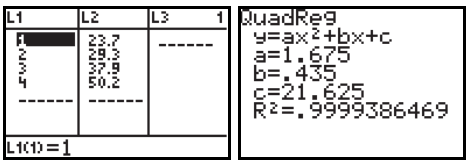
$$x = \frac{-b}{2a} = \frac{-1800}{2(-3)} = \frac{-1800}{-6} = 300$$

Thus, profit is maximized when 300 units are installed per month. The maximum profit is found by evaluating $P(300)$.

$$P(300) = -3(300)^2 + 1800(300) - 202{,}500$$
$$= \$67{,}500$$

Therefore, the maximum profit is \$67,500.

53. **a.**



$y = 1.675x^2 + 0.435x + 21.625$ (rounded)

b. For year 2020, $x = 12$.

$$y = 1.675(12)^2 + 0.435(12) + 21.6$$
$$\approx \$268 \text{ billion}$$

54. **a.** $f(-1) = \dfrac{3}{(-1)(-1-2)} = \dfrac{3}{3} = 1$

b. Domain = $\{x \mid x \neq 0, x \neq 2\}$

c. Range = $\{y \mid y > 0 \text{ or } y \leq -3\}$

55. **a.** $f(-8) = \dfrac{16}{(-8)(-8+4)} = \dfrac{16}{32} = \dfrac{1}{2}$

b. Domain = $\{x \mid x \neq 0, x \neq -4\}$

c. Range = $\{y \mid y > 0 \text{ or } y \leq -4\}$

56. **a.** $g(-4) = |-4+2| - 2 = 2 - 2 = 0$

b. Domain = $\Re$

c. Range = $\{y \mid y \geq 0\}$

57. **a.** $g(-5) = (-5) - |-5| = -5 - 5 = -10$

b. Domain = $\Re$

c. Range = $\{y \mid y \geq 0\}$

58.

$$5x^4 + 10x^3 = 15x^2$$
$$5x^4 + 10x^3 - 15x^2 = 0$$
$$5x^2\left(x^2 + 2x - 3\right) = 0$$
$$x^2(x+3)(x-1) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = -3$; Equals 0 at $x = 1$

$x = 0$, $x = -3$, and $x = 1$

59.

$$4x^5 + 8x^4 = 32x^3$$
$$4x^5 + 8x^4 - 32x^3 = 0$$
$$4x^3\left(x^2 + 2x - 8\right) = 0$$
$$x^3(x+4)(x-2) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = -4$; Equals 0 at $x = 2$

$x = 0$, $x = -4$, and $x = 2$

60.

$$2x^{5/2} - 8x^{3/2} = 10x^{1/2}$$
$$2x^{5/2} - 8x^{3/2} - 10x^{1/2} = 0$$
$$2x^{1/2}(x^2 - 4x - 5) = 0$$
$$x^{1/2}(x-5)(x+1) = 0$$

Equals 0 at $x = 0$; Equals 0 at $x = 5$; Equals 0 at $\cancel{x = -1}$

$x = 0$, $x = 5$ and $\cancel{x = -1}$

Only $x = 0$ and $x = 5$ are solutions.

61.

$$3x^{5/2} + 3x^{3/2} = 18x^{1/2}$$
$$3x^{5/2} + 3x^{3/2} - 18x^{1/2} = 0$$
$$3x^{1/2}(x^2 + x - 6) = 0$$
$$x^{1/2}(x+3)(x-2) = 0$$

Equals 0 at $x = 0$; Equals 0 at $\cancel{x = -3}$; Equals 0 at $x = 2$

$x = 0$, $\cancel{x = -3}$ and $x = 2$

Only $x = 0$ and $x = 2$ are solutions.

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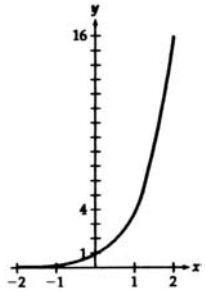
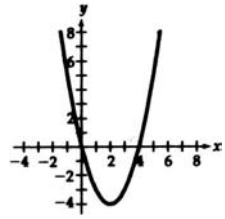
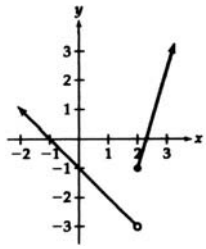
62.



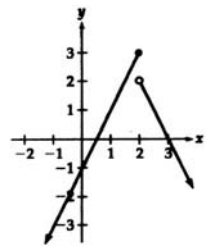
63.



64.



65.



66. **a.**

$$f\left(g\left(x\right)\right)=\left[g\left(x\right)\right]^2+1$$
$$=\left(\frac{1}{x}\right)^2+1$$
$$=\frac{1}{x^2}+1$$

b. $g\left(f\left(x\right)\right)=\dfrac{1}{f\left(x\right)}=\dfrac{1}{x^2+1}$

67. **a.** $f\left(g\left(x\right)\right)=\sqrt{g\left(x\right)}=\sqrt{5x-4}$

b. $g\left(f\left(x\right)\right)=5\left[f\left(x\right)\right]-4=5\sqrt{x}-4$

68. **a.** $f\left(g\left(x\right)\right)=\dfrac{g\left(x\right)+1}{g\left(x\right)-1}=\dfrac{x^3+1}{x^3-1}$

b. $g\left(f\left(x\right)\right)=\left[f\left(x\right)\right]^3=\left(\dfrac{x+1}{x-1}\right)^3$

69. **a.** $f\left(g\left(x\right)\right)=\left|\,g\left(x\right)\,\right|=\left|\,x+2\,\right|$

b. $g\left(f\left(x\right)\right)=\left[f\left(x\right)\right]+2=\left|\,x\,\right|+2$

70.

$$f(x)=2x^2-3x+1$$
$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h}$$
$$=\frac{2(x^2+2xh+h^2)-3(x+h)+1-(2x^2-3x+1)}{h}$$
$$=\frac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h}$$
$$=\frac{4xh+2h^2-3h}{h}$$
$$=\frac{h(4x+2h-3)}{h}$$
$$=4x+2h-3$$

71.

$$f(x)=\frac{5}{x}$$
$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{5}{x+h}-\frac{5}{x}}{h}$$
$$=\frac{\frac{5}{x+h}-\frac{5}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}$$
$$=\frac{5x-5(x+h)}{hx(x+h)}$$
$$=\frac{5x-5x-5h}{hx(x+h)}$$
$$=\frac{-5h}{hx(x+h)}$$
$$=\frac{-5}{x(x+h)}$$

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72. The advertising budget A as a function of t is the composition of $A(p)$ and $p(t)$.

$$A\big(p(t)\big) = 2\big[p(t)\big]^{0.15} = 2(18+2t)^{0.15}$$

$$A(4) = 2\big[18+2(4)\big]^{0.15} = 2(26)^{0.15}$$

$$\approx \$3.26 \text{ million}$$

73. **a.**

$$x^4 - 2x^3 - 3x^2 = 0$$

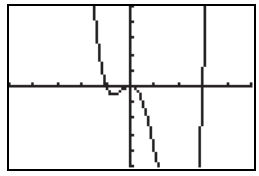
$$x^2(x^2 - 2x - 3) = 0$$

$$x^2(x-3)(x+1) = 0$$

Equals 0 at $x = 0$ Equals 0 at $x = 3$ Equals 0 at $x = -1$

$x = 0$, $x = 3$ and $x = -1$

b.



on [–5, 5] by [–5, 5]

74. **a.**

$$x^3 + 2x^2 - 3x = 0$$

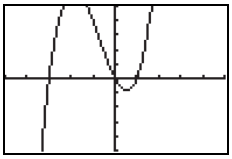
$$x(x^2 + 2x - 3) = 0$$

$$x(x+3)(x-1) = 0$$

Equals 0 at $x = 0$ Equals 0 at $x = -3$ Equals 0 at $x = 1$

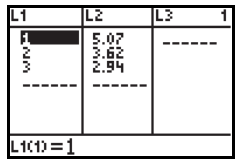
$x = 0$, $x = -3$, and $x = 1$

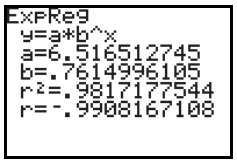
b.



on [–5, 5] by [–5, 5]

75. **a.**





$y = 6.52 \cdot 0.761^x$

b. For year 2020, $x = 4$.

$y = 6.52 \cdot 0.761^4 \approx 2.2$

2.2 crimes per 100,000

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Chapter 2: Derivatives and Their Uses

EXERCISES 2.1

1.

x	$5x-7$
1.9	2.500
1.99	2.950
1.999	2.995

x	$5x-7$
2.1	3.500
2.01	3.050
2.001	3.005

a. $\lim_{x\to 2^-}(5x-7)=3$

b. $\lim_{x\to 2^+}(5x-7)=3$

c. $\lim_{x\to 2}(5x-7)=3$

2.

x	$2x+1$
3.9	8.800
3.99	8.980
3.999	8.998

x	$2x+1$
4.100	9.200
4.010	9.020
4.001	9.002

a. $\lim_{x\to 4^-}(2x+1)=9$

b. $\lim_{x\to 4^+}(2x+1)=9$

c. $\lim_{x\to 4}(2x+1)=9$

3.

x	$\frac{x^3-1}{x-1}$
0.900	2.710
0.990	2.970
0.999	2.997

x	$\frac{x^3-1}{x-1}$
1.1	3.310
1.01	3.030
1.001	3.003

a. $\lim_{x\to 1^-}\frac{x^3-1}{x-1}=3$

b. $\lim_{x\to 1^+}\frac{x^3-1}{x-1}=3$

c. $\lim_{x\to 1}\frac{x^3-1}{x-1}=3$

4.

x	$\frac{x^4-1}{x-1}$
0.9	3.439
0.99	3.940
0.999	3.994

x	$\frac{x^4-1}{x-1}$
1.1	4.641
1.01	4.060
1.001	4.006

a. $\lim_{x\to 1^-}\frac{x^4-1}{x-1}=4$

b. $\lim_{x\to 1^+}\frac{x^4-1}{x-1}=4$

c. $\lim_{x\to 1}\frac{x^4-1}{x-1}=4$

5.

x	$(1+2x)^{1/x}$
–0.1	9.313
–0.01	7.540
–0.001	7.404

x	$(1+2x)^{1/x}$
0.1	6.192
0.01	7.245
0.001	7.374

$\lim_{x\to 0}(1+2x)^{1/x}\approx 7.4$

6.

x	$(1-x)^{1/x}$
–0.1	0.386
–0.01	0.370
–0.001	0.368

x	$(1-x)^{1/x}$
0.1	0.349
0.01	0.366
0.001	0.368

$\lim_{x\to 0}(1-x)^{1/x}\approx 0.368$

7.

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
1.9	–0.263
1.99	–0.251
1.999	–0.250

x	$\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$
2.1	-0.238
2.01	-0.249
2.001	-0.250

$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}=-0.25$

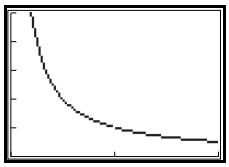
8.

x	$\frac{\sqrt{x}-1}{x-1}$
0.9	0.513
0.99	0.501
0.999	0.500

x	$\frac{\sqrt{x}-1}{x-1}$
1.1	0.488
1.01	0.499
1.001	0.500

$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=0.5$

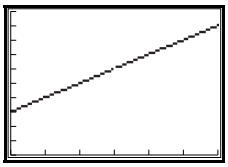
9.



on [0, 2] by [0, 5]

$\lim_{x\to 1}\frac{\frac{1}{x}-1}{1-x}=1$

10.



on [0, 3] by [0, 10]

$\lim_{x\to 1.5}\frac{2x^2-4.5}{x-1.5}=6$

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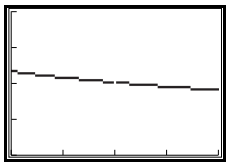
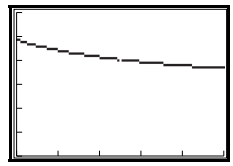
11. 

on [3, 5] by [1, 3]

$$\lim_{x\to 4}\frac{x^{1.5}-4x^{0.5}}{x^{1.5}-2x}=2$$

12. 

on [0.5, 1.5] by [0, 3]

$$\lim_{x\to 1}\frac{x-1}{x-\sqrt{x}}=2$$

13. $\lim_{x\to 3} 4x^2-10x+2=4(3)^2-10(3)+2=8$

14. $\lim_{x\to 7}\frac{x^2-x}{2x-7}=\frac{(7)^2-7}{2(7)-7}=6$

15. $\lim_{x\to 5}\frac{3x^2-5x}{7x-10}=\frac{3(5)^2-5(5)}{7(5)-10}=2$

16. $\lim_{t\to 3}\sqrt[3]{t^2+t-4}=\sqrt[3]{(3)^2+3-4}=2$

17. $\lim_{x\to 3}\sqrt{2}=\sqrt{2}$ because the limit of a constant is just the constant.

18. $\lim_{q\to 9}\frac{8+2\sqrt{q}}{8-2\sqrt{q}}=\frac{8+2\sqrt{9}}{8-2\sqrt{9}}=\frac{14}{2}=7$

19. $\lim_{t\to 25}\left[(t+5)t^{-1/2}\right]=(25+5)(25)^{-1/2}=6$

20. $\lim_{s\to 4}\left(s^{3/2}-3s^{1/2}\right)=\left[4^{3/2}-3(4)^{1/2}\right]=2$

21. $\lim_{h\to 0}(5x^3+2x^2h-xh^2)=5x^3+2x^2\cdot 0-x(0)^2=5x^3$

22. $\lim_{h\to 0}(2x^2+4xh+h^2)=2x^2+4x\cdot 0+(0)^2=2x^2$

23. $\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}$
$=\lim_{x\to 2}(x+2)=2+2=4$

24. $\lim_{x\to 1}\frac{x-1}{x^2+x-2}=\lim_{x\to 1}\frac{x-1}{(x+1)(x-1)}$
$=\lim_{x\to 1}\frac{1}{x+2}=\frac{1}{1+2}=\frac{1}{3}$

25. $\lim_{x\to -3}\frac{x+3}{x^2+8x+15}=\lim_{x\to -3}\frac{x+3}{(x+3)(x+5)}$
$=\lim_{x\to -3}\frac{1}{x+5}=\frac{1}{-3+5}=\frac{1}{2}$

26. $\lim_{x\to -4}\frac{x^2+9x+20}{x+4}=\lim_{x\to -4}\frac{(x+5)(x+4)}{x+4}$
$=\lim_{x\to -4}x+5=-4+5=1$

27. $\lim_{x\to -1}\frac{3x^3-3x^2-6x}{x^2+x}=\lim_{x\to -1}\frac{3x(x^2-x-2)}{x(x+1)}$
$=\lim_{x\to -1}\frac{3x(x-2)(x+1)}{x(x+1)}$
$=\lim_{x\to -1}3(x-2)=3(-1-2)=-9$

28. $\lim_{x\to 0}\frac{x^2-x}{x^2+x}=\lim_{x\to 0}\frac{x(x-1)}{x(x+1)}=\lim_{x\to 0}\frac{x-1}{x+1}=-1$

29. $\lim_{h\to 0}\frac{2xh-3h^2}{h}=\lim_{h\to 0}(2x-3h)=2x-3(0)=2x$

30. $\lim_{h\to 0}\frac{5x^4h-9xh^2}{h}=\lim_{h\to 0}(5x^4-9xh)$
$=5x^4-9x(0)=5x^4$

31. $\lim_{h\to 0}\frac{4x^2h+xh^2-h^3}{h}=\lim_{h\to 0}(4x^2+xh-h^2)$
$=4x^2+x(0)-(0)^2=4x^2$

32. $\lim_{h\to 0}\frac{x^2h-xh^2+h^3}{h}=\lim_{h\to 0}(x^2-xh+h^2)$
$=x^2-x(0)+(0)^2=x^2$

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33. **a.** $\lim_{x\to 2^-} f(x) = 1$

b. $\lim_{x\to 2^+} f(x) = 3$

c. $\lim_{x\to 2} f(x)$ does not exist.

34. **a.** $\lim_{x\to 2^-} f(x) = 1$

b. $\lim_{x\to 2^+} f(x) = 2$

c. $\lim_{x\to 2} f(x)$ does not exist.

35. **a.** $\lim_{x\to 2^-} f(x) = -1$

b. $\lim_{x\to 2^+} f(x) = -1$

c. $\lim_{x\to 2} f(x) = -1$

36. **a.** $\lim_{x\to 2^-} f(x) = 3$

b. $\lim_{x\to 2^+} f(x) = 3$

c. $\lim_{x\to 2} f(x) = 3$

37. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (3-x)$
$= 3-(4) = -1$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (10-2x)$
$= 10-2(4) = 2$

c. $\lim_{x\to 4} f(x)$ does not exist.

38. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (5-x)$
$= 5-4 = 1$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (2x-5)$
$= 2(4)-5 = 3$

c. $\lim_{x\to 4} f(x)$ does not exist.

39. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (2-x)$
$= 2-4 = -2$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (x-6)$
$= 4-6 = -2$

c. $\lim_{x\to 4} f(x) = -2$

40. **a.** $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (2-x)$
$= 2-4 = -2$

b. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (2x-10)$
$= 2(4)-10 = -2$

c. $\lim_{x\to 4} f(x) = -2$

41. **a.** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x)$
$= -0 = 0$

b. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x)$
$= 0$

c. $\lim_{x\to 0} f(x) = 0$

42. **a.** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} [-(-x)]$
$= -(-0) = 0$

b. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -(x)$
$= -0 = 0$

c. $\lim_{x\to 0} f(x) = 0$

43. **a.**

x	$\frac{\lvert x\rvert}{x}$
–0.1	–1
–0.01	–1
–0.001	–1

$\lim_{x\to 0} f(x) = -1$

b.

x	$\frac{\lvert x\rvert}{x}$
0.1	1
0.01	1
0.001	1

$\lim_{x\to 0} f(x) = 1$

c. $\lim_{x\to 0} f(x)$ does not exist.

44. **a.**

x	$-\frac{\lvert x\rvert}{x}$
–0.1	1
–0.01	1
–0.001	1

$\lim_{x\to 0} f(x) = 1$

b.

x	$-\frac{\lvert x\rvert}{x}$
0.1	–1
0.01	–1
0.001	–1

$\lim_{x\to 0} f(x) = -1$

c. $\lim_{x\to 0} f(x)$ does not exist.

45. $\lim_{x\to -\infty} f(x) = 0$; $\lim_{x\to 3^-} f(x) = \infty$ and $\lim_{x\to 3^+} f(x) = -\infty$,
so $\lim_{x\to 3} f(x)$ does not exist and $\lim_{x\to \infty} f(x) = 0$.

46. $\lim_{x\to -\infty} f(x) = 0$; $\lim_{x\to -3^-} f(x) = \infty$ and $\lim_{x\to -3^+} f(x) = \infty$,
so $\lim_{x\to -3} f(x) = \infty$ and $\lim_{x\to \infty} f(x) = 0$.

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47. $\lim_{x\to-\infty} f(x)=0$; $\lim_{x\to0^-} f(x)=\infty$ and $\lim_{x\to0^+} f(x)=\infty$,
so $\lim_{x\to0} f(x)=\infty$ and $\lim_{x\to\infty} f(x)=0$.

48. $\lim_{x\to-\infty} f(x)=0$; $\lim_{x\to0^-} f(x)=-\infty$ and $\lim_{x\to0^+} f(x)=\infty$,
so $\lim_{x\to0} f(x)$ does not exist and $\lim_{x\to\infty} f(x)=0$.

49. $\lim_{x\to-\infty} f(x)=2$; $\lim_{x\to1^-} f(x)=-\infty$ and $\lim_{x\to1^+} f(x)=\infty$,
so $\lim_{x\to1} f(x)$ does not exist and $\lim_{x\to\infty} f(x)=2$.

50. $\lim_{x\to-\infty} f(x)=1$; $\lim_{x\to-3^-} f(x)=\infty$ and $\lim_{x\to-3^+} f(x)=-\infty$,
so $\lim_{x\to-3} f(x)$ does not exist and $\lim_{x\to\infty} f(x)=1$.

51. $\lim_{x\to-\infty} f(x)=1$; $\lim_{x\to-2^-} f(x)=\infty$ and $\lim_{x\to-2^+} f(x)=\infty$,
so $\lim_{x\to-2} f(x)=\infty$ and $\lim_{x\to\infty} f(x)=1$.

52. $\lim_{x\to-\infty} f(x)=2$; $\lim_{x\to1^-} f(x)=\infty$ and $\lim_{x\to1^+} f(x)=\infty$,
so $\lim_{x\to1} f(x)=\infty$ and $\lim_{x\to\infty} f(x)=2$.

53. Continuous

54. Discontinuous at c because $f(c)$ is not defined.

55. Discontinuous at c because $\lim_{x\to c} f(x)\neq f(c)$.

56. Discontinuous at c because $\lim_{x\to c} f(x)$ does not exist.

57. Discontinuous at c because $f(c)$ is not defined.

58. Continuous

59. Discontinuous at c because $\lim_{x\to c} f(x)$ does not exist.

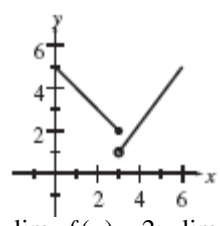
60. Discontinuous at c because $\lim_{x\to c} f(x)\neq f(c)$.

61. **a.**

b. $\lim_{x\to3^-} f(x)=3$; $\lim_{x\to3^+} f(x)=3$

c. Continuous

62. **a.**



b. $\lim_{x\to3^-} f(x)=2$; $\lim_{x\to3^+} f(x)=1$

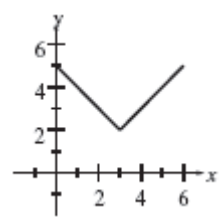
c. Discontinuous because $\lim_{x\to3} f(x)$ does not exist.

63. **a.**

b. $\lim_{x\to3^-} f(x)=3$; $\lim_{x\to3^+} f(x)=4$

c. Discontinuous because $\lim_{x\to3} f(x)$ does not exist.

64. **a.**



b. $\lim_{x\to3^-} f(x)=2$; $\lim_{x\to3^+} f(x)=2$

c. Continuous

65. Continuous

66. Continuous

67. Discontinuous at $x = 1$

68. Discontinuous at $x = -7$ and $x = 2$

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69. $f(x)=\dfrac{12}{5x^3-5x}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$5x^3-5x=0$$

$$5x(x^2-1)=0$$

$5x$ equals zero at $x=0$ and x^2-1 equals zero at $x=\pm 1$.
Thus, the function is discontinuous at $x=0$, $x=-1$, and $x=1$.

70. $f(x)=\dfrac{x+2}{x^4-3x^3-4x^2}$ is discontinuous at values of x for which the denominator is zero. Thus, consider

$$x^4-3x^3-4x^2=0$$
$$x^2\left(x^2-3x-4\right)=0$$
$$x^2(x-4)(x+1)=0$$

x^2 equals zero at $x=0$, $x-4$ equals zero at $x=4$, and $x+1$ equals zero at $x=-1$.
Thus, the function is discontinuous at $x=0$, $x=4$, and $x=-1$.

71. From Exercise 37, we know $\lim\limits_{x\to 4} f(x)$ does not exist. Therefore, the function is discontinuous at $x=4$.

72. From Exercise 38, we know $\lim\limits_{x\to 4} f(x)$ does not exist. Therefore, the function is discontinuous at $x=4$.

73. From Exercise 39, we know $\lim\limits_{x\to 4} f(x)=-2=f(4)$. Therefore, the function is continuous.

74. From Exercise 40, we know $\lim\limits_{x\to 4} f(x)=-2=f(4)$. Therefore, the function is continuous.

75. From Exercise 41, we know $\lim\limits_{x\to 0} f(x)=0=f(0)$. Therefore, the function is continuous.

76. From Exercise 43, we know $\lim\limits_{x\to 0} f(x)$ does not exist. Therefore the function is discontinuous at $x=0$.

77. From the graph, we can see that $\lim\limits_{x\to 6} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x=6$.

78. From the graph, we can see that $\lim\limits_{x\to 7} f(x)$ does not exist because the left and right limits do not agree. $f(x)$ is discontinuous at $x=7$.

