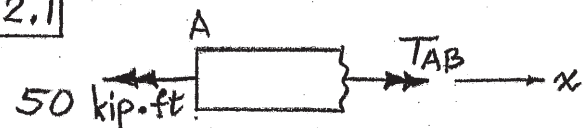


***Mechanics of Materials With Applications in Excel
1st edition Muvdi Solutions Manual***

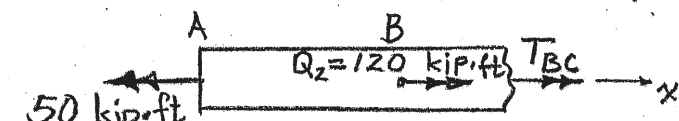
SKU: 9781466570719-SOLUTIONS

2.1



$$\sum T_x = 0: T_{AB} - 50 = 0$$

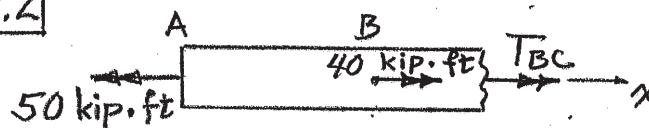
$$T_{AB} = 50 \text{ kip·ft} \quad \text{ANS.}$$



$$\sum T_x = 0: T_{BC} + 120 - 50 = 0$$

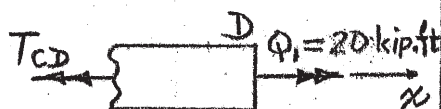
$$T_{BC} = -70 \text{ kip·ft} \quad \text{ANS.}$$

2.2



$$\sum T_x = 0: T_{BC} + 40 - 50 = 0$$

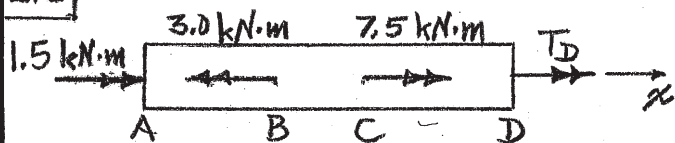
$$T_{BC} = 10 \text{ kip·ft} \quad \text{ANS.}$$



$$\sum T_x = 0: 20 - T_{CD} = 0$$

$$T_{CD} = 20 \text{ kip·ft} \quad \text{ANS.}$$

2.3



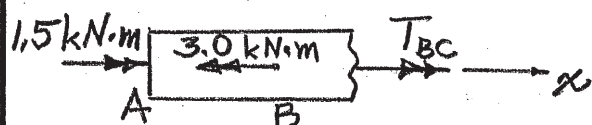
$$\sum T_x = 0: T_D + 7.5 - 3.0 + 1.5 = 0$$

$$T_D = -6.0 \text{ kN·m} \quad \text{ANS.}$$



$$\sum T_x = 0: T_{AB} + 1.5 = 0$$

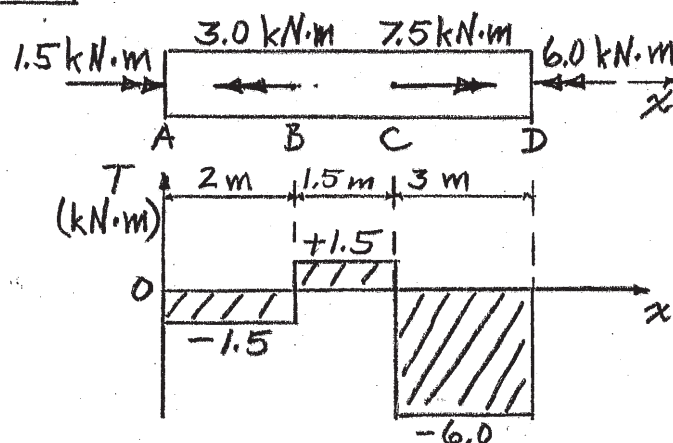
$$T_{AB} = -1.5 \text{ kN·m} \quad \text{ANS.}$$



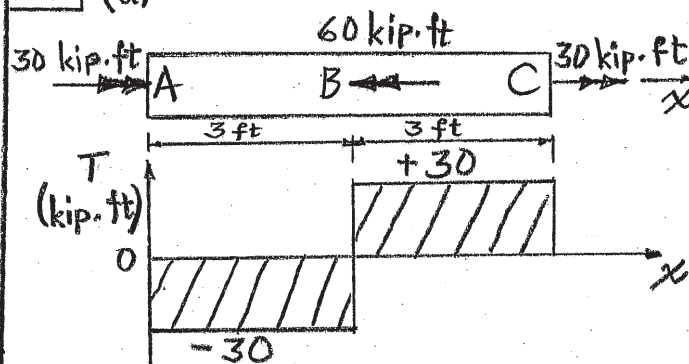
$$\sum T_x = 0: T_{BC} - 3.0 + 1.5 = 0$$

$$T_{BC} = 1.5 \text{ kN·m} \quad \text{ANS.}$$

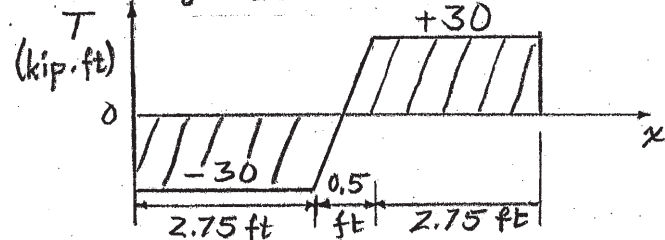
2.4



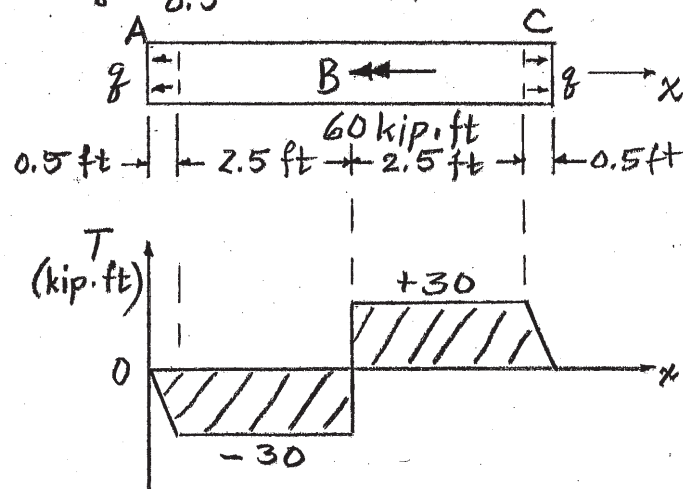
2.5 (a)



$$(b) \quad q = \frac{60}{0.5} = 120 \text{ kip/ft}$$



$$2.6 \quad q = \frac{30}{0.5} = 60 \text{ kip/ft}$$



2.7 $q = 20 \text{ kN/m}$

(a) $\sum T_x = 0: T_{BC} + \int_{1.5}^x 20 dx - 50 = 0$
 $T_{BC} = 80 - 20x$ ANS.
 $T_C = 80 - 20(2.5) = 30 \text{ kN·m}$ ANS.

(b)

2.8 REFER TO THE F.B.D. IN PROB. 2.7.
 MAKE $q = 100(x-1.5)^2 \text{ kN/m}$.

(a) $\sum T_x = 0: T_{BC} + \int_{1.5}^x 100(x-1.5)^2 dx - 50 = 0$
 $T_{BC} = 50 - \frac{100}{3}(x-1.5)^3$ ANS.
 $T_C = 50 - \frac{100}{3}(2.5-1.5)^3 = 16.7 \text{ kN·m}$ ANS.

(b)

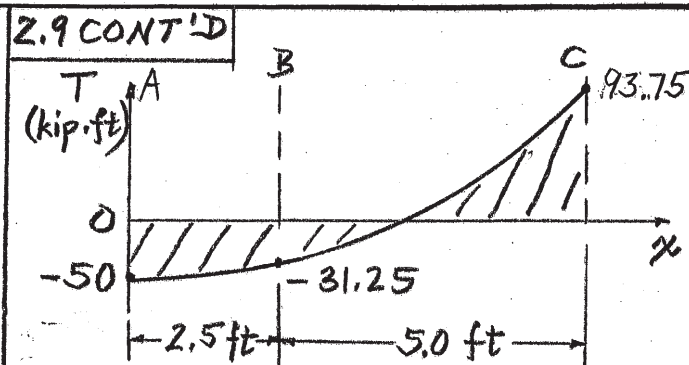
2.9

$q_1 = 6x_1$

$\sum T_x = 0: T_{AB} - \int_0^{x_1} 6x_1 dx_1 + 50 = 0$
 $T_{AB} = 3x_1^2 - 50$ ANS.

$q_2 = 3x_2^2$

$T_B = 3(2.5^2) - 50 = -31.25 \text{ kip·ft}$
 $\sum T_x = 0: T_{BC} - \int_0^{x_2} 3x_2^2 dx_2 + 31.25 = 0$
 $T_{BC} = x_2^3 - 31.25$ ANS.



2.10

$q_1 = 6x_1$

$\sum T_x = 0: T_{AB} - \int_0^{x_1} 6x_1^2 dx_1 + 150 = 0$
 $T_{AB} = 2x_1^3 - 40$ ANS.

$q_2 = 10x_2$

$T_B = 2(2.0^3) - 40 = -24.0 \text{ kN·m}$
 $\sum T_x = 0: T_{BC} - \int_0^{x_2} 10x_2 dx_2 + 24.0 = 0$
 $T_{BC} = 5x_2^2 - 24$ ANS.
 $T_C = 101 = 5b^2 - 24$ ANS.
 $b = 5.0 \text{ m}$

2.11

$q = 250 + 3x^2$

$\sum T_x = 0: T + \int_0^x (250 + 3x^2) dx = 0$
 $T = -(250x + x^3)$ ANS.

2.12 REFER TO F.B.D. IN PROB. 2.11

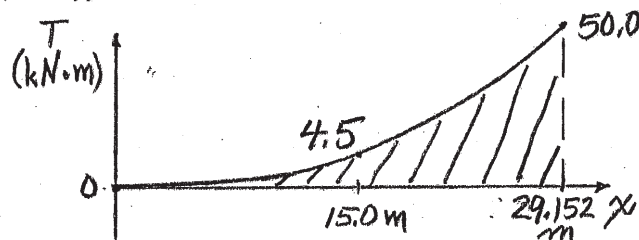
$$q = 1.2 - 0.2x$$

$$\sum T_x = 0: T + \int_0^x (1.2 - 0.2x) dx = 0$$

$$T = 0.1x^2 - 1.2x \quad \text{ANS.}$$

$$T_B = 50.0 = 0.1L^2 - 1.2L$$

$$L^2 - 12L - 500 = 0; L = 29.152 \text{ m} \quad \text{ANS.}$$



2.13

$$(a) G = \frac{TL}{J\theta} = \frac{24.85 \times 10^3 (0.25)}{(\frac{\pi}{32})(0.1^4 - 0.092^4)(1.4\pi/180)}$$

$$G = 91.3 \text{ GPa} \quad \text{ANS.}$$

$$\tau_u = \frac{T\rho R}{J} = \frac{46.25 \times 10^3 (0.05)}{(\pi/32)(0.1^4 - 0.092^4)}$$

$$\tau_u = 830.6 \text{ MPa} \quad \text{ANS.}$$

$$(b) \tau_p = \frac{TR}{J} = \frac{24.85 \times 10^3 (0.05)}{(\pi/32)(0.1^4 - 0.092^4)}$$

$$\tau_p = 446.3 \text{ MPa} \quad \text{ANS.}$$

$$\gamma_p = \frac{\tau_p}{G} = \frac{446.3 \times 10^6}{91.3 \times 10^9}$$

$$\gamma_p = 4.89 \times 10^{-3} \quad \text{ANS.}$$

2.14

$$(a) G = \frac{TL}{J\theta} = \frac{62.8 \times 10^3 (6.0)}{(\frac{\pi}{32})(2.0^4)(1.14\pi/180)}$$

$$G = 12.056 \times 10^6 \text{ psi} \quad \text{ANS.}$$

$$(b) u = \frac{1}{2} \tau_p \gamma_p = \frac{1}{2} \left(\frac{TR}{J} \right) \left(\frac{R\theta}{L} \right)$$

$$= \frac{1}{2} \left[\frac{62.8 \times 10^3 (1.0^2)(1.14\pi/180)}{(\pi/32)(2.0^4)(6.0)} \right]$$

$$u = 66.3 \text{ lb. in. / in.}^3 \quad \text{ANS.}$$

2.15

$$(a) G = \frac{TL}{J\theta} = \frac{220 (10.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)(1.4\pi/180)}$$

$$G = 10.4 \times 10^3 \text{ ksi} \quad \text{ANS.}$$

$$\tau_u = \frac{T\rho R}{J} = \frac{456 (2.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)}$$

$$\tau_u = 105.5 \text{ ksi} \quad \text{ANS.}$$

$$(b) \tau_p = \frac{TR}{J} = \frac{220 (2.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)}$$

$$\tau_p = 50.9 \text{ ksi}$$

$$\gamma_p = \frac{\tau_p}{G} = \frac{50.9}{10.4 \times 10^3} = 4.894 \times 10^{-3}$$

$$u = \frac{1}{2} \tau_p \gamma_p = \left(\frac{1}{2} \right) (50.9) (4.894 \times 10^{-3})$$

$$u = 0.125 \text{ kip. in. / in.} \quad \text{ANS.}$$

2.16

$$(a) G = \frac{TL}{J\theta} = \frac{7.35 \times 10^3 (0.150)}{(\frac{\pi}{32})(0.04^4)(1.2\pi/180)}$$

$$G = 209.4 \text{ GPa} \quad \text{ANS.}$$

$$(b) u = \frac{1}{2} \tau_p \gamma_p = \frac{1}{2} \left(\frac{TR}{J} \right) \left(\frac{R\theta}{L} \right)$$

$$= \frac{1}{2} \left[\frac{7.35 \times 10^3 (0.04^2)(1.2\pi/180)}{(\frac{\pi}{32})(0.04^4)(0.150)} \right]$$

$$u = 3266.7 \text{ kN. m / m}^3 \quad \text{ANS.}$$

$$(c) \tau_u = \frac{T\rho R}{J} = \frac{18.25 \times 10^3 (0.04)}{(\frac{\pi}{32})(0.04^4)}$$

$$\tau_u = 2904.6 \text{ MPa} \quad \text{ANS.}$$

2.17

$$T_{AB} = 5 \text{ kip. in.}; T_{BC} = 10 \text{ kip. in.}$$

$$(a) \tau_{\text{MAX}} = \frac{TR}{J} = \frac{10 (1.0)}{(\frac{\pi}{32})(2.0^4)(1.25^4)}$$

$$\tau_{\text{MAX}} = 7.513 \text{ ksi} \quad \text{ANS.}$$

$$(b) \tau_{\text{MAX}} = \frac{TR}{J} = \frac{10 (1.0)}{(\frac{\pi}{32})(2.0^4)}$$

$$\tau_{\text{MAX}} = 6.366 \text{ ksi} \quad \text{ANS.}$$

2.18 $T_{AB} = 5 \text{ kip}\cdot\text{in.}; T_{BC} = 10 \text{ kip}\cdot\text{in.}$

(a) $\theta_A = \sum \frac{TL}{JG} = \theta_{AB} + \theta_{BC}$

$$= \frac{5 \times 10^3 (10)}{\left(\frac{\pi}{32}\right) (1.5^4) (6 \times 10^6)} - \frac{10 \times 10^3 (15)}{\left(\frac{\pi}{32}\right) (1.5^4) (6 \times 10^6)}$$

$$= 17.885 \times 10^{-3} - 53.654 \times 10^{-3}$$

$$\theta_A = -35.769 \times 10^{-3} \text{ rad. CCW AS VIEWED FROM A TO C. } \underline{\text{ANS.}}$$

(b) $\theta_A = \sum \frac{TL}{JG} = \theta_{AB} + \theta_{BC}$

$$= \frac{5 \times 10^3 (10)}{\left(\frac{\pi}{32}\right) (1.5^4) (6 \times 10^6)} - \frac{10 \times 10^3 (15)}{\left(\frac{\pi}{32}\right) (1.5^4) (6 \times 10^6)}$$

$$= 16.767 \times 10^{-3} - 50.301 \times 10^{-3}$$

$$\theta_A = -33.534 \times 10^{-3} \text{ rad. CCW AS VIEWED FROM A TO C. } \underline{\text{ANS.}}$$