79. Since the function $\dfrac{(x-1)(x+2)}{x-1}$ is not defined at $x=1$ and the function $x+2$ equals 3 at $x=1$, the functions are not equal.

80. $\lim\limits_{v\to c}\sqrt{1-\left(\frac{v}{c}\right)^2}=\sqrt{1-\left(\frac{c}{c}\right)^2}=0$

81.

x	$\left(1+\frac{x}{10}\right)^{1/x}$	x	$\left(1+\frac{x}{10}\right)^{1/x}$
–0.1	1.11	0.1	1.10
–0.01	1.11	0.01	1.11
–0.001	1.11	0.001	1.11
–0.0001	1.11	0.0001	1.11

$\lim\limits_{x\to 0}\left(1+\frac{x}{10}\right)^{1/x}\approx \$1.11$

82.

x	$\left(1+\frac{x}{20}\right)^{1/x}$	x	$\left(1+\frac{x}{20}\right)^{1/x}$
–0.1	1.05	0.1	1.05
–0.01	1.05	0.01	1.05
–0.001	1.05	0.001	1.05
–0.0001	1.05	0.0001	1.05

$\lim\limits_{x\to 0}\left(1+\frac{x}{20}\right)^{1/x}\approx \$1.05$

83. $\lim\limits_{x\to 0}\dfrac{100}{1+.001x^2}=\dfrac{100}{1+.001(0)^2}=\dfrac{100}{1}$
$=100$

84. The left and right limits at 1 ounce, 2 ounces, and at 3 ounces do not agree. This function is discontinuous at 1 ounce, 2 ounces and at 3 ounces.

85. As x approaches c, the function is approaching $\lim\limits_{x\to c} f(x)$ even if the value of the function at c is different, so the limit is where the function is "going".

86. In a continuous function, when x equals c, the function equals $\lim\limits_{x\to c} f(x)$. This is not true for a discontinuous function.

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85. As x approaches c, the function is approaching $\lim_{x\to c} f(x)$ even if the value of the function at c is different, so the limit is where the function is "going".

86. In a continuous function, when x equals c, the function equals $\lim_{x\to c} f(x)$. This is not true for a discontinuous function.

87. False: The value of the function at 2 has nothing to do with the limit as x approaches 2.

88. False: There could be a "hole" or "jump" at $x = 2$.

89. False: Both one-sided limits would have to exist and agree to guarantee that the limit exists.

90. True: If $\lim_{x\to 2} f(x) = 7$, then $\lim_{x\to 2^+} f(x) = 7$ and $\lim_{x\to 2^-} f(x) = 7$.

91. False: On the left side of the limit exists and equals 2 (as we saw in Example 4), but on the right side of the denominator of the fraction is zero. Therefore one side of the equation is defined and the other is not.

92. True: A function must be defined at $x = c$ to be continuous at $x = c$.

93. True: The third requirement for continuity at $x = 2$ is that the limit and the value at 2 must agree, so if one is 7 the other must be 7.

94. True: If a function is continuous at every x-value, then its graph has no jumps or breaks. The jumps or breaks would make it discontinuous.

95. $\lim_{x\to 0^+} f(x)$ does not exist; $\lim_{x\to 0^-} f(x)$ does not exist; $\lim_{x\to 0} f(x)$ does not exist

EXERCISES 2.2

1. The slope is positive at P_1.
The slope is negative at P_2.
The slope is zero at P_3.

2. The slope is zero at P_1.
The slope is positive at P_2.
The slope is negative at P_3.

3. The slope is positive at P_1.
The slope is negative at P_2.
The slope is zero at P_3.

4. The slope is negative at P_1.
The slope is zero at P_2.
The slope is negative at P_3.

5. The tangent line at P_1 contains the points (0, 2) and (1, 5). The slope of this line is

$$m = \frac{5-2}{1-0} = 3.$$

The slope of the curve at P_1 is 3.
The tangent line at P_2 contains the points (3, 5) and (5, 4). The slope of this line is

$$m = \frac{4-5}{5-3} = -\frac{1}{2}$$

The slope of the curve at P_2 is $-\frac{1}{2}$.

6. The tangent line at P_1 contains the points (1, 4) and (4, 3). The slope of this line is

$$m = \frac{3-4}{4-1} = -\frac{1}{3}.$$

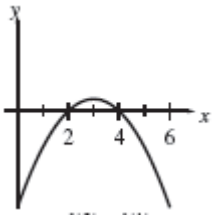
The slope of the curve at P_1 is $-\frac{1}{3}$.
The tangent line at P_2 contains the points (5, 3) and (6, 5). The slope of this line is

$$m = \frac{5-3}{6-5} = 2$$

The slope of the curve at P_2 is 2.

7. Your graph should look roughly like the following:

8. Your graph should look roughly like the following:



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9. **a.** $\frac{f(3)-f(1)}{2}=\frac{12-2}{2}=5$

b. $\frac{f(2)-f(1)}{1}=\frac{6-2}{1}=4$

c. $\frac{f(1.5)-f(1)}{.5}=\frac{3.75-2}{.5}=3.5$

d. $\frac{f(1.1)-f(1)}{.1}=\frac{2.31-2}{.1}=3.1$

e. $\frac{f(1.01)-f(1)}{.01}=\frac{2.0301-2}{.01}=3.01$

f. Answers seem to be approaching 3.

10. **a.** $\frac{f(3)-f(1)}{2}=\frac{23-7}{2}=8$

b. $\frac{f(2)-f(1)}{1}=\frac{13-7}{1}=6$

c. $\frac{f(1.5)-f(1)}{.5}=\frac{9.5-7}{.5}=5$

d. $\frac{f(1.1)-f(1)}{.1}=\frac{7.42-7}{.1}=4.2$

e. $\frac{f(1.01)-f(1)}{.01}=\frac{7.0402-7}{.01}=4.02$

f. Answers seem to be approaching 4.

11. **a.** $\frac{f(4)-f(2)}{2}=\frac{34-8}{2}=13$

b. $\frac{f(3)-f(2)}{1}=\frac{19-8}{1}=11$

c. $\frac{f(2.5)-f(2)}{.5}=\frac{13-8}{.5}=10$

d. $\frac{f(2.1)-f(2)}{.1}=\frac{8.92-8}{.1}=9.2$

e. $\frac{f(2.01)-f(1)}{.01}=\frac{8.0902-8}{.01}=9.02$

f. Answers seen to be approaching 9.

12. **a.** $\frac{f(4)-f(2)}{2}=\frac{23-7}{2}=8$

b. $\frac{f(3)-f(2)}{1}=\frac{14-7}{1}=7$

c. $\frac{f(2.5)-f(2)}{.5}=\frac{10.25-7}{.5}=6.5$

d. $\frac{f(2.1)-f(2)}{.1}=\frac{7.61-7}{.1}=6.1$

e. $\frac{f(2.01)-f(1)}{.01}=\frac{7.0601-7}{.01}=6.01$

f. Answers seem to be approaching 6.

13. **a.** $\frac{f(5)-f(3)}{2}=\frac{26-16}{2}=5$

b. $\frac{f(4)-f(3)}{1}=\frac{21-16}{1}=5$

c. $\frac{f(3.5)-f(3)}{.5}=\frac{18.5-16}{.5}=5$

d. $\frac{f(3.1)-f(3)}{.1}=\frac{16.5-16}{.1}=5$

e. $\frac{f(3.01)-f(3)}{.01}=\frac{16.05-16}{.01}=5$

f. Answers seem to be approaching 5.

14. **a.** $\frac{f(5)-f(3)}{2}=\frac{33-19}{2}=7$

b. $\frac{f(4)-f(3)}{1}=\frac{26-19}{1}=7$

c. $\frac{f(3.5)-f(3)}{.5}=\frac{22.5-19}{.5}=7$

d. $\frac{f(3.1)-f(3)}{.1}=\frac{19.7-19}{.1}=7$

e. $\frac{f(3.01)-f(3)}{.01}=\frac{19.07-19}{.01}=7$

f. Answers seem to be approaching 7.

15. **a.** $\frac{f(6)-f(4)}{2}=\frac{2.4495-2}{2}=0.2247$

b. $\frac{f(5)-f(4)}{1}=\frac{2.236-2}{1}=0.2361$

c. $\frac{f(4.5)-f(4)}{.5}=\frac{2.121-2}{.5}=0.2426$

d. $\frac{f(4.1)-f(4)}{.1}=\frac{2.025-2}{.1}=0.2485$

e. $\frac{f(4.01)-f(4)}{.01}=\frac{2.0025-2}{.01}=0.2498$

f. Answers seem to approach 0.25.

16. **a.** $\frac{f(6)-f(4)}{2}=\frac{.6667-1}{2}=-0.1667$

b. $\frac{f(5)-f(4)}{1}=\frac{.8-1}{1}=-0.2000$

c. $\frac{f(4.5)-f(4)}{.5}=\frac{.889-1}{.5}=-0.2222$

d. $\frac{f(4.1)-f(4)}{.1}=\frac{.976-1}{.1}=-0.2439$

e. $\frac{f(4.01)-f(4)}{.01}=\frac{.9975-1}{.01}=-0.2494$

f. Answers seem to approach –0.25.

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17. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$

$=\lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$

$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$

$=\lim_{h\to 0}2x+h+1=2x+1$

Evaluating at $x=1$ gives $2(1)+1=3$, which matches the answer from Exercise 9.

18. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2+2(x+h)-1-(x^2+2x-1)}{h}$

$=\lim_{h\to 0}\frac{x^2+2xh+h^2+2x+2h-1-x^2-2x+1}{h}$

$=\lim_{h\to 0}\frac{2xh+h^2+2h}{h}$

$=\lim_{h\to 0}2x+h+2=2x+2$

Evaluating at $x=2$ gives $2(2)+2=6$, which matches the answer from Exercise 12.

19. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{5(x+h)+1-(5x+1)}{h}$

$=\lim_{h\to 0}\frac{5x+5h+1-5x-1}{h}$

$=\lim_{h\to 0}\frac{5h}{h}$

$=\lim_{h\to 0}5=5$

Evaluating at x=3 gives 5, which matches the answer from Exercise 13.

20. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\frac{4}{(x+h)}-\frac{4}{x}}{h}$

$=\lim_{h\to 0}\frac{1}{h}\left(\frac{4}{x+h}-\frac{4}{x}\right)$

$=\lim_{h\to 0}\frac{1}{h}\left(\frac{4x-4(x+h)}{(x+h)x}\right)$

$=\lim_{h\to 0}\frac{4x-4x-4h}{h(x^2+xh)}$

$=\lim_{h\to 0}\frac{-4}{x^2+xh}=-\frac{4}{x^2}$

Evaluating at x=4 gives $-\frac{4}{4^2}$=-.25, which matches the answer from Exercise 16.

21. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{2(x+h)^2+(x+h)-2-(2x^2+x-2)}{h}$

$=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+x+h-2-2x^2-x+2}{h}$

$=\lim_{h\to 0}\frac{4xh+2h^2+h}{h}$

$=\lim_{h\to 0}4x+2h+1=4x+1$

The slope of the tangent line is at $x=2$ is $4(2)+1=9$, which matches the answer from Exercise 11.

22. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2(x+h)^2+5-(2x^2+5)}{h}$

$=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+5-2x^2-5}{h}$

$=\lim_{h\to 0}\frac{4xh+2h^2}{h}$

$=\lim_{h\to 0}4x+2h=4x$

The slope of the tangent line at $x=1$ is $4(1)=4$, which matches the answer from Exercise 10.

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23. $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$=\lim\limits_{h\to 0}\left(\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$

$=\lim\limits_{h\to 0}\dfrac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$

$=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{x+h}+\sqrt{x}}=\dfrac{1}{\sqrt{x+0}+\sqrt{x}}=\dfrac{1}{2\sqrt{x}}$

The slope of the tangent line at $x=4$ is $\dfrac{1}{2\sqrt{4}}=0.25$, which matches the answer from Exercise 15.

24. $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{7(x+h)-2-(7x-2)}{h}$

$=\lim\limits_{h\to 0}\dfrac{7x+7h-2-7x+2}{h}$

$=\lim\limits_{h\to 0}\dfrac{7h}{h}$

$=\lim\limits_{h\to 0}7=7$

The slope of the tangent line at $x=3$ is 7, which matches the answer from Exercise 14.

25. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{(x+h)^2-3(x+h)+5-(x^2-3x+5)}{h}$

$=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2-3x-3h+5-x^2+3x-5}{h}$

$=\lim\limits_{h\to 0}(2x+h-3)$

$=2x-3$

26. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{2(x+h)^2-5(x+h)+1-(2x^2-5x+1)}{h}$

$=\lim\limits_{h\to 0}\dfrac{2x^2+4xh+2h^2-5x-5h+1-2x^2+5x-1}{h}$

$=\lim\limits_{h\to 0}(4x+2h-5)$

$=4x-5$

27. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{1-(x+h)^2-\left(1-x^2\right)}{h}$

$=\lim\limits_{h\to 0}\dfrac{1-x^2-2xh-h^2-1+x^2}{h}$

$=\lim\limits_{h\to 0}\left(-2x-h\right)$

$=-2x$

28. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\frac{1}{2}(x+h)^2+1-\left(\frac{1}{2}x^2+1\right)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2+1-\frac{1}{2}x^2-1}{h}$

$=\lim\limits_{h\to 0}x+\frac{1}{2}h=x$

29. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{9(x+h)-2-(9x-2)}{h}$

$=\lim\limits_{h\to 0}\dfrac{9x+9h-2-9x+2}{h}=9$

30. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{-3(x+h)+5-(-3x+5)}{h}$

$=\lim\limits_{h\to 0}\dfrac{-3x-3h+5+3x-5}{h}=-3$

31. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\frac{x+h}{2}-\frac{x}{2}}{h}=\lim\limits_{h\to 0}\dfrac{\frac{h}{2}}{h}$

$=\lim\limits_{h\to 0}\dfrac{h}{2h}=\dfrac{1}{2}$

32. $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{0.01(x+h)+0.05-(0.01x+0.05)}{h}$

$=\lim\limits_{h\to 0}\dfrac{0.01x+0.01h+0.05-0.01x-0.05}{h}$

$=\lim\limits_{h\to 0}\dfrac{0.01h}{h}=0.01$

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33. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{4-4}{h}=0$

34. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\pi-\pi}{h}=0$

35.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}\\&=\lim_{h\to 0}\frac{ax^2+2axh+ah^2+bx+bh+c-ax^2-bx-c}{h}\\&=\lim_{h\to 0}(2ax+ah+b)=2ax+b\end{aligned}$$

36. Use $(x+a)^2=x^2+2ax+a^2$.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+2a(x+h)+a^2-(x^2+2ax+a^2)}{h}\\&=\lim_{h\to 0}\frac{x^2+2xh+h^2+2ax+2ah+a^2-x^2-2ax-a^2}{h}\\&=\lim_{h\to 0}(2x+h+2a)\\&=2x+2a\end{aligned}$$

37.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{(x+h)^5-x^5}{h}\\&=\lim_{h\to 0}\frac{x^5+5x^4h+10x^3h^2+5xh^4+h^5-x^5}{h}\\&=\lim_{h\to 0}5x^4+10x^3h+5xh^3+h^4=5x^4\end{aligned}$$

38.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}\\&=\lim_{h\to 0}\frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}\\&=\lim_{h\to 0}4x^3+6x^2h+4xh^2+h^3=4x^3\end{aligned}$$

39.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{\frac{2}{x+h}-\frac{2}{x}}{h}\\&=\lim_{h\to 0}\frac{\frac{2x}{x(x+h)}-\frac{2(x+h)}{x(x+h)}}{h}\\&=\lim_{h\to 0}\frac{2x-2x-2h}{x(x+h)}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2h}{x(x+h)}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2}{x(x+h)}\\&=-\frac{2}{x^2}\end{aligned}$$

40.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\&=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\&=\lim_{h\to 0}\frac{\frac{x^2}{x^2(x+h)^2}-\frac{(x+h)^2}{x^2(x+h)^2}}{h}\\&=\lim_{h\to 0}\frac{\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}}{h}\\&=\lim_{h\to 0}-\frac{2xh+h^2}{x^2(x+h)^2}\cdot\frac{1}{h}\\&=\lim_{h\to 0}-\frac{2x+h}{x^2(x+h)^2}=-\frac{2x}{x^4}=-\frac{2}{x^3}\end{aligned}$$

41.
$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\\&=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\\&=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\&=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\&=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}\end{aligned}$$

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42. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x-(x+h)}{\left(\sqrt{x}\sqrt{x+h}\right)\left(\sqrt{x}+\sqrt{x+h}\right)}}{h}=\lim_{h\to 0}\frac{-h}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=-\frac{1}{\sqrt{x}\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}=-\frac{1}{x\left(2\sqrt{x}\right)}$$

$$=-\frac{1}{2x\sqrt{x}}$$

43. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$$

$$=\lim_{h\to 0}\frac{(x^3+3x^2h+3xh^2+h^3)+(x^2+2xh+h^2)-x^3-x^2}{h}$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+2xh+h^2}{h}=\lim_{h\to 0}3x^2+3xh+h^2+2x+h$$

$$=3x^2+2x$$

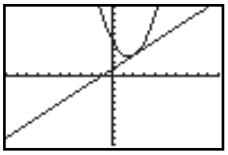
44. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}=\lim_{h\to 0}\frac{\frac{1}{2x+2h}-\frac{1}{2x}}{h}=\lim_{h\to 0}\frac{\frac{2x-(2x+2h)}{2x(2x+2h)}}{h}=\lim_{h\to 0}\frac{\frac{-2h}{2x(2x+2h)}}{h}$$

$$=\lim_{h\to 0}\frac{-h}{x(2x+2h)}\cdot\frac{1}{h}=\lim_{h\to 0}-\frac{1}{x(2x+2h)}=-\frac{1}{2x^2}$$

45. **a.** The slope of the tangent line at $x=2$ is $f'(2)=2(2)-3=1$. To find the point of the curve at $x=2$, we calculate $y=f(2)=2^2-3(2)+5=3$. Using the point-slope form with the point (2, 3), we have

$$y-3=1(x-2)$$
$$y-3=x-2$$
$$y=x+1$$

b.



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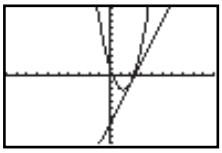
46. **a.** The slope of the tangent line at $x=2$ is $f'(2)=4(2)-5=3$. To find the point of the curve at $x=2$, we calculate $y=f(2)=2(2)^2-5(2)+1=-1$. Using the point-slope form with the point (2, –1), we have

$$y-(-1)=3(x-2)$$
$$y+1=3x-6$$
$$y=3x-7$$

b.



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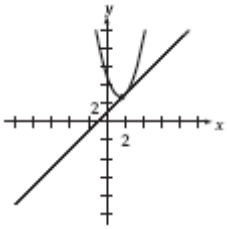
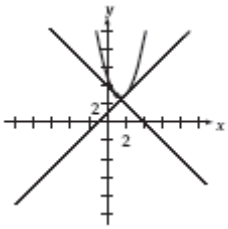
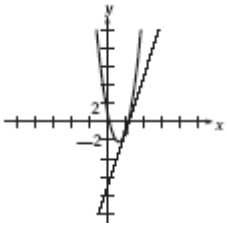
47. **a.**



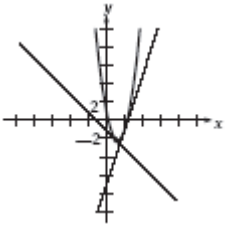
b.



48. **a.**



b.



49. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)-4-(3x-4)}{h} = \lim_{h\to 0} \frac{3x+3h-4-3x+4}{h} = \lim_{h\to 0} 3 = 3$

b. The graph of $f(x) = 3x - 4$ is a straight line with slope 3.

50. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)-9-(2x-9)}{h} = \lim_{h\to 0} \frac{2x+2h-9-2x+9}{h} = \lim_{h\to 0} 2 = 2$

b. The graph of $f(x) = 2x - 9$ is a straight line with slope 2.

51. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{5-5}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = 5$ is a straight line with slope 0.

52. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{12-12}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = 12$ is a straight line with slope 0.

53. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{m(x+h)+b-(mx+b)}{h} = \lim_{h\to 0} \frac{mx+mh+b-mx-b}{h} = \lim_{h\to 0} \frac{mh}{h}$
$= \lim_{h\to 0} m = m$

b. The graph of $f(x) = mx + b$ is a straight line with slope m.

54. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{b-b}{h} = \lim_{h\to 0} 0 = 0$

b. The graph of $f(x) = b$ is a straight line with slope 0.

55. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 8(x+h)+110-(x^2-8x+110)}{h}$
$= \lim_{h\to 0} \frac{x^2+2hx+h^2-8x-8h+110-x^2+8x-110}{h}$
$= \lim_{h\to 0} 2x+h-8 = 2x-8$

b. $f'(2) = 2(2)-8 = -4$. The temperature is decreasing at a rate of 4 degrees per minute after 2 minutes.

c. $f'(5) = 2(5)-8 = 2$. The temperature is increasing at a rate of 2 degrees per minute after 5 minutes.

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56. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 12(x+h) + 200 - (3x^2 - 12x + 200)}{h}$

$= \lim_{h\to 0} \frac{3x^2 + 6hx + 3h^2 - 12x - 12h + 200 - 3x^2 + 12x - 200}{h}$

$= \lim_{h\to 0} 6x + 3h - 12 = 6x - 12$

b. $f'(1) = 6(1) - 12 = -6$. The population is decreasing by 6 people per week.

c. $f'(5) = 6(5) - 12 = 18$. The population is increasing by 18 people per week.

57. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$

$= \lim_{h\to 0} \frac{2x^2 + 4hx + 2h^2 - x - h - 2x^2 + x}{h} = \lim_{h\to 0} 4x + 2h - 1 = 4x - 1$

b. $f'(5) = 4(5) - 1 = 19$. When 5 words have been memorized, the memorization time is increasing at a rate of 19 seconds per word.

58. **a.** $S'(x) = \lim_{h\to 0} \frac{S(x+h)-S(x)}{h} = \lim_{h\to 0} \frac{-(x+h)^2 + 10(x+h) - (-x^2 + 10x)}{h}$

$= \lim_{h\to 0} \frac{-x^2 - 2hx - h^2 + 10x + 10h + x^2 - 10x}{h}$

$= \lim_{h\to 0} -2x - h + 10 = -2x + 10$

b. $S'(3) = -2(3) + 10 = 4$. The number of cars sold on the third day of the advertising campaign is increasing at a rate of 4 cars per day.

c. $S'(6) = -2(6) + 10 = -2$. The number of cars sold on the sixth day of the advertising campaign is decreasing at a rate of 2 cars per day.

59. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2}(x+h)^2 - 3.7(x+h) + 12 - \left(\frac{1}{2}x^2 - 3.7x + 12\right)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 3.7x - 3.7h + 12 - \left(\frac{1}{2}x^2 - 3.7x + 12\right)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - 3.7x - 3.7h + 12 - \frac{1}{2}x^2 + 3.7x - 12}{h}$

$= \lim_{h\to 0} \left(x + \frac{1}{2}h - 3.7\right) = x - 3.7$

b. $f'(1) = 1 - 3.7 = -2.7$. In 1940, the percentage of immigrants was decreasing by 2.7 percentage points per decade (which is about 0.27 of a percentage point decrease per year).

c. $f'(8) = 8 - 3.7 = 4.3$. Increasing by 4.3 percentage points per decade (so about 0.43 of a percentage point per year).

60. **a.** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{6(x+h)^2 - 16(x+h) + 109 - \left(6x^2 - 16x + 109\right)}{h}$

$= \lim_{h\to 0} \frac{6x^2 + 12xh + 6h^2 - 16x - 16h + 109 - \left(6x^2 - 16x + 109\right)}{h}$

$= \lim_{h\to 0} \frac{6x^2 + 12xh + 6h^2 - 16x - 16h + 109 - 6x^2 + 16x - 109}{h}$

$= \lim_{h\to 0} \frac{(12x - 6h - 16)h}{h}$

$= 12x - 16$

b. $f'(0) = 12(0) - 16 = -16$. In 2010, net income was decreasing at the rate of \$16 million per year.

c. $f'(2) = 12(2) - 16 = 8$. In 2012, net income was increasing at the rate of \$8 million per year.

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61. The average rate of change requires two x-values and is the change in the function - values divided by the change in the x-values. The instantaneous rate of change is at a single x-value, and is found from the formula $f(x) = \lim_{x\to\infty} \frac{f(x+h)-f(x)}{h}$.

62. A secant line crosses the function twice while a tangent line crosses only once. To find the slope of a secant line, use the formula for average rate of change. For the slope of a tangent line, use the formula for instantaneous rate of change.

63. Substitution $h = 0$ would make the denominator zero, and we can't divide by zero. That's why we need to do some algebra on the difference quotient, to cancel out the terms that are zero so that afterwards we can evaluate by direct substitution.

64. The units of the derivative $f'(x)$ is in widgets per blivet because the derivative is equivalent to the slope $f(x)$ which is the change in f over the change in x.

65. The units of x are blargs and the units of f are prendles because the derivative $f'(x)$ is equivalent to the slope of $f(x)$, which is the change in f over the change in x.

66. The population is decreasing because the negative derivative implies a negative slope.

67. The patient's health is deteriorating during the first day (temperature is rising above normal). The patient's health is improving during the second day (temperature is falling).

68. The temperature at 6 a.m. is the lowest temperature throughout the first half of the day because the temperature falls until 6 a.m. and rises after 6 a.m.

EXERCISES 2.3

1. $f'(x) = \frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$

2. $f'(x) = \frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$

3. $f'(x) = \frac{d}{dx}(x^{500}) = 500x^{500-1} = 500x^{499}$

4. $f'(x) = \frac{d}{dx}(x^{1000}) = 1000x^{1000-1} = 1000x^{999}$

5. $f'(x) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

6. $f'(x) = \frac{d}{dx}(x^{1/3}) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$

7. $g'(x) = \frac{d}{dx}\left(\frac{1}{4}x^4\right) = \frac{1}{2}\cdot 4x^{4-1} = 2x^3$

8. $g'(x) = \frac{d}{dx}\left(\frac{1}{3}x^9\right) = \frac{1}{3}\cdot 9x^{9-1} = 3x^8$

9. $g'(w) = \frac{d}{dw}(6w^{1/3}) = 6\cdot\frac{1}{3}w^{1/3-1} = 2w^{-2/3}$

10. $g'(w) = \frac{d}{dw}(12w^{1/2}) = 12\cdot\frac{1}{2}w^{1/2-1} = 6w^{-1/2}$

11. $h'(x) = \frac{d}{dx}(3x^{-2}) = 3(-2)x^{-2-1} = -6x^{-3}$

12. $h'(x) = \frac{d}{dx}(4x^{-3}) = 4(-3)x^{-3-1} = -12x^{-4}$

13. $f'(x) = \frac{d}{dx}(4x^2 - 3x + 2)$
$= 4\cdot 2x^{2-1} - 3\cdot 1x^{1-1} + 0$
$= 8x - 3$

14. $f'(x) = \frac{d}{dx}(3x^2 - 5x + 4)$
$= 3\cdot 2x^{2-1} - 5\cdot 1x^{1-1} + 0$
$= 6x - 5$

15. $f'(x) = \frac{d}{dx}\left(\frac{1}{x^{1/2}}\right) = \frac{d}{dx}\left(x^{-1/2}\right)$
$= -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$

16. $f'(x) = \frac{d}{dx}\left(\frac{1}{x^{2/3}}\right) = \frac{d}{dx}\left(x^{-2/3}\right)$
$= -\frac{2}{3}x^{-5/3} = -\frac{2}{3x^{5/3}}$

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17. $f'(x)=\frac{d}{dx}\left(\frac{6}{\sqrt[3]{x}}\right)=\frac{d}{dx}\left(6x^{-1/3}\right)$
$=6\left(-\frac{1}{3}\right)x^{-4/3}=-\frac{2}{x^{4/3}}$

18. $f'(x)=\frac{d}{dx}\left(\frac{4}{\sqrt{x}}\right)=\frac{d}{dx}\left(4x^{-1/2}\right)$
$=4\left(-\frac{1}{2}\right)x^{-3/2}=-\frac{2}{x^{3/2}}$

19. $f'(r)=\frac{d}{dr}\left(\pi r^2\right)=\pi(2r)=2\pi r$

20. $f'(r)=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi(3r^2)=4\pi r^2$

21. $f'(x)=\frac{d}{dx}\left(\frac{1}{6}x^3+\frac{1}{2}x^2+x+1\right)$
$=\frac{1}{6}(3x^2)+\frac{1}{2}(2x)+1$
$=\frac{1}{2}x^2+x+1$

22. $f'(x)=\frac{d}{dx}\left(\frac{1}{24}x^4+\frac{1}{6}x^3+\frac{1}{2}x^2+x+1\right)$
$=\frac{1}{24}(4x^3)+\frac{1}{6}(3x^2)+\frac{1}{2}(2x)+1$
$=\frac{1}{6}x^3+\frac{1}{2}x^2+x+1$

23. $g'(x)=\frac{d}{dx}(x^{1/2}-x^{-1})=\frac{1}{2}x^{1/2-1}-(-1)x^{-1-1}$
$=\frac{1}{2}x^{-1/2}+x^{-2}$

24. $g'(x)=\frac{d}{dx}(x^{1/3}-x^{-1})=\frac{1}{3}x^{1/3-1}-(-1)x^{-1-1}$
$=\frac{1}{3}x^{-2/3}+x^{-2}$

25. $h'(x)=\frac{d}{dx}\left(6x^{2/3}-12x^{-1/3}\right)=6\cdot\frac{3}{2}x^{2/3-1}-12\left(-\frac{1}{3}\right)x^{-1/3-1}$
$=4x^{-1/3}+4x^{-4/3}$

26. $h'(x)=\frac{d}{dx}\left(8x^{3/2}-8x^{-1/4}\right)=8\cdot\frac{3}{2}x^{3/2-1}-8\left(-\frac{1}{4}\right)x^{-1/4-1}$
$=12x^{1/2}+2x^{-5/4}$

27. $f'(x)=\frac{d}{dx}\left(10x^{-1/2}-\frac{9}{5}x^{5/3}+17\right)=10\left(-\frac{1}{2}\right)x^{-1/2-1}-\frac{9}{5}\left(\frac{5}{3}\right)x^{5/3-1}+0$
$=-5x^{-3/2}-3x^{2/3}$

28. $f'(x)=\frac{d}{dx}\left(\frac{9}{2}x^{-2/3}-16x^{5/2}-14\right)=\frac{9}{2}\left(-\frac{2}{3}\right)x^{-2/3-1}-16\left(\frac{5}{2}\right)x^{5/2-1}-0$
$=-3x^{-5/3}-40x^{3/2}$

29. $f'(x)=\frac{d}{dx}\left(\frac{x^2+x^3}{x}\right)=\frac{d}{dx}(x+x^2)=1+2x$

30. $f'(x)=\frac{d}{dx}\left(x^2(x+1)\right)=\frac{d}{dx}(x^3+x^2)=3x^2+2x$

31. $f'(x)=\frac{d}{dx}(x^5)=5x^4;\ f'(-2)=5(-2)^4=80$

32. $f'(x)=\frac{d}{dx}(x^4)=4x^3;\ f'(-3)=4(-3)^3=-108$

33. $f'(x)=\frac{d}{dx}(6x^{2/3}-48x^{-1/3})=6\left(\frac{2}{3}\right)x^{-1/3}-48\left(-\frac{1}{3}\right)x^{-4/3}$
$=4x^{-1/3}+16x^{-4/3}$
$f'(8)=4(8)^{-1/3}+16(8)^{-4/3}=\frac{4}{\sqrt[3]{8}}+\frac{16}{\sqrt[3]{8^4}}=\frac{4}{2}+\frac{16}{\sqrt[3]{8^3}\cdot\sqrt[3]{8}}=2+\frac{16}{8\cdot 2}=3$

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34. $f'(x) = \frac{d}{dx}\left(12x^{2/3} + 48x^{-1/3}\right) = 12\left(\frac{2}{3}\right)x^{-1/3} + 48\left(-\frac{1}{3}\right)x^{-4/3}$

$= 4x^{-1/3} + 16x^{-4/3}$

$f'(8) = 4(8)^{-1/3} + 16(8)^{-4/3} = \frac{4}{\sqrt[3]{8}} + \frac{16}{\sqrt[3]{8^4}} = \frac{4}{2} + \frac{16}{\sqrt[3]{8^3}\cdot\sqrt[3]{8}} = 2 + \frac{16}{8\cdot 2} = 3$

35. $\frac{df}{dx} = \frac{d}{dx}(x^3) = 3x^2; \left.\frac{df}{dx}\right|_{x=-3} = 3(-3)^2 = 27$

36. $\frac{df}{dx} = \frac{d}{dx}(x^4) = 4x^3; \left.\frac{df}{dx}\right|_{x=-2} 4(-2)^3 = -32$

37. $\frac{df}{dx} = \frac{d}{dx}\left(16x^{-1/2} + 8x^{1/2}\right) = 16\left(-\frac{1}{2}\right)x^{-3/2} + 8\left(\frac{1}{2}\right)x^{-1/2}$

$= 8x^{-3/2} + 4x^{-1/2}$

$\left.\frac{df}{dx}\right|_{x=4} = -8(4)^{-3/2} + 4(4)^{-1/2} = -\frac{8}{\sqrt{4^3}} + \frac{4}{\sqrt{4}} = -\frac{8}{8} + \frac{4}{2} = 1$

38. $\frac{df}{dx} = \frac{d}{dx}\left(54x^{-1/2} + 12x^{1/2}\right) = 54\left(-\frac{1}{2}\right)x^{-3/2} + 12\left(\frac{1}{2}\right)x^{-1/2}$

$= -27x^{-3/2} + 6x^{-1/2}$

$\left.\frac{df}{dx}\right|_{x=9} = -27(9)^{-3/2} + 6(9)^{-1/2} = -\frac{27}{\sqrt{9^3}} + \frac{6}{\sqrt{9}} = -\frac{27}{27} + \frac{6}{3} = 1$

39. **a.** $f'(x) = 2\cdot x^1 - 2 + 0 = 2x + 2$

$f'(3) = 2(3) - 2 = 4$

The slope of the tangent line is 4.

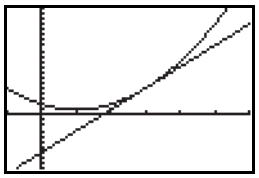
$y - 5 = 4(x - 3)$

$y - 5 = 4x - 12$

$y = 4x - 7$

The equation for the tangent line is $y = 4x - 7$.

b.



on [–1, 6] by [–10, 20]

40. **a.** $f'(x) = 2\cdot x^1 - 4 + 0 = 2x - 4$

$f'(1) = 2(1) - 4 = -2$

The slope of the tangent line is –2.

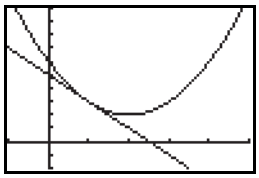
$y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$y = -2x + 5$

The equation for the tangent line is $y = -2x + 5$.

b.



on [–1, 5] by [–2, 10]

41. **a.** $f'(x) = 3\cdot x^2 - 3(2)x^1 + 2 + 0 = 3x^2 - 6x + 2$

$f'(2) = 3(2)^2 - 6(2) + 2 = 2$

The slope of the tangent line is 2.

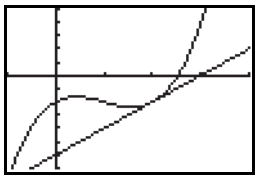
$y + 2 = 2(x - 2)$

$y + 2 = 2x - 4$

$y = 2x - 6$

The equation for the tangent line is $y = 2x - 6$.

b.



on [–1, 4] by [–7, 5]

42. **a.** $f'(x) = 3(2)x^1 - 3\cdot x^2 = 5x - 3x^2$

$f'(1) = 6(1) - 3(1)^2 = 3$

The slope of the tangent line is 3.

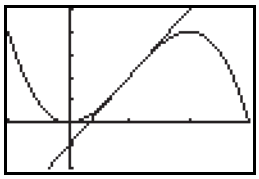
$y - 2 = 3(x - 1)$

$y - 2 = 3x - 3$

$y = 3x - 1$

The equation for the tangent line is $y = 3x - 1$.

b.



on [–1, 3] by [–2, 5]

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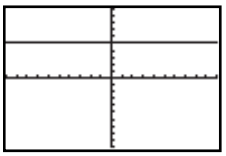
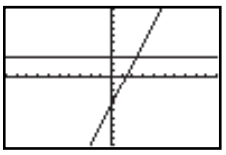
43. For $y_1 = 5$ and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:



44. For $y_1 = 3x - 4$ and viewing rectangle [–10, 10] by [–10, 10], your graph should look roughly like the following:



45. $y' = 9.638(-1.394)x^{-2.394} = -13.435372x^{-2.394}$

a. $y'(2) = -13.435372(2)^{-2.394} \approx -2.6$

b. $y'(5) = -13.435372(5)^{-2.394} \approx -0.29$

Interpretation: Near full employment, inflation greatly decreases with small increases in unemployment but with higher unemployment, inflation hardly decreases with increases in unemployment.

46. $y' = 45.4(-1.54)x^{-2.54} = -69.916x^{-2.54}$

a. $y'(3) = -69.916(3)^{-2.54} \approx -4.3$

b. $y'(8) = -69.916(8)^{-2.54} \approx -0.36$

Interpretation: Near full employment, inflation greatly decreases with small increases in unemployment but with higher unemployment, inflation hardly decreases with increases in unemployment.

47. **a.** $C'(x) = \dfrac{16}{\sqrt[3]{x}}$

$C'(8) = \dfrac{16}{\sqrt[3]{8}} = \dfrac{16}{2} = 8$

When 8 items are purchased, the cost of the last item is about \$8.

b. $C'(64) = \dfrac{16}{\sqrt[3]{64}} = \dfrac{16}{4} = 4$

When 8 items are purchased, the cost of the last item is about \$4.

48. **a.** $C'(x) = 140x^{-1/6}$

$C'(1) = \dfrac{140}{\sqrt[6]{1}} = 140$

When 1 license is purchased, the cost is about \$140.

b. $C'(64) = \dfrac{140}{\sqrt[6]{64}} = \dfrac{140}{2} = 70$

When 64 licenses are purchased, the cost is about \$70.

49. $C(64) - C(63) = 24(64)^{2/3} - 24(63)^{2/3} \approx 4.01$

The answer is close to \$4.

50. $C(64) - C(63) = 168(64)^{5/6} - 168(63)^{5/6} \approx 70.09$

The answer is close to \$70.

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51. **a.** The rate of change of the population in x years is the derivative of the population function

$P'(x) = -x^2 + 5x - 3$

To find the rate of change of the population 20 years from now, evaluate $P'(x)$ for $x = 2$.

$P'(2) = -2^2 + 5(2) - 3 = 3$

In 2030, this population group will be increasing by 3 million per decade.

b. To find the rate of change of the population 0 years from now, evaluate $P'(x)$ for $x = 0$.

$P'(0) = -0^2 + 5(0) - 3 = -3$

In 2010, this population group will be decreasing by 3 million per decade.

52. **a.** The rate of change of the number of newly infected people is the derivative of the function

$f(t) = 13t^2 - t^3$

$f'(t) = 2 \cdot 13t - 3t^2 = 26t - 3t^2$

The rate of change on day 5 is found by evaluating $f'(t)$ for $t = 5$.

$f'(5) = 26(5) - 3(5)^2 = 55$

The number of newly infected people on day 5 is increasing by about 55 people per day.

b. The rate of change on day 10 is found by evaluating $f'(t)$ for $t = 10$.

$f'(10) = 26(10) - 3(10)^2 = -40$

The number of newly infected people on day 10 is decreasing by about 40 people per day.

53. The rate of change of the pool of potential customers is the derivative of the function

$N(x) = 400,000 - \frac{200,000}{x}$.

$N'(x) = \frac{d}{dx}(400,000 - 200,000x^{-1})$

$N'(x) = 0 - (-1)200,000x^{-2} = \frac{200,000}{x^2}$

To find the rate of change of the pool of potential customers when the ad has run for 5 days, evaluate $N'(x)$ for $x = 5$.

$N'(5) = \frac{200,000}{5^2} = 8000$

The pool of potential customers is increasing by about 8000 people per additional day.

54. $A(t) = 0.01t^2 \quad 1 \le t \le 5$

The instantaneous rate of change of the cross-sectional area t hours after administration of nitroglycerin is given by

$A'(t) = 2(0.01)t^{2-1} = 0.02t$

$A'(4) = 0.02(4) = 0.08$

After 4 hours the cross-sectional area is increasing by about 0.08 cm^2 per hour.

55. The rate of change of 3D movies is the derivative of the function $f(x) = \frac{1}{2}x^2 + 11x + 8$.

$f'(x) = x + 11$

The rate of change of number of 3D movies in 10 years is found by evaluating $f'(x)$ at $x = 10$.

$f'(10) = 10 + 11 = 21$

In 2020, the number of 3D movies will be increasing by 21 per year.

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56. **a.** The rate of change of the function is the derivative of the function:

$f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$.

Find the rate of change for $x = 4$.

$f'(4) = -\frac{1}{2\sqrt{4^3}} = -\frac{1}{16}$

In 2005, the number of fatalities is decreasing by $\frac{1}{16}$ per hundred million vehicle miles traveled every five years.

b. Find the rate of change for $x = 4$.

$f'(9) = -\frac{1}{2\sqrt{9^3}} = -\frac{1}{54}$

In 2030, the number of fatalities is decreasing by $\frac{1}{54}$ per hundred million vehicle miles traveled every five years.

57. The instantaneous rate of change of the number of phrases students can memorize is the derivative of the function $p(t) = 24\sqrt{t}$.

$p'(t) = \frac{d}{dt}(24t^{1/2})$

$= \frac{1}{2}(24t^{-1/2}) = 12t^{-1/2}$

$p'(4) = 12(4)^{-1/2} = \frac{12}{\sqrt{4}} = 6$

The number of phrases students can memorize after 4 hours is increasing by 6.

58. **a.** The instantaneous rate of change of the amount of dissolved oxygen x miles downstream is the derivative of the function $D(x) = 0.2x^2 - 2x + 10$.

$D'(x) = 2(0.2)x - 2 + 0 = 0.4x - 2$

$D'(1) = 0.4(1) - 2 = -1.6$

The amount of dissolved oxygen 1 mile downstream is decreasing by about 1.6 mpl per mile.

b. $D'(10) = 0.4(10) - 2 = 2$

The amount of dissolved oxygen 10 miles downstream is increasing by about 2 mpl per mile.

59. **a.** $U(x) = 100\sqrt{x} = 100x^{1/2}$

$MU(x) = U'(x) = \frac{1}{2}(100)x^{1/2-1} = 50x^{-1/2}$

b. $MU(1) = U'(1) = 50(1)^{-1/2} = 50$

The marginal utility of the first dollar is 50.

c. $MU(1{,}000{,}000) = U'(1{,}000{,}000) = 50(10^6)^{-1/2} = 50(10)^{-3} = \frac{50}{1000} = 0.05$

The marginal utility of the millionth dollar is 0.05.

60. **a.** $U(x) = 12\sqrt[3]{x} = 12x^{1/3}$

$MU(x) = U'(x) = \frac{1}{3}(12x^{-2/3}) = 4x^{-2/3}$

b. $MU(1) = U'(1) = 4(1)^{-2/3} = 4$

The marginal utility of the first dollar is 4.

c. $MU(1{,}000{,}000) = U'(1{,}000{,}000) = 4(1{,}000{,}000)^{-2/3} = \frac{4}{(\sqrt[3]{1{,}000{,}000})^2} = \frac{4}{100^2} = 0.0004$

The marginal utility of the millionth dollar is 0.0004.

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61. **a.** $f(12) = 0.831(12)^2 - 18.1(12) + 137.3 = 39.764$

A smoker who is a high school graduate has a 39.8% chance of quitting.

$f'(x) = 0.831(2)x - 18.1 = 1.662x - 18.1$

$f'(12) = 1.662(12) - 18.1 = 1.844$

When a smoker has a high school diploma, the chance of quitting is increasing at the rate of 1.8% per year of education.

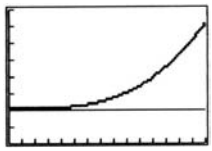
b. $f(16) = 0.831(16)^2 - 18.1(16) + 137.3 = 60.436$

A smoker who is a college graduate has a 60.4% chance of quitting.

$f'(16) = 1.662(16) - 18.1 = 8.492$

When a smoker has a college degree, the chance of quitting is increasing at the rate of 8.5% per year of education.

62. **a.**



on [0, 15] by [–10, 30]

b. $N(10) = 0.00437(10)^{3.2} \approx 6.9$

A group with 10 years of exposure to asbestos will have 7 cases of lung cancer.

$N'(t) = 0.00437(3.2)t^{3.2-1} = 0.013984t^{2.2}$

$N'(10) = 0.013984(10)^{2.2} \approx 2.2$

When a group has 10 years of exposure to asbestos, the number of cases of lung cancer is increasing at the rate of 2.2 cases per year.

63. **a.** $f(5) = 650(5)^2 + 3000(5) + 12{,}000$
$= \$43{,}250$

b. $f'(x) = 1300x + 3000$

$f'(5) = 1300(5) + 3000 = \$9500$

In 2020-2021, private college tuition will be increasing by \$9500 every 5 years.

c. \$1900

64. **a.** $f(5) = 400(5)^2 + 500(5) + 2700$
$= \$15{,}200$

b. $f'(x) = 800x + 500$

$f'(5) = 800(5) + 500 = \$4500$

In 2020-2021, public college tuition will be increasing by \$4500 every 5 years.

c. \$900

65. $f(x) = 2$ will have a graph that is a horizontal line at height 2, and the slope (the derivative) of a horizontal line is zero. A function that stays constant ill have a rate of change of zero, so its derivative (instantaneous rate of change) will be zero.

66. $f(x) = 3x - 5$ is a line with a slope (derivative) of 3. A function that has a slope of 3 will have a rate of change of 3, so its derivative (instantaneous rate of change) will be three.

67. If *f* has a particular rate of change, then the rate of change of $2 \cdot f(x)$ will be twice as large, and the rate of change of $c \cdot f(x)$ will be $c \cdot f'(x)$, which is just the constant multiple rule.

68. If *f* and *g* have particular rates of change, then the rate of change of $f(x) + g(x)$ would be the rate of change of $f(x)$ plus the rate of change of $g(x)$. For example, if $f(x) = 2x$ and $g(x) = 3x$, then the rate of change of $2x + 3x = 5x$ is the same as the rate of change of 2*x* plus the rate of change of 3*x*. The derivative of $f(x) + g(x)$ will be $f'(x) + g'(x)$, which is just the sum rate.

69. Since –*f* slopes down by the same amount that *f* slopes up, the slope of –*f* should be the negative of the slope of *f*. The constant multiple rule with $c = -1$ also says that the slope of –*f* will be the negative of the slope of *f*.

70. The slopes of the *f* and $f + 10$ will be the same because $f + 10$ is just *f* raised 10 units. Using the sum rule with $g(x) = 10$, also says that the slope will just be *f* because $g'(x) = 0$.

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71. Evaluating first would give a constant and the derivative of a constant is zero, so evaluating and the differentiating would always give zero, regardless of the function and number. This supports the idea that we should always differentiate and then evaluate to obtain anything meaningful.

72. To find a function that is positive but does not have a positive slope at a particular x-value, we need to find an equation with a negative slope. $y=-5x$ will work at $x=-1$.