2.19 $T_{AB} = 5 \text{ kN}\cdot\text{m}; T_{BC} = 10 \text{ kN}\cdot\text{m}$

(a) $\tau_{AB} = \frac{TR}{J} = \frac{5 \times 10^3 (0.03)}{\left(\frac{\pi}{32}\right) (0.06^4)}$

$$= 117.9 \text{ MPa}$$

$$\tau_{BC} = \frac{TR}{J} = \frac{10 \times 10^3 (0.04)}{\left(\frac{\pi}{32}\right) (0.08^4)}$$

$$= 99.5 \text{ MPa}$$

$\tau_{\text{MAX}} = \tau_{AB} = 117.9 \text{ MPa} \quad \underline{\text{ANS.}}$

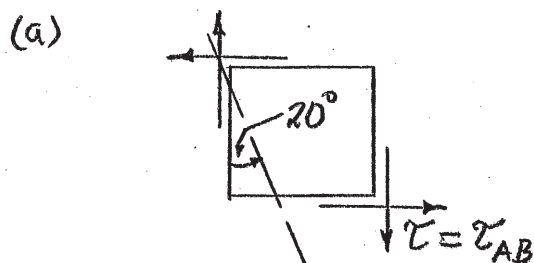
(b) $\theta_{A/C} = \sum \frac{TL}{JG} = \theta_{A/B} + \theta_{B/C}$

$$= \frac{5 \times 10^3 (0.30)}{\left(\frac{\pi}{32}\right) (0.06^4) (40 \times 10^9)} - \frac{10 \times 10^3 (0.50)}{\left(\frac{\pi}{32}\right) (0.08^4) (25 \times 10^9)}$$

$$= 2.947 \times 10^{-2} - 4.974 \times 10^{-2}$$

$$\theta_{A/C} = -2.027 \times 10^{-2} \text{ rad. CCW AS VIEWED FROM A TO C. } \underline{\text{ANS.}}$$

2.20 $T_{AB} = 5 \text{ kN}\cdot\text{m}; T_{BC} = 10 \text{ kN}\cdot\text{m}$
 FROM PROB. 2.19, $\tau_{AB} = 117.9 \text{ MPa}$

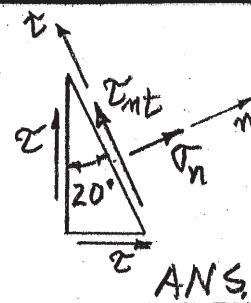


2.20 CONT'D (b) BY EQS. (2.10a) & (2.10b),

$\tau_n = \tau \sin 2\theta$

$$= -117.9 \sin 40^\circ$$

$$= -75.8 \text{ MPa}$$



$\tau_{nt} = \tau \cos 2\theta$

$$= -117.9 \cos 40^\circ = -90.3 \text{ MPa} \quad \underline{\text{ANS.}}$$

2.21 $\tau = \frac{TP}{J}$

$$25 = \frac{50(D_1/2)}{\left(\frac{\pi}{32}\right)(D_1^4)}; D_1^3 = 10.186$$

$$D_1 = 2.168 \text{ in.}$$

AS DEDUCED FROM EQS. (2.10),
 $\tau_{\text{MAX}} \text{ AT } \theta = 0^\circ = (\tau_n)_{\text{MAX}} \text{ AT } \theta = 45^\circ = 40 \text{ ksi.}$

$$\therefore 40 = \frac{50(D_2/2)}{\left(\frac{\pi}{32}\right)(D_2^4)}; D_2^3 = 6.366$$

$$D_2 = 1.853 \text{ in.}$$

$D_{\text{MIN}} = D_1 = 2.168 \text{ in.} \quad \underline{\text{ANS.}}$

2.22 EQS. (2.10) LEAD TO THE CONCLUSION THAT $\tau_{\text{MAX}} \text{ AT } \theta = 0^\circ = (\tau_n)_{\text{MAX}} \text{ AT } \theta = 45^\circ$

$$\tau_{\text{MAX}} = \frac{\tau J}{R} = \frac{100 \times 10^6 \left(\frac{\pi}{32}\right) (0.05^4 - 0.03^4)}{0.025}$$

$$= 100 \text{ MPa.}$$

$$T_{\text{MAX}} = 2.136 \text{ kN}\cdot\text{m} \quad \underline{\text{ANS.}}$$

2.23 $J = \frac{\pi}{32} (d_o^4 - d_i^4) = \frac{\pi}{32} (2.0^4 - d_i^4) = \frac{TL}{G\theta}$

$$\frac{\pi}{32} (2.0^4 - d_i^4) = \frac{40(20)}{4 \times 10^3 (10\pi/180)}$$

$$d_i^4 = 4.32780; d_i = 1.442 \text{ in.} \quad \underline{\text{ANS.}}$$

2.24 AB: $\tau = \frac{TP}{J}$

$$25 = \frac{Q_1(1.0)}{\left(\frac{\pi}{32}\right)(2.0^4)}; Q_1 = 39.270 \text{ kip}\cdot\text{in.}$$

$$\therefore T_{BC} = 59.270 \text{ kip}\cdot\text{in.}$$

 BC: $\tau = \frac{TR}{J}$

$$25 = \frac{59.270(D/2)}{\left(\frac{\pi}{32}\right) D^4}; D^3 = 12.07439$$

$$D = 2.294 \text{ in.} \quad \underline{\text{ANS.}}$$

2.25 $T_{AB} = 25 \text{ kip}\cdot\text{in}; T_{BC} = 35 \text{ kip}\cdot\text{in}$

$$\theta_{B/C} = \left(\frac{TL}{JG} \right)_{BC} = \frac{35(10)}{(\pi/32)(3.5^4)(4 \times 10^3)}$$

$$= 5.93931 \times 10^{-3} \text{ rad.}$$

$$\theta_{A/B} = \left(\frac{TL}{JG} \right) = \frac{25(6)}{(\pi/32)D_{AB}^4(10 \times 10^3)}$$

$$= 5.93931 \times 10^{-3}$$

$$D_{AB} = 2.252 \text{ in.} \quad \text{ANS.}$$

2.26

$$\tau = \frac{TR}{J}$$

$$= \frac{20 \times 10^3(0.2)}{(\pi/32)(0.4^4 - 0.38^4)}$$

$$\tau = 8.580 \text{ MPa}$$

BY EQS. (2.10a) & (2.10b),

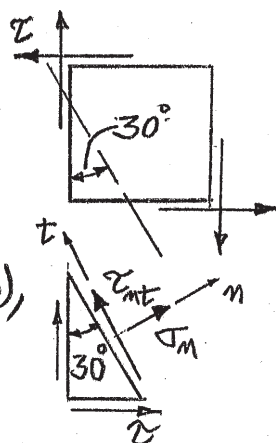
$$\tau_n = \tau \sin 2\theta$$

$$= 8.580 \sin 60^\circ$$

$$= -7.4 \text{ MPa}$$

$$\tau_{nt} = \tau \cos 2\theta$$

$$= -8.580 \cos 60^\circ = -4.3 \text{ MPa} \quad \text{ANS.}$$



2.27 BY EQ. (2.10b)

$$\tau = \frac{10}{\cos 2\theta} = 57.588$$

$$57.588 = \frac{50(12)(\frac{1}{2})(D_o + \frac{1}{2})}{(\pi/32)(D_o + \frac{1}{2})^4}$$

$$(D_o + \frac{1}{2})^3 = 53.06270$$

$$D_o = 3.258 \text{ in.}$$

$$\approx 3.3 \text{ in.}$$

BY EQ. (2.10a)

$$\tau = \frac{\tau_n}{\sin 2\theta}$$

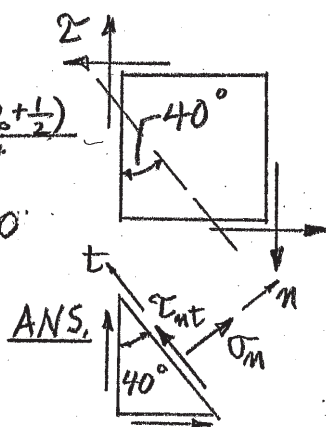
$$= \frac{15}{\sin 80^\circ} = 15.231$$

$$15.231 = \frac{50(12)(\frac{1}{2})(D_o + \frac{1}{2})}{(\pi/32)(D_o + \frac{1}{2})^4}$$

$$(D_o + \frac{1}{2})^3 = 200.62865$$

$$D_o = 5.354 \text{ in.}$$

$$(D_o)_{\text{MAX}} = 5.354 \approx 5.4 \text{ in.}$$



2.28 $\tau = \frac{TR}{J}; T_{AB} = Q_1$

$$AB: 100 \times 10^6 = \frac{Q_1(0.015/2)}{(\pi/32)(0.015^4)}$$

$$Q_1 = 66.267 \approx 66.3 \text{ N}\cdot\text{m} \quad \text{ANS.}$$

$$BC: T_{BC} = Q_2 - 66.267$$

$$100 \times 10^6 = \frac{(Q_2 - 66.267)(0.03/2)}{(\pi/32)(0.03^4)}$$

$$Q_2 = 596.411 \approx 596.4 \text{ N}\cdot\text{m} \quad \text{ANS.}$$

$$CD: T_{CD} = 530.144 - Q_3$$

$$100 \times 10^6 = \frac{(530.144 - Q_3)(0.045/2)}{(\pi/32)(0.045^4)}$$

$$Q_3 = -1259.091 \approx -1259.1 \text{ N}\cdot\text{m} \quad \text{ANS.}$$

2.29 $T_{AB} = -5 \text{ kip}\cdot\text{in}; T_{BC} = 5 \text{ kip}\cdot\text{in}$

$$T_{CD} = -15 \text{ kip}\cdot\text{in}$$

$$\theta = \sum \frac{TL}{JG} = \frac{15\pi}{180} = \frac{-5(18)}{(\pi/32)D_1^4(10 \times 10^3)}$$

$$+ \frac{5(12)}{(\pi/32)(\frac{3}{2}D_1)^4(5 \times 10^3)} + \frac{-15(8)}{(\pi/32)(3D_1)^4(3 \times 10^3)}$$

$$D_1 = 0.726 \text{ in.}; D_2 = 1.088 \text{ in. AND}$$

$$D_3 = 2.177 \text{ in.} \quad \text{ANS.}$$

2.30 BY INSPECTION, CONCLUDE THAT

$\tau_{BC} < \tau_{AB}$ AND $\tau_{CD} < \tau_{DE}$
 $\therefore$ THE CHOICE IS BETWEEN SEGMENTS AB AND DE.

$$AB: T_{AB} = Q_1 = 50 \text{ kN}\cdot\text{m}$$

$$\tau_{AB} = \left(\frac{TR}{J} \right)_{AB} = \frac{50 \times 10^3(0.04)}{(\pi/32)(0.08^4 - 0.04^4)}$$

$$= 530.5 \text{ MPa}$$

$$DE: T_{DE} = Q_1 + Q_2 - Q_3 = -50 \text{ kN}\cdot\text{m}$$

$$|\tau_{DE}| = \left(\frac{TR}{J} \right)_{DE} = \frac{50 \times 10^3(0.06)}{(\pi/32)(0.12^4 - 0.04^4)}$$

$$= 149.2 \text{ MPa}$$

$$\tau_{\text{MAX}} = \tau_{AB} = 530.5 \text{ MPa} \quad \text{ANS.}$$

2.31 $T_{AB} = 50 \text{ kN}\cdot\text{m}$; $T_{BC} = 75 \text{ kN}\cdot\text{m}$;
 $T_{CD} = T_{DE} = -50 \text{ kN}\cdot\text{m}$
 $\theta_A = \sum \frac{TL}{JG} = \theta_{A/B} + \theta_{B/C} + \theta_{C/E} + \theta_{E/F}$

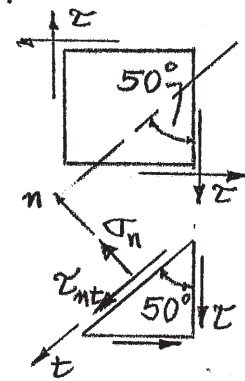
$$= \frac{50 \times 10^3 (0.2)}{\left(\frac{\pi}{32}\right)(0.08^4 - 0.04^4)(70 \times 10^9)} + \frac{75 \times 10^3 (0.1)}{\left(\frac{\pi}{32}\right)(0.08^4)(70 \times 10^9)}$$

$$- \frac{50 \times 10^3 (0.15)}{\left(\frac{\pi}{32}\right)(0.12^4)(70 \times 10^9)} - \frac{50 \times 10^3 (0.25)}{\left(\frac{\pi}{32}\right)(0.12^4 - 0.04^4)(70 \times 10^9)}$$

$$= 0.05039 \text{ rad ccw AS VIEWED FROM F TO A} \quad \text{ANS.}$$

2.32 $T_{AB} = 20 \text{ kip}\cdot\text{in.}$; $T_{BC} = 10 \text{ kip}\cdot\text{in.}$;
 $T_{CD} = 5 \text{ kip}\cdot\text{in.}$; $T_{DE} = 0$.
 $\tau_{AB} = \left(\frac{TR}{J}\right)_{AB} = \frac{20(1.25)}{\left(\frac{\pi}{32}\right)(2.5^4)} = 6.5 \text{ ksi}$
 $\tau_{BC} = \left(\frac{TR}{J}\right)_{BC} = \frac{10(1.0)}{\left(\frac{\pi}{32}\right)(2.0^4)} = 6.4 \text{ ksi}$
 $\tau_{CD} = \left(\frac{TR}{J}\right)_{CD} = \frac{5(0.875)}{\left(\frac{\pi}{32}\right)(1.75^4)} = 4.8 \text{ ksi}$
 $\tau_{DE} = 0$
 $\tau_{\text{MAX}} = \tau_{AB} = 6.5 \text{ ksi} \quad \text{ANS.}$

2.33 $T_{BC} = 1.5 \text{ kN}\cdot\text{m}$
 $\tau = \frac{1.5 \times 10^3 (0.0325)}{\left(\frac{\pi}{32}\right)(0.065^4)} = 27.8 \text{ MPa}$
 BY EQS. (2.10a) & (2.10b),
 $\sigma_m = \tau \sin 2\theta$
 $= -27.8 \sin(-100^\circ)$
 $\tau = 27.4 \text{ MPa} \quad \text{ANS.}$
 $\tau_{mt} = \tau \cos 2\theta$
 $= -27.8 \cos(-100^\circ)$
 $= 4.8 \text{ MPa} \quad \text{ANS.}$



2.34 USE TORQUE STATED IN PROB. 2.32.
 $\theta_{E/A} = \theta_{E/D} + \theta_{D/C} + \theta_{C/B} + \theta_{B/A}$

$$= \frac{20(3)(12)}{\left(\frac{\pi}{32}\right)(2.5^4)(10 \times 10^3)} + \frac{10(3)(12)}{\left(\frac{\pi}{32}\right)(2.0^4)(10 \times 10^3)}$$

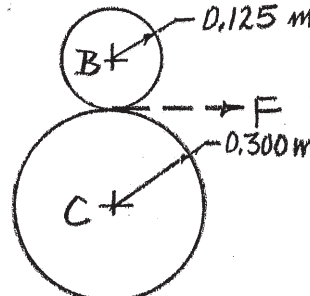
$$+ \frac{5(3)(12)}{\left(\frac{\pi}{32}\right)(1.75^4)(10 \times 10^3)} + 0$$

$$= 0.06124 \text{ rad ccw AS VIEWED FROM E TO A.} \quad \text{ANS.}$$

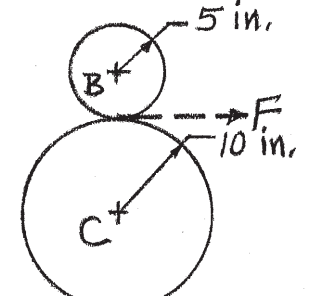
2.35
 $Q = 10 = F(0.125)$
 $F = 80 \text{ kN}$
 $T_{CD} = 80(0.3) = 24 \text{ kN}\cdot\text{m}$
 $\tau_{\text{MAX}} = \tau_{CD}$

$$= \left(\frac{TP}{J}\right)_{CD} = \frac{24 \times 10^3 (0.0375)}{\left(\frac{\pi}{32}\right)(0.075^4 - 0.045^4)}$$

$$\tau_{\text{MAX}} = 332.9 \text{ MPa} \quad \text{ANS.}$$



2.36
 $Q = F(5)$
 $F = \frac{Q}{5}$
 $T_{CD} = F(10) = 2Q$
 $\theta_{C/D} = \left(\frac{TL}{JG}\right)_{CD} = \frac{2Q(12)}{\left(\frac{\pi}{32}\right)(1.75^4)(4 \times 10^3)} = 6.51627 \times 10^{-3} Q$
 $\theta_B = 2\theta_{C/D} = 13.03254 \times 10^{-3} Q$
 $\theta_{A/B} = \left(\frac{TL}{JG}\right)_{AB} = \frac{Q(12)}{\left(\frac{\pi}{32}\right)(1.75^4)(4 \times 10^3)} = 3.25814 \times 10^{-3} Q$
 $\theta_{A/D} = \theta_B + \theta_{A/B} = 16.29068 \times 10^{-3} Q$
 $\therefore 16.29068 \times 10^{-3} Q = 5\pi/180$
 $Q = 5.4 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$



2.37 $\sum T_x = 0$:

$$T - \int_0^x \left(\frac{3}{4}\right)x dx = 0$$

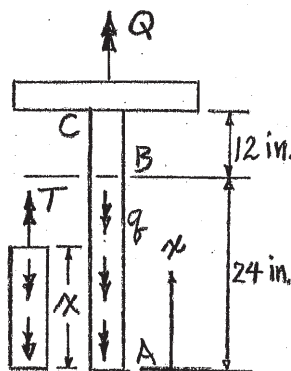
$$T = \left(\frac{3}{8}\right)x^2$$

$$T_{MAX} = T_{x=24} = 216 \text{ lb.in.}$$

$$\tau_{MAX} = \frac{T_{MAX} R}{J}$$

$$= \frac{216(1)}{\left(\frac{\pi}{32}\right)(2^4 - 1.5^4)} = 201.2 \text{ psi}$$

ANY PLACE BETWEEN B & C ON OUTSIDE SURFACE ANS.