73. Each additional year of education increases life expectancy by 1.7 years.

74. The probability of an accident increases by 13% as the speed exceeds the limit.

75. **a.** If $D(c)=0$, then $f(c)-L(c)=0$ so $f(c)=L(c)$.

b. Since the line $L(x)$ must pass through the point $(c, L(c))$, part (a) shows that this point is $(c, f(c))$. Then the point-slope form $y-y_1=m(x-x_1)$ with $x_1=c$ and $y_1=f(c)$ becomes

$$y-f(c)=m(x-c)$$

or $y=m(x-c)+f(c)$, giving the line

$$L(x)=m(x-c)+f(c)$$

c. $D(x)=f(x)-L(x)=f(x)-(m(x-c)+f(c))=f(x)-m(x-c)-f(c)$

d. Differentiating the last expression for $D(x)$ above gives

$$D'(x)=f'(x)-m$$

which at $x=c$ becomes $D'(c)=f'(c)-m$, and setting this equal to zero gives $m=f'(c)$.

e. $L(x)=f'(c)(x-c)+f(c)$ so $y=L(x)$ is the same as $y-f(c)=f'(c)(x-c)$, which is the point-slope form of the equation of the tangent line to the curve $y=f(x)$ at $x=c$.

f. The tangent line has zero error at the point and errors near the point that differ as little as possible from zero. So the tangent line is the best linear approximation to the function since it has the smallest possible differences between a line and the curve at and near the point.

EXERCISES 2.4

1. **a.** Using the product rule:

$$\frac{d}{dx}(x^4\cdot x^6)=4x^3\cdot x^6+x^4(6x^5)=4x^9+6x^9=10x^9$$

b. Using the power rule:

$$\frac{d}{dx}(x^4\cdot x^6)=\frac{d}{dx}(x^{10})=10x^9$$

2. **a.** Using the product rule:

$$\frac{d}{dx}(x^7\cdot x^2)=7x^6\cdot x^2+x^7\cdot 2x=7x^8+2x^8=9x^8$$

b. Using the power rule:

$$\frac{d}{dx}(x^7\cdot x^2)=\frac{d}{dx}(x^9)=9x^8$$

3. **a.** Using the product rule:

$$\frac{d}{dx}[x^4(x^5+1)]=4x^3(x^5+1)+x^4(5x^4)=4x^8+4x^3+5x^8=9x^8+4x^3$$

b. Using the power rule:

$$\frac{d}{dx}[x^4(x^5+1)]=\frac{d}{dx}(x^9+x^4)=9x^8+4x^3$$

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4. **a.** Using the product rule:
$\frac{d}{dx}[x^5(x^4+1)] = 5x^4(x^4+1) + x^5(4x^3) = 5x^8 + 5x^4 + 4x^8 = 9x^8 + 5x^4$

b. Using the power rule:
$\frac{d}{dx}[x^5(x^4+1)] = \frac{d}{dx}(x^9 + x^5) = 9x^8 + 5x^4$

5. $f'(x) = 2x(x^3+1) + x^2(3x^2) = 2x^4 + 2x + 3x^4 = 5x^4 + 2x$

6. $f'(x) = 3x^2(x^2+1) + x^3(2x) = 3x^4 + 3x^2 + 2x^4 = 5x^4 + 3x^2$

7. $f'(x) = 1(5x^2-1) + x(10x) = 5x^2 - 1 + 10x^2 = 15x^2 - 1$

8. $f'(x) = 2(x^4+1) + 2x(4x^3) = 2x^4 + 2 + 8x^4 = 10x^4 + 2$

9. $f'(x) = \frac{1}{2}x^{-1/2}(6x+2) + x^{1/2}(6) = 3x^{1/2} + x^{-1/2} + 6x^{1/2} = 9x^{1/2} + x^{-1/2} = 9\sqrt{x} + \frac{1}{\sqrt{x}}$

10. $f'(x) = 6\left(\frac{1}{3}x^{-2/3}\right)(2x+1) + 6x^{1/3}(2) = 4x^{1/3} + 2x^{-2/3} + 12x^{1/3} = 16x^{1/3} + 2x^{-2/3} = 16\sqrt[3]{x} + \frac{2}{\sqrt[3]{x^2}}$

11. $f'(x) = (2x)(x^2-1) + (x^2+1)(2x) = 2x^3 - 2x + 2x^3 + 2x = 4x^3$

12. $f'(x) = 3x^2(x^3+1) + (x^3-1)(3x^2) = 3x^5 + 3x^2 + 3x^5 - 3x^2 = 6x^5$

13. $f'(x) = (2x+1)(3x+1) + (x^2+x)(3) = 6x^2 + 5x + 1 + 3x^2 + 3x = 9x^2 + 8x + 1$

14. $f'(x) = (2x+2)(2x+1) + (x^2+2x)(2) = 4x^2 + 6x + 2 + 2x^2 + 4x = 6x^2 + 10x + 2$

15. $f'(x) = 2x\left(x^2+3x-1\right) + x^2(2x+3) = 2x^3 + 6x^2 - 2x + 2x^3 + 3x^2 = 4x^3 + 9x^2 - 2x$

16. $f'(x) = 3x^2\left(x^2-4x+3\right) + x^3(2x-4) = 3x^4 - 12x^3 + 9x^2 + 2x^4 - 4x^3 = 5x^4 - 16x^3 + 9x^2$

17. $f'(x) = (4x)(1-x) + (2x^2+1)(1) = 4x - 4x^2 - 2x^2 - 1 = -6x^2 + 4x - 1$

18. $f'(x) = (2x)(1-x^2) + (2x-1)(-2x) = 2 - 2x^2 - 4x^2 + 2x = -6x^2 + 2x + 2$

19. $f'(x) = \left(\frac{1}{2}x^{-1/2}\right)x^{1/2} + 1 + \left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2} + \frac{1}{2}x^{-1/2} + \frac{1}{2} - \frac{1}{2}x^{-1/2} = 1$

20. $f'(x) = \left(\frac{1}{2}x^{-1/2}\right)\left(x^{1/2}-2\right) + \left(x^{1/2}+2\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2} - x^{-1/2} + \frac{1}{2} + x^{-1/2} = 1$

21. $f'(t) = 8t^{1/3}(3t^{2/3}+1) + 6t^{4/3}(2t^{-1/3}) = 24t + 8t^{1/3} + 12t = 36t + 8t^{1/3}$

22. $f'(t) = 6t^{1/2}(2t^{1/2}-1) + 4t^{3/2}(t^{-1/2}) = 12t - 6t^{1/2} + 4t = 16t - 6t^{1/2}$

23. $f'(z) = (4z^3+2z)(z^3-z) + (z^4+z^2+1)(3z^2-1)$
$= 4z^6 - 2z^4 - 2z^2 + 3z^6 + 3z^4 + 3z^2 - z^4 - z^2 - 1$
$= 7z^6 - 1$

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24. $f'(z)=\left(\frac{1}{4}z^{-3/4}+\frac{1}{2}z^{-1/2}\right)(z^{1/4}-z^{1/2})+(z^{1/4}+z^{1/2})\left(\frac{1}{4}z^{-3/4}-\frac{1}{2}z^{-1/2}\right)$

$=\frac{1}{4}z^{-1/2}-\frac{1}{4}z^{-1/4}+\frac{1}{2}z^{-1/4}-\frac{1}{2}+\frac{1}{4}z^{-1/2}-\frac{1}{2}z^{-1/4}+\frac{1}{4}z^{-1/4}-\frac{1}{2}$

$=\frac{1}{4}z^{-1/2}+\frac{1}{4}z^{-1/2}-\frac{1}{2}-\frac{1}{2}=\frac{1}{2}z^{-1/2}-1$

25. $f'(x)=\left(2+4\cdot\frac{1}{2}z^{-1/2}\right)\left(2z^{1/2}+1\right)+\left(2z+4z^{1/2}-1\right)\left(2\cdot\frac{1}{2}z^{-1/2}\right)$

$=4z^{1/2}+2+4+2z^{-1/2}+2z^{1/2}+4-z^{-1/2}$

$=6z^{1/2}+z^{-1/2}+10$

$=6\sqrt{z}+\frac{1}{\sqrt{z}}+10$

26. $f'(x)=\left(1+6\cdot\frac{1}{2}z^{-1/2}\right)\left(z-2z^{1/2}+1\right)+\left(z+6z^{1/2}\right)\left(1-2\cdot\frac{1}{2}z^{-1/2}\right)$

$=z+3z^{1/2}-2z^{1/2}-6+1+3z^{-1/2}+z-z^{1/2}+6z^{1/2}-6$

$=2z+6z^{1/2}+3z^{-1/2}-11$

$=2z+6\sqrt{z}+\frac{3}{\sqrt{z}}-11$

27. **a.** Using the quotient rule:

$\frac{d}{dx}\left(\frac{x^8}{x^2}\right)=\frac{x^2(8x^7)-2x(x^8)}{(x^2)^2}=\frac{8x^9-2x^9}{x^4}=6x^5$

b. Using the power rule:

$\frac{d}{dx}\left(\frac{x^8}{x^2}\right)=\frac{d}{dx}(x^6)=6x^5$

28. **a.** Using the quotient rule:

$\frac{d}{dx}\left(\frac{x^9}{x^3}\right)=\frac{x^3(9x^8)-3x^2(x^9)}{(x^3)^2}=\frac{9x^{11}-3x^{11}}{x^6}=6x^5$

b. Using the power rule:

$\frac{d}{dx}\left(\frac{x^9}{x^3}\right)=\frac{d}{dx}(x^6)=6x^5$

29. **a.** Using the quotient rule:

$\frac{d}{dx}\left(\frac{1}{x^3}\right)=\frac{x^3(0)-3x^2(1)}{(x^3)^2}=-\frac{3x^2}{x^6}=-\frac{3}{x^4}$

b. Using the power rule:

$\frac{d}{dx}\left(\frac{1}{x^3}\right)=\frac{d}{dx}(x^{-3})=-3x^{-4}=-\frac{3}{x^4}$

30. **a.** Using the quotient rule:

$\frac{d}{dx}\left(\frac{1}{x^4}\right)=\frac{x^4(0)-4x^3(1)}{(x^4)^2}=-\frac{4x^3}{x^8}=-\frac{4}{x^5}$

b. Using the power rule:

$\frac{d}{dx}\left(\frac{1}{x^4}\right)=\frac{d}{dx}(x^{-4})=-4x^{-5}=-\frac{4}{x^5}$

31. $f'(x)=\frac{x^3(4x^3)-3x^2(x^4+1)}{(x^3)^2}=\frac{4x^6-3x^6-3x^2}{x^6}=\frac{x^6-3x^2}{x^6}=1-\frac{3}{x^4}$

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32. $f'(x)=\dfrac{x^2(5x^4)-2x(x^5-1)}{(x^2)^2}=\dfrac{5x^6-2x^6+2x}{x^4}=\dfrac{3x^6+2x}{x^4}=3x^2+\dfrac{2}{x^3}$

33. $f'(x)=\dfrac{(x-1)(1)-(1)(x+1)}{(x-1)^2}=\dfrac{x-1-x-1}{(x-1)^2}=-\dfrac{2}{(x-1)^2}$

34. $f'(x)=\dfrac{(x+1)(1)-(1)(x-1)}{(x+1)^2}=\dfrac{x+1-x+1}{(x+1)^2}=\dfrac{2}{(x+1)^2}$

35. $f'(x)=\dfrac{(2+x)(3)-(1)(3x+1)}{(2+x)^2}=\dfrac{6+3x-3x-1}{(2+x)^2}=\dfrac{5}{(2+x)^2}$

36. $f'(x)=\dfrac{(2x^2+1)(1)-(4x)(x+1)}{(2x^2+1)^2}=\dfrac{2x^2+1-4x^2-4x}{(2x^2+1)^2}=\dfrac{-2x^2-4x+1}{(2x^2+1)^2}$

37. $f'(t)=\dfrac{(t^2+1)(2t)-(2t)(t^2-1)}{(t^2+1)^2}=\dfrac{2t^3+2t-2t^3+2t}{(t^2+1)^2}=\dfrac{4t}{(t^2+1)^2}$

38. $f'(t)=\dfrac{(t^2-1)(2t)-(2t)(t^2+1)}{(t^2-1)^2}=\dfrac{2t^3-2t-2t^3-2t}{(t^2-1)^2}=-\dfrac{4t}{(t^2-1)^2}$

39. $f'(s)=\dfrac{(s+1)(3s^2)-(1)(s^3-1)}{(s+1)^2}=\dfrac{3s^3+3s^2-s^3+1}{(s+1)^2}=\dfrac{2s^3+3s^2+1}{(s+1)^2}$

40. $f'(s)=\dfrac{(s-1)(3s^2)-(1)(s^3+1)}{(s-1)^2}=\dfrac{3s^3-3s^2-s^3-1}{(s-1)^2}=\dfrac{2s^3-3s^2-1}{(s-1)^2}$

41. $f'(x)=\dfrac{(x+1)(2x-2)-(1)(x^2-2x+3)}{(x+1)^2}=\dfrac{2x^2-2-x^2+2x-3}{(x+1)^2}=\dfrac{x^2+2x-5}{(x+1)^2}$

42. $f'(x)=\dfrac{(x-1)(2x+3)-(1)(x^2+3x-1)}{(x-1)^2}=\dfrac{2x^2+x-3-x^2-3x+1}{(x-1)^2}=\dfrac{x^2-2x-2}{(x-1)^2}$

43. $f'(x)=\dfrac{(x^2+1)(4x^3+2x)-(2x)(x^4+x^2+1)}{(x^2+1)^2}=\dfrac{4x^3(x^2+1)+2x(x^2+1)-2x(x^4)-2x(x^2+1)}{(x^2+1)^2}$

$=\dfrac{4x^5+4x^3-2x^5}{(x^2+1)^2}=\dfrac{2x^5+4x^3}{(x^2+1)^2}$

44. $f(x)=\dfrac{x^5+x^3+x}{x^3+x}=\dfrac{x(x^4+x^2+1)}{x(x^2+1)}=\dfrac{x^4+x^2+1}{x^2+1}$

Thus, $f'(x)=\dfrac{2x^5+4x^3}{(x^2+1)^2}$.

45. $f'(t)=\dfrac{(t^2+t-3)(2t+2)-(t^2+2t-1)(2t+1)}{(t^2+t-3)^2}=\dfrac{2t^3+4t^2-4t-6-2t^3-5t^2+1}{(t^2+t-3)^2}=-\dfrac{t^2+4t+5}{(t^2+t-3)^2}$

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46. $f'(t)=\dfrac{(t^2-t+2)(4t+1)-(2t^2+t-5)(2t-1)}{(t^2-t+2)^2}=\dfrac{4t^3-3t^2+7t+2-4t^3+11t-5}{(t^2-t+2)^2}=\dfrac{-3t^2+18t-3}{(t^2-t+2)^2}$

47. Rewrite $y=3x^{-1}$ Differentiate $\dfrac{dy}{dx}=-3x^{-2}$ Rewrite $\dfrac{dy}{dx}=-\dfrac{3}{x^2}$

48. Rewrite $y=\dfrac{1}{4}x^2$ Differentiate $\dfrac{dy}{dx}=\dfrac{1}{2}x$ Rewrite $\dfrac{dy}{dx}=\dfrac{x}{2}$

49. Rewrite $y=\dfrac{3}{8}x^4$ Differentiate $\dfrac{dy}{dx}=\dfrac{3}{2}x^3$ Rewrite $\dfrac{dy}{dx}=\dfrac{3x^3}{2}$

50. Rewrite $y=\dfrac{3}{2}x^{-2}$ Differentiate $\dfrac{dy}{dx}=-3x^{-3}$ Rewrite $\dfrac{dy}{dx}=-\dfrac{3}{x^3}$

51. Rewrite $y=\dfrac{1}{3}x^2-\dfrac{5}{3}x$ Differentiate $\dfrac{dy}{dx}=\dfrac{2}{3}x-\dfrac{5}{3}$ Rewrite $\dfrac{dy}{dx}=\dfrac{2x-5}{3}$

52. Rewrite $y=4x^{-1/2}$ Differentiate $\dfrac{dy}{dx}=-2x^{-3/2}$ Rewrite $\dfrac{dy}{dx}=\dfrac{-2}{\sqrt{x^3}}$

53.
$$\begin{aligned}\frac{d}{dx}\left[(x^3+2)\frac{x^2+1}{x+1}\right]&=\frac{d}{dx}(x^3+2)\cdot\frac{x^2+1}{x+1}+(x^3+2)\cdot\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right)\\&3x^2\left(\frac{x^2+1}{x+1}\right)+(x^3+2)\frac{(x+1)(2x)-(1)(x^2+1)}{(x+1)^2}\\&=3x^2\left(\frac{x^2+1}{x+1}\right)+(x^3+2)\frac{x^2+2x-1}{(x+1)^2}\end{aligned}$$

54.
$$\begin{aligned}\frac{d}{dx}\left[(x^5+1)\frac{x^3+2}{x+1}\right]&=\frac{d}{dx}(x^5+1)\cdot\frac{x^3+2}{x+1}+(x^5+1)\cdot\frac{d}{dx}\left(\frac{x^3+2}{x+1}\right)\\&=5x^4\left(\frac{x^3+2}{x+1}\right)+(x^5+1)\frac{(x+1)(3x^2)-(1)(x^3+2)}{(x+1)^2}\\&=5x^4\left(\frac{x^3+2}{x+1}\right)+(x^5+1)\frac{3x^3+3x^2-x^3-2}{(x+1)^2}\\&=5x^4\left(\frac{x^3+2}{x+1}\right)+(x^5+1)\frac{2x^3+3x^2-2}{(x+1)^2}\end{aligned}$$

55.
$$\begin{aligned}\frac{d}{dx}\frac{(x^2+3)(x^3+1)}{x^2+2}&=\frac{(x^2+2)\frac{d}{dx}\left[(x^2+3)(x^3+1)\right]-\left[\frac{d}{dx}(x^2+2)\right]\left[(x^2+3)(x^3+1)\right]}{(x^2+2)^2}\\&=\frac{(x^2+2)\left\{\left[\frac{d}{dx}(x^2+3)\right](x^3+1)+(x^2+3)\frac{d}{dx}(x^3+1)\right\}-2x\left[(x^2+3)(x^3+1)\right]}{(x^2+2)^2}\\&=\frac{(x^2+2)\left[(2x)(x^3+1)+(x^2+3)(3x^2)\right]-2x(x^2+3)(x^3+1)}{(x^2+2)^2}\\&=\frac{(x^2+2)(2x^4+2x+3x^4+9x^2)-(2x)(x^5+3x^3+x^2+3)}{(x^2+2)^2}\\&=\frac{2x^6+2x^3+3x^6+9x^4+4x^4+4x+6x^4+18x^2-2x^6-6x^4-2x^3-6x}{(x^2+2)^2}\\&=\frac{3x^6+13x^4+18x^2-2x}{(x^2+2)^2}\end{aligned}$$

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56. $$\frac{d}{dx}\frac{(x^3+2)(x^2+2)}{x^3+1}=\frac{(x^3+1)\frac{d}{dx}\left[(x^3+2)(x^2+2)\right]-\left[\frac{d}{dx}(x^3+1)\right]\left[(x^3+2)(x^2+2)\right]}{(x^3+1)^2}$$
$$=\frac{(x^3+1)\left[(3x^2)(x^2+2)+(x^3+2)(2x)\right]-3x^2(x^3+2)(x^2+2)}{(x^3+1)^2}$$
$$=\frac{(x^3+1)(3x^2)(x^2+2)+(x^3+1)(x^3+2)(2x)-3x^2(x^3+2)(x^2+2)}{(x^3+1)^2}$$
$$=\frac{3x^2(x^2+2)\left[(x^3+1)-(x^3+2)\right]+(x^3+1)(x^3+2)(2x)}{(x^3+1)^2}$$
$$=\frac{-3x^4-6x^2+2x^7+6x^4+4x}{(x^3+1)^2}$$
$$=\frac{2x^7+3x^4-6x^2+4x}{(x^3+1)^2}$$

57. $$\frac{d}{dx}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)=\frac{d}{dx}\left(\frac{x^{1/2}-1}{x^{1/2}+1}\right)=\frac{(x^{1/2}+1)(\frac{1}{2}x^{-1/2})-\frac{1}{2}x^{-1/2}(x^{1/2}-1)}{(x^{1/2}+1)^2}$$
$$=\frac{\frac{1}{2}+\frac{1}{2}x^{-1/2}-\frac{1}{2}+\frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2}=\frac{x^{-1/2}}{(x^{1/2}+1)^2}=\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

58. $$\frac{d}{dx}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)=\frac{d}{dx}\left(\frac{x^{1/2}+1}{x^{1/2}-1}\right)=\frac{(x^{1/2}-1)(\frac{1}{2}x^{-1/2})-\frac{1}{2}x^{-1/2}(x^{1/2}+1)}{(x^{1/2}-1)^2}$$
$$=\frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{(x^{1/2}-1)^2}=-\frac{x^{-1/2}}{(x^{1/2}-1)^2}=-\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$$

59. $$\frac{d}{dx}\left[\frac{R(x)}{x}\right]=\frac{x\cdot R'(x)-1\cdot R(x)}{x^2}=\frac{xR'(x)-R(x)}{x^2}$$

60. $$\frac{d}{dx}\left[\frac{P(x)}{x}\right]=\frac{x\cdot P'(x)-1\cdot P(x)}{x^2}=\frac{xP'(x)-P(x)}{x^2}$$

61. **a.** The instantaneous rate of change of cost with respect to purity is the derivative of the cost function $C(x)=\frac{100}{100-x}$ on $50 \bullet x < 100$.

$$C'(x)=\frac{(100-x)\cdot 0-(-1)(100)}{(100-x)^2}=\frac{100}{(100-x)^2} \quad \text{on } 50 \bullet x < 100$$

b. To find the rate of change for a purity of 95%, evaluate $C'(x)$ at $x = 95$.

$$C'(95)=\frac{100}{(100-95)^2}=\frac{100}{5^2}=4$$

The cost is increasing by 4 cents per additional percent of purity.

c. To find the rate of change for a purity of 98%, evaluate $C'(x)$ at $x = 98$.

$$C'(98)=\frac{100}{(100-98)^2}=\frac{100}{2^2}=25$$

The cost is increasing by 25 cents per additional percent of purity.

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62. **a.** $AC(x)=\dfrac{C(x)}{x}=\dfrac{6x+50}{x}$

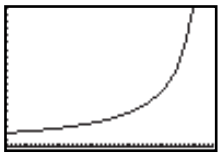
b. The marginal average cost function $MAC(x)$ is the derivative of the average cost function $AC(x)=\dfrac{C(x)}{x}=\dfrac{6x+50}{x}$.

$$MAC(x)=\frac{d}{dx}\left(\frac{6x+50}{x}\right)=\frac{x(6)-1(6x+50)}{x^2}$$
$$=\frac{6x-6x-50}{x^2}=\frac{-50}{x^2}$$

c. $MAC(25)=\dfrac{-50}{25^2}=-0.08$

The average cost is decreasing at the rate of 8 cents per additional truck.

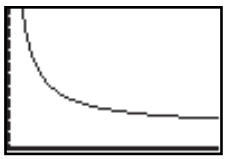
63. **a.**



on [50, 100] by [0, 20]

b. Rate of change of cost is 4 for $x = 95$; rate of change of cost is 25 for $x = 98$.

64. **a.**



on [0, 400] by [0,50]

b. Rate of change is –0.03 for $x = 200$.

65. **a.** $AC(x)=\dfrac{C(x)}{x}=\dfrac{8x+45}{x}$

b. The marginal average cost function $MAC(x)$ is the derivative of the average cost function $AC(x)=\dfrac{C(x)}{x}=\dfrac{8x+45}{x}$, or $8+\dfrac{45}{x}$.

$$MAC(x)=\frac{d}{dx}\left(\frac{8x+45}{x}\right)=\frac{x(8)-1(8x+45)}{x^2}=\frac{8x-8x-45}{x^2}=\frac{-45}{x^2}$$

c. $MAC(30)=\dfrac{-45}{30^2}=-\dfrac{1}{20}=-0.05$

The average cost is decreasing at the rate of 5 cents per additional clock.

66. To find the rate of change of the number of bottles sold, find $N'(p)$.

$$N'(p)=\frac{(p+7)\cdot 0-(1)(2250)}{(p+7)^2}=-\frac{2250}{(p+7)^2}$$

When $p = 8$, $N'(8)=-\dfrac{2250}{(8+7)^2}=-\dfrac{2250}{225}=-10$.

At \$8 per bottle, the number of bottles of whiskey sold will decrease by about 10 bottles for each \$1 increase in price.

67. To find the rate of change of temperature, find $T'(x)$.

$$T'(x)=3x^2(4-x^2)+x^3(-2x)$$
$$=12x^2-3x^4-2x^4=12x^2-5x^4$$

For $x = 1$, $T'(1)=12(1)^2-5(1)^4=12-5=7$.

After 1 hour, the person's temperature is increasing by 7 degrees per hour.

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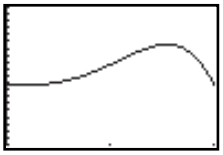
68. To find the rate of change of the sales, find $S'(x)$.

$$S'(x) = 2x(8 - x^3) + x^2(-3x^2)$$
$$= 16x - 2x^4 - 3x^4 = 16x - 5x^4$$

For $x = 1$, $S'(1) = 16(1) - 5(1)^4 = 16 - 5 = 11$.

After 1 month, sales are increasing by 11 thousand per month.

69. **a.**

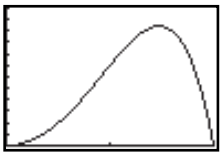


on [0, 2] by [90, 110]

b. The rate of change at $x = 1$ is 7.

c. The maximum temperature is about 104.5 degrees.

70. **a.**



on [0, 2] by [0, 12]

b. The rate of change at $x = 1$ is 11.

c. The maximum sales are about 10.4 thousand.

71. **a.** $y_3(8) = \dfrac{-73.1(8)^2 + 1270(8) + 16,280}{2.3(8) + 314} \approx 65.468$

The per capita national debt in 2020 would be \$65,468.

$y_3(13) = \dfrac{-73.1(13)^2 + 1270(13) + 16,280}{2.3(13) + 314} \approx 59.425$

The per capita national debt in 2025 would be \$59,425.

b. $y_3'(8) = -0.151$

In 2020 the per capita national debt will be shrinking by \$151 per year.

$y_3'(13) = -2.231$

In 2025 the per capita national debt will be shrinking by \$2231 per year.

72. **a.** $g'(x) = \dfrac{(x^2 - 110x + 3500)(-30x + 1125) - (2x - 110)(-15x^2 + 1125x)}{(x^2 - 110x + 3500)^2}$

$$= \frac{525x^2 - 105,000x + 3,937,500}{(x^2 - 110x + 3500)^2}$$

b. $g'(40) = \dfrac{525(40)^2 - 105,000(40) + 3,937,500}{\left((40)^2 - 110(40) + 3500\right)^2}$

$$= 1.1785714 \ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 40 mph, your gas mileage increases by 1.1785714 for each additional mile per hour.

$g'(50) = \dfrac{525(50)^2 - 105,000(50) + 3,937,500}{\left((50)^2 - 110(50) + 3500\right)^2}$

$$= 0 \ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

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At 50 mph, your gas mileage will not change for each additional mile per hour.

$$g'(60)=\frac{525(60)^2-105{,}000(60)+3{,}937{,}500}{\left((60)^2-110(60)+3500\right)^2}$$
$$=-1.89\ \frac{\text{mi/gallon}}{\text{mi/hour}}$$

At 60 mph, your gas mileage decreases by 1.89 for each additional mile per hour.

c. The positive sign of $g'(40)$ tells you that gas mileage increases with speed when driving at 40 mph. The negative sign of $g'(60)$ tells you that gas mileage decreases with speed when driving at 60 mph. The fact that $g'(50)$ is zero tells you that gas mileage neither increases nor decreases with speed when driving at 50 mph. This means it is the most economical speed.

73. $I(x)=0.45(x-1.7)(x^2-12.5x+43)$

$$I'(x)=0.45\left[1(x^2-12.5x+43)+(x-1.7)(2x-12.5)\right]$$
$$I'(5)=0.45\left[(5)^2-12.5(5)+43+(5-1.7)(2(5)-12.5)\right]$$
$$=-1.2375$$
$$I'(6)=0.45\left[(6)^2-12.5(6)+43+(6-1.7)(2(6)-12.5)\right]$$
$$=0.8325$$

Interest rates were declining by about 1.24 percent per year in 2010 and rising by 0.83 percent per year in 2011.

74. $I(x)=0.106(x^2-6.85x+14)(x^2-14.5x+56)$

$$I'(x)=0.106\left[(2x-6.85)(x^2-14.5x+56)+(x^2-6.85x+14)(2x-14.5)\right]$$
$$I'(3)=0.106\left[(2(3)-6.85)((3)^2-14.5(3)+56)+((3)^2-6.85(3)+14)(2(3)-14.5)\right]$$
$$=-4.1446$$
$$I'(5)=0.106\left[(2(5)-6.85)((5)^2-14.5(5)+56)+((5)^2-6.85(5)+14)(2(5)-14.5)\right]$$
$$=0.5724$$

Interest rates were declining by about 4.14 percent per year in 2008 and rising by 0.57 percent per year in 2010.

75. $E(x)=\dfrac{261x}{x+8.84}+647$

$$E'(x)=\frac{(x+8.84)(261)-(1)(261x)}{(x+8.84)^2}=\frac{2307.24}{(x+8.84)^2}$$
$$E'(4)=\frac{2307.24}{(4+8.84)^2}\approx 13.99$$
$$E'(8)=\frac{2307.24}{(8+8.84)^2}\approx 8.14$$

Median weekly earnings were rising by \$13.99 per year in 2009 and by \$8.14 per year in 2013.

76. $C(x)=\dfrac{75(44-5x)}{x^2-18.3x+85}+115x+1605$

$$=75\left[\frac{44-5x}{x^2-18.3x+85}\right]+115x+1605$$
$$C'(x)=75\left[\frac{(x^2-18.3x+85)(-5)-(2x-18.3)(44-5x)}{(x^2-18.3x+85)^2}\right]+115$$
$$=75\left[\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2}\right]+115$$

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$$C'(7)=75\left[\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2}\right]+115\approx 134.82$$
$$C'(9)=75\left[\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2}\right]+115\approx -186.78$$
$$C'(11)=75\left[\frac{5x^2-88x+380.2}{(x^2-18.3x+85)^2}\right]+115\approx 173.34$$

Outstanding credit was rising by about \$135 million per year in 2007, falling by \$187 million per year in 2009, and rising by \$173 million per year in 2011.

77. **a.** $\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}(x^{-2})=-2x^{-3}=-\frac{2}{x^3}$

$\left.\frac{d}{dx}\right|_{x=0}=-\frac{2}{0^3}$ Undefined

b. Answers will vary.

78. **a.** $\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{d}{dx}(x^{-1})=-x^{-2}=-\frac{1}{x^2}$

$\left.\frac{d}{dx}\right|_{x=0}=-\frac{1}{0^2}$ Undefined

b. Answers will vary.

79. False: the product rule gives the correct right-hand side.

80. False: the quotient rule gives the correct right-hand side.

81. True: $\frac{d}{dx}(x\cdot f)=\frac{d}{dx}x\cdot f+x\cdot f'=f+x\cdot f'$

82. True: $\frac{d}{dx}\left(\frac{f}{x}\right)=\frac{x\cdot f'-\frac{d}{dx}x\cdot f}{(x)^2}=\frac{x\cdot f'-f}{x^2}$

83. $\frac{d}{dx}(f\cdot g)=\frac{(f\cdot g)f'}{f}+\frac{(f\cdot g)g'}{g}=g\cdot f'+f\cdot g'$

The right-hand side multiplies out to $g\cdot f'+f\cdot g'$ which agrees with the product rule.

84. $\frac{d}{dx}\left(\frac{f}{g}\right)=\left(\frac{f}{g}\right)\left(\frac{f'}{f}\right)-\left(\frac{f}{g}\right)\left(\frac{g'}{g}\right)$

$=\frac{f'}{g}-\frac{f\cdot g'}{g^2}$

$=\frac{g\cdot f'+f\cdot g'}{g^2}$

The right-hand side multiplies out to $\frac{g\cdot f'+f\cdot g'}{g^2}$ which agrees with the quotient rule.

85. False: This would be the same as saying that the derivative (instantaneous rate of change) of a product is a product of the derivatives. The product rule gives the correct way of finding the derivative of a product.

86. False: This would be the same as saying that the derivative (instantaneous rate of change) of a quotient is a quotient of the derivatives. The quotient rule gives the correct way of finding the derivative of a quotient.

87. $\frac{d}{dx}(f\cdot g\cdot h)=\frac{d}{dx}[f\cdot(g\cdot h)]=\frac{df}{dx}\cdot(g\cdot h)+f\cdot\frac{d}{dx}(g\cdot h)$

$=\frac{df}{dx}\cdot(g\cdot h)+f\left(\frac{dg}{dx}\cdot h+g\cdot\frac{dh}{dx}\right)$

$=\frac{df}{dx}\cdot g\cdot h+f\cdot\frac{dg}{dx}\cdot h+f\cdot g\cdot\frac{dh}{dx}$

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88. **a.** $Q(x) = \dfrac{f(x)}{g(x)}$

b. $g(x)\cdot Q(x) = \dfrac{f(x)}{g(x)}\cdot g(x)$

$Q(x)\cdot g(x) = f(x)$

c. $Q'(x)\cdot g(x) + Q(x)\cdot g'(x) = f'(x)$

d. $Q'(x)\cdot g(x) = f'(x) - Q(x)\cdot g'(x)$

$$Q'(x) = \frac{f'(x) - Q(x)\cdot g'(x)}{g(x)}$$

e. $$Q'(x) = \frac{f'(x) - \frac{f(x)}{g(x)}\cdot g'(x)}{g(x)} = \frac{\frac{1}{g(x)}[g(x)\cdot f'(x) - f(x)\cdot g'(x)]}{g(x)} = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

89. $$\begin{aligned}\frac{d}{dx}[f(x)]^2 &= \frac{d}{dx}[f(x)\cdot f(x)]\\ &= \left[\frac{d}{dx}f(x)\right]f(x) + f(x)\left[\frac{d}{dx}f(x)\right]\\ &= f'(x)\cdot f(x) + f(x)\cdot f'(x) = 2f(x)\cdot f'(x)\end{aligned}$$

90. Rewrite $[f(x)]^{-1} = \frac{1}{f(x)}$ and find $\frac{d}{dx}\left[\frac{1}{f(x)}\right]$.

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{f(x)\cdot 0 - f'(x)\cdot 1}{[f(x)]^2} = -\frac{f'(x)}{[f(x)]^2}$$

91. $$\frac{dy}{dx}\left(\frac{Rx}{1+\left(\frac{R-1}{K}\right)x}\right) = \frac{\left(1+\left(\frac{R-1}{K}\right)x\right)R - Rx\left(\frac{R-1}{K}\right)}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2} = \frac{R + Rx\left(\frac{R-1}{K}\right) - Rx\left(\frac{R-1}{K}\right)}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2} = \frac{R}{\left(1+\left(\frac{R-1}{K}\right)x\right)^2}$$

To show $y' > 0$, show that both the numerator and denominator are greater than zero.

$R > 1 > 0 \Rightarrow$ numerator is greater than zero.

$R > 1 \Rightarrow R - 1 > 0.$

$R - 1 > 0;\ K > 0 \Rightarrow \dfrac{R-1}{K} > 0$

$\dfrac{R-1}{K} > 0,\ x > 0 \Rightarrow \left(1+\left(\dfrac{R-1}{K}\right)x\right) > 0 \Rightarrow$ denominator is greater than zero.

$\therefore y' > 0.$

This means the density of the offspring is always increasing faster than the density of the parents.

92. $w > 5$ and $3.5 > 0,\ (w-1.5)^2 > 0$ and $w - 1.5 > 0 \Rightarrow R'(w) = \dfrac{w-5}{w-1.5} > 0.$

A task that expends w kcal/min of work for $w > 5$ requires more than w minutes of rest.

EXERCISES 2.5

1. **a.** $f'(x) = 4x^3 - 3(2)x^2 - 2(3)x + 5 - 0$

$= 4x^3 - 6x^2 - 6x + 5$

b. $f''(x) = 3(4)x^2 - 2(6)x - 6 + 0 = 12x^2 - 12x - 6$

c. $f'''(x) = 2(12)x - 12 - 0 = 24x - 12$

d. $f^{(4)}(x) = 24$

2. **a.** $f'(x) = 4x^3 - 9x^2 + 4x - 8$

b. $f''(x) = 12x^2 - 18x + 4$

c. $f'''(x) = 24x - 18$

d. $f^{(4)}(x) = 24$

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3. **a.** $f'(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4$

b. $f''(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$

c. $f'''(x)=1+x+\frac{1}{2}x^2$

d. $f^{(4)}(x)=1+x$

4. **a.** $f'(x)=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$

b. $f''(x)=1+x+\frac{1}{2}x^2$

c. $f'''(x)=1+x$

d. $f^{(4)}(x)=1$

5. $f(x)=\sqrt{x^5}=x^{5/2}$

a. $f'(x)=\frac{5}{2}x^{3/2}$

b. $f''(x)=\frac{3}{2}\left(\frac{5}{2}\right)x^{1/2}=\frac{15}{4}\sqrt{x}$

c. $f'''(x)=\frac{d}{dx}\left(\frac{15}{4}x^{1/2}\right)=\frac{1}{2}\left(\frac{15}{4}\right)x^{-1/2}$
$=\frac{15}{8}x^{-1/2}=\frac{15}{8\sqrt{x}}$

d. $f^{(4)}(x)=\frac{d}{dx}\left(\frac{15}{8}x^{-1/2}\right)=-\frac{1}{2}\left(\frac{15}{8}\right)x^{-3/2}$
$=-\frac{15}{16}x^{-3/2}$

6. $f(x)=\sqrt{x^3}=x^{3/2}$

a. $f'(x)=\frac{3}{2}x^{1/2}=\frac{3}{2}\sqrt{x}$

b. $f''(x)=\frac{3}{4}x^{-1/2}=\frac{3}{4\sqrt{x}}$

c. $f'''(x)=-\frac{3}{8}x^{-3/2}$

d. $f^{(4)}(x)=\frac{9}{16}x^{-5/2}$

7. $f(x)=\frac{x-1}{x}=1-\frac{1}{x}$

a. $f'(x)=\frac{1}{x^2}$

$f''(x)=\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}\left(x^{-2}\right)=-2x^{-3}$
$=-\frac{2}{x^3}$

b. $f''(3)=-\frac{2}{3^3}=-\frac{2}{27}$

8. $f(x)=\frac{x+2}{x}=1+\frac{2}{x}$

a. $f'(x)=-\frac{2}{x^2}$

$f''(x)=\frac{d}{dx}\left(-\frac{2}{x^2}\right)=\frac{d}{dx}\left(-2x^{-2}\right)=4x^{-3}$
$=\frac{4}{x^3}$

b. $f''(3)=\frac{4}{3^3}=\frac{4}{27}$

9. $f(x)=\frac{x+1}{2x}=\frac{1}{2}+\frac{1}{2x}$

a. $f'(x)=-\frac{1}{2x^2}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{2x^2}\right)=\frac{d}{dx}\left(-\frac{1}{2}x^{-2}\right)=\frac{1}{x^3}$

b. $f''(3)=\frac{1}{3^3}=\frac{1}{27}$

10. $f(x)=\frac{x-2}{4x}=\frac{1}{4}-\frac{1}{2x}$

a. $f'(x)=\frac{1}{2x^2}$

$f''(x)=\frac{d}{dx}\left(\frac{1}{2x^2}\right)=\frac{d}{dx}\left(\frac{1}{2}x^{-2}\right)=-\frac{1}{x^3}$

b. $f''(3)=-\frac{1}{3^3}=-\frac{1}{27}$

11. $f(x)=\frac{1}{6x^2}$

a. $f'(x)=-2\left(\frac{1}{6}\right)x^{-3}=-\frac{1}{3x^3}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{3}x^{-3}\right)=x^{-4}=\frac{1}{x^4}$

b. $f''(3)=\frac{1}{3^4}=\frac{1}{81}$

12. $f(x)=\frac{1}{12x^3}$

a. $f'(x)=-\frac{1}{4}x^{-4}$

$f''(x)=\frac{d}{dx}\left(-\frac{1}{4}x^{-4}\right)=x^{-5}=\frac{1}{x^5}$

b. $f''(3)=\frac{1}{3^5}=\frac{1}{243}$

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13. $f(x)=(x^2-2)(x^2+3)=x^4+x^2-6$
$f'(x)=4x^3+2x$
$f''(x)=12x^2+2$

14. $f(x)=(x^2-1)(x^2+2)=x^4+x^2-2$
$f'(x)=4x^3+2x$
$f''(x)=12x^2+2$

15. $f(x)=\frac{27}{\sqrt[3]{x}}=27x^{-1/3}$
$f'(x)=\left(-\frac{1}{3}\right)(27x^{-4/3})=-9x^{-4/3}$
$f''(x)=\left(-\frac{4}{3}\right)(-9x^{-7/3})=12x^{-7/3}$

16. $f(x)=\frac{32}{\sqrt[4]{x}}=32x^{-1/4}$
$f'(x)=\left(-\frac{1}{4}\right)(32x^{-5/4})=-8x^{-5/4}$
$f''(x)=\left(-\frac{5}{4}\right)(-8x^{-9/4})=10x^{-9/4}$

17. $f(x)=\frac{x}{x-1}$

$$f'(x)=\frac{(x-1)\frac{dx}{dx}-\left[\frac{d}{dx}(x-1)\right](x)}{(x-1)^2}=\frac{(x-1)(1)-(1)(x)}{(x-1)^2}=-\frac{1}{(x-1)^2}=-\frac{1}{x^2-2x+1}$$

$$f''(x)=\frac{(x^2-2x+1)\frac{d}{dx}(-1)-\left[\frac{d}{dx}(x^2-2x+1)\right](-1)}{[(x-1)^2]^2}=\frac{0-(2x-2)(-1)}{(x-1)^4}=\frac{2x-2}{(x-1)^4}=\frac{2(x-1)}{(x-1)^4}=\frac{2}{(x-1)^3}$$

18. $f(x)=\frac{x}{x-2}$

$$f'(x)=\frac{(x-2)(1)-(1)(x)}{(x-2)^2}=\frac{x-2-x}{(x-2)^2}=-\frac{2}{(x-2)^2}=-\frac{2}{x^2-4x+4}$$

$$f''(x)=\frac{(x^2-4x+4)\cdot 0-(2x-4)(-2)}{(x^2-4x+4)^2}=\frac{4x-8}{[(x-2)^2]^2}=\frac{4(x-2)}{(x-2)^4}=\frac{4}{(x-2)^3}$$

19. $\frac{d}{dr}(\pi r^2)=2\pi r$
$\frac{d^2}{dr^2}(\pi r^2)=\frac{d}{dr}(2\pi r)=2\pi$

20. $\frac{d}{dr}(\frac{4}{3}\pi r^3)=4\pi r^2$
$\frac{d^2}{dr^2}(\frac{4}{3}\pi r^3)=\frac{d}{dr}(4\pi r^2)=8\pi r$
$\frac{d^3}{dr^3}(\frac{4}{3}\pi r^3)=\frac{d}{dr}(8\pi r)=8\pi$

21. $\frac{d}{dx}x^{10}=10x^9$
$\frac{d^2}{dx^2}x^{10}=\frac{d}{dx}(10x^9)=90x^8$
$\left.\frac{d^2}{dx^2}x^{10}\right|_{x=-1}=90(-1)^8=90$

22. $\frac{d}{dx}x^{11}=11x^{10}$
$\frac{d^2}{dx^2}x^{11}=\frac{d}{dx}(11x^{10})=110x^9$
$\left.\frac{d^2}{dx^2}x^{11}\right|_{x=-1}=110(-1)^9=-110$

23. From Exercise 21, we know
$\frac{d^2}{dx^2}x^{10}=90x^8$
$\frac{d^3}{dx^3}x^{10}=\frac{d}{dx}(90x^8)=720x^7$
Thus, $\left.\frac{d^3}{dx^3}x^{10}\right|_{x=-1}=720(-1)^7=-720$

24. From Exercise 22, we know
$\frac{d^2}{dx^2}x^{11}=110x^9$
$\frac{d^3}{dx^3}x^{11}=\frac{d}{dx}(110x^9)=990x^8$
Thus, $\left.\frac{d^3}{dx^3}x^{11}\right|_{x=-1}=990(-1)^8=990$

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25. $\frac{d}{dx}\sqrt{x^3} = \frac{d}{dx}x^{3/2} = \frac{3}{2}x^{1/2}$

$\frac{d^2}{dx^2}\sqrt{x^3} = \frac{d}{dx}\left(\frac{3}{2}x^{1/2}\right) = \frac{3}{4}x^{-1/2}$

$\left.\frac{d^2}{dx^2}\sqrt{x^3}\right|_{x=1/16} = \frac{3}{4}\left(\frac{1}{16}\right)^{-1/2} = \frac{3}{4}\sqrt{16} = 3$

26. $\frac{d}{dx}\sqrt[3]{x^4} = \frac{d}{dx}x^{4/3} = \frac{4}{3}x^{1/3}$

$\frac{d^2}{dx^2}\sqrt[3]{x^4} = \frac{d}{dx}\left(\frac{4}{3}x^{1/3}\right) = \frac{4}{9}x^{-2/3}$

$\left.\frac{d^2}{dx^2}\left(\sqrt[3]{x^4}\right)\right|_{x=1/27} = \frac{4}{9}\left(\frac{1}{27}\right)^{-2/3} = \frac{4}{9}(27)^{2/3} = 4$

27.
$$\begin{aligned}\frac{d}{dx}[(x^2-x+1)(x^3-1)] &= (2x-1)(x^3-1)+(x^2-x+1)(3x^2)\\ &= 2x^4-x^3-2x+1+3x^4-3x^3+3x^2\\ &= 5x^4-4x^3+3x^2-2x+1\end{aligned}$$
$$\begin{aligned}\frac{d^2}{dx^2}[(x^2-x+1)(x^3-1)] &= \frac{d}{dx}(5x^4-4x^3+3x^2-2x+1)\\ &= 20x^3-12x^2+6x-2\end{aligned}$$

28.
$$\begin{aligned}\frac{d}{dx}[(x^3+x-1)(x^3+1)] &= (3x^2+1)(x^3+1)+(x^3+x-1)(3x^2)\\ &= 3x^5+3x^2+x^3+1+3x^5+3x^3-3x^2 = 6x^5+4x^3+1\end{aligned}$$
$$\frac{d^2}{dx^2}[(x^3+x-1)(x^3+1)] = \frac{d}{dx}(6x^5+4x^3+1) = 30x^4+12x^2$$

29.
$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{(x^2+1)\left(\frac{dx}{dx}\right)-x\frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \frac{x^2+1-x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{x^4+2x^2+1}$$
$$\begin{aligned}\frac{d^2}{dx^2}\left(\frac{x}{x^2+1}\right) &= \frac{d}{dx}\left(\frac{1-x^2}{x^4+2x^2+1}\right) = \frac{(x^4+2x^2+1)\frac{d}{dx}(1-x^2)-(1-x^2)\frac{d}{dx}(x^4+2x^2+1)}{[(x^2+1)^2]^2}\\ &= \frac{(x^4+2x^2+1)(-2x)-(1-x^2)(4x^3+4x)}{(x^2+1)^4} = \frac{(x^2+1)[(x^2+1)(-2x)-(1-x^2)(4x)]}{(x^2+1)^4}\\ &= \frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}\end{aligned}$$

30.
$$\frac{d}{dx}\left(\frac{x}{x^2-1}\right) = \frac{(x^2-1)\cdot 1-(2x)x}{(x^2-1)^2} = \frac{x^2-2x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2} = \frac{-x^2-1}{x^4-2x^2+1}$$
$$\begin{aligned}\frac{d^2}{dx^2}\left(\frac{x}{x^2-1}\right) &= \frac{(x^2-1)^2(-2x)-(4x^3-4x)(-x^2-1)}{(x^2-1)^4}\\ &= \frac{-2x^5+4x^3-2x-(-4x^5-4x^3+4x^3+4x)}{(x^2-1)^4}\\ &= \frac{-2x^5+4x^3-2x+4x^5-4x}{(x^2-1)^4} = \frac{2x^5+4x^3-6x}{(x^2-1)^4}\end{aligned}$$

31.
$$\frac{d}{dx}\left(\frac{2x-1}{2x+1}\right) = \frac{(2x+1)\cdot 2-2(2x-1)}{(2x+1)^2} = \frac{4x+2-4x+2}{4x^2+4x+1} = \frac{4}{4x^2+4x+1}$$
$$\begin{aligned}\frac{d^2}{dx^2}\left(\frac{2x-1}{2x+1}\right) &= \frac{(4x^2+4x+1)\cdot 0-(8x+4)(4)}{(4x^2+4x+1)^2} = -\frac{32x+16}{[(2x+1)^2]^2} = -\frac{32x+16}{(2x+1)^4}\\ &= -\frac{16(2x+1)}{(2x+1)^4} = -\frac{16}{(2x+1)^3}\end{aligned}$$

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32. $\frac{d}{dx}\left(\frac{3x+1}{3x-1}\right)=\frac{(3x-1)(3)-3(3x+1)}{(3x-1)^2}=\frac{9x-3-9x-3}{9x^2-6x+1}=-\frac{6}{9x^2-6x+1}$

$\frac{d^2}{dx^2}\left(\frac{3x+1}{3x-1}\right)=-\frac{(9x^2-6x+1)\cdot 0-6(18x-6)}{(9x^2-6x+1)^2}=\frac{36(3x-1)}{[(3x-1)^2]^2}=\frac{36}{(3x-1)^3}$

33. To find velocity, we differentiate the distance function $s(t)=18t^2-2t^3$.

$v(t)=s'(t)=36t-6t^2$

a. For $t=3$, $v(3)=s'(3)=36(3)-6(3)^2=108-54=54$ miles per hour

b. For $t=7$, $v(7)=s'(7)=36(7)-6(7)^2=252-294=-42$ miles per hour

c. To find the acceleration, we differentiate the velocity function $s'(t)=36t-6t^2$.
$a(t)=s''(t)=36-12t$.
For $t=1$, $a(1)=s''(1)=36-12(1)=24$ mi / hr^2

34. To find velocity, we differentiate the distance function $s(t)=24t^2-2t^3$.

$v(t)=s'(t)=48t-6t^2$

a. For $t=4$, $v(4)=s'(4)=48(4)-6(4)^2=96$ miles per hour

b. For $t=10$, $v(10)=s'(10)=48(10)-6(10)^2=-120$ miles per hour

c. To find the acceleration, we differentiate the velocity function $s'(t)=48t-6t^2$.
$a(t)=s''(t)=48-12t$.
For $t=1$, $a(1)=s''(1)=48-12(1)=36$ mi / hr^2

35. $v(t)=h'(t)=3t^2+2(0.5)t=3t^2+t$

$v(10)=3(10)^2+10=310$ feet per second

$a(t)=v'(t)=\frac{d}{dt}(3t^2+t)=6t+1$

$a(10)=6(10)+1=61$ ft / sec^2

36. $v(t)=s'(t)=60+\frac{(t+3)\cdot 0-(1)(100)}{(t+3)^2}$

$=60-\frac{100}{(t+3)^2}$

$v(2)=60-\frac{100}{(2+3)^2}=60-\frac{100}{25}=56$ miles per hour

37. **a.** To find when the steel ball will reach the ground, we need to determine what value of t produces $s(t)=0$. Thus, set $s(t)=0$ and solve the equation.