2.38 REFER TO THE F.B.D. IN PROB. 2.37.

(a) $\sum T_x = 0$: $48 - q(0.7) = 0$

$$q = 68.571 \text{ N.m/m} \quad \text{ANS.}$$

(b) $\sum T_x = 0$: $T - \int_0^x (68.571) dx = 0$

$$T = 68.571x$$

$$\theta_{A/C} = \int \frac{T dx}{JG} = \int_0^{0.7} \frac{68.571x dx}{\left(\frac{\pi}{32}\right)(0.055^4 - 0.045^4)(25 \times 10^9)}$$

$$= \frac{68.571(x^2)}{1.2394 \times 10^{-4}} \bigg|_0^{0.7} = 27.108 \times 10^{-4} \text{ rad.} \quad \text{ANS.}$$

2.39

$$\sum T_x = 0$$

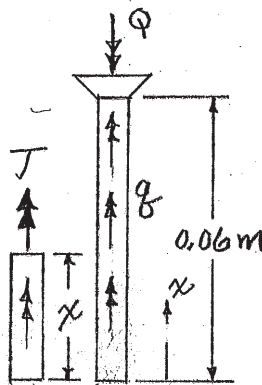
$$\int_0^x 3x^2 dx - T = 0$$

$$T = x^3$$

$$T_{MAX} = T_{x=0.06} = Q$$

$$= 2.16 \times 10^{-4} \text{ kN.m}$$

$$\tau_{MAX} = \frac{T_{MAX} R}{J} = \frac{2.16 \times 10^{-4} \times 10^3 (0.003)}{\left(\frac{\pi}{32}\right)(0.006^4)} = 5.1 \text{ MPa} \quad \text{ANS.}$$



2.40 REFER TO THE F.B.D. IN PROB. 2.39.

$$\sum T_x = 0$$

$$T + qx = 0; T = -qx$$

$$|T_{MAX}| = |T_{x=0.06}| = 0.06q = Q \quad (1)$$

$$\tau_{MAX} = \frac{T_{MAX} R}{J} = \frac{T_{MAX} (0.04)}{\left(\frac{\pi}{32}\right)(0.08^4)} = 10 \times 10^6$$

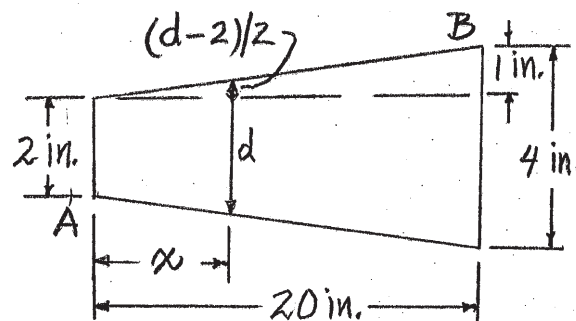
$$T_{MAX} = 1.005 \text{ kN.m} = Q \quad (2) \quad \text{ANS.}$$

2.40 CONT'D

(2) INTO (1) $\Rightarrow$

$$q = \frac{1.005}{0.06} = 16.75 \text{ kN.m/m} \quad \text{ANS.}$$

2.41



USING SIMILAR TRIANGLES, WE HAVE

$$\frac{(d-2)/2}{x} = \frac{1}{20}; d = \frac{20+x}{10}$$

(a) $\tau_{MAX} = \tau_A = \frac{TR}{J} = \frac{30(1)}{\left(\frac{\pi}{32}\right)(2^4)}$

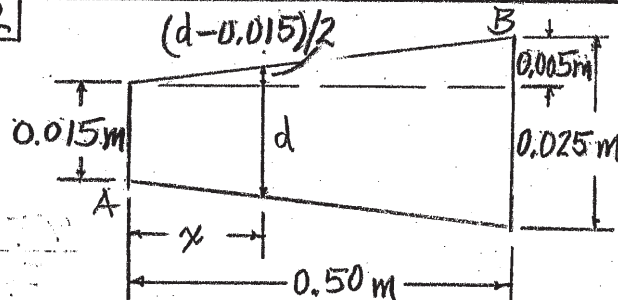
$$(b) \theta_{A/B} = \frac{T}{G} \int_0^{20} \frac{dx}{J}$$

$$= \frac{30}{4 \times 10^6} \int_0^{20} \frac{dx}{\left(\frac{\pi}{32}\right)\left(\frac{20+x}{10}\right)^4}$$

$$= \frac{30(32)(10^4)}{\pi(4 \times 10^6)} \int_0^{20} (20+x)^{-4} dx$$

$$= 2.785 \times 10^{-5} \text{ rad} \quad \text{ANS.}$$

2.42



USING SIMILAR TRIANGLES, WE HAVE

$$\frac{(d-0.015)/2}{x} = \frac{0.005}{0.50}; d = \frac{x+0.75}{50}$$

$$\tau = \frac{TR}{J}; T = Q$$

$$70 \times 10^6 = \frac{Q(0.0075)}{\left(\frac{\pi}{32}\right)(0.015^4)}$$

$$Q = 46.387 \text{ N.m}$$

2.42 CONT'D

$$\theta = \int \frac{T dx}{JG}; T = Q$$

$$\frac{4\pi}{180} = \frac{Q}{50 \times 10^9} \int_{0.32}^{0.5} \frac{dx}{\left(\frac{\pi}{32}\right) \left(\frac{x+0.75}{50}\right)^4}$$

$$= \frac{32(50^4)Q}{\pi(50 \times 10^9)} \int_{0.32}^{0.5} \frac{dx}{(x+0.75)^4}$$

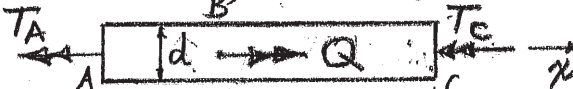
$$Q = 88.5 \text{ N}\cdot\text{m}$$

$$\therefore Q_{\text{MAX}} = 46.4 \text{ N}\cdot\text{m}$$

ANS.

2.43

$$a = 10 \text{ in.}; b = 20 \text{ in.}; d = 3 \text{ in.}$$



$$Q = 75 \text{ kip}\cdot\text{in.}; a = 10 \text{ in.}; b = 20 \text{ in.}; d = 3 \text{ in.}$$

$$\sum T_x = 0: 75 - T_A - T_C = 0 \quad (1)$$

$$\theta_{B/A} = \theta_{B/C} = \frac{T_A(10)}{\left(\frac{\pi}{32}\right)(3^4)(4 \times 10^3)} = \frac{T_C(20)}{\left(\frac{\pi}{32}\right)(3^4)(4 \times 10^3)}$$

$$T_A = 2T_C \quad (2)$$

SOLVE (1) & (2) TO GET

$$T_A = 50 \text{ kip}\cdot\text{in.}; T_C = 25 \text{ kip}\cdot\text{in.}$$

$$(a) \tau_{\text{MAX}} = (\tau)_{AB} = \frac{50(1.5)}{\left(\frac{\pi}{32}\right)(3^4)} = 9.4 \text{ ksi} \quad \text{ANS.}$$

$$(b) \theta_B = \theta_{B/C} = \frac{50(10)}{\left(\frac{\pi}{32}\right)(3^4)(4 \times 10^3)} = 0.01572 \text{ rad} \quad \text{ANS.}$$

2.44 REFER TO THE F.B.D. IN PROB. 2.43.

$$a = 0.5 \text{ m}; b = 1.0 \text{ m}; d = 0.07 \text{ m}$$

$$\sum T_x = 0: Q - T_A - T_C = 0 \quad (1)$$

$$\theta_{B/A} = \theta_{B/C} = \frac{T_A(0.5)}{\left(\frac{\pi}{32}\right)(0.07^4)(25 \times 10^9)} = \frac{T_C(1.0)}{\left(\frac{\pi}{32}\right)(0.07^4)(40 \times 10^9)}$$

$$T_A = \left(\frac{5}{4}\right)T_C \quad (2)$$

 $\therefore (\tau_{\text{MAX}})_A$ CONTROLS. THUS,

$$150 \times 10^6 = \frac{T_A(0.035)}{\left(\frac{\pi}{32}\right)(0.07^4)}$$

$$T_A = 10.102 \text{ kN}\cdot\text{m}$$

$$(2) \Rightarrow T_C = \left(\frac{4}{5}\right)(10.102) = 8.082 \text{ kN}\cdot\text{m}$$

$$(1) \Rightarrow Q = T_A + T_C$$

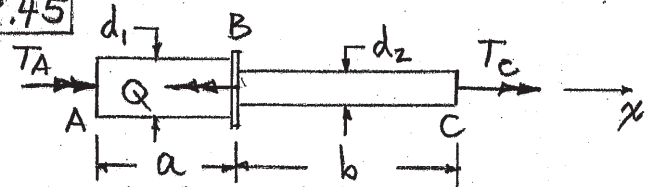
$$= 10.102 + 8.082$$

$$= 18.184$$

$$\approx 18.2 \text{ kN}\cdot\text{m}$$

ANS.

2.45



$$Q = 0.2(100) = 20 \text{ kN}\cdot\text{m}$$

$$a = 0.3 \text{ m}; b = 0.5 \text{ m}; d_1 = 0.050 \text{ m}$$

$$d_2 = 0.030 \text{ m.}$$

$$\sum T_x = 0: T_A + T_C - 20 = 0 \quad (1)$$

$$\theta_{B/A} = \theta_{B/C} = \frac{T_A(0.3)}{\left(\frac{\pi}{32}\right)(0.05^4)(40 \times 10^9)} = \frac{T_C(0.5)}{\left(\frac{\pi}{32}\right)(0.03^4)(40 \times 10^9)}$$

$$T_A = 12.860 T_C \quad (2)$$

SOLVE (1) & (2) TO GET

$$T_A = 18.557 \text{ kN}\cdot\text{m}; T_C = 1.443 \text{ kN}\cdot\text{m}$$

$$\tau_{\text{MAX}} = (\tau)_{AB} = \frac{18.557(0.025)}{\left(\frac{\pi}{32}\right)(0.05^4)}$$

$$= 756.1 \text{ MPa}$$

ANS.

$$\theta_{\text{MAX}} = \theta_{B/A} = \frac{18.557 \times 10^3(0.3)}{\left(\frac{\pi}{32}\right)(0.05^4)(40 \times 10^9)}$$

$$= 0.22682 \text{ rad}$$

ANS.

2.46 REFER TO THE F.B.D. IN PROB. 2.45.

$$Q = 8 \text{ F}; a = 12 \text{ in.}; b = 20 \text{ in.}$$

$$d_1 = 2.0 \text{ in.}; d_2 = 1.0 \text{ in.}$$

$$\sum T_x = 0: T_A + T_C - 8 \text{ F} = 0 \quad (1)$$

$$\theta_B = \theta_{B/A} = \frac{5\pi}{180} = \frac{T_A(12)}{\left(\frac{\pi}{32}\right)(2^4)(4 \times 10^6)}$$

$$T_A = 45.693 \text{ kip}\cdot\text{in.} \quad (2)$$

$$\theta_B = \theta_{B/C} = \frac{5\pi}{180} = \frac{T_C(20)}{\left(\frac{\pi}{32}\right)(1^4)(10 \times 10^6)}$$

$$T_C = 4.284 \text{ kip}\cdot\text{in.} \quad (3)$$

(2) & (3) INTO (1) $\Rightarrow$

$$F = 6.247 \approx 6.2 \text{ kips}$$

ANS.

2.47 REFER TO THE F.B.D. IN PROB. 2.45.

$$Q = 10 \text{ F}; a = 15 \text{ in.}; b = 25 \text{ in.}$$

$$d_1 = 3.0 \text{ in.}; d_2 = 2.0 \text{ in.}$$

$$\sum T_x = 0: T_A + T_C - 10 \text{ F} = 0 \quad (1)$$

$$\theta_{B/A} = \theta_{B/C} = \frac{T_A(15)}{\left(\frac{\pi}{32}\right)(3^4)(4 \times 10^3)} = \frac{T_C(25)}{\left(\frac{\pi}{32}\right)(2^4)(10 \times 10^3)}$$

$$T_A = 3.375 T_C \quad (2)$$

$$T_A = 10 = \frac{T_A(1.5)}{\left(\frac{\pi}{32}\right)(3^4)}; T_A = 53.014 \text{ kip}\cdot\text{in.} \quad (3)$$

$$\tau_C = 15 = \frac{T_C(1.0)}{\left(\frac{\pi}{32}\right)(2^4)}; T_C = 23.562 \text{ kip}\cdot\text{in.} \quad (4)$$

2.47 CONT'D

ASSUME $T_A = 53.014$ kip.in. CONTROLS.
THEN, BY EQ. (2),

$$T_C = \frac{53.014}{3.375} = 15.708 \text{ kip.in.} \quad (5)$$

$$\tau_A = \frac{53.014(1.5)}{(\pi/32)(3^4)} = 10.0 \text{ ksi} = (\tau_{ALL})_A \quad \text{OK}$$

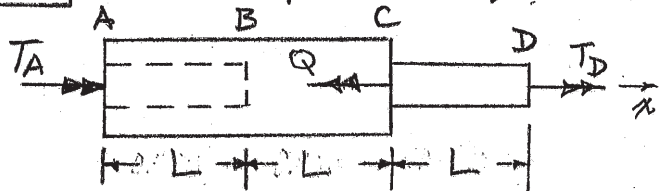
$$\tau_C = \frac{15.708(1.0)}{(\pi/32)(2^4)} = 10.0 \text{ ksi} < (\tau_{ALL})_S \quad \text{OK}$$

(3) $\frac{1}{2}$ (5) INTO (1) $\Rightarrow$

$$F = 6.872 \approx 6.9 \text{ kips} \quad \text{ANS.}$$

2.48

$$Q = 10 \text{ kN.m}; L = 0.4 \text{ m}$$



$$\sum T_x = 0: T_A + T_D - 10 = 0 \quad (1)$$

$$\theta_{C/A} = \theta_{C/D}: \frac{T_A(0.4)}{(\frac{\pi}{32})(0.075^4 - 0.04^4)(25 \times 10^9)} + \frac{T_A(0.4)}{(\frac{\pi}{32})(0.04^4)(25 \times 10^9)} = \frac{T_D(0.4)}{(\frac{\pi}{32})(0.04^4)(25 \times 10^9)}$$

$$T_A = 5.919 T_D \quad (2)$$

SOLVE (1) $\frac{1}{2}$ (2) TO GET

$$T_A = 8.555 \text{ kN.m}; T_D = 1.445 \text{ kN.m}$$

$$(a) \tau_{MAX} = \tau_{CD} = \frac{1.445 \times 10^3(0.02)}{(\frac{\pi}{32})(0.04^4)} = 115.0 \text{ MPa} \quad \text{ANS.}$$

$$(b) \theta_{MAX} = \theta_{C/D} = \frac{1.445 \times 10^3(0.4)}{(\frac{\pi}{32})(0.04^4)(25 \times 10^9)} = 0.0920 \text{ rad} \quad \text{ANS.}$$

2.49 REFER TO F.B.D. IN PROB. 2.48.

$$L = 12 \text{ in.}$$

$$\sum T_x = 0: T_A + T_D - Q = 0 \quad (1)$$

$$\theta_{C/A} = \theta_{C/D}: \frac{T_A(12)}{(\frac{\pi}{32})(4^4 - 2^4)(4 \times 10^3)} + \frac{T_A(12)}{(\frac{\pi}{32})(4^4)(4 \times 10^3)} = \frac{T_D(12)}{(\frac{\pi}{32})(2^4)(6 \times 10^3)}$$

$$T_A = 5.161 T_D \quad (2)$$

$$\tau_{AB} = 30 = \frac{T_A(2)}{(\frac{\pi}{32})(4^4 - 2^4)}$$

$$T_A = 353.429 \text{ kip.in.}$$

2.49 CONT'D

$$\tau_{CD} = 60 = \frac{T_D(1)}{(\frac{\pi}{32})(2^4)}; T_D = 94.248 \text{ kip.in.}$$

ASSUME $T_A = 353.429$ kip.in. CONTROLS.