$0=s(t)=2717-16t^2$

$0=2717-16t^2$

$16t^2=2717$

$t^2=169.8125$

$t\approx 13.0312$ sec

The steel ball will reach the ground after about 13.03 seconds.

b. $v(t)=s'(t)=-32t$

$v(13.03)=s'(13.03)\approx -417$ feet per second

c. $a(t)=v'(t)=s''(t)$

$=-32$ feet per second per second

38. **a.** To find how long it will take to reach the ground, we need to determine what value of t produces $s(t)=0$. Thus, set $s(t)=0$ and solve the equation.

$s(t)=1451-16t^2=0$

$t^2=\frac{1451}{16}$

$t\approx 9.52$ sec

It will take about 9.52 seconds to reach the ground.

b. $v(t)=s'(t)=-32t$

$v(9.52)=-32(9.52)\approx -305$ feet per second

c. $a(t)=v'(t)=s''(t)$

$=-32$ feet per second per second

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39. **a.** $v(t) = s'(t) = -32t + 1280$

b. When $s(t)$ is a maximum, $s'(t) = 0$.

$$-32t + 1280 = 0$$
$$32t = 1280$$
$$t = 40 \text{ seconds}$$

c. At $t = 40$,

$$s(40) = -16(40)^2 + 1280(40)$$
$$= -25,600 + 51,200$$
$$= 25,600 \text{ feet}$$

40. $T(2) = 98 + \frac{8}{\sqrt{2}} \approx 103.7$

The temperature of a patient 2 hours after taking the medicine is 103.7 degrees.

$$T(t) = 98 + 8t^{-1/2}$$
$$T'(t) = -4t^{-3/2} = -\frac{4}{t^{3/2}}$$
$$T'(2) = -\frac{4}{2^{3/2}} \approx -1.4$$

Two hours after taking the medicine, the patient's temperature is decreasing at the rate of 1.4 degrees per hour.

$$T''(t) = 6t^{-5/2} = \frac{6}{t^{5/2}}$$
$$T''(2) = \frac{6}{2^{5/2}} \approx 1.1$$

After 2 hours, the rate of decrease of the patient's temperature is increasing by about 1.1 degree per hour each hour.

41. $D'(t) = \frac{4}{3}\left(9t^{1/3}\right) = 12t^{1/3}$

$D'(8) = 12(8)^{1/3} = 12(2) = 24$

The national debt is increasing by 24 billion dollars per year after 8 years.

$$D''(t) = \frac{d}{dt}\left(12t^{1/3}\right) = \frac{1}{3}\left(12t^{-2/3}\right) = 4t^{-2/3}$$
$$D''(8) = 4(8)^{-2/3} = 4\left(\frac{1}{8}\right)^{2/3} = 4\left(\frac{1}{4}\right) = 1$$

The rate of growth of the national debt is increasing by 1 billion dollars per year each year after 8 years.

42. $T(10) \approx 6.3$

In 2100 global temperatures will have risen by about 6.3°F.

$$T'(t) = 0.35t^{0.4}$$
$$T'(10) \approx 0.88$$

In 2100 global temperatures will be rising by 0.88 degrees per decade, or about a tenth of a degree per year.

$$T''(t) = 0.14t^{-0.6}$$
$$T''(10) \approx 0.035$$

The temperature increases will be speeding up (by about 0.035 degrees per decade per decade).

43. $L(10) = 9.3$

By 2100, sea levels may have risen by 93 cm (about 3 feet).

$$L'(t) = 0.06x^4 - 0.14x + 8$$
$$L'(10) = 12.6$$

In 2100 sea levels will be rising by about 12.6 centimeters per decade, or about 1.26 cm (about half an inch) per year.

$$L''(x) = 0.12x - 0.14$$
$$L''(10) = 1.06$$

The rise in the sea level will be speeding up (by about 1 cm per decade per decade).

44. $P'(x) = 1.581x^{-0.7} - 0.70376x^{0.52}$

$P''(x) \approx -1.1067x^{-1.7} - 0.3660x^{-0.48}$

$P(3) \approx 4.87$

The profit 3 years from now will be \$4.87 million.

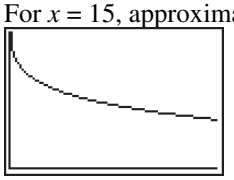
$P'(3) \approx -0.51$

The profit will be decreasing by about \$0.51 million per year 3 years from now.

$P''(3) \approx -0.39$

In 3 years, the rate of decline of profit will be accelerating by about \$0.39 million per year each year.

45. **a.** For $x = 15$, approximately 21.589°; for $x = 30$, approximately 17.597°.



on [0,50] by [0,40]

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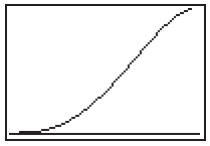
b. Each 1-mph increase in wind speed lowers the wind-chill index. As wind speed increases, the rate with which the wind-chill index decreases slows.

c. For $x = 15$, $y' = -0.363$. For a wind speed of 15 mph, each additional mile per hour decreases the wind-chill index by about 0.363. For $x = 30$, $y' = -0.203$. For a wind speed of 30 mph, each additional mile per hour decreases the wind-chill index by about 0.203°.

46. **a.**



on [1, 20] by [0, 800]

b. The second derivative is $f''(x) = -0.2184x^2 + 3.156x - 2.6$, which equals zero when x is approximately equal to 13.57. The AIDS epidemic began to slow in 1993.

47. **a.** The first derivative is negative because the temperature is dropping.

b. The second derivative is negative because the temperature is dropping increasingly rapidly making the change in temperature speed up in a negative direction.

48. **a.** The first derivative is positive because the economy is growing.

b. The second derivative is negative because the growth is slowing.

49. **a.** The first derivative is negative, because the stock market is declining.

b. The second derivative is positive because the change in the stock market is slowing down in a negative direction.

50. **a.** The first derivative is positive because the population is growing.

b. The second derivative is positive because the change in population is speeding up.

51. True: For example $\frac{d^4}{dx^4}x^3 = 0$.

52. **a.** $f'(1)$ will be positive because the altitude is increasing.
$f''(1)$ will be positive because acceleration is positive.

b. $f'(59)$ will be negative because the altitude is decreasing.
$f''(59)$ will be negative because acceleration is negative.

53. **a.** iii (showing a stop, then a slower velocity)

b. i (showing a stop, then a negative velocity)

c. ii (showing stops and starts and then a higher velocity)

54. **a.** ii (showing positive and increasing slope)

b. iii (showing negative but increasing slope)

c. i (showing positive but decreasing slope)

55. $\frac{d^{100}}{dx^{100}}(x^{99} - 4x^{98} + 3x^{50} + 6) = 0$

56.

$$\frac{d}{dx}(x^{-1}) = -x^{-2}$$
$$\frac{d^2}{dx^2}(x^{-1}) = 2x^{-3} = \frac{2!}{x^3}$$
$$\frac{d^3}{dx^3}(x^{-1}) = -6x^{-4} = -\frac{3!}{x^4}$$
$$\frac{d^4}{dx^4}(x^{-1}) = 24x^{-5} = \frac{4!}{x^5}$$
$$\vdots$$
$$\frac{d^n}{dx^n}(x^{-1}) = (-1)^n \frac{n!}{x^{n+1}}$$

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57. $\frac{d^2}{dx^2}(f\cdot g)=\frac{d}{dx}\left(\frac{df}{dx}\cdot g+f\cdot\frac{dg}{dx}\right)\quad\left[\text{or }\frac{d}{dx}(f'\cdot g+f\cdot g')\right]$

$=\frac{d}{dx}\left(\frac{df}{dx}\cdot g\right)+\frac{d}{dx}\left(f\cdot\frac{dg}{dx}\right)\quad\left[\text{or }\frac{d}{dx}(f'\cdot g+\frac{d}{dx}(f\cdot g')\right]$

$\frac{d^2f}{dx^2}\cdot g+\frac{df}{dx}\cdot\frac{dg}{dx}+\frac{df}{dx}\cdot\frac{dg}{dx}+f\cdot\frac{d^2g}{dx^2}\quad[\text{or } f''\cdot g+f'\cdot g'+f'\cdot g'+f\cdot g'']$

$=f''\cdot g+2f'\cdot g'+f\cdot g''$

58. From Exercise 31, we know that $\frac{d^2}{dx^2}(f\cdot g)=f''\cdot g+2f'\cdot g'+f\cdot g''$.

$\frac{d^3}{dx^3}(f\cdot g)=\frac{d}{dx}\left[\frac{d^2}{dx^2}(f\cdot g)\right]=\frac{d}{dx}(f''\cdot g+2f'\cdot g'+f\cdot g'')=f'''\cdot g+f''\cdot g'+2f''\cdot g'+2f'\cdot g''+f'\cdot g''+f\cdot g'''$

$=f'''\cdot g+3f''\cdot g'+3f'\cdot g''+f\cdot g'''$

EXERCISES 2.6

1. $f\left(g(x)\right)=\sqrt{x^2-3x+1}$

The outside function is $\sqrt{x}$ and the inside function is x^2-3x+1.

Thus, we take $\begin{cases} f(x)=\sqrt{x} \\ g(x)=x^2-3x+1 \end{cases}$

2. $f(g(x))=(5x^2-x+2)^4$

The outside function is x^4 and the inside function is $5x^2-x+2$.

Thus, we take $\begin{cases} f(x)=x^4 \\ g(x)=5x^2-x+2 \end{cases}$

3. $f(g(x))=(x^2-x)^{-3}$

The outside function is x^{-3} and the inside function is x^2-x.

Thus, we take $\begin{cases} f(x)=x^{-3} \\ g(x)=x^2-x \end{cases}$

4. $f(g(x))=\frac{1}{x^2+x}$

The outside function is $\frac{1}{x}$ and the inside function is x^2+x.

Thus, we take $\begin{cases} f(x)=\frac{1}{x} \\ g(x)=x^2+x \end{cases}$

5. $f(g(x))=\frac{x^3+1}{x^3-1}$

$f(x)=\frac{x+1}{x-1}$

$g(x)=x^3$

6. $f\left(g(x)\right)=\frac{\sqrt{x}-1}{\sqrt{x}+1}$

$f(x)=\frac{x-1}{x+1}$

$g(x)=\sqrt{x}$

7. $f\left(g(x)\right)=\left(\frac{x+1}{x-1}\right)^4$

$f(x)=x^4$

$g(x)=\frac{x+1}{x-1}$

8. $f\left(g(x)\right)=\sqrt{\frac{x-1}{x+1}}$

$f(x)=\sqrt{x}$

$g(x)=\frac{x-1}{x+1}$

9. $f\left(g(x)\right)=\sqrt{x^2-9}+5$

$f(x)=\sqrt{x}+5$

$g(x)=x^2-9$

10. $f\left(g(x)\right)=\sqrt[3]{x^3+8}-5$

$f(x)=\sqrt[3]{x}-5$

$g(x)=x^3+8$

11. $f(x)=(x^2+1)^3$

$f'(x)=3(x^2+1)^2(2x)=6x(x^2+1)^2$

12. $f(x)=(x^3+1)^4$

$f'(x)=4(x^3+1)^3(3x^2)=12x^2(x^3+1)^3$

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13. $g(x)=\left(2x^2-7x+3\right)^4$
$g'(x)=4\left(2x^2-7x+3\right)^3(4x-7)$

14. $g(x)=\left(3x^3-x^2+1\right)^5$
$g'(x)=5\left(3x^3-x^2+1\right)^4(9x^2-2x)$

15. $h(z)=(3z^2-5z+2)^4$
$h'(z)=4(3z^2-5z+2)^3(6z-5)$

16. $h(z)=(5z^2+3z-1)^3$
$h'(z)=3(5z^2+3z-1)^2(10z+3)$

17. $f(x)=\sqrt{x^4-5x+1}=\left(x^4-5x+1\right)^{1/2}$
$f'(x)=\frac{1}{2}\left(x^4-5x+1\right)^{-1/2}\left(4x^3-5\right)$

18. $f(x)=\sqrt{x^6+3x-1}=\left(x^6+3x-1\right)^{1/2}$
$f'(x)=\frac{1}{2}\left(x^6+3x-1\right)^{-1/2}\left(6x^5+3\right)$

19. $w(z)=\sqrt[3]{9z-1}=(9z-1)^{1/3}$
$w'(z)=\frac{1}{3}(9z-1)^{-2/3}(9)=3(9z-1)^{-2/3}$

20. $w(z)=\sqrt[5]{10z-4}=(10z-4)^{1/5}$
$w'(z)=\frac{1}{5}(10z-4)^{-4/5}(10)=2(10z-4)^{-4/5}$

21. $y=(4-x^2)^4$
$y'=4(4-x^2)^3(-2x)=-8x(4-x^2)^3$

22. $y=(1-x)^{50}$
$y'=50(1-x)^{49}(-1)=-50(1-x)^{49}$

23. $y=\left(\frac{1}{w^3-1}\right)^4=[(w^3-1)^{-1}]^4=(w^3-1)^{-4}$
$y'=-4(w^3-1)^{-5}(3w^2)=-12w^2(w^3-1)^{-5}$

24. $y=\left(\frac{1}{w^4+1}\right)^5=[(w^4+1)^{-1}]^5=(w^4+1)^{-5}$
$y'=-5(w^4+1)^{-6}(4w^3)=-20w^3(w^4+1)^{-6}$

25. $y=x^4+(1-x)^4$
$y'=4x^3+4(1-x)^3(-1)=4x^3-4(1-x)^3$

26. $f(x)=(x^2+4)^3-(x^2+4)^2$
$f'(x)=3(x^2+4)^2(2x)-2(x^2+4)(2x)$
$=6x(x^2+4)^2-4x(x^2+4)$

27. $f(x)=\frac{1}{\sqrt{3x^2-5x+1}}=\left(3x^2-5x+1\right)^{-1/2}$
$f'(x)=-\frac{1}{2}\left(3x^2-5x+1\right)^{-3/2}(6x-5)$

28. $f(x)=\frac{1}{\sqrt{2x^2-7x+1}}=\left(2x^2-7x+1\right)^{-1/2}$
$f'(x)=-\frac{1}{2}\left(2x^2-7x+1\right)^{-3/2}(4x-7)$

29. $f(x)=\frac{1}{\sqrt[3]{(9x+1)^2}}=\frac{1}{(9x+1)^{2/3}}=(9x+1)^{-2/3}$
$f'(x)=-\frac{2}{3}(9x+1)^{-5/3}(9)=-6(9x+1)^{-5/3}$

30. $f(x)=\frac{1}{\sqrt[3]{(3x-1)^2}}=(3x-1)^{-2/3}$
$f'(x)=-\frac{2}{3}(3x-1)^{-5/3}(3)=-2(3x-1)^{-5/3}$

31. $f(x)=\frac{1}{\sqrt[3]{\left(2x^2-3x+1\right)^2}}=\left(2x^2-3x+1\right)^{-2/3}$
$f'(x)=-\frac{2}{3}\left(2x^2-3x+1\right)^{-5/3}(4x-3)$

32. $f(x)=\frac{1}{\sqrt[3]{\left(x^2+x-9\right)^2}}=\left(x^2+x-9\right)^{-2/3}$
$f'(x)=-\frac{2}{3}\left(x^2+x-9\right)^{-5/3}(2x+1)$

33. $f(x)=[(x^2+1)^3+x]^3$
$f'(x)=3[(x^2+1)^3+x]^2[3(x^2+1)^2(2x)+1]$
$=3[(x^2+1)^3+x]^2[6x(x^2+1)^2+1]$

34. $f(x)=[(x^3+1)^2-x]^4$
$f'(x)=4[(x^3+1)^2-x]^3[2(x^3+1)(3x^2)-1]$
$=4[(x^3+1)^2-x]^3[6x^2(x^3+1)-1]$

35. $f(x)=3x^2(2x+1)^5$
$f'(x)=6x(2x+1)^5+3x^2[5(2x+1)^4(2)]$
$=6x(2x+1)^5+30x^2(2x+1)^4$

36. $f(x)=2x(x^3-1)^4$
$f'(x)=2(x^3-1)^4+2x[4(x^3-1)^3(3x^2)]$
$=2(x^3-1)^4+24x^3(x^3-1)^3$

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37. $f(x)=(2x+1)^3(2x-1)^4$

$f'(x)=3(2x+1)^2(2)(2x-1)^4+(2x+1)^3[4(2x-1)^3(2)]$

$=6(2x+1)^2(2x-1)^4+8(2x+1)^3(2x-1)^3$

38. $f(x)=(2x-1)^3(2x+1)^4$

$f'(x)=3(2x-1)^2(2)(2x+1)^4+(2x-1)^3[4(2x+1)^3(2)]$

$=6(2x-1)^2(2x+1)^4+8(2x-1)^3(2x+1)^3$

39. $g(z)=2z\left(3z^2-z+1\right)^4$

$g'(z)=2\left(3z^2-z+1\right)^4+2z(4)\left(3z^2-z+1\right)^3(6z-1)$

40. $g(z)=z^2\left(2z^3-z+5\right)^4$

$g'(z)=2z\left(2z^2-z+5\right)^4+2^2(4)\left(2z^2-z+5\right)^3\left(6z^2-1\right)$

$=2z\left(2z^2-z+5\right)^3\left(14z^3-3z+5\right)$

41. $f(x)=\left(\frac{x+1}{x-1}\right)^3$

$f'(x)=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{(x-1)(1)-(1)(x+1)}{(x-1)^2}\right]$

$=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{x-1-x-1}{(x-1)^2}\right]=3\left(\frac{x+1}{x-1}\right)^2\left[\frac{-2}{(x-1)^2}\right]$

$=-6\frac{(x+1)^2}{(x-1)^4}$

42. $f(x)=\left(\frac{x-1}{x+1}\right)^5$

$f'(x)=5\left(\frac{x-1}{x+1}\right)^4\left[\frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}\right]$

$=5\left(\frac{x-1}{x+1}\right)^4\left[\frac{x+1-x+1}{(x+1)^2}\right]=10\frac{(x-1)^4}{(x+1)^6}$

43. $f(x)=x^2\sqrt{1+x^2}=x^2\left(1+x^2\right)^{1/2}$

$f'(x)=2x\left(1+x^2\right)^{1/2}+x^2\left[\frac{1}{2}\left(1+x^2\right)^{-1/2}(2x)\right]$

$=2x\left(1+x^2\right)^{1/2}+x^3\left(1+x^2\right)^{-1/2}$

44. $f(x)=x^2\sqrt{x^2-1}=x^2\left(x^2-1\right)^{1/2}$

$f'(x)=2x\left(x^2-1\right)^{1/2}+x^2\left[\frac{1}{2}\left(x^2-1\right)^{-1/2}(2x)\right]$

$=2x\left(x^2-1\right)^{1/2}+x^3\left(x^2-1\right)^{-1/2}$

45. $f(x)=\sqrt{1+\sqrt{x}}=\left(1+x^{1/2}\right)^{1/2}$

$f'(x)=\frac{1}{2}\left(1+x^{1/2}\right)^{-1/2}\left(\frac{1}{2}x^{-1/2}\right)$

$=\frac{1}{4}x^{-1/2}\left(1+x^{1/2}\right)^{-1/2}$

46. $f(x)=\sqrt[3]{1+\sqrt[3]{x}}=\left(1+x^{1/3}\right)^{1/3}$

$f'(x)=\frac{1}{3}\left(1+x^{1/3}\right)^{-2/3}\left(\frac{1}{3}x^{-2/3}\right)$

$=\frac{1}{9}x^{-2/3}\left(1+x^{1/3}\right)^{-2/3}$

47. $f(x)=(2x+1)^4$

$f(-1)=(2(-1)+1)^4=1$

The point is (–1, 1).

$f'(x)=4(2x+1)^3(2)=8(2x+1)^3$

$f'(-1)=8(2(-1)+1)^3=-8$

The tangent line has slope –8 at (–1, 1).

$y-1=-8(x+1)$

$y=-8x-7$

48. $f(x)=\sqrt{x^2+3}=\left(x^2+3\right)^{1/2}$

$f(1)=\sqrt{(1)^2+3}=2$

The point is (1, 2).

$f'(x)=\frac{1}{2}\left(x^2+3\right)^{-1/2}(2x)=\frac{2x}{2\sqrt{x^2+3}}$

$f'(1)=\frac{2(1)}{2\sqrt{(1)^2+3}}=\frac{1}{2}$

The tangent line has slope $\frac{1}{2}$ at (1, 2).