THEN, BY EQ. (2),

$$T_D = \frac{353.429}{5.161} = 68.481 \text{ kip.in.}$$

$$\tau_A = \frac{353.429(2)}{(\frac{\pi}{32})(4^4 - 2^4)} = 30 \text{ ksi} = (\tau_{ALL})_A \quad \text{OK}$$

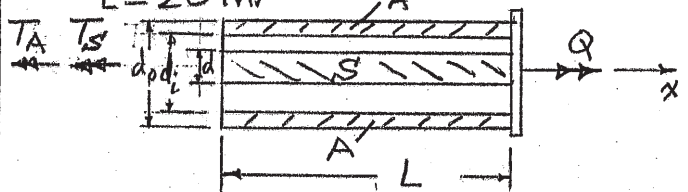
$$\tau_D = \frac{68.481(1)}{(\frac{\pi}{32})(2^4)} = 43.6 \text{ ksi} < (\tau_{ALL})_T \quad \text{OK}$$

$$\therefore (1) \Rightarrow Q = 353.429 + 68.481 = 421.91$$

$$Q \approx 421.9 \text{ kip.in.} \quad \text{ANS.}$$

2.50

$$Q = 50 \text{ kip.in.}; d_o = 5 \text{ in.}; d_i = 3 \text{ in.}; d = 2 \text{ in.}; L = 20 \text{ in.}$$



$$\sum T_x = 0: 50 - T_S - T_A = 0 \quad (1)$$

$$\theta_S = \theta_A: \frac{T_S(20)}{(\frac{\pi}{32})(2^4)(10 \times 10^3)} = \frac{T_A(20)}{(\frac{\pi}{32})(5^4 - 3^4)(4 \times 10^3)}$$

$$T_A = 13.546 T_S \quad (2)$$

SOLVE (1) $\frac{1}{2}$ (2) TO GET

$$T_A = 46.563 \text{ kip.in.}; T_S = 3.437 \text{ kip.in.}$$

$$(a) \tau_A = \frac{46.563(2.5)}{(\frac{\pi}{32})(5^4 - 3^4)} \approx 2.2 \text{ ksi} \quad \text{ANS.}$$

$$\tau_S = \frac{3.437(1)}{(\frac{\pi}{32})(2^4)} \approx 2.2 \text{ ksi} \quad \text{ANS.}$$

$$(b) \theta_{PL} = \theta_S = \frac{3.437(20)}{(\frac{\pi}{32})(2^4)(10 \times 10^3)} = 0.00438 \text{ rad} \quad \text{ANS.}$$

2.51 REFER TO THE F.B.D. IN PROB. 2.50.

$$d_o = 120 \text{ mm}; d_i = 80 \text{ mm}; d = 45 \text{ mm}; L = 0.5 \text{ m}$$

$$\sum T_x = 0: Q - T_S - T_A = 0 \quad (1)$$

$$\theta_S = \theta_A: \frac{T_S(0.5)}{(\frac{\pi}{32})(0.045^4)(75 \times 10^9)} = \frac{T_A(0.5)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)(25 \times 10^9)}$$

$$T_A = 13.526 T_S \quad (2)$$

$$\tau_A = 60 \times 10^6 = \frac{T_A(0.06)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)}; T_A = 16.336 \text{ kN.m}$$

$$\tau_S = 100 \times 10^6 = \frac{T_S(0.0225)}{(\frac{\pi}{32})(0.045^4)}; T_S = 1.789 \text{ kN.m}$$

2.51 CONT'D

ASSUME $T_A = 16.336 \text{ kN}\cdot\text{m}$ CONTROLS.
THEN, BY EQ. (2)

$$T_S = \frac{16.336}{13.526} = 1.208 \text{ kN}\cdot\text{m}$$

$$\tau_A = \frac{16.336 \times 10^3 (0.06)}{\left(\frac{\pi}{32}\right) (0.12^4 - 0.08^4)} = 60 \text{ MPa} = (\tau_{\text{ALL}})_A \quad \text{OK}$$

$$\tau_S = \frac{1.208 \times 10^3 (0.0225)}{\left(\frac{\pi}{32}\right) (0.045^4)} = 67.5 \text{ MPa} < (\tau_{\text{ALL}})_S \quad \text{OK}$$

BY EQ. (1) $Q = T_S + T_A = 1.208 + 16.336$

$$Q \approx 17.5 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.52 AS IN PROB. 2.51

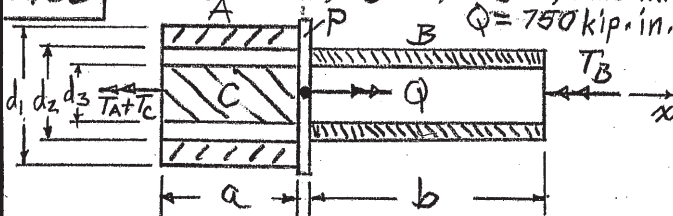
$$Q - T_S - T_A = 0 \quad (1)$$

$$\theta = \frac{TL}{JG}$$

$$\frac{5\pi}{180} = \frac{T_S (0.5)}{\left(\frac{\pi}{32}\right) (0.045^4) (75 \times 10^9)}; T_S = 5.269 \text{ kN}\cdot\text{m}$$

$$\frac{5\pi}{180} = \frac{T_A (0.5)}{\left(\frac{\pi}{32}\right) (0.12^4 - 0.08^4) (25 \times 10^9)}; T_A = 71.283 \text{ kN}\cdot\text{m}$$

$$(1) \Rightarrow Q = 5.269 + 71.283 = 76.552 \approx 76.6 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.53 $d_1 = 8 \text{ in.}; d_2 = 6 \text{ in.}; d_3 = 3 \text{ in.}; a = 6 \text{ in.}; b = 10 \text{ in.}$ 

$$\sum T_x = 0: 750 - T_A - T_B - T_C = 0 \quad (1)$$

AT P, $\theta_A = \theta_B = \theta_C$

$$\frac{T_A (6)}{\left(\frac{\pi}{32}\right) (8^4 - 5^4) (4 \times 10^6)} = \frac{T_B (10)}{\left(\frac{\pi}{32}\right) (5^4 - 3^4) (6 \times 10^6)} = \frac{T_C (6)}{\left(\frac{\pi}{32}\right) (3^4) (10 \times 10^6)}$$

$$T_B = 0.14105 T_A \quad (2)$$

$$T_C = 0.05834 T_A \quad (3)$$

$$(1) \Rightarrow T_A + 0.14105 T_A + 0.05834 T_A = 750$$

$$T_A = 625.318; T_B = 88.201; T_C = 36.481$$

$$\tau_{\text{MAX}} = \tau_A = \left(\frac{TR}{J}\right)_A = \frac{625.318 (4)}{\left(\frac{\pi}{32}\right) (8^4 - 5^4)}$$

$$= 7.340 \approx 7.3 \text{ ksi} \quad \text{ANS.}$$

$$\theta_P = \theta_C = \frac{36.481 \times 10^3 (6)}{\left(\frac{\pi}{32}\right) (3^4) (10 \times 10^6)} = 0.00273 \text{ rad} \quad \text{ANS.}$$

2.54 REFER TO THE F.B.D. IN PROB. 2.53.

$$a = 0.2 \text{ m}; b = 0.3 \text{ m}$$

$$d_1 = 250 \text{ mm}; d_2 = 150 \text{ mm}; d_3 = 70 \text{ mm}$$

$$\sum T_x = 0 \Rightarrow Q = T_A + T_B + T_C \quad (1)$$

AT P, $\theta_A = \theta_B = \theta_C$

$$\frac{T_A (0.2)}{\left(\frac{\pi}{32}\right) (0.25^4 - 0.15^4) (25 \times 10^9)} = \frac{T_B (0.3)}{\left(\frac{\pi}{32}\right) (0.15^4 - 0.07^4) (40 \times 10^9)}$$

$$= \frac{T_C (0.2)}{\left(\frac{\pi}{32}\right) (0.07^4) (75 \times 10^9)}$$

$$T_B = 0.1513 T_A \quad (2)$$

$$T_C = 0.0212 T_A \quad (3)$$

ASSUME $\tau_A = 70 \text{ MPa}$ CONTROLS. THEN

$$70 \times 10^6 = \frac{T_A (0.125)}{\left(\frac{\pi}{32}\right) (0.25^4 - 0.15^4)}$$

$$T_A = 186.925 \text{ kN}\cdot\text{m}$$

$$(2) \Rightarrow T_B = 28.282 \text{ kN}\cdot\text{m}$$

$$(3) \Rightarrow T_C = 3.963 \text{ kN}\cdot\text{m}$$

$$\tau_B = \frac{28.282 \times 10^3 (0.075)}{\left(\frac{\pi}{32}\right) (0.15^4 - 0.07^4)} = 44.8 \text{ MPa} < (\tau_{\text{ALL}})_B \quad \text{OK}$$

$$\tau_C = \frac{3.963 \times 10^3 (0.035)}{\left(\frac{\pi}{32}\right) (0.07^4)} = 58.8 \text{ MPa} < (\tau_{\text{ALL}})_C \quad \text{OK}$$

$$(1) \Rightarrow Q = 186.925 + 28.282 + 3.963 = 219.170 \approx 219.2 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.55 REFER TO THE F.B.D. IN PROB. 2.53.

$$a = 0.2 \text{ m}; b = 0.3 \text{ m}$$

$$d_1 = 250 \text{ mm}; d_2 = 150 \text{ mm}; d_3 = 70 \text{ mm}$$

$$\sum T_x = 0 \Rightarrow Q = T_A + T_B + T_C \quad (1)$$

$$\theta = \frac{TL}{JG}$$

$$\frac{0.5\pi}{180} = \frac{T_A (0.2)}{\left(\frac{\pi}{32}\right) (0.25^4 - 0.15^4) (25 \times 10^9)}$$

$$T_A = 364.113 \text{ kN}\cdot\text{m}$$

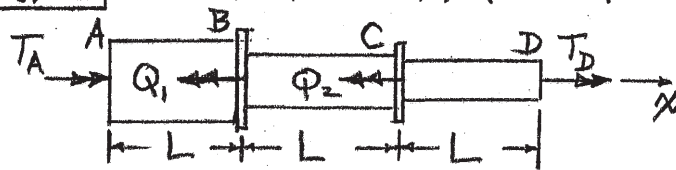
$$\frac{0.5\pi}{180} = \frac{T_B (0.3)}{\left(\frac{\pi}{32}\right) (0.15^4 - 0.07^4) (40 \times 10^9)}$$

$$T_B = 55.087 \text{ kN}\cdot\text{m}$$

$$\frac{0.5\pi}{180} = \frac{T_C (0.2)}{\left(\frac{\pi}{32}\right) (0.07^4) (75 \times 10^9)}; T_C = 7.713 \text{ kN}\cdot\text{m}$$

$$(1) \Rightarrow Q = 364.113 + 55.087 + 7.713 = 426.913 \approx 426.9 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.56 $L = 1.5 \text{ ft}$; $Q_1 = 10 \text{ kip}\cdot\text{ft}$; $Q_2 = 15 \text{ kip}\cdot\text{ft}$



$$\sum T_x = 0: T_A + T_D - 25 = 0 \quad (1)$$

$$\text{ALSO, } T_{BC} = 10 - T_A = T_D - 15 \quad (2)$$

$$\theta_{B/A} = \theta_{B/C} + \theta_{C/D} \quad (2) \Rightarrow T_D = 25 - T_A \quad (3)$$

$$\frac{T_A (1.5)}{(\pi/32)(2.5^4)(4 \times 10^3)} = \frac{(10 - T_A)(1.5)}{(\pi/32)(1.5^4)(6 \times 10^3)} + \frac{(25 - T_A)(1.5)}{(\pi/32)(1.0^4)(10 \times 10^3)}$$

$$T_A = 20.307; T_D = 4.693; T_{BC} = -10.307$$

$$\tau_{\text{MAX}} = \tau_{CD} = \frac{4.693 \times 10^3 (0.5)}{(\pi/32)(1.0^4)} = 23.901$$

$$\tau_{\text{MAX}} = \tau_{CD} \approx 23.9 \text{ ksi} \quad \text{ANS.}$$

$$\theta_C = \theta_{C/D} = \frac{4.693 \times 10^3 (1.5 \times 12)}{(\pi/32)(1.0^4)(10 \times 10^6)} = 86.045 \times 10^{-3}$$

$$\theta_C = \theta_{C/D} \approx 0.086 \text{ rad} \quad \text{ANS.}$$

2.57 REFER TO THE F.B.D. IN PROB. 2.56

$$L = 1.0 \text{ m}; Q_1 = 50 \text{ kN}\cdot\text{m}; Q_2 = 75 \text{ kN}\cdot\text{m}$$

$$\sum T_x = 0: T_A + T_D - 125 = 0 \quad (1)$$

$$\text{ALSO, } T_{BC} = 50 - T_A = T_D - 75 \quad (2)$$

$$(2) \Rightarrow T_D = 125 - T_A \quad (3)$$

$$\theta_{B/A} = \theta_{B/C} + \theta_{C/D}$$

$$\frac{T_A (1.0)}{(\pi/32)d^4(30 \times 10^9)} = \frac{(50 - T_A)(1.0)}{(\pi/32)d^4(30 \times 10^9)} + \frac{(125 - T_A)(1.0)}{(\pi/32)d^4(30 \times 10^9)}$$

$$T_A = 58.333; T_D = 66.667; T_{BC} = -8.333$$

MINIMUM DIA. CONTROLLED BY SHEARING STRESS IN SEGMENT CD. THUS,

$$120 \times 10^6 = \frac{66.667 \times 10^3 (d/2)}{(\pi/32)d^4}; d = 0.14144 \text{ m}$$

$$d_{\text{MIN}} \approx 141.4 \text{ mm} \quad \text{ANS.}$$

2.58 $\sum T_{BA} = 0:$

$$T_{BA} + 3F - 20 = 0 \quad (1)$$

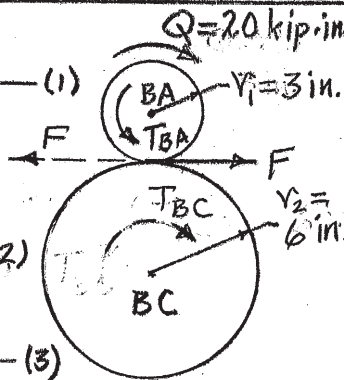
$$\sum T_{BC} = 0:$$

$$6F - T_{BC} = 0$$

$$F = \left(\frac{1}{6}\right) T_{BC} \quad (2)$$

$$(2) \text{ INTO } (1) \Rightarrow$$

$$T_{BA} + \left(\frac{1}{2}\right) T_{BC} = 20 \quad (3)$$



2.58 CONT'D

$$3\theta_{B/A} = 6\theta_{B/C} \Rightarrow \theta_{B/A} = 2\theta_{B/C} \quad (4)$$

$$(4) \Rightarrow \frac{T_{BA} (15)}{(\pi/32)(1.0^4)(10 \times 10^3)} = 2 \left[\frac{T_{BC} (15)}{(\pi/32)(2.0^4)(4 \times 10^3)} \right]$$

$$T_{BC} = 3.2 T_{BA} \quad (5)$$

SOLVE (3) & (5) TO GET

$$T_{BC} = 24.614; T_{BA} = 7.692$$

$$\tau_{AB} = \frac{7.692 (0.5)}{(\pi/32)(1.0^4)} \approx 39.2 \text{ ksi} \quad \text{ANS.}$$

$$\tau_{BC} = \frac{24.614 (1.0)}{(\pi/32)(2.0^4)} \approx 15.7 \text{ ksi} \quad \text{ANS.}$$