$y-2=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x+\frac{3}{2}$

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49.
$$f(x)=\left[(x-1)^2-x\right]^2$$
$$f(2)=\left[(2-1)^2-2\right]^2=1$$
The point is (2, 1).
$$f'(x)=2\left[(x-1)^2-x\right]\left[2(x-1)(1)-1\right]$$
$$=2\left[(x-1)^2-x\right](2x-3)$$
$$f'(2)=2\left[(2-1)^2-2\right](2(2)-3)=-2$$
The tangent line has slope –2 at (2, 1).
$$y-1=-2(x-2)$$
$$y=-2x+5$$

50.
$$f(x)=(2x+3)^3+(3x+2)^2+4x-5$$
$$f(-2)=(2(-2)+3)^3+(3(-2)+2)^2+4(-2)-5$$
$$=2$$
The point is (–2, 2).
$$f'(x)=3(2x+3)^2(2)+2(3x+2)(3)+4$$
$$=6(2x+3)^2+6(3x+2)+4$$
$$f'(-2)=6(2(-2)+3)^2+6(3(-2)+2)+4$$
$$=-14$$
The tangent line has slope –14 at (–2, 2).
$$y-2=-14(x+2)$$
$$y=-14x-26$$

51. **a.** $\frac{d}{dx}[(x^2+1)^2]=2(x^2+1)(2x)=4x^3+4x$

b. $\frac{d}{dx}[(x^2+1)^2]=\frac{d}{dx}(x^4+2x^2+1)=4x^3+4x$

52. **a.** $\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{x^2\cdot 0-(2x)(1)}{(x^2)^2}=\frac{-2x}{x^4}=-\frac{2}{x^3}$

b. $\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}[(x^2)^{-1}]=-1(x^2)^{-2}(2x)$
$$=-\frac{2}{x^3}$$

c. $\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{d}{dx}(x^{-2})=-2x^{-3}=-\frac{2}{x^3}$

53. **a.** $\frac{d}{dx}\left(\frac{1}{3x+1}\right)=\frac{(3x+1)0-3(1)}{(3x+1)^2}=-\frac{3}{(3x+1)^2}$

b. $\frac{d}{dx}[(3x+1)^{-1}]=-(3x+1)^{-2}(3)$
$$=-\frac{3}{(3x+1)^2}$$

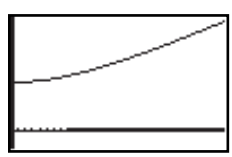
54. $\frac{d}{dx}f(g(h(x)))=f'(g(h(x)))g'(h(x))h'(x)$

55.
$$f(x)=(x^2+1)^{10}$$
$$f'(x)=10(x^2+1)^9(2x)=20x(x^2+1)^9$$
$$f''(x)=20(x^2+1)^9+20x[9(x^2+1)^8(2x)]$$
$$=20(x^2+1)^9+40x^2[9(x^2+1)^8]$$
$$=20(x^2+1)^9+360x^2(x^2+1)^8$$

56.
$$f(x)=(x^3-1)^5$$
$$f'(x)=5(x^3-1)^4(3x^2)=15x^2(x^3-1)^4$$
$$f''(x)=30x(x^3-1)^4+15x^2[4(x^3-1)^3(3x^2)]$$
$$=30x(x^3-1)^4+180x^4(x^3-1)^3$$

57. The marginal cost function is the derivative of the function $C(x)=\sqrt{4x^2+900}=(4x^2+900)^{1/2}$.
$$MC(x)=\tfrac{1}{2}(4x^2+900)^{-1/2}(8x)$$
$$=4x(4x^2+900)^{-1/2}$$
$$MC(20)=4(20)\left[4(20)^2+900\right]^{-1/2}$$
$$=80\left[4(400)+900\right]^{-1/2}$$
$$=80(2500)^{-1/2}=\frac{80}{50}=1.60$$

58.



on [0, 30] by [–10, 70]

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59. $x = 27$

60. $S(e) = 0.22(e+4)^{2.1}$

$S'(e) = 0.462(e+4)^{1.1}$

$S'(12) = 0.462(12+4)^{1.1} \approx 9.75$

At a level of 12 units, a person's social status increases by about 9.75 units per additional year of education.

61. $S(i) = 17.5(i-1)^{0.53}$

$S'(i) = 9.275(i-1)^{-0.47}$

$S'(25) = 9.275(25-1)^{-0.47} \approx 2.08$

At an income of \$25,000 social status increases by about 2.08 units per additional \$1000 of income.

62. $V(r) = 1000(1+0.01r)^5$

$V'(r) = 5000(1+0.01r)^4(0.01)$

$= 50(1+0.01r)^4$

$V'(6) = 50[1+0.01(6)]^4 \approx 63.12$

At a rate of 6%, the value increases by about \$63.12 for each additional percentage point of interest.

63. $R(x) = 4x\sqrt{11+0.5x} = 4x(11+0.5x)^{1/2}$

$R'(x) = 4x\left(\frac{1}{2}\right)(11+0.5x)^{-1/2}(0.5) + 4(11+0.5x)^{1/2}$

$= \frac{x}{\sqrt{11+0.5x}} + 4\sqrt{11+0.5x}$

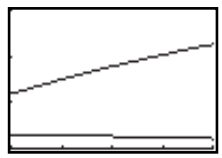
The sensitivity to a dose of 50 mg is

$R'(50) = \frac{x}{\sqrt{11+0.5(50)}} + 4\sqrt{11+0.5(50)}$

$= \frac{50}{\sqrt{11+25}} + 4\sqrt{11+25} = \frac{50}{\sqrt{36}} + 4\sqrt{36}$

$= \frac{50}{6} + 24 = \frac{25+72}{3} = \frac{97}{3} = 32\frac{1}{3}$

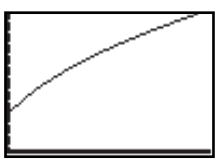
64.



on [1, 5] by [0, 3]

$x = 3.6$ years

65.



on [0, 140] by [0, 50]

$x \approx 26$ mg

66. $R(x) = 0.25(1+x)^4$

$R'(x) = 4(0.25)(1+x)^4 = (1+x)^3$

a. $R'(0) = (1+0)^3 = 1$

b. $R'(1) = (1+1)^3 = 8$

67. $P(t) = 0.02(12+2t)^{3/2} + 1$

$P'(t) = 0.03(12+2t)^{1/2}(2)$

$= 0.06(12+2t)^{1/2}$

$P'(2) = 0.06[12+2(2)]^{1/2} = 0.24$

68. $T(p) = 36(p+1)^{-1/3}$

$T'(p) = -\frac{1}{3}(36)(p+1)^{-4/3} = -12(p+1)^{-4/3}$

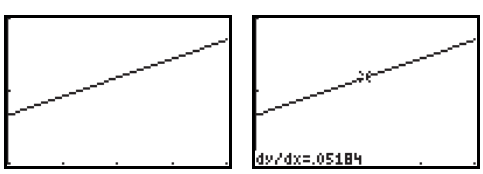
$T'(7) = -12(7+1)^{-4/3} = -\frac{3}{4}$

The time is decreasing by about $\frac{3}{4}$ minute for each additional practice session.

c. $\frac{1.8}{x} = 0.05184$

$x \approx 34.7$, so about 35 years

69. **b.**



on [0, 20] by [60, 62]

slope = 0.05184

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70. $h(40) = 8.2 - (0.01(40) - 2.8)^2 = 2.44$
$h'(x) = -2(0.01t - 2.8)(0.01) = -0.0002t + 0.056$
$h'(40) = 0.048$
At a temperature of 40 degrees, happiness approximately 2.4, and each additional degree of temperature would raise happiness by about 0.05.

71. False: There should not be a prime of the first g on the right-hand side.

72. False: There should not be a prime of the first g on the right-hand side.

73. The generalized power rule is a special case of the chain rule, when the outer function is just a power of the variable.

74. The power rule has only x as the inner function, but the generalized power rule can have any inner function.

75. True: $\frac{d}{dx} f(5x) = \frac{d}{dx} 5x \cdot f'(5x) = 5 \cdot f'(5x)$

76. True: $\frac{d}{dx} f\left(\frac{x}{2}\right) = \frac{d}{dx}\frac{x}{2} \cdot f'\left(\frac{x}{2}\right) = \frac{f'\left(\frac{x}{2}\right)}{2}$

77. False: The outer function, $\sqrt{x}$, was not differentiated. The correct right-hand side is $\frac{1}{2}[g(x)]^{-1/2} \cdot g'(x)$.

78. True: $\frac{d}{dx} f(x+5) = f'(x+5) \cdot (1) = f'(x+5)$

79. No: Since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^2 = [f'(x)]^2$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^2$.

80. No: since instantaneous rates of change are derivatives, this would be saying that $\frac{d}{dx}[f(x)]^3 = [f'(x)]^3$, where $f(x)$ is the length of a side. The chain rule gives the correct derivative of $[f(x)]^3$.

81. $\frac{d}{dx} L(g(x)) = L'(g(x))g'(x)$
But since $L'(x) = \frac{1}{x}$, $L'(g(x)) = \frac{1}{g(x)}$.
Thus, $\frac{d}{dx} L(g(x)) = \frac{g'(x)}{g(x)}$.

82. $\frac{d}{dx} E(g(x)) = E'(g(x)) \cdot g'(x)$
Since $E'(x) = E(x)$, $E'(g(x)) = E(g(x))$.
Thus, $\frac{d}{dx} E(g(x)) = E(g(x)) \cdot g'(x)$.

83. Let $G(x) = g(h(x))$, then the chain rule gives $G'(x) = g'(h(x)) \cdot h'(x)$. We use the chain rule to find $\frac{d}{dx} f(G(x))$ and substitute the expressions for $G(x)$ and $G'(x)$.

$$\begin{aligned}\frac{d}{dx} f(G(x)) &= f'(G(x)) \cdot G'(x) \\ &= f'(g(h(x)) \cdot g'(h(x)) \cdot h'(x)\end{aligned}$$

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84. **a.** Use $\frac{d}{dx}f(g(h(x)) = f'(g(h(x))\cdot g'(h(x))\cdot h'(x)$ to find $\frac{d}{dx}((x^2)^3)^4$.

$$\frac{d}{dx}((x^2)^3)^4 = 4((x^2)^3)^3\cdot 3(x^2)^2\cdot 2x$$
$$= 4x^{18}\cdot 3x^4\cdot 2x$$
$$= 24x^{23}$$

b. $((x^2)^3)^4 = (x^6)^4 = x^{24}$

$\frac{d}{dx}x^{24} = 24x^{23}$

which is the same result as in part (a)

85. Solve $f(x+h) - f(x) = F(h)\cdot h$ for $F(h)$ to find $F(h) = \frac{f(x+h)-f(x)}{h}$ and use the continuity of F at 0 to obtain

$F(0) = \lim_{h\to 0} F(h) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$,

which is the definition of the derivative·

86. **a.** Apply Caratheodory's definition since g is differentiable.

b. Apply Caratheodory's definition since f is differentiable.

c. In the first step add and subtract $g(x)$.

In the second step, use the associative property to group $g(x+h) - g(x)$.

In the third step, use Caratheodory's definition as given in part (b), where "h" is $g(x+h) - g(x)$.

In the last step, use Caratheodory's definition as given in part (a).

d. The derivative of $f(g(x))$,

$$\frac{d}{dx}f(g(x))$$
$$= \lim_{h\to 0}\frac{F(g(x+h)-g(x))\cdot G(h)\cdot h}{h}$$
$$= \lim_{h\to 0}\left[F(g(x+h)-g(x))\cdot G(h)\right] \quad \text{Remove a factor of } h$$
$$= F(g(x+0)-g(x))\cdot G(0) \quad \text{Apply the limit}$$
$$= F(0)\cdot G(0) \quad \text{Simplify } g(x)-g(x)=0$$
$$= f'(g(x))\cdot g'(x) \quad \text{Substitute } F(0) = f'(g(x)) \text{ and } G(0) = g'(x)$$

EXERCISES 2.7

1. The derivative does not exist at the corner points $x = -2,\ 0,\ 2$.

2. The derivative does not exist at the discontinuous points $x = -3,\ 0,\ 3$.

3. The derivative does not exist at the discontinuous points $x = -3,\ 3$.

4. The derivative does not exist at the vertical tangents at $x = -4,\ -2,\ 2,\ 4$.

5. For positive h, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|2x+2h|-|2x|}{h}$. For $x = 0$, this becomes

$\lim_{h\to 0}\frac{|0+2h|-|0|}{h} = \frac{|2h|}{h} = 2$ because h is positive. For h negative,

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|2x+2h|-|2x|}{h}$. Since $x = 0$, we get

$\lim_{h\to 0}\frac{|0+2h|-|0|}{h} = \frac{|2h|}{h} = -2$. Thus, the derivative does not exist.

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6. For positive h and $x = 0$, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|0+3h|-|0|}{h} = \frac{3h}{h} = 3$.

For negative h and $x = 0$, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|0+3h|-|0|}{h} = \frac{-3h}{h} = -3$.

Thus, the derivative does not exist.

7. For $x = 0$, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(0+h)^{2/5}-0}{h} = \lim_{h\to 0}\frac{h^{2/5}}{h} = \lim_{h\to 0}\frac{1}{h^{3/5}}$ which does not exist. Thus, the derivative does not exist at $x = 0$.

8. For $x = 0$, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(0+h)^{4/5}-0}{h} = \lim_{h\to 0}\frac{h^{4/5}}{h} = \lim_{h\to 0}\frac{1}{h^{1/5}}$ which does not exist.
Thus, the derivative does not exist at $x = 0$.

9. If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

10. If you get a numerical answer, it is wrong because the function is undefined at $x = 0$. Thus, the derivative at $x = 0$ does not exist.

11. **a.** For $x = 0$,

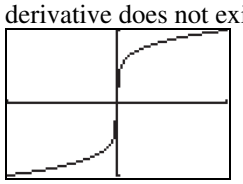
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt[5]{0+h}-\sqrt[5]{0}}{h} = \lim_{h\to 0}\frac{\sqrt[5]{h}}{h}$$

b.

h	$\frac{\sqrt[5]{h}}{h}$
0.1	6.3
0.001	251.2
0.00001	10,000

c. No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.



On [–1, 1] by [–1, 1]

12. **a.** For $x = 0$,

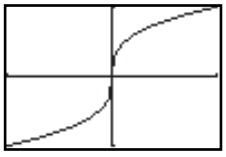
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt[3]{0+h}-\sqrt[3]{0}}{h} = \lim_{h\to 0}\frac{\sqrt[3]{h}}{h}$$

b.

h	$\frac{\sqrt[3]{h}}{h}$
0.1	4.64
0.0001	464.16
0.0000001	46,415.89

c. No, the limit does not exist. No, the derivative does not exist at $x = 0$.

d.



on [–1, 1] by [–1, 1]

13. At a corner point, a proposed tangent line can tip back and forth and so there is no well-defined slope.

14. A vertical tangent has an undefined slope, or derivative.

15. At a discontinuity, the values of the function take a sudden jump, and so a (steady) rate of change cannot be defined.

16. False: A function can have a corner or vertical tangent and still be continuous.

17. True: For a function to be differentiable, it cannot jump or break.

18. False: A function could have a vertical tangent.

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REVIEW EXERCISES AND CHAPTER TEST FOR CHAPTER 2

1.

x	$4x+2$
1.9	9.6
1.99	9.96
1.999	9.996

x	$4x+2$
2.1	10.4
2.01	10.04
2.001	10.004

a. $\lim_{x\to 2^-} 4x+2=10$

b. $\lim_{x\to 2^+} 4x+2=10$

c. $\lim_{x\to 2} 4x+2=10$

2.

x	$\frac{\sqrt{x+1}-1}{x}$
–0.1	0.513
–0.01	0.501
–0.0001	0.500

x	$\frac{\sqrt{x+1}-1}{x}$
0.1	0.488
0.01	0.499
0.001	0.500

a. $\lim_{x\to 0^-} \frac{\sqrt{x+1}-1}{x}=0.5$

b. $\lim_{x\to 0^+} \frac{\sqrt{x+1}-1}{x}=0.5$

c. $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}=0.5$

3. **a.** $\lim_{x\to 5^-} f(x)=\lim_{x\to 5^-}(2x-7)$
$=2(5)-7)=3$

b. $\lim_{x\to 5^+} f(x)=\lim_{x\to 5+}(3-x)$
$=3-5=-2$

c. $\lim_{x\to 5} f(x)$ does not exist.

4. **a.** $\lim_{x\to 5^-} f(x)=\lim_{x\to 5^-}(4-x)$
$=4-5=-1$

b. $\lim_{x\to 5^+} f(x)=\lim_{x\to 5^+}(2x-11)$
$=2(5)-11=-1$

c. $\lim_{x\to 5} f(x)=-1$

5. $\lim_{x\to 4}\sqrt{x^2+x+5}=\sqrt{4^2+4+5}=\sqrt{16+9}$
$=\sqrt{25}=5$

6. $\lim_{x\to 0}\pi=\pi$

7. $\lim_{s\to 16}\left(\frac{1}{2}s-s^{1/2}\right)=\frac{1}{2}(16)-16^{1/2}=8-4=2$

8. $\lim_{r\to 8}\frac{r}{r^2-30\sqrt[3]{r}}=\frac{8}{8^2-30\sqrt[3]{8}}=\frac{8}{64-30(2)}=2$

9. $\lim_{x\to 1}\frac{x^2-x}{x^2-1}=\lim_{x\to 1}\frac{x(x-1)}{(x+1)(x-1)}$
$=\lim_{x\to 1}\frac{x}{x+1}=\frac{1}{2}$

10. $\lim_{x\to -1}\frac{3x^3-3x}{2x^2+2x}=\lim_{x\to -1}\frac{3x(x^2-1)}{2x(x+1)}$
$=\lim_{x\to -1}\frac{3x(x-1)(x+1)}{2x(x+1)}$
$=\lim_{x\to -1}\frac{3(x-1)}{2}=\frac{3(-1-1)}{2}=-3$

11. $\lim_{h\to 0}\frac{2x^2h-xh^2}{h}=\lim_{h\to 0}\frac{xh(2x-h)}{h}$
$=\lim_{h\to 0}x(2x-h)$
$=x(2x-0)=2x^2$

12. $\lim_{h\to 0}\frac{6xh^2-x^2h}{h}=\lim_{h\to 0}\frac{xh(6h-x)}{h}$
$=\lim_{h\to 0}x(6h-x)$
$=x(0-x)=-x^2$

13. $\lim_{x\to -\infty} f(x)=0$; $\lim_{x\to -2^-} f(x)=\infty$ and $\lim_{x\to -2^+} f(x)=\infty$,
so $\lim_{x\to -2} f(x)=\infty$ and $\lim_{x\to\infty} f(x)=0$.

14. $\lim_{x\to -\infty} f(x)=3$; $\lim_{x\to 2^-} f(x)=-\infty$ and $\lim_{x\to 2^+} f(x)=\infty$,
so $\lim_{x\to 2} f(x)$ does not exist and $\lim_{x\to\infty} f(x)=3$.

15. Continuous

16. Continuous

17. Discontinuous at $x=-1$

18. Continuous

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19. The function is discontinuous at values of x for which the denominator is zero. Thus, we consider $x^2 + x = 0$ and solve.

$$x^2 + x = 0$$
$$x(x+1) = 0$$
$$x = 0, -1$$

Discontinuous at $x = 0, -1$

20. Discontinuous at $x = -3, 3$

21. From Exercise 3, we know $\lim_{x\to 5} f(x)$ does not exist. Therefore, the function is discontinuous at $x = 5$.

22. From Exercise 4, we know $\lim_{x\to 5} f(x) = -1 = f(5)$. Therefore, the function is continuous.

23.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-1-(2x^2+3x-1)}{h} = \lim_{h\to 0}\frac{2(x^2+2xh+h^2)+3x+3h-1-2x^2-3x+1}{h}$$
$$= \lim_{h\to 0}\frac{2x^2+4xh+2h^2+3x-3x+3h-1+1-2x^2}{h} = \lim_{h\to 0}\frac{4xh+2h^2+3h}{h} = \lim_{h\to 0} 4x+2h+3 = 4x+3$$

24.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2+2(x+h)-3-(3x^2+2x-3)}{h} = \lim_{h\to 0}\frac{3x^2+6xh+3h^2+2x+2h-3-3x^2-2x+3}{h}$$
$$= \lim_{h\to 0}\frac{6xh+3h^2+2h}{h} = \lim_{h\to 0} 6x+3h+2 = 6x+2$$

25.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h} = \lim_{h\to 0}\frac{\frac{3x-3(x+h)}{x(x+h)}}{h} = \lim_{h\to 0}\frac{3x-3x-3h}{x(x+h)}\cdot\frac{1}{h} = \lim_{h\to 0}\frac{-3h}{x(x+h)h} = \lim_{h\to 0} -\frac{3}{x(x+h)} = -\frac{3}{x^2}$$

26.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{4\sqrt{x+h}-4\sqrt{x}}{h} = \lim_{h\to 0} 4\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} 4\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$= \lim_{h\to 0} 4\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0}\frac{4}{\sqrt{x+h}+\sqrt{x}} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$$

27.
$$f(x) = 6\sqrt[3]{x^5} - \frac{4}{\sqrt{x}} + 1 = 6x^{5/3} - 4x^{-1/2} + 1$$
$$f'(x) = \frac{5}{3}\left(6x^{2/3}\right) - \left(-\frac{1}{2}\right)\left(4x^{-3/2}\right) + 0$$
$$= 10x^{2/3} + 2x^{-3/2}$$

28.
$$f(x) = 4\sqrt{x^5} - \frac{6}{\sqrt[3]{x}} + 1 = 4x^{5/2} - 6x^{-1/3} + 1$$
$$f'(x) = \frac{5}{2}\left(4x^{3/2}\right) - \left(-\frac{1}{3}\right)\left(6x^{-4/3}\right) + 0$$
$$= 10x^{3/2} + 2x^{-4/3}$$

29.
$$f(x) = \frac{1}{x^2} = x^{-2}$$
$$f'(x) = -2x^{-3}$$
$$f'\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^{-3} = -2(2)^3 = -16$$

30.
$$f(x) = \frac{1}{x} = x^{-1}$$
$$f'(x) = -x^{-2}$$
$$f'\left(\frac{1}{3}\right) = -\left(\frac{1}{3}\right)^{-2} = -(3)^2 = -9$$

31.
$$f(x) = 12\sqrt[3]{x} = 12x^{1/3}$$
$$f'(x) = \frac{1}{3}\left(12x^{-2/3}\right) = 4x^{-2/3}$$
$$f'(8) = 4(8)^{-2/3} = 4\left(\frac{1}{8}\right)^{2/3} = 4\left(\frac{1}{8^{2/3}}\right)$$
$$= 4\left(\frac{1}{\sqrt[3]{8^2}}\right) = 4\left(\frac{1}{\sqrt[3]{64}}\right) = 1$$

32.
$$f(x) = 6\sqrt[3]{x} = 6x^{1/3}$$
$$f'(x) = \frac{1}{3}\left(6x^{-2/3}\right) = 2x^{-2/3}$$
$$f'(-8) = 2(-8)^{-2/3} = 2\left(-\frac{1}{8}\right)^{2/3} = 2\left(\frac{1}{\sqrt[3]{(-8)^2}}\right) = \frac{1}{2}$$

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33. $y = \frac{1}{x} + x^2 = x^{-1} + x^2$

$y(1) = \frac{1}{1} + 1^2 = 2$

The point is (1, 2).

$y' = -1x^{-2} + 2x = -\frac{1}{x^2} + 2x$

$y'(1) = -\frac{1}{(1)^2} + 2(1) = 1$

The tangent line has slope 1 at (1, 2).

$y - 2 = 1(x - 1)$

$y = x + 1$

34. **a.** $C'(x) = 20x^{-1/6}$

$C'(1) = \frac{20}{\sqrt[6]{1}} = 20$

Costs are increasing by about \$20 per additional license.

b. $C'(64) = \frac{20}{\sqrt[6]{64}} = \frac{20}{2} = 10$

Costs are increasing by about \$10 per additional license.

35. $f(x) = 150x^{-0.322}$

$f'(x) = -0.322(150x^{-1.322}) = -48.3x^{-1.322}$

$f'(10) = -48.3(10)^{-1.322} \approx -2.3$

After 10 planes, construction time is decreasing by about 2300 hours for each additional plane built.