2.59 REFER TO THE F.B.D. IN PROB. 2.58

$$\sum T_{BA} = 0: T_{BA} + 0.1F - Q = 0 \quad (1)$$

$$\sum T_{BC} = 0: 0.15F - T_{BC} = 0 \quad (2)$$

$$F = \left(\frac{1}{0.15}\right) T_{BC} \quad (2)$$

$$(2) \text{ INTO } (1) \Rightarrow$$

$$T_{BA} + \left(\frac{1}{1.5}\right) T_{BC} = Q \quad (3)$$

$$0.1\theta_{B/A} = 0.15\theta_{B/C}; \theta_{B/A} = 1.5\theta_{B/C} \quad (4)$$

$$\theta_{B/A} = \frac{T_{BA} (0.75)}{(\pi/32)(0.075^4)(50 \times 10^9)} = 4.829 \times 10^{-6} T_{BA} \quad (5)$$

$$\theta_{B/C} = \frac{T_{BC} (0.75)}{(\pi/32)(0.075^4)(50 \times 10^9)} = 4.829 \times 10^{-6} T_{BC} \quad (6)$$

$$(5) \& (6) \text{ INTO } (4) \Rightarrow T_{BA} = 1.5 T_{BC} \quad (7)$$

$$(7) \text{ INTO } (3) \Rightarrow T_{BC} = 0.462Q$$

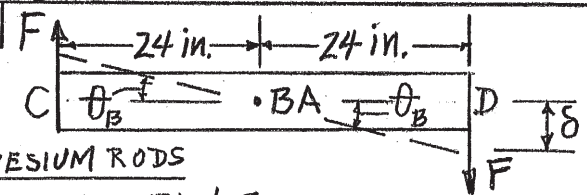
$$(7) \Rightarrow T_{BA} = 0.692Q$$

$\therefore$ SHEARING STRESS IN AB CONTROLS. THUS

$$120 \times 10^6 = \frac{0.692Q (0.0375)}{(\pi/32)(0.075^4)}$$

$$Q = 14364 \text{ N}\cdot\text{m} = 14.4 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.60



MAGNESIUM RODS

$$\delta = \alpha \Delta T L - FL/AE$$

$$= 14.5 \times 10^{-6} (150)(18) - F(18)/(1)(6.5 \times 10^6)$$

$$\delta = (39,150 - 2.769F) \times 10^{-6}$$

$$\theta_B = \delta/24 = (1,631.25 - 0.11538F) \times 10^{-6} \quad (1)$$

2.60 CONT'D

$$\text{ALSO, } \theta_B = \left(\frac{TL}{JG} \right)_{BA} = \frac{T_{BA}(36)}{\left(\frac{\pi}{32} \right) (4.0^4 - 3.5^4) (6 \times 10^6)}$$

$$\theta_B = 0.5769 T_{BA} \times 10^{-6} \quad (2)$$

$$\sum T_{BA} = 0: T_{BA} - 48F = 0; F = \frac{T_{BA}}{48} \quad (3)$$

$$(3) \text{ INTO } (1) \Rightarrow \theta_B = (1631.25 - 0.00240 T_{BA}) \times 10^{-6} \quad (4)$$

$$(2) = (4) \Rightarrow 0.5769 T_{BA} \times 10^{-6} = (1631.25 - 0.00240 T_{BA}) \times 10^{-6}$$

$$T_{BA} = 2815.89 \text{ lb.in.} = 2.816 \text{ kip.in.}$$

$$\tau_{BA} = \frac{2.816(2)}{\left(\frac{\pi}{32} \right) (4.0^4 - 3.5^4)} = 0.542 \text{ ksi} \quad \text{ANS.}$$

$$(2) \Rightarrow \theta_B = 0.5769 (2815.89) \times 10^{-6}$$

$$\theta_B = 0.00162 \text{ rad} \quad \text{ANS.}$$

2.61 REFER TO THE F.B.D. IN PROB. 2.60.

$$\sum T_{BA} = 0: T_{BA} - 48F = 0; F = \frac{T_{BA}}{48} \quad (1)$$

ASSUME $\tau = 7$ ksi CONTROLS

$$\tau = 7 = \frac{T_{BA}(1.5)}{\left(\frac{\pi}{32} \right) (3.0^4 - 2.5^4)}; T_{BA} = 19.214 \text{ kip.in.}$$

$$(1) \Rightarrow F = \frac{19.214}{48} = 0.400 \text{ kip}$$

$$\tau_{\text{ROD}} = \frac{F}{A} = \frac{0.400}{\left(\frac{\pi}{4} \right) (0.5^2)} = 2.04 < (\tau_{\text{ALL}}) \quad \text{OK}$$

K. ASSUMPTION IS CORRECT.

$$\delta = \alpha \Delta T L - FL/AE$$

$$= 13.0 \times 10^{-6} \Delta T (15) - \frac{0.400 \times 10^3 (15)}{\left(\frac{\pi}{4} \right) (0.5^2) (10 \times 10^6)}$$

$$\delta = (195 \Delta T - 3055.775) \times 10^{-6}$$

$$\theta_B = \frac{\delta}{24} = (8.125 \Delta T - 127.324) \times 10^{-6}$$

$$\text{ALSO, } \theta_B = \left(\frac{TL}{JG} \right)_{BA} = \frac{19.214 \times 10^3 (15)}{\left(\frac{\pi}{32} \right) (3.0^4 - 2.5^4) (4 \times 10^6)}$$

$$\theta_B = 17.5 \times 10^{-3} \text{ rad}$$

$$\therefore 17.5 \times 10^{-3} = (8.125 \Delta T - 127.324) \times 10^{-6}$$

$$\Delta T_{\text{MAX}} = 2170^\circ \text{F} \quad \text{ANS.}$$

$$2.62 \quad P = Tw; 120 = T \left(\frac{900 \times 2\pi}{60} \right)$$

$$T = 1.273 \text{ kN.m}$$

$$\tau_{\text{MAX}} = \frac{T\gamma}{J} = \frac{1.273 \times 10^3 (0.015)}{\left(\frac{\pi}{32} \right) (0.03^4)}$$

$$= 240.1 \text{ MPa} \quad \text{ANS.}$$

$$\theta = \frac{TL}{JG} = \frac{1.273 \times 10^3 (1.0)}{\left(\frac{\pi}{32} \right) (0.03^4) (70 \times 10^9)}$$

$$= 0.229 \text{ rad} \quad \text{ANS.}$$

2.63 SHEARING STRESS LIMITATION

$$\tau_{\text{ALL}} = \frac{T\gamma}{J}; T = \frac{\tau_{\text{ALL}} J}{\gamma} = \frac{8 \left(\frac{\pi}{32} \right) (2.0^4 - 1.5^4)}{1.0}$$

$$T = 8.590 \text{ kip.in.}$$

ANGLE OF TWIST LIMITATION

$$\theta_{\text{ALL}} = \frac{TL}{JG}; T = \frac{\theta_{\text{ALL}} JG}{L}$$

$$T = \frac{(4\pi/180) (\pi/32) (2.0^4 - 1.5^4) (4 \times 10^6)}{3 \times 12}$$

$$= 8.329 \text{ kip.in.}$$

∴ ALLOWABLE ANGLE OF TWIST CONTROLS.

$$P = Tw = 8.329 \times 10^3 \left(\frac{600 \times 2\pi}{60} \right)$$

$$= 523,327 \times 10^3 \text{ lb.in./s}$$

$$= 79.3 \text{ hp} \quad \text{ANS.}$$

$$2.64 \quad P = Tw; T = \frac{P}{\omega} = \frac{30 \times 10^3 \times 60}{1200 (2\pi)}$$

$$T = 238.732 \text{ N.m}$$

SHEARING STRESS LIMITATION

$$\tau_{\text{ALL}} = \frac{TR}{J}; \frac{J}{R} = \frac{T}{\tau_{\text{ALL}}}$$

$$\frac{(\pi/32) d^4}{d/2} = \frac{238.732}{50 \times 10^6}; d = 0.02897 \text{ m}$$

ANGLE OF TWIST LIMITATION

$$\theta_{\text{ALL}} = \frac{TL}{JG}; J = \frac{TL}{\theta_{\text{ALL}} G}$$

$$\frac{(\pi/32) d^4}{d/2} = \frac{238.732 (0.50)}{(0.08) (75 \times 10^9)}$$

$$d = 0.0212 \text{ m}$$

$$\therefore d_{\text{MIN}} = 0.02897 \text{ m} \approx 29.0 \text{ mm} \quad \text{ANS.}$$

$$2.65 \quad P = T\omega; T = \frac{P}{\omega} = \frac{20 \times 10^3 \times 60}{1000 \times 2\pi}$$

$$T = 190.986 \text{ N}\cdot\text{m}$$

SHEARING STRESS LIMITATION

$$\tau_{ALL} = \frac{TR}{J}; J = \frac{TR}{\tau_{ALL}}$$

$$\left(\frac{\pi}{32}\right)(0.05^4 - d_i^4) = \frac{190.986(0.025)}{65 \times 10^6}$$

$$d_i = 0.0484 \text{ m} = 48.4 \text{ mm}$$

ANGLE OF TWIST LIMITATION

$$\theta_{ALL} = \frac{TL}{JG}; J = \frac{TL}{\theta_{ALL}G}$$

$$\left(\frac{\pi}{32}\right)(0.05^4 - d_i^4) = \frac{190.986(0.75)}{0.07(75 \times 10^9)}$$

$$d_i = 0.0494 \text{ m} = 49.4 \text{ mm}$$

$$\therefore (d_i)_{MAX} = 48.4 \text{ mm}$$

ANS.

$$2.66 \quad P = T\omega; T = \frac{P}{\omega} = \frac{20(6600)60}{600 \times 2\pi}$$

$$T = 2100.845 \text{ lb}\cdot\text{in.}$$

$$\tau_{ALL} = \frac{Tr}{J}; J = \frac{Tr}{\tau_{ALL}}$$

$$\left(\frac{\pi}{32}\right)d^4 = \frac{2100.845(d/2)}{15 \times 10^3}$$

$$d_{MIN} = 0.893 \text{ in.}$$

ANS.

$$2.67 \quad P = T\omega; \omega = P/T \text{ --- (1)}$$

SHEARING STRESS LIMITATION

$$\tau_{ALL} = \frac{TR}{J}; T = \frac{\tau_{ALL}J}{R}$$

$$T = \frac{25 \times 10^6 (\pi/32)(0.055^4)}{(0.055/2)}$$

$$T = 816.691 \text{ N}\cdot\text{m}$$

ANGLE OF TWIST LIMITATION

$$\theta_{ALL} = \frac{TL}{JG}; T = \frac{\theta_{ALL}JG}{L}$$

$$T = \frac{(3\pi/180)(\pi/32)(0.055^4)(26 \times 10^9)}{(1)}$$

$$T = 1,222 \text{ N}\cdot\text{m}$$

$\therefore$ SHEARING STRESS CONTROLS. THUS

$$(1) \Rightarrow \omega = \frac{150 \times 10^3}{816.691} = 183.668 \text{ rad/s}$$

$$= 1753.9 \text{ rpm}$$

ANS.

$$2.68 \quad P = T\omega; \gamma_1 = 2.5 \text{ in.}; \gamma_2 = 4.0 \text{ in.}$$

$$T_{BA} = \frac{P}{\omega} = \frac{20(6600)(60)}{300(2\pi)}$$

$$T_{BA} = 4201.690 \text{ lb}\cdot\text{in.}$$

$$(\tau_{MAX})_{BA} = \frac{4201.690(0.75)}{BA \left(\frac{\pi}{32}\right)(1.5^4)}$$

$$(\tau_{MAX})_{BA} = 6.3 \text{ ksi} \text{ ANS.}$$

$$\theta_{BA} = \frac{4201.690(4 \times 12)}{\left(\frac{\pi}{32}\right)(1.5^4)(4 \times 10^6)}$$

$$\theta_{BA} = 0.101 \text{ rad}$$

ANS.

$$F = \frac{T_{BA}}{r_1} = \frac{4201.690}{2.5} = 1680.676 \text{ lb}$$

$$\therefore \frac{T_{CD}}{r_2} = 1680.676; T_{CD} = 1680.676(4.0)$$

$$T_{CD} = 6722.704 \text{ lb}\cdot\text{in.}$$

$$(\tau_{MAX})_{CD} = \frac{6722.704(0.75)}{\left(\frac{\pi}{32}\right)(1.5^4)} = 10.1 \text{ ksi} \text{ ANS.}$$

$$\theta_{CD} = \frac{6722.704(3 \times 12)}{\left(\frac{\pi}{32}\right)(1.5^4)(4 \times 10^6)} = 0.12174 \text{ rad} \text{ ANS.}$$

2.69 REFER TO THE F.B.D. IN PROB. 2.68.

$$P = T\omega; T_{BA} = \frac{P}{\omega} = \frac{10 \times 10^3 \times 60}{400(2\pi)}$$

$$T_{BA} = 2.387 \times 10^2 \text{ N}\cdot\text{m.}$$

$$(\tau_{ALL})_{BA} = \left(\frac{Tr}{J}\right)_{BA}; \frac{J}{r} = \frac{T}{\tau_{ALL}} = \frac{2.387 \times 10^2}{50 \times 10^6}$$

$$\left(\frac{\pi}{32}\right)\left(\frac{d^4}{d/2}\right) = 4.774 \times 10^{-6}$$

$$d = 0.02897 \text{ m} \approx 29.0 \text{ mm} \text{ ANS.}$$

$$F = \frac{T_{BA}}{r_1} = \frac{2.387 \times 10^2}{0.04} = 5967.5 \text{ N}$$

$$\therefore \frac{T_{CD}}{r_2} = 5967.5; T_{CD} = 5967.5(0.075)$$

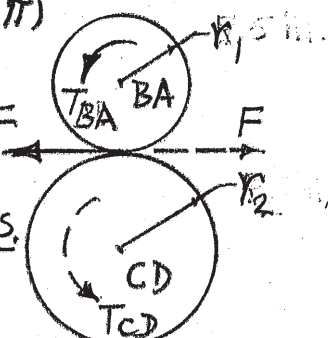
$$T_{CD} = 447.6 \text{ N}\cdot\text{m}$$

$$(\tau_{ALL})_{CD} = \left(\frac{Tr}{J}\right)_{CD}; \frac{J}{r} = \frac{T}{\tau_{ALL}} = \frac{447.6}{50 \times 10^6}$$

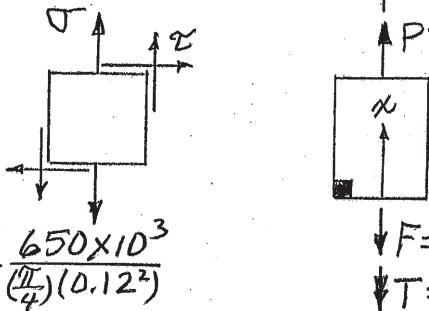
$$\frac{J}{r} = 8.952 \times 10^{-6}$$

$$\left(\frac{\pi}{32}\right)\left(\frac{d^4}{d/2}\right) = 8.952 \times 10^{-6}$$

$$d_{CD} = 0.03572 \text{ m} \approx 35.7 \text{ mm} \text{ ANS.}$$



2.70 $P = 650 \text{ kN}; Q = 20 \text{ kN}\cdot\text{m}$



$$\sigma = \frac{F}{A} = \frac{650 \times 10^3}{\left(\frac{\pi}{4}\right)(0.12^2)}$$

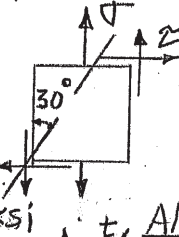
$$\sigma = 57.473 \approx 57.5 \text{ MPa} \quad \text{ANS.}$$

$$\tau = \frac{TR}{J} = \frac{20 \times 10^3 (0.06)}{\left(\frac{\pi}{32}\right)(0.12^4)} = 58.946 \approx 58.9 \text{ MPa} \quad \text{ANS.}$$

2.71 REFER TO THE F.B.D. IN PROB. 2.70.

$P = 150 \text{ kips}; Q = 150 \text{ kip}\cdot\text{in.}$

(a) $\sigma = \frac{F}{A} = \frac{150}{\left(\frac{\pi}{4}\right)(5^2)}$



$$\sigma = 7.639 \approx 7.6 \text{ ksi} \quad \text{ANS.}$$

$$\tau = \frac{TR}{J} = \frac{150 (2.5)}{\left(\frac{\pi}{32}\right)(5^4)} = 6.112 \approx 6.1 \text{ ksi} \quad \text{ANS.}$$

(b) LET A_n BE AREA OF INCLINED PLANE.