36. **a.** $A = \pi r^2$

$A' = 2\pi r$

b. As the radius increases, the area grows by "a circumference."

37. **a.** $V = \frac{4}{3}\pi r^3$

$A' = 3\left(\frac{4}{3}\pi r^2\right) = 4\pi r^2$

b. As the radius increases, the volume grows by "a surface area."

38. $f(x) = 2x(5x^3 + 3)$

$f'(x) = 2(5x^3 + 3) + 2x(15x^2)$

$= 2(5x^3 + 3) + 30x^3 = 40x^3 + 6$

39. $f(x) = x^2(3x^3 - 1)$

$f'(x) = 2x(3x^3 - 1) + x^2(9x^2)$

$= 2x(3x^3 - 1) + 9x^4 = 15x^4 - 2x$

40. $f(x) = (x^2 + 5)(x^2 - 5)$

$f'(x) = 2x(x^2 - 5) + (x^2 + 5)(2x)$

$= 2x^3 - 10x + 2x^3 + 10x = 4x^3$

41. $f(x) = (x^2 + 3)(x^2 - 3)$

$f'(x) = 2x(x^2 - 3) + (x^2 + 3)(2x)$

$= 2x^3 - 6x + 2x^3 + 6x = 4x^3$

42. $y = (x^4 + x^2 + 1)(x^5 - x^3 + x)$

$y' = (4x^3 + 2x)(x^5 - x^3 + x) + (x^4 + x^2 + 1)(5x^4 - 3x^2 + 1)$

$= 4x^8 - 4x^6 + 4x^4 + 2x^6 - 2x^4 + 2x^2 + 5x^8 - 3x^6 + x^4 + 5x^6 - 3x^4 + x^2 + 5x^4 - 3x^2 + 1$

$= 9x^8 + 5x^4 + 1$

43. $y = (x^5 + x^3 + x)(x^4 - x^2 + 1)$

$y' = (5x^4 + 3x^2 + 1)(x^4 - x^2 + 1) + (x^5 + x^3 + x)(4x^3 - 2x)$

$= 5x^8 - 5x^6 + 5x^4 + 3x^6 - 3x^4 + 3x^2 + x^4 - x^2 + 1 + 4x^8 + 4x^6 + 4x^4 - 2x^6 - 2x^4 - 2x^2$

$= 5x^8 - 2x^6 + 3x^4 + 2x^2 + 1 + 4x^8 + 2x^6 + 2x^4 - 2x^2$

$9x^8 + 5x^4 + 1$

44. $y = \frac{x-1}{x+1}$

$y' = \frac{(x+1)(1) - (1)(x-1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$

45. $y = \frac{x+1}{x-1}$

$y' = \frac{(x-1)(1) - (1)(x+1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = -\frac{2}{(x-1)^2}$

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46. $y = \dfrac{x^5+1}{x^5-1}$

$$y' = \frac{(x^5-1)(5x^4)-(5x^4)(x^5+1)}{(x^5-1)^2}$$

$$= \frac{5x^9-5x^4-5x^9-5x^4}{(x^5-1)^2} = -\frac{10x^4}{(x^5-1)^2}$$

47. $y = \dfrac{x^6-1}{x^6+1}$

$$y' = \frac{(x^6+1)(6x^5)-(6x^5)(x^6-1)}{(x^6+1)^2}$$

$$= \frac{6x^{11}+6x^5-6x^{11}+6x^5}{(x^6+1)^2} = \frac{12x^5}{(x^6+1)^2}$$

48. **a.** $f(x) = \dfrac{2x+1}{x}$

$$f'(x) = \frac{x(2)-(1)(2x+1)}{x^2}$$

$$= \frac{2x-2x-1}{x^2} = -\frac{1}{x^2}$$

b. $f(x) = (2x+1)(x^{-1})$

$$f'(x) = 2(x^{-1}) + (2x+1)(-1)x^{-2}$$

$$= \frac{2}{x} - \frac{2x+1}{x^2} = \frac{2x}{x^2} - \frac{2x+1}{x^2} = -\frac{1}{x^2}$$

c. Dividing $2x+1$ by x, we get

$$f(x) = 2 + \frac{1}{x} = 2 + x^{-1}$$

$$f'(x) = 0 + (-1)x^{-2} = -\frac{1}{x^2}$$

49. $y = \dfrac{x^3-1}{x^4+1}$

$$y(-1) = \frac{(-1)^3-1}{(-1)^4+1} = -1$$

The point is $(-1, -1)$.

$$y' = \frac{(x^4+1)(3x^2)-(4x^3)(x^3-1)}{(x^4+1)^2}$$

$$= \frac{-x^6+4x^3+3x^2}{(x^4+1)^2}$$

$$y'(-1) = \frac{-(-1)^6+4(-1)^3+3(-1)^2}{((-1)^4+1)^2} = \frac{-1-4+3}{4}$$

$$= -\frac{1}{2}$$

The tangent line has slope $-\dfrac{1}{2}$ at $(-1, -1)$.

$$y+1 = -\frac{1}{2}(x+1)$$

$$y = -\frac{1}{2}x - \frac{3}{2}$$

50. $S(x) = \dfrac{2250}{x+9} = 2250(x+9)^{-1}$

$$S'(x) = -2250(x+9)^{-2} = -\frac{2250}{(x+9)^2}$$

$$S'(6) = -\frac{2250}{(6+9)^2 = -10}$$

At \$6 per flash drive, the number of drives sold is decreasing by 10 per dollar increase in price.

51. **a.** To find the average profit function, divide $P(x) = 6x - 200$ by x.

$$AP(x) = \frac{6x-200}{x}$$

b. To find the marginal average profit, take the derivative of $AP(x)$.

$$MAP(x) = \frac{x(6)-(1)(6x-200)}{x^2} = \frac{200}{x^2}$$

c. $MAP(10) = \dfrac{200}{(10)^2} = \dfrac{200}{100} = 2$

Average profit is increasing by about \$2 for each additional unit.

52. **a.** $AC(x) = \dfrac{C(x)}{x} = \dfrac{7.5x+50}{x} = 7.5 + \dfrac{50}{x}$

b. $MAC(x) = \dfrac{d}{dx}\left(7.5 + \dfrac{50}{x}\right) = \dfrac{-50}{x^2}$

c. $MAC(50) = \dfrac{-50}{50^2} = -\dfrac{1}{50} = -0.02$

Average cost is decreasing by about \$0.02 per additional mouse.

53. $f(x) = 12\sqrt{x^3} - 9\sqrt[3]{x} = 12x^{3/2} - 9x^{1/3}$

$$f'(x) = \frac{3}{2}(12x^{1/2}) - \frac{1}{3}(9x^{-2/3}) = 18x^{1/2} - 3x^{-2/3}$$

$$f''(x) = \frac{1}{2}(18x^{-1/2}) - \frac{2}{3}(-3x^{-5/3})$$

$$= 9x^{-1/2} + 2x^{-5/3}$$

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54. $f(x) = 18\sqrt[3]{x^2} - 4\sqrt{x^3} = 18x^{2/3} - 4x^{3/2}$
$f'(x) = 12x^{-1/3} - 6x^{1/2}$
$f''(x) = -4x^{-4/3} - 3x^{-1/2}$

55. $f(x) = \frac{1}{3x^2} = \frac{1}{3}x^{-2}$
$f'(x) = -2\left(\frac{1}{3}x^{-3}\right) = -\frac{2}{3}x^{-3}$
$f''(x) = -3\left(-\frac{2}{3}x^{-4}\right) = 2x^{-4}$

56. $f(x) = \frac{1}{2x^3} = \frac{1}{2}x^{-3}$
$f'(x) = -\frac{3}{2}x^{-4}$
$f''(x) = 6x^{-5}$

57. $f(x) = \frac{2}{x^3} = 2x^{-3}$
$f'(x) = -3\left(2x^{-4}\right) = -6x^{-4} = -\frac{6}{x^4}$
$f''(x) = -4\left(-6x^{-5}\right) = 24x^{-5} = \frac{24}{x^5}$
$f''(-1) = \frac{24}{(-1)^5} = -24$

58. $f(x) = \frac{3}{x^4} = 3x^{-4}$
$f'(x) = -12x^{-5}$
$f''(x) = 60x^{-6} = \frac{60}{x^6}$
$f''(-1) = \frac{60}{(-1)^6} = 60$

59. $\frac{d}{dx}x^6 = 6x^5$
$\frac{d^2}{dx^2}x^6 = \frac{d}{dx}6x^5 = 30x^4$
$\left.\frac{d^2}{dx^2}\right|_{x=-2} x^6 = 30(-2)^4 = 480$

60. $\frac{d}{dx}x^{-2} = -2x^{-3}$
$\frac{d^2}{dx^2}x^{-2} = \frac{d}{dx}(-2x^{-3}) = 6x^{-4} = \frac{6}{x^4}$
$\left.\frac{d^2}{dx^2}\right|_{x=-2} x^{-2} = \frac{6}{(-2)^4} = \frac{6}{16} = \frac{3}{8}$

61. $\frac{d}{dx}\sqrt{x^5} = \frac{d}{dx}x^{5/2} = \frac{5}{2}x^{3/2}$
$\frac{d^2}{dx^2}\sqrt{x^5} = \frac{d}{dx}\left(\frac{5}{2}x^{3/2}\right) = \frac{15}{4}x^{1/2}$
$\left.\frac{d^2}{dx^2}\right|_{x=16} \sqrt{x^5} = \frac{15}{4}(16)^{1/2} = 15$

62. $\frac{d}{dx}\sqrt{x^7} = \frac{d}{dx}x^{7/2} = \frac{7}{2}x^{5/2}$
$\frac{d^2}{dx^2}\sqrt{x^7} = \frac{d}{dx}\left(\frac{7}{2}x^{5/2}\right) = \frac{35}{4}x^{3/2}$
$\left.\frac{d^2}{dx^2}\right|_{x=4} \sqrt{x^7} = \frac{35}{4}(4)^{3/2} = 70$

63. $P(t) = 0.25t^3 - 3t^2 + 5t + 200$
$P'(t) = 3(0.25t^2) - 2(3t) + 1(5) + 0 = 0.75t^2 - 6t + 5$
$P''(t) = 2(0.75t) - 6(1) + 0 = 1.5t - 6$
$P(10) = 0.25(10)^3 - 3(10)^2 + 5(10) + 200 = 250 - 300 + 50 + 200 = 200$
In 10 years, the population will be 200,000.
$P'(10) = 0.75(10)^2 - 6(10) + 5 = 75 - 60 + 5 = 20$
In 10 years, the population will be increasing by about 20,000 per year.
$P''(10) = 1.5(10) - 6 = 9$
In 10 years, the rate of growth of the increase will be 9000 per year each year.

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64. $s(t) = 8t^{5/2}$

$v(t) = s'(t) = \frac{5}{2}(8t^{3/2}) = 20t^{3/2}$

$a(t) = v'(t) = \frac{3}{2}(20t^{1/2}) = 30t^{1/2}$

$v(25) = 20(25)^{3/2} = 2500 \text{ ft / sec}$

$a(25) = 30(25)^{1/2} = 150 \text{ ft / sec}^2$

65. **a.** When the height is a maximum, the velocity is zero. Thus, to find the maximum height, set $v(t) = 0$ and solve. First, we find $v(t) = s'(t)$.

$s(t) = -16t^2 + 148t + 5$

$v(t) = s'(t) = -32t + 148$

Now set $v(t) = 0$ and solve.

$v(t) = -32t + 148 = 0$

$-32t = -148$

$t = 4.625$

To find the height when $t = 4.625$, evaluate $s(4.625)$.

$s(4.625) = -16(4.625)^2 + 148(4.625) + 5$

$= 347.25$ feet

b.

66. $h(z) = (4z^2 - 3z + 1)^3$

$h'(z) = 3(4z^2 - 3z + 1)^2(8z - 3)$

67. $h(z) = (3z^2 - 5z - 1)^4$

$h'(z) = 4(3z^2 - 5z - 1)^3(6z - 5)$

68. $g(x) = (100 - x)^5$

$g'(x) = 5(100 - x)^4(-1) = -5(100 - x)^4$

69. $g(x) = (1000 - x)^4$

$g'(x) = 4(1000 - x)^3(-1) = -4(1000 - x)^3$

70. $f(x) = \sqrt{x^2 - x + 2} = \left(x^2 - x + 2\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(x^2 - x + 2\right)^{-1/2}(2x - 1)$

71. $f(x) = \sqrt{x^2 - 5x - 1} = \left(x^2 - 5x - 1\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(x^2 - 5x - 1\right)^{-1/2}(2x - 5)$

72. $w(z) = \sqrt[3]{6z - 1} = (6z - 1)^{1/3}$

$w'(z) = \frac{1}{3}(6z - 1)^{-2/3}(6) = 2(6z - 1)^{-2/3}$

73. $w(z) = \sqrt[3]{3z + 1} = (3z + 1)^{1/3}$

$w'(z) = \frac{1}{3}(3z + 1)^{-2/3}(3) = (3z + 1)^{-2/3}$

74. $h(x) = \dfrac{1}{\sqrt[5]{(5x + 1)^2}} = (5x + 1)^{-2/5}$

$h'(x) = -\frac{2}{5}(5x + 1)^{-7/5}(5) = -2(5x + 1)^{-7/5}$

75. $h(x) = \dfrac{1}{\sqrt[5]{(10x + 1)^3}} = (10x + 1)^{-3/5}$

$h'(x) = -\frac{3}{5}(10x + 1)^{-8/5}(10) = -6(10x + 1)^{-8/5}$

76. $g(x) = x^2(2x - 1)^4$

$g'(x) = 2x(2x - 1)^4 + x^2[4(2x - 1)^3(2)]$

$= 2x(2x - 1)^4 + 8x^2(2x - 1)^3$

77. $g(x) = 5x(x^3 - 2)^4$

$g'(x) = 5(x^3 - 2)^4 + 5x[4(x^3 - 2)^3(3x^2)]$

$= 5(x^3 - 2)^4 + 60x^3(x^3 - 2)^3$

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78. $y = x^3\sqrt[3]{x^3+1} = x^3(x^3+1)^{1/3}$

$y' = 3x^2(x^3+1)^{1/3} + x^3\left[\frac{1}{3}(x^3+1)^{-2/3}(3x^2)\right]$

$= 3x^2(x^3+1)^{1/3} + x^5(x^3+1)^{-2/3}$

79. $y = x^4\sqrt{x^2+1} = x^4(x^2+1)^{1/2}$

$y' = 4x^3(x^2+1)^{1/2} + x^4\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]$

$= 4x^3(x^2+1)^{1/2} + x^5(x^2+1)^{-1/2}$

80. $f(x) = [(2x^2+1)^4 + x^4]^3$

$f'(x) = 3[(2x^2+1)^4 + x^4]^2[4(2x^2+1)^3(4x) + 4x^3]$

$= 3[(2x^2+1)^4 + x^4]^2[16x(2x^2+1)^3 + 4x^3]$

81. $f(x) = [(3x^2-1)^3 + x^3]^2$

$f'(x) = 2[(3x^2-1)^3 + x^3][3(3x^2-1)^2(6x) + 3x^2]$

$= 2[(3x^2-1)^3 + x^3][18x(3x^2-1)^2 + 3x^2]$

82. $f(x) = \sqrt{(x^2+1)^4 - x^4} = [(x^2+1)^4 - x^4]^{1/2}$

$f'(x) = \frac{1}{2}[(x^2+1)^4 - x^4]^{-1/2}[4(x^2+1)^3(2x) - 4x^3]$

$= \frac{1}{2}[(x^2+1)^4 - x^4]^{-1/2}[8x(x^2+1)^3 - 4x^3]$

83. $f(x) = \sqrt{(x^3+1)^2 + x^2} = [(x^3+1)^2 + x^2]^{1/2}$

$f'(x) = \frac{1}{2}[(x^3+1)^2 + x^2]^{-1/2}[2(x^3+1)(3x^2) + 2x]$

$= \frac{1}{2}[(x^3+1)^2 + x^2]^{-1/2}[6x^2(x^3+1) + 2x]$

$= [(x^3+1)^2 + x^2]^{-1/2}[3x^2(x^3+1) + x]$

84. $f(x) = (3x+1)^4(4x+1)^3$

$f'(x) = 4(3x+1)^3(3)(4x+1)^3 + (3x+1)^4(3)(4x+1)^2(4)$

$= 12(3x+1)^3(4x+1)^3 + 12(3x+1)^4(4x+1)^2$

$= 12(3x+1)^3(4x+1)^2[(4x+1) + (3x+1)]$

$= 12(3x+1)^3(4x+1)^2(7x+2)$

85. $f(x) = (x^2+1)^3(x^2-1)^4$

$f'(x) = 3(x^2+1)^2(2x)(x^2-1)^4 + (x^2+1)^3[4(x^2-1)^3(2x)]$

$= 6x(x^2+1)^2(x^2-1)^4 + 8x(x^2+1)^3(x^2-1)^3$

86. $f(x) = \left(\frac{x+5}{x}\right)^4$

$f'(x) = 4\left(\frac{x+5}{x}\right)^3\left[\frac{x(1)-(1)(x+5)}{x^2}\right]$

$= 4\left(\frac{x+5}{x}\right)^3\left(\frac{x-x-5}{x^2}\right) = -\frac{20}{x^2}\frac{(x+5)^3}{x^3}$

$= -\frac{20(x+5)^3}{x^5}$

87. $f(x) = \left(\frac{x+4}{x}\right)^5$

$f'(x) = 5\left(\frac{x+4}{x}\right)^4\left[\frac{x(1)-(1)(x+4)}{x^2}\right]$

$= 5\left(\frac{x+4}{x}\right)^4\left(\frac{x-x-4}{x^2}\right) = -\frac{20}{x^2}\left(\frac{x+4}{x}\right)^4$

$= -\frac{20(x+4)^4}{x^6}$

88. $y = \sqrt[3]{x^2+4} = (x^2+4)^{1/3}$

$y(2) = \sqrt[3]{(2)^2+4} = 2$

The point is (2, 2).

$y' = \frac{1}{3}(x^2+4)^{-2/3}(2x) = \frac{2x}{3(x^2+4)^{2/3}}$

$y'(2) = \frac{2(2)}{3((2)^2+4)^{2/3}} = \frac{1}{3}$

The tangent line has slope $\frac{1}{3}$ at (2, 2).

$y - 2 = \frac{1}{3}(x-2)$

$y = \frac{1}{3}x + \frac{4}{3}$

89. $h(w) = (2w^2-4)^5$

$h'(w) = 5(2w^2-4)^4(4w)$

$h''(w) = 20(2w^2-4)^3(4w)(4w) + 5(2w^2-4)^4(4)$

$= 20(16w^2)(2w^2-4)^3 + 20(2w^2-4)^4$

$= 320w^2(2w^2-4)^3 + 20(2w^2-4)^4$

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90. $h(w) = (3w^2+1)^4$

$h'(w) = 4(3w^2+1)^3(6w)$

$h''(w) = 12(3w^2+1)^2(6w)(6w) + 4(3w^2+1)^3(6)$

$= 432w^2(3w^2+1)^2 + 24(3w^2+1)^3$

91. $g(z) = z^3(z+1)^3$

$g'(z) = 3z^2(z+1)^3 + z^3[3(z+1)^2] = 3z^2(z+1)^3 + 3z^3(z+1)^2$

$g''(z) = 6z(z+1)^3 + 3z^2[3(z+1)^2] + 9z^2(z+1)^2 + 3z^3[2(z+1)]$

$= 6z(z+1)^3 + 18z^2(z+1)^2 + 6z^3(z+1)$

92. $g(z) = z^4(z+1)^4$

$g'(z) = 4z^3(z+1)^4 + 4z^4(z+1)^3$

$g''(z) = 12z^2(z+1)^4 + 4z^3[4(z+1)^3] + 16z^3(z+1)^3 + 4z^4[3(z+1)^2]$

$= 12z^2(z+1)^4 + 32z^3(z+1)^3 + 12z^4(z+1)^2$

93. **a.** $\frac{d}{dx}(x^3-1)^2 = 2(x^3-1)(3x^2) = 6x^2(x^3-1) = 6x^5 - 6x^2$

b. $\frac{d}{dx}(x^3-1)^2 = \frac{d}{dx}(x^6 - 2x^3 + 1) = 6x^5 - 6x^2$

94. **a.** $\frac{d}{dx}\left(\frac{1}{x^3+1}\right) = \frac{(x^3+1)(0)-(3x^2)(1)}{(x^3+1)^2} = -\frac{3x^2}{(x^3+1)^2}$

b. $\frac{d}{dx}\left(\frac{1}{x^3+1}\right) = \frac{d}{dx}(x^3+1)^{-1} = -1(x^3+1)^{-2}(3x^2) = -\frac{3x^2}{(x^3+1)^2}$

95. $P(x) = \sqrt{x^3 - 3x + 34} = (x^3 - 3x + 34)^{1/2}$

$P'(x) = \frac{1}{2}(x^3 - 3x + 34)^{-1/2}(3x^2 - 3) = \frac{3x^2 - 3}{2\sqrt{x^3 - 3x + 34}}$

$P'(5) = \frac{3(5)^2 - 3}{2\sqrt{(5)^3 - 3(5) + 34}} = \frac{72}{2\sqrt{144}} = \frac{72}{24} = 3$

When 5 tons is produced, profit is increasing at about \$3000 for each additional ton.

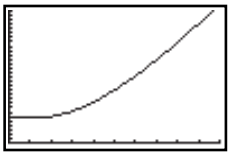
96. $V(r) = 500(1 + 0.01r)^3$

$V'(r) = 1500(1 + 0.01r)^2(0.01) = 15(1 + 0.01r)^2$

$V'(8) = 15[1 + 0.01(8)]^2 \approx 17.50$

For 8 percent interest, the value increases by about \$17.50 for each additional percent interest.

97.



on [0, 10] by [0, 30]

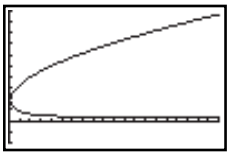
a. $P(5) - P(4) \approx 12 - 9.27 = 2.73$

$P(6) - P(5) \approx 15.23 - 12 = 3.23$

Both values are near 3.

b. $x \approx 7.6$

98.



on [0, 20] by [–2, 10]

$x \approx 16$

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99.
$$R(x) = 0.25(0.01x+1)^4$$
$$R'(x) = 0.25[4(0.01x+1)^3(0.01)]$$
$$= 0.01(0.01x+1)^3$$
$$R'(100) = 0.01[0.01(100)+1)]^3 = 0.08$$

100.
$$N(x) = 1000\sqrt{100-x} = 1000(100-x)^{1/2}$$
$$N'(x) = 500(100-x)^{-1/2}(-1) = -\frac{500}{\sqrt{100-x}}$$
$$N'(96) = -\frac{500}{\sqrt{100-96}} = -250$$
At age 96, the number of survivors is decreasing by 250 people per year.

101. The derivative does not exist at corner points $x=-3$ and $x=3$ and at the discontinuous point $x=1$.

102. The derivative does not exist at the corner point $x=2$ and at the discontinuous point $x=-2$.

103. The derivative does not exist at the corner point $x=3.5$ and the discontinuous point $x=0$.

104. The derivative does not exist at the corner points $x=0$ and $x=3$.

105. For positive h,
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|5x+5h|-|5x|}{h} = \lim_{h\to 0}\frac{|5(0)+5h|-|5\cdot 0|}{h} \quad \text{for} \quad x=0$$
$$= \lim_{h\to 0}\frac{5h}{h} = 5$$
For negative h,
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{|5x+5h|-|5x|}{h} = \lim_{h\to 0}\frac{|5(0)+5h|-|5(0)|}{h} \quad \text{for} \quad x=0$$
$$= \lim_{h\to 0}-\frac{5h}{h} = -5$$
Thus, the limit does not exist, and so the derivative does not exist.

106.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^{3/5}-x^{3/5}}{h} = \lim_{h\to 0}\frac{h^{3/5}}{h} \quad \text{for} \quad x=0$$
$$= \lim_{h\to 0}\frac{1}{h^{2/5}}$$
which does not exist. Thus, the derivative does not exist.

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