$$\sum F_n = 0$$

$$\sigma_n A + \tau (A \cos 30) \sin 30 + \tau (A \sin 30) \cos 30 - \sigma (A \sin 30) \sin 30 = 0$$

$$\sigma_n = \sigma \sin^2 30 - 2\tau \sin 30 \cos 30$$

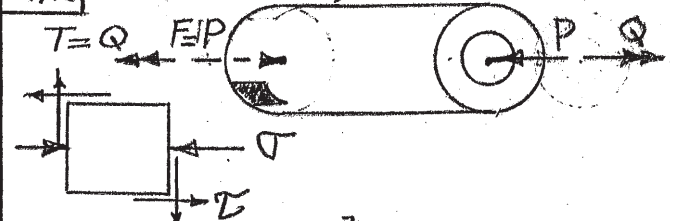
$$\sigma_n = -3.383 \approx -3.4 \text{ ksi} \quad \text{ANS.}$$

$$\sum F_t = 0; \tau_{nt} A_n - \tau (A \cos 30) \cos 30 + \tau (A \sin 30) \sin 30 + \sigma (A \sin 30) \cos 30 = 0$$

$$\tau_{nt} = \tau (\cos^2 30 - \sin^2 30) - \sigma \sin 30 \cos 30$$

$$\tau_{nt} = -0.252 \approx -0.3 \text{ ksi} \quad \text{ANS.}$$

2.72 $P = 500 \text{ kN}; Q = 30 \text{ kN}\cdot\text{m}$



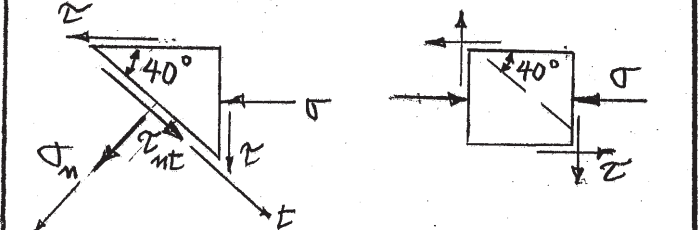
$$\sigma = \frac{F}{A} = \frac{500 \times 10^3}{\left(\frac{\pi}{4}\right)(0.15^2 - 0.006^2)} = 28,340$$

$$\sigma \approx 28.3 \text{ MPa} \quad \text{ANS.}$$

$$\tau = \frac{TR}{J} = \frac{30 \times 10^3 (0.075)}{\left(\frac{\pi}{32}\right)(0.15^4 - 0.006^4)} = 45.271$$

$$\tau \approx 45.3 \text{ MPa} \quad \text{ANS.}$$

2.73 REFER TO THE F.B.D. IN PROB. 2.72. FROM PROB. 2.72; $\sigma = 28,340 \text{ MPa}; \tau = 45.271 \text{ MPa}$.



LET A_n BE AREA OF INCLINED PLANE.

$$\sum F_n = 0; \sigma_n A_n + \tau (A_n \cos 40) \sin 40 + \tau (A_n \sin 40) \cos 40 + \sigma (A_n \sin 40) \sin 40 = 0$$

$$\sigma_n = -2\tau \sin 40 \cos 40 - \sigma \sin^2 40$$

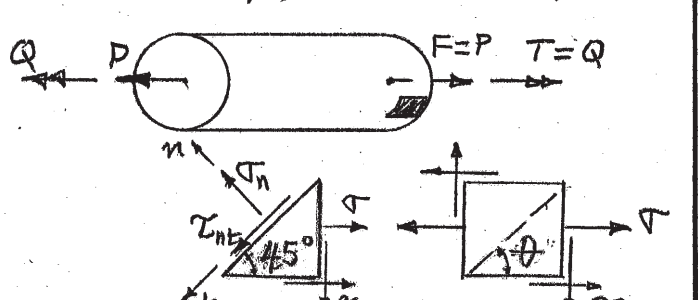
$$\sigma_n = -56.293 \approx -56.3 \text{ MPa} \quad \text{ANS.}$$

$$\sum F_t = 0; \tau_{nt} A_n + \tau (A_n \cos 40) \cos 40 + \tau (A_n \sin 40) \sin 40 - \sigma (A_n \sin 40) \cos 40 = 0$$

$$\tau_{nt} = \sigma \sin 40 \cos 40 + \tau (\cos^2 40 - \sin^2 40)$$

$$\tau_{nt} = 21.816 \approx 21.8 \text{ MPa} \quad \text{ANS.}$$

2.74 $P = 125 \text{ kips}; Q = 150 \text{ kip}\cdot\text{in.}; \theta = 45^\circ$



$$\sigma = \frac{F}{A} = \frac{125}{\left(\frac{\pi}{4}\right)(3.5^2)} = 12.992 \text{ ksi}$$

$$\tau = \frac{TR}{J} = \frac{150 (1.75)}{\left(\frac{\pi}{32}\right)(3.5^4)} = 17.818 \text{ ksi}$$

LET A_n BE AREA OF INCLINED PLANE.

$$\sum F_n = 0; \sigma_n A_n - \sigma (A_n \sin 45) \sin 45 - \tau (A_n \sin 45) \cos 45 - \tau (A_n \cos 45) \sin 45 = 0$$

$$\sigma_n = \sigma \sin^2 45 + 2\tau \sin 45 \cos 45$$

$$\sigma_n = 24.314 \approx 24.3 \text{ ksi} \quad \text{ANS.}$$

$$\sum F_t = 0; \tau_{nt} A_n - \sigma (A_n \sin 45) \cos 45 + \tau (A_n \sin 45) \sin 45 - \tau (A_n \cos 45) \cos 45 = 0$$

$$\tau_{nt} = \sigma \sin 45 \cos 45 + \tau (\cos^2 45 - \sin^2 45)$$

$$\tau_{nt} = 6.496 \approx 6.5 \text{ ksi} \quad \text{ANS.}$$

2.75 REFER TO THE F.B.D. IN PROB. 2.74.

$$P = 600 \text{ kN}; Q = 20 \text{ kN}\cdot\text{m}; \theta = 60^\circ$$

$$\sigma = \frac{F}{A} = \frac{600 \times 10^3}{\frac{\pi}{4}(0.1^2)} = 76.394 \text{ MPa}$$

$$\tau = \frac{TR}{J} = \frac{20 \times 10^3(0.05)}{\frac{\pi}{32}(0.1^4)} = 101.859 \text{ MPa}$$

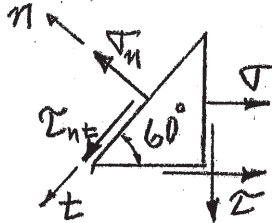
LET A_n BE AREA OF INCLINED PLANE.

$$\Sigma F_n = 0: \sigma_n A_n - \sigma(A_n \sin 60) \sin 60 - \tau(A_n \cos 60) \sin 60 = 0$$

$$\sigma_n = \sigma \sin^2 60 + \tau \sin 60 \cos 60 = 145.508 \approx 145.5 \text{ MPa} \quad \text{ANS.}$$

$$\Sigma F_t = 0: \tau_{nt} A_n - \sigma(A_n \sin 60) \cos 60 + \tau(A_n \cos 60) \cos 60 = 0$$

$$\tau_{nt} = \sigma \sin 60 \cos 60 + \tau(\cos^2 60 - \sin^2 60) = -17.850 \approx -17.9 \text{ MPa} \quad \text{ANS.}$$



2.77 REFER TO F.B.D. IN PROB. 2.76.

$$P = 550 \text{ kN}; P_1 = 8 \text{ kN}; P_2 = 12 \text{ kN}; r_1 = 0.375 \text{ m}; r_2 = 0.25 \text{ m}; d = 120 \text{ mm}; \theta = 30^\circ; T = 12 \text{ kN}\cdot\text{m}$$

$$\sigma = \frac{F}{A} = \frac{550 \times 10^3}{\frac{\pi}{4}(0.12^2)} = 48.631 \text{ MPa}$$

$$\tau = \frac{TR}{J} = \frac{12(10^3)(0.06)}{\frac{\pi}{32}(0.12^4)} = 35.368 \text{ MPa}$$

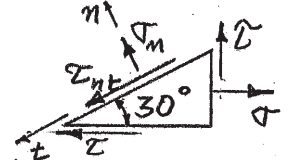
LET A_n BE AREA OF INCLINED PLANE.

$$\Sigma F_n = 0: \sigma_n A_n - \sigma(A_n \sin 30) \sin 30 + \tau(A_n \sin 30) \cos 30 + \tau(A_n \cos 30) \sin 30 = 0$$

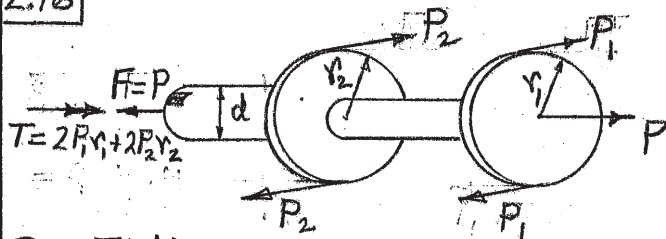
$$\sigma_n = \sigma \sin^2 30 - 2\tau \sin 30 \cos 30 = -18.472 \approx -18.5 \text{ MPa} \quad \text{ANS.}$$

$$\Sigma F_t = 0: \tau_{nt} A_n - \sigma(A_n \sin 30) \cos 30 - \tau(A_n \sin 30) \sin 30 + \tau(A_n \cos 30) \cos 30 = 0$$

$$\tau_{nt} = \sigma \sin 30 \cos 30 + \tau(\sin^2 30 - \cos^2 30) = 3.374 \approx 3.4 \text{ MPa} \quad \text{ANS.}$$



2.76



$$P = 150 \text{ kips}; P_1 = 2 \text{ kips}; P_2 = 3 \text{ kips}; r_1 = 0.75 \text{ ft}; r_2 = 1.0 \text{ ft}; d = 4 \text{ in.}; \theta = 25^\circ; T = 9 \text{ kip}\cdot\text{ft}$$

$$\sigma = \frac{F}{A} = \frac{150}{\frac{\pi}{4}(4^2)} = 11.937 \text{ ksi}$$

$$\tau = \frac{TR}{J} = \frac{9(12)(2)}{\frac{\pi}{32}(4^4)} = 8.594 \text{ ksi}$$

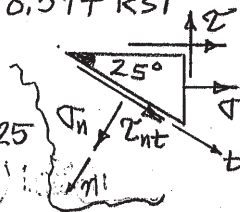
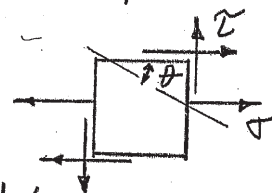
LET A_n BE AREA OF INCLINED PLANE.

$$\Sigma F_n = 0: \sigma_n A_n - \tau(A_n \cos 25) \sin 25 - \tau(A_n \sin 25) \cos 25 = 0$$

$$\sigma_n = 2\tau \sin 25 \cos 25 + \tau \sin^2 25 = 8.715 \text{ ksi} \approx 8.7 \text{ ksi} \quad \text{ANS.}$$

$$\Sigma F_t = 0: \tau_{nt} A_n + \tau(A_n \cos 25) \cos 25 - \tau(A_n \sin 25) \sin 25 + \sigma(A_n \sin 25 \cos 25) = 0$$

$$\tau_{nt} = \tau(\sin^2 25 - \cos^2 25) - \sigma \sin^2 25 = -7.656 \approx -7.7 \text{ ksi} \quad \text{ANS.}$$



2.78

$$\sigma_n = 15 \text{ ksi}$$

$$\tau_{nt} = 10 \text{ ksi}$$

$$\theta = 35^\circ$$

LET A_n BE AREA OF INCLINED PLANE.

$$\Sigma F_n = 0: 15 A_n - \sigma(A_n \cos 35) \cos 35 - \tau(A_n \cos 35) \sin 35 - \tau(A_n \sin 35) \cos 35 = 0$$

$$0.6710 \sigma + 0.9397 \tau = 15 \quad (1)$$

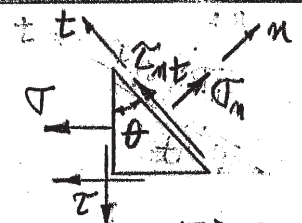
$$\Sigma F_t = 0: 10 A_n + \sigma(A_n \cos 35) \sin 35 - \tau(A_n \cos 35) \cos 35 + \tau(A_n \sin 35) \sin 35 = 0$$

$$-0.4698 \sigma + 0.3420 \tau = 10 \quad (2)$$

SOLVE (1) & (2) TO GET

$$\sigma = -6.357 \approx -6.4 \text{ ksi} \quad \text{ANS.}$$

$$\tau = 20.504 \approx 20.5 \text{ ksi} \quad \text{ANS.}$$

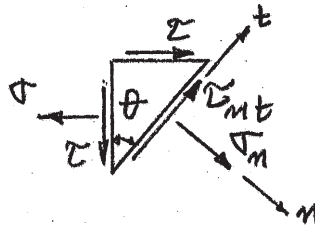


2.79

$$\sigma_n = 150 \text{ MPa}$$

$$\tau_{nt} = 100 \text{ MPa}$$

$$\theta = 40^\circ$$



LET A_n BE AREA OF INCLINED PLANE.

$$\begin{aligned} \sum F_n = 0: & 150 A_n - \sigma (A_n \cos 40) \cos 40 \\ & + \tau (A_n \cos 40) \sin 40 + \tau (A_n \sin 40) \cos 40 = 0 \\ 0.5868 \sigma - 0.9848 \tau & = 150 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \sum F_t = 0: & 100 A_n - \sigma (A_n \cos 40) \sin 40 \\ & - \tau (A_n \cos 40) \cos 40 + \tau (A_n \sin 40) \sin 40 = 0 \\ 0.4924 \sigma + 0.1736 \tau & = 100 \quad \text{--- (2)} \end{aligned}$$

SOLVE (1) & (2) TO GET

$$\sigma = 212.207 \approx 212.2 \text{ MPa} \quad \text{ANS.}$$

$$\tau = -25.869 \approx -25.9 \text{ MPa} \quad \text{ANS.}$$

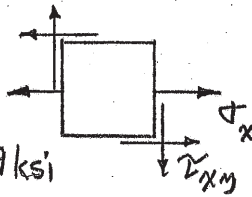
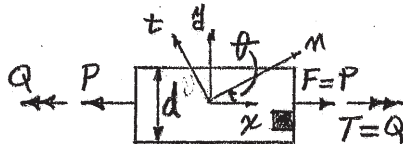
2.80

$$P = 60 \text{ kips}$$

$$Q = 30 \text{ kip}\cdot\text{in.}$$

$$d = 2 \text{ in.}$$

$$\theta = 30^\circ$$



$$\sigma_x = \frac{F}{A} = \frac{60}{\frac{\pi}{4}(2^2)} = 19.099 \text{ ksi}$$

$$\tau_{xy} = \frac{TR}{J} = \frac{30(1)}{\frac{\pi}{32}(2^4)} = 19.099 \text{ ksi}$$

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \mu \sigma_y) = \frac{1}{30 \times 10^3} (19.099 - 0) \\ &= 0.6366 \times 10^{-3} = 636.6 \mu \quad \text{ANS.} \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (\sigma_y - \mu \sigma_x) = \frac{1}{30 \times 10^3} (0 - 0.33 \times 19.099) \\ &= -0.2101 \times 10^{-3} = -210.1 \mu \quad \text{ANS.} \end{aligned}$$

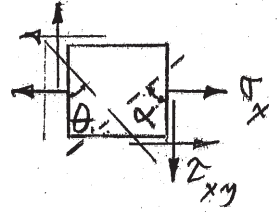
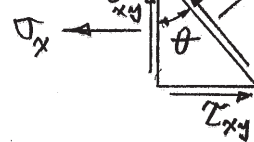
$$\begin{aligned} \gamma_{xy} &= \frac{\tau_{xy}}{G} = \frac{\tau_{xy}}{E/2(1+\mu)} = \frac{19.099}{30 \times 10^3 / 2(1+0.33)} \\ &= 1.6934 \times 10^{-3} = 1693.4 \mu \quad \text{ANS.} \end{aligned}$$

2.81 REFER TO THE F.B.D. IN PROB. 2.80.

$$P = 150 \text{ kN}; Q = 5 \text{ kN}\cdot\text{m}; d = 60 \text{ mm};$$

$$\theta = 40^\circ$$

$$\alpha = 50^\circ$$



$$\sigma_x = \frac{F}{A} = \frac{150 \times 10^3}{\frac{\pi}{4}(0.06^2)} = 53.052 \text{ MPa}$$

$$\tau_{xy} = \frac{TR}{J} = \frac{5 \times 10^3(0.03)}{(\pi/32)(0.06^4)} = 117.893 \text{ MPa}$$

LET A_n BE AREA OF INCLINED PLANE.

$$\begin{aligned} \sum F_n = 0: & \sigma_n A_n - \sigma_x (A_n \cos^2 40) + \tau_{xy} (A_n \cos^2 40) \sin 40 \\ & + \tau_{xy} (A_n \sin^2 40) \cos 40 = 0 \end{aligned}$$

$$\sigma_n = \sigma_x \cos^2 40 - 2 \tau_{xy} \sin 40 \cos 40$$

$$\sigma_n = -84.970 \text{ MPa}$$

$$\begin{aligned} \sum F_t = 0: & \tau_{nt} A_n + \tau_{xy} (A_n \cos^2 40) - \tau_{xy} (A_n \sin^2 40) \\ & + \sigma_x (A_n \cos 40) \sin 40 = 0 \end{aligned}$$

$$\tau_{nt} = \sigma_x \sin 40 \cos 40 + \tau_{xy} (\sin^2 40 - \cos^2 40)$$

$$\tau_{nt} = -46.595 \text{ MPa}$$

$$\begin{aligned} \sum F_t = 0: & \sigma_t A_i - \sigma_x (A_i \sin^2 40) \\ & - \tau_{xy} (A_i \sin 40) \cos 40 \\ & - \tau_{xy} (A_i \cos 40) \sin 40 = 0 \end{aligned}$$

$$\begin{aligned} \sigma_t &= \sigma_x \sin^2 40 + 2 \tau_{xy} \sin 40 \cos 40 \\ &= 138.022 \text{ MPa} \end{aligned}$$

$$\begin{aligned} (a) \epsilon_n &= \frac{1}{E} (\sigma_n - \mu \sigma_t) \\ &= \frac{1}{75 \times 10^9} [-84.970 - (0.28)(138.022)] \times 10^6 \\ &= -1.648 \times 10^{-3} = -1648 \mu \quad \text{ANS.} \end{aligned}$$

$$\begin{aligned} \epsilon_t &= \frac{1}{E} (\sigma_t - \mu \sigma_n) \\ &= \frac{1}{75 \times 10^9} [138.022 - (0.28)(-84.970)] \times 10^6 \\ &= 2.158 \times 10^{-3} = 2158 \mu \quad \text{ANS.} \end{aligned}$$

$$\begin{aligned} (b) \gamma_{nt} &= \frac{\tau_{nt}}{G} = \frac{\tau_{nt}}{E/2(1+\mu)} = \frac{-46.595 \times 10^6}{75 \times 10^9 / 2(1+0.28)} \\ &= -1.590 \times 10^{-3} = -1590 \mu \quad \text{ANS.} \end{aligned}$$

2.82 REFER TO THE F.B.D. IN PROB. 2.80.

$$\epsilon_x = 400 \mu; \epsilon_y = -120 \mu; \gamma_{xy} = 800 \mu$$

$$\sigma_x = \frac{E}{1-\mu^2} (\epsilon_x + \mu \epsilon_y)$$

$$= \frac{(10 \times 10^6)}{1-0.3^2} [400 + 0.3(-120)] \times 10^{-6}$$

$$= 4,000 \text{ psi}$$

$$\sigma_y = \frac{E}{1-\mu^2} (\epsilon_y + \mu \epsilon_x)$$

$$= \frac{(10 \times 10^6)}{1-0.3^2} [-120 + 0.3(400)] \times 10^{-6}$$

$$= 0$$

$$P = F = \sigma_x A = 4,000 \left(\frac{\pi}{4} \right) (2.5^2)$$

$$= 19,634.95 \text{ lb} \approx 19.6 \text{ kips} \text{ ANS.}$$

$$\tau_{xy} = \gamma_{xy} G = \gamma_{xy} \left(\frac{E}{2(1+\mu)} \right)$$

$$= 800 \times 10^{-6} \left(\frac{10 \times 10^6}{2(1+0.3)} \right) = 3,077 \text{ psi}$$

$$Q = T = \frac{\tau_{xy} J}{R} = \frac{3,077 (\pi/32) (2.5^4)}{1.25}$$

$$= 9440.1 \text{ lb}\cdot\text{in.} \approx 9.4 \text{ kip}\cdot\text{in.} \text{ ANS.}$$

2.83 REFER TO THE F.B.D. IN PROB. 2.80.

$$\epsilon_m = -336.3 \mu; \epsilon_t = 858.6 \mu; \gamma_{mt} = -1,939.8 \mu;$$

$$\theta = 30^\circ$$

$$\sigma_m = \frac{E}{1-\mu^2} (\epsilon_m + \mu \epsilon_t) = \frac{200 \times 10^9}{1-0.3^2} [-336.3 + 0.3(858.6)] \times 10^{-6}$$

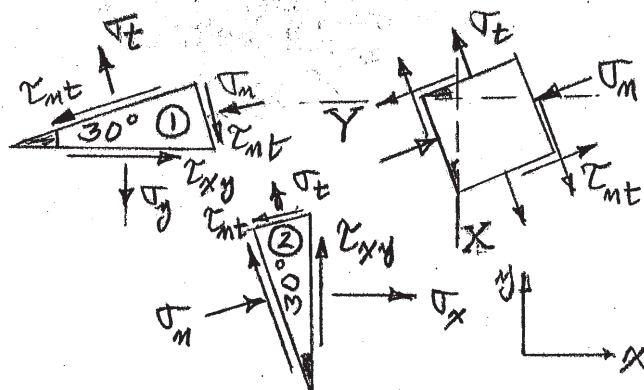
$$= -17.301 \text{ MPa}$$

$$\sigma_t = \frac{E}{1-\mu^2} (\epsilon_t + \mu \epsilon_m) = \frac{200 \times 10^9}{1-0.3^2} [858.6 + 0.3(-336.3)] \times 10^{-6}$$

$$= 166.530 \text{ MPa}$$

$$\tau_{mt} = G \gamma_{mt} = \left(\frac{E}{2(1+\mu)} \right) \gamma_{mt}$$

$$= \left(\frac{200 \times 10^9}{2(1+0.3)} \right) (-1,939.8) \times 10^{-6} = -149.215 \text{ MPa}$$



2.83 CONT'D

F.B.D. (1): A IS AREA OF Y PLANE.

$$\sum F_y = 0$$

$$\sigma_y A + \tau_{mt} (A \cos 30) \sin 30 - \sigma_t (A \cos^2 30)$$

$$+ \tau_m (A \sin^2 30) + \tau_{mt} (A \cos 30) \sin 30 = 0$$

$$\sigma_y = \sigma_t \cos^2 30 - \tau_m \sin^2 30 - 2\tau_{mt} \sin 30 \cos 30$$

$$= 0.004 \approx 0 \text{ OK}$$

F.B.D. (2): A IS AREA OF X PLANE.

$$\sum F_x = 0: \sigma_x A + \sigma_m (A \cos^2 30) - \tau_{mt} (A \cos 30) \sin 30$$

$$- \tau_{mt} (A \sin 30) \cos 30 - \sigma_t (A \sin^2 30) = 0$$

$$\sigma_x = 2\tau_{mt} \sin 30 \cos 30 + \sigma_t \sin^2 30 - \sigma_m \cos^2 30$$

$$= -74.616 \text{ MPa}$$

$$\sum F_y = 0: \tau_{xy} A + \sigma_t (A \sin 30) \cos 30 - \tau_{mt} (A \sin^2 30)$$

$$+ \sigma_m (A \cos 30) \sin 30 + \tau_{mt} (A \cos^2 30) = 0$$

$$\tau_{xy} = \tau_{mt} (\sin^2 30 - \cos^2 30) - \sigma_t \sin 30 \cos 30$$

$$- \sigma_m \sin 30 \cos 30$$

$$= -9.989 \text{ MPa}$$

$$P = F = \sigma_x A = -74.616 \left(\frac{\pi}{4} \right) (0.08^2) (10^6)$$

$$= -375.061 \text{ N} \approx -375.1 \text{ kN} \text{ ANS.}$$

$$Q = T = \frac{\tau_{xy} J}{R} = \frac{-9.989 (10^6) (\pi/32) (0.08^4)}{0.04}$$

$$= -1.004 \approx -1.0 \text{ kN}\cdot\text{m} \text{ ANS.}$$

2.84

$$\sigma_m = 60 \text{ ksi}; \sigma_t = -30 \text{ ksi};$$

$$\tau_{mt} = 20 \text{ ksi}; E = 10 \times 10^3 \text{ ksi};$$

$$\mu = 0.3; \theta = 30^\circ$$

$$\epsilon_m = \frac{1}{E} (\sigma_m - \mu \sigma_t)$$

$$= \frac{1}{10 \times 10^3} [60 - 0.3(-30)] = 0.0069$$

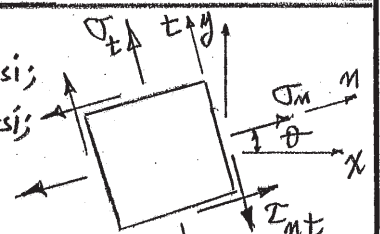
$$= 6,900 \mu \text{ ANS.}$$

$$\epsilon_t = \frac{1}{E} (\sigma_t - \mu \sigma_m) = \frac{1}{10 \times 10^3} [-30 - 0.3(60)]$$

$$= -0.0048 = -4,800 \mu \text{ ANS.}$$

$$\gamma_{mt} = \frac{\tau_{mt}}{G} = \frac{\tau_{mt}}{E/2(1+\mu)} = \frac{20}{10 \times 10^3 / 2(1+0.3)}$$

$$= 0.0052 = 5,200 \mu \text{ ANS.}$$



2.85 REFER TO THE STRESS ELEMENT

IN PROB. 2.84.

$$\epsilon_m = 500 \mu; \epsilon_t = 400 \mu; \gamma_{nt} = 0; \theta = 30^\circ$$

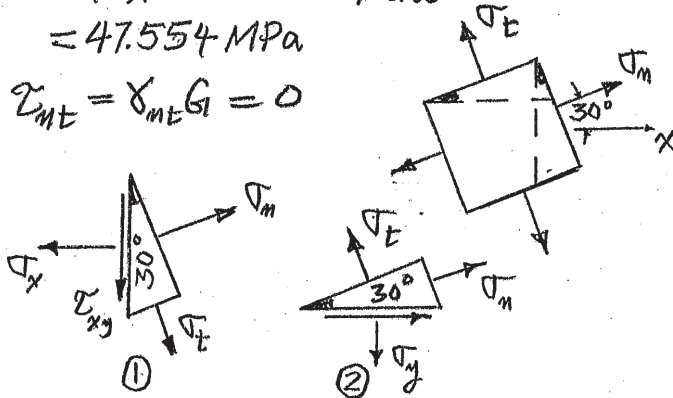
$$\sigma_m = \frac{E}{1-\mu^2} (\epsilon_m + \mu \epsilon_t) = \frac{75 \times 10^9}{1-0.33^2} [500 + 0.33(400)] \times 10^{-6}$$

$$= 53.193 \text{ MPa}$$

$$\sigma_t = \frac{E}{1-\mu^2} (\epsilon_t + \mu \epsilon_m) = \frac{75 \times 10^9}{1-0.33^2} [400 + 0.33(500)] \times 10^{-6}$$

$$= 47.554 \text{ MPa}$$

$$\tau_{nt} = \gamma_{nt} G = 0$$



F.B.D. ①:

$$\sum F_x = 0: \sigma_x A_x - \sigma_m (A_x \cos^2 30^\circ) - \tau_t (A_x \sin^2 30^\circ) = 0$$

$$\sigma_x = \sigma_m \cos^2 30^\circ + \sigma_t \sin^2 30^\circ$$

$$= 51.783 \approx 51.8 \text{ MPa}$$

ANS.

$$\sum F_y = 0: \tau_{xy} A_x - \sigma_m (A_x \sin 30^\circ) \cos 30^\circ + \sigma_t (A_x \sin 30^\circ) \cos 30^\circ = 0$$

$$\tau_{xy} = (\sigma_m - \sigma_t) \sin 30^\circ \cos 30^\circ$$

$$= 2.442 \approx 2.4 \text{ MPa}$$

ANS.

F.B.D. ②:

$$\sum F_y = 0: \sigma_y A_y - \sigma_t (A_y \cos^2 30^\circ) - \sigma_m (A_y \sin^2 30^\circ) = 0$$

$$\sigma_y = \sigma_t \cos^2 30^\circ + \sigma_m \sin^2 30^\circ$$

$$= 48.964 \approx 49.0 \text{ MPa}$$

ANS.

2.86 REFER TO THE STRESS ELEMENT

IN PROB. 2.84.

$$\epsilon_x = 450 \mu; \epsilon_y = 600 \mu; \gamma_{xy} = -500 \mu; \theta = 20^\circ$$

$$\sigma_x = \frac{E}{1-\mu^2} (\epsilon_x + \mu \epsilon_y) = \frac{30 \times 10^3}{1-0.3^2} [450 + 0.3(600)] \times 10^{-6}$$

$$= 20.769 \text{ ksi}$$

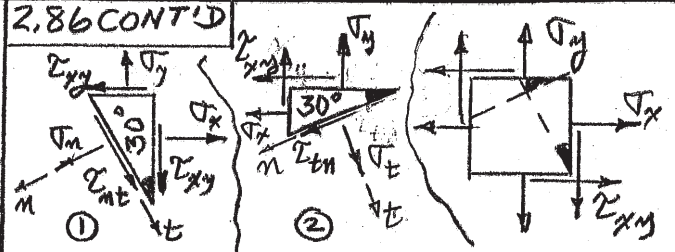
$$\sigma_y = \frac{E}{1-\mu^2} (\epsilon_y + \mu \epsilon_x) = \frac{30 \times 10^3}{1-0.3^2} [600 + 0.3(450)] \times 10^{-6}$$

$$= 24.231 \text{ ksi}$$

$$\tau_{xy} = \gamma_{xy} G = \gamma_{xy} \left(\frac{E}{2(1+\mu)} \right) = -500(10) \left(\frac{30 \times 10^3}{2(1+0.3)} \right)$$

$$= -5.769 \text{ ksi}$$

2.86 CONT'D

LET A_n BE AREA OF INCLINED PLANE,

F.B.D. ①:

$$\sum F_n = 0: \sigma_m A_n - \sigma_x (A_n \cos^2 20^\circ) + \tau_{xy} (A_n \cos 20^\circ) \sin 20^\circ + \tau_{xy} (A_n \sin 20^\circ) \cos 20^\circ - \sigma_y (A_n \sin^2 20^\circ) = 0$$

$$\sigma_m = \sigma_x \cos^2 20^\circ + \sigma_y \sin^2 20^\circ - 2\tau_{xy} \sin 20^\circ \cos 20^\circ$$

$$= 24.882 \approx 24.9 \text{ ksi}$$

ANS.

$$\sum F_t = 0: \tau_{nt} A_n + \tau_{xy} (A_n \cos^2 20^\circ) + \sigma_x (A_n \cos 20^\circ) \sin 20^\circ - \sigma_y (A_n \sin 20^\circ) \cos 20^\circ - \tau_{xy} (A_n \sin^2 20^\circ) = 0$$

$$\tau_{nt} = (\sigma_y - \sigma_x) \sin 20^\circ \cos 20^\circ + \tau_{xy} (\sin^2 20^\circ - \cos^2 20^\circ)$$

$$= 5.532 \approx 5.5 \text{ ksi}$$

ANS.

F.B.D. ②:

$$\sum F_t = 0: \tau_t A_n - \tau_{xy} (A_n \cos^2 20^\circ) - \tau_{xy} (A_n \cos 20^\circ) \sin 20^\circ - \tau_{xy} (A_n \sin 20^\circ) \cos 20^\circ - \sigma_x (A_n \sin^2 20^\circ) = 0$$

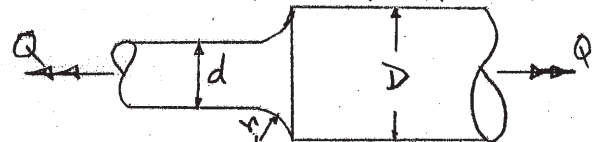
$$\tau_t = \sigma_x \sin^2 20^\circ + \sigma_y \cos^2 20^\circ + 2\tau_{xy} \sin 20^\circ \cos 20^\circ$$

$$= 20.118 \approx 20.1 \text{ ksi}$$

ANS.

2.87

$$D = 3.0 \text{ in.}; d = 2.0 \text{ in.}; T = Q = 5 \text{ kip} \cdot \text{in.}$$



$$(a) r = 0.15 \text{ in.}; T = 5 \text{ kip} \cdot \text{in.}$$

$$\frac{D}{d} = 1.5; \frac{r}{d} = 0.075 \Rightarrow k \approx 1.5$$

$$\tau_{max} = k \left(\frac{TR}{J} \right) = 1.5 \left(\frac{5(1)}{\frac{\pi}{32}(2^4)} \right)$$

$$= 4.775 \approx 4.8 \text{ ksi}$$

ANS.

$$(b) r = 0.05 \text{ in.}; T = 5 \text{ kip} \cdot \text{in.}$$

$$\frac{D}{d} = 1.5; \frac{r}{d} = 0.025 \Rightarrow k \approx 2.0$$

$$\tau_{max} = k \left(\frac{TR}{J} \right) = 2.0 \left(\frac{5(1)}{\frac{\pi}{32}(2^4)} \right)$$

$$= 6.366 \approx 6.4 \text{ ksi}$$

ANS.

2.88 REFER TO THE SKETCH IN PROB. 2.87.

$$D = 100 \text{ mm}; d = 50 \text{ mm}; \tau_{\text{ALL}} = 45 \text{ MPa};$$

$$\omega = 1200 \text{ rpm};$$

$$(a) r = 3.0 \text{ mm}$$

$$\frac{D}{d} = 2; \frac{r}{d} = 0.06 \Rightarrow k \approx 1.65$$

$$\tau_{\text{MAX}} = k \left(\frac{TR}{J} \right); T = \frac{\tau_{\text{MAX}} J}{k R}$$

$$T = \frac{45 \times 10^6 (\pi/32) (0.05^4)}{1.65 (0.025)} = 0.669 \text{ kN}\cdot\text{m}$$

$$P = T\omega = 0.669 \left(\frac{1200 (2\pi)}{60} \right) = 84.1 \text{ kW ANS.}$$

$$(b) r = 1.0 \text{ mm}$$

$$\frac{D}{d} = 2; \frac{r}{d} = 0.02 \Rightarrow k \approx 3.20$$

$$T = \frac{45 \times 10^6 (\pi/32) (0.05^4)}{3.20 (0.025)} = 0.345 \text{ kN}\cdot\text{m}$$

$$P = T\omega = 0.345 \left(\frac{1200 (2\pi)}{60} \right) = 43.4 \text{ kW ANS.}$$

2.89 REFER TO THE SKETCH IN PROB. 2.87.

$$D = 2.0 \text{ in.}; d = 1.0 \text{ in.}$$

$$\tau_{\text{MAX}} = k \left(\frac{TR}{J} \right); T = \frac{\tau_{\text{MAX}} J}{k R}$$

$$T = \frac{10 \times 10^3 (\pi/32) (1.0^4)}{k (0.5)} = \frac{1963.5}{k} \quad (1)$$

$$P = T\omega; T = \frac{P}{\omega} = \frac{15 (6600)}{900 (2\pi/60)}$$

$$T = 1050.4 \text{ lb}\cdot\text{in.} \quad (2)$$

EQUATE (1) & (2) TO GET $k = 1.87$.

$$\text{ALSO, } \frac{D}{d} = 2.0 \Rightarrow \frac{r}{d} \approx 0.03$$

$$\therefore r \approx 0.03 d \approx 0.03 (1.0)$$

$$\approx 0.03 \text{ in. ANS.}$$

2.90 $d = 70 \text{ mm}; r = 4 \text{ mm}$ 

$$P = T\omega; T = \frac{P}{\omega} = \frac{50 \times 10^3}{180 (2\pi/60)}$$

$$T = 2652.6 \text{ N}\cdot\text{m}$$

$$\tau_{\text{MAX}} = k \left(\frac{TR}{J} \right); k = \frac{\tau_{\text{MAX}} J}{TR}$$

$$k = \frac{60 \times 10^6 (\pi/32) (0.07^4)}{2652.6 (0.035)} = 1.52$$

2.90 CONT'D

$$\frac{r}{d} = 0.057 \Rightarrow \frac{D}{d} \approx 1.25$$

$$D = 1.25 d = 87.5 \text{ mm ANS.}$$

2.91 REFER TO THE SKETCH IN PROB. 2.90.

$$D = 3.0 \text{ in.}; d = 2.0 \text{ in.}$$

$$P = T\omega; T = \frac{P}{\omega} = \frac{50 (6600)}{600 (2\pi/60)}$$

$$T = 5252.1 \text{ lb}\cdot\text{in.}$$

$$\tau_{\text{MAX}} = k \left(\frac{TR}{J} \right); k = \frac{\tau_{\text{MAX}} J}{TR}$$

$$k = \frac{6 \times 10^3 (\pi/32) (2.0^4)}{5252.1 (1.0)} = 1.79$$

$$\frac{D}{d} = 1.5 \Rightarrow \frac{r}{d} \approx 0.04$$

$$r \approx 0.04 d \approx 0.08 \text{ in. ANS.}$$

2.92 REFER TO THE SKETCH IN PROB. 2.90.

$$D = 100 \text{ mm}; d = 70 \text{ mm}; r = 5 \text{ mm}$$

$$\frac{D}{d} = 1.43; \frac{r}{d} = 0.071 \Rightarrow k \approx 1.55$$

$$\tau_{\text{MAX}} = k \left(\frac{TR}{J} \right); T = \frac{\tau_{\text{MAX}} J}{k R}$$

$$T = \frac{70 \times 10^6 (\pi/32) (0.07^4)}{1.55 (0.035)} = 3042 \text{ N}\cdot\text{m}$$

$$P = T\omega; \omega = \frac{P}{T} = \frac{100 \times 10^3}{3042}$$

$$\omega = 32.873 \text{ rad/s} \approx 314 \text{ rpm ANS.}$$

$$2.93 \Delta = \theta b = \left(\frac{TL}{JG} \right) b = \frac{F b^2 L}{JG}$$

$$\Delta = \frac{F (15^2) (20)}{(\pi/32) (4.0^4 - 3.0^4) (4 \times 10^6)} = 65.481 \times 10^{-6} \text{ F}$$

$$0.85 W (h + \Delta) = \frac{1}{2} F \Delta$$

$$0.85 (100) (1.2 + 65.481 \times 10^{-6} \text{ F}) = \frac{1}{2} \text{ F } (65.481 \times 10^{-6} \text{ F})$$

$$F^2 - 169.997 \text{ F} - 3.115 \times 10^6 = 0$$

$$F = 1851.981 \text{ lb}$$

$$(a) \tau_{\text{MAX}} = \frac{TR}{J} = \frac{F b R}{J} = \frac{1851.981 (15) (2)}{(\pi/32) (4.0^4 - 3.0^4)}$$

$$= 3233.8 \text{ psi} \approx 3.2 \text{ ksi ANS.}$$

$$(b) \theta = \frac{TL}{JG} = \frac{1851.981 (20)}{(\pi/32) (4.0^4 - 3.0^4) (4 \times 10^6)}$$

$$= 5.340 \times 10^{-4} \approx 0.00053 \text{ rad ANS.}$$

2.94

$$\tau_{MAX} = \frac{TR}{J}; T_1 = \frac{\tau_{MAX} J}{R}$$

$$T_1 = \frac{100 \times 10^6 (\pi/32) (0.05)^4}{0.025} = 2454.4 \text{ N}\cdot\text{m}$$

$$\theta_{MAX} = \frac{TL}{JG}; T_2 = \frac{\theta_{MAX} JG}{L}$$

$$T_2 = \frac{(5\pi/180) (\pi/32) (0.05)^4 (75 \times 10^9)}{0.75} = 5354 \text{ N}\cdot\text{m}$$

$\therefore T_1$ CONTROLS,

NOW, $T = Fb$; $F = \frac{T}{b}$

$$F = \frac{2454.4}{1.25} = 1963.5 \text{ N}$$

$$\Delta = \theta b = \left(\frac{TL}{JG} \right) b = \left(\frac{(Fb)L}{JG} \right) b = \frac{Fb^2 L}{JG}$$

$$W(h + \Delta) = \frac{1}{2} F \Delta; W(h + \frac{Fb^2 L}{JG}) = \frac{1}{2} \left(\frac{F^2 b^2 L}{JG} \right)$$

$$W \left[0.075 + \frac{1963.5 (1.25^2) (0.75)}{(\pi/32) (0.05^4) (75 \times 10^9)} \right]$$

$$= \frac{1}{2} \left[\frac{1963.5^2 (1.25^2) (0.75)}{(\pi/32) (0.05^4) (75 \times 10^9)} \right]$$

$$W = 392.7 \text{ N}$$

ANS.

2.95

$$\tau_{MAX} = \frac{TR}{J}; T_1 = \frac{\tau_{MAX} J}{R}$$

$$T_1 = \frac{12 (\pi/32) (6.0^4 - 5.5^4)}{3} = 149.6 \text{ kip}\cdot\text{in.}$$

$$\theta_{MAX} = \frac{TL}{JG}; T_2 = \frac{\theta_{MAX} JG}{L}$$

$$T_2 = \frac{0.2 (\pi/32) (6.0^4 - 5.5^4) (4 \times 10^3)}{3(12)} = 831.1 \text{ kip}\cdot\text{in.}$$

$\therefore T_1$ CONTROLS,

NOW, $T = Fb$; $F = \frac{T}{b}$

$$F = \frac{149.6}{5(12)} = 2.493 \text{ kips} \approx 2493 \text{ lb}$$

AS IN PROB. 2.94, $\Delta = \frac{Fb^2 L}{JG}$

$$W(h + \frac{Fb^2 L}{JG}) = \frac{1}{2} \left(\frac{F^2 b^2 L}{JG} \right)$$

$$150 \left[h + \frac{2493 (60^2) (36)}{(\pi/32) (6.0^4 - 5.5^4) (4 \times 10^6)} \right]$$

$$= \frac{1}{2} \left[\frac{2493^2 (60^2) (36)}{(\pi/32) (6.0^4 - 5.5^4) (4 \times 10^6)} \right]$$

$$h = 15.788 \approx 15.8 \text{ in.}$$

ANS.

2.96

$$W(h + \Delta) = \frac{1}{2} F \Delta$$

AS IN PROB. 2.94, $\Delta = \frac{Fa^2 (L/2)}{JG}$

$$\Delta = \frac{F(2.5^2)(1.0)}{(\pi/32)(0.15^4 - 0.13^4)(75 \times 10^9)} = 3.847 \times 10^{-6} F$$

$$\therefore 800(0.125 + 3.847 \times 10^{-6} F) = \frac{1}{2} F(3.847 \times 10^{-6} F)$$

$$F^2 - 1600F - 51.989 \times 10^6 = 0$$

$$F = 8055 \text{ N}$$

$$\tau_{MAX} = \frac{TR}{J} = \frac{(Fa)R}{J} = \frac{(8055 \times 2.5)(0.075)}{(\pi/32)(0.15^4 - 0.13^4)}$$

$$= 69.724 \text{ MPa} \approx 69.7 \text{ MPa} \quad \text{ANS.}$$

$$\theta = \frac{TL}{JG} = \frac{(Fa)L}{JG} = \frac{(8055 \times 2.5)(1.0)}{(\pi/32)(0.15^4 - 0.13^4)(75 \times 10^9)}$$

$$= 0.0124 \text{ rad} \quad \text{ANS.}$$

2.97

$$W(h + \Delta) = \frac{1}{2} F \Delta$$

AS IN PROB. 2.94, $\Delta = \frac{Fa^2 (L/2)}{JG}$

$$\Delta = \frac{F(10 \times 12)^2 (8 \times 12)}{(\pi/32) d^4 (4 \times 10^6)} = 3.520 (F/d^4)$$

$$\therefore 185[6 + 3.520 (F/d^4)] = \frac{1}{2} F[3.520 (F/d^4)]$$

$$1110 + 651.2 \left(\frac{F}{d^4} \right) = 1.76 \left(\frac{F^2}{d^4} \right) \quad (1)$$

$$\tau_{ALL} = \frac{TR}{J} = \frac{(Fa)R}{J}$$

$$\therefore 12 \times 10^3 = \frac{F(10 \times 12)(d/2)}{(\pi/32) d^4}$$

$$F = 19.635 d^3 \quad (2)$$

(2) INTO (1) $\Rightarrow d^3 - 1.636d - 18.844 = 0$

A TRIAL-AND-ERROR SOLUTION LEADS TO

$$d = 2.865 \approx 2.9 \text{ in.} \quad \text{ANS.}$$

2.98

BECAUSE OF SYMMETRY, THE APPLIED TORQUE IS DIVIDED EQUALLY BY THE TWO SEGMENTS OF THE SHAFT. THUS,

$$W(h + \Delta) = \frac{1}{2} F \Delta \quad \text{WHERE}$$

$$\Delta = \left[\frac{(\frac{1}{2}T)L}{JG} \right] a = \frac{\frac{1}{2}Fa^2 (L/2)}{JG}$$

$$= \frac{F(10 \times 12)^2 (10 \times 12)}{2 \left(\frac{\pi}{32} \right) (4.0^4 - 3.5^4) (10 \times 10^6)} = 8.307 \times 10^{-3} F$$

$$\therefore 180(5 + 8.307 \times 10^{-3} F) = \frac{1}{2} F(8.307 \times 10^{-3} F)$$

$$F^2 - 360F - 216,685 = 0$$

$$F = 679.084 \text{ lb}$$

2.98 CONT'D

$$\tau = \frac{\frac{1}{2}TR}{J} = \frac{\frac{1}{2}(679.084)(10 \times 12)(2.0)}{(\pi/32)(4.0^4 - 3.5^4)}$$

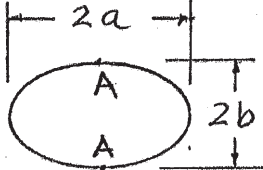
$$\tau = 7.835 \approx 7.8 \text{ ksi} \quad \text{ANS.}$$

$$\theta = \frac{\frac{1}{2}T(L/2)}{JG} = \frac{\frac{1}{2}(679.084)(10 \times 12)(10 \times 12)}{(\pi/32)(4.0^4 - 3.5^4)(10 \times 10^6)}$$

$$\theta = 0.047 \text{ rad} \quad \text{ANS.}$$

2.99 $a = 2 \text{ in.}; b = 1 \text{ in.}$

(a)

$$\tau_{\text{MAX}} = \frac{2T}{\pi a b^2} \text{ AT A.}$$


$$= \frac{2(30)}{\pi(2)(1)^2} = 9.549 \approx 9.5 \text{ ksi} \quad \text{ANS}$$

(b)

$$K = \frac{\pi a^3 b^3}{a^2 + b^2} = \frac{\pi(2^3)(1^3)}{2^2 + 1^2} = 5.0265$$

$$\theta = \frac{TL}{KG} = \frac{30(1)}{5.0265(4 \times 10^3)} = 0.00149 \text{ rad/in.} \quad \text{ANS}$$

2.100 REFER TO THE SKETCH IN PROB. 2.99.

$$a = 60 \text{ mm}; b = 40 \text{ mm}; L = 1.5 \text{ m}$$

$$\tau_{\text{MAX}} = \frac{2T}{\pi a b^2}; T_1 = \frac{\pi a b^2 \tau_{\text{MAX}}}{2}$$

$$T_1 = \frac{\pi(0.06)(0.04^2)(60 \times 10^6)}{2} = 9.048 \text{ kN}\cdot\text{m}$$

$$K = \frac{\pi a^3 b^3}{a^2 + b^2} = \frac{\pi(0.06^3)(0.04^3)}{0.06^2 + 0.04^2} = 8.351 \times 10^{-6}$$

$$\theta_{\text{MAX}} = \frac{TL}{KG}; T_2 = \frac{KG\theta}{L}$$

$$T_2 = \frac{8.351 \times 10^{-6}(75 \times 10^9)(12\pi/180)}{1.5} = 7.288 \text{ kN}\cdot\text{m}$$

 $\therefore \theta$ CONTROLS AND

$$Q = T_2 = 7.288 \approx 7.3 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

2.101 REFER TO THE SKETCH IN PROB. 2.99.

$$a = 3.0 \text{ in.}; b = ?; L = 3.5 \text{ ft}$$

$$\tau_{\text{MAX}} = \frac{2T}{\pi a b^2}; b = \sqrt{\frac{2T}{\pi a \tau_{\text{MAX}}}}$$

$$b = \sqrt{\frac{2(40)}{\pi(3.0)8}} = 1.030 \text{ in.} \quad \text{ANS.}$$

2.102 $T = 2V(\text{CONST.})$

$$= 2(3.0 \times 10^{-6})(80 \times 10^6) = 480 \text{ N}\cdot\text{m}$$

$$\tau_{\text{MAX}} = \frac{2T}{\pi a b^2} = \frac{2(480)}{\pi(0.035)(0.025^2)}$$

$$\tau_{\text{MAX}} = 13.969 \approx 14.0 \text{ MPa} \quad \text{ANS.}$$

2.102 CONT'D $K = \frac{\pi a^3 b^3}{a^2 + b^2}$

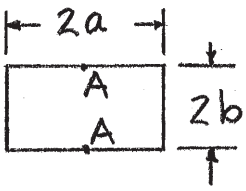
$$K = \frac{\pi(0.035^3)(0.025^3)}{0.035^2 + 0.025^2} = 1.137 \times 10^{-6}$$

$$\theta = \frac{TL}{KG} = \frac{480(1.5)}{1.137 \times 10^{-6}(25 \times 10^9)}$$

$$\theta = 0.02533 \text{ rad} \quad \text{ANS.}$$

2.103

$$a = 50 \text{ mm}; b = 30 \text{ mm}$$

$$(a) \tau_{\text{MAX}} = \frac{T(3a + 1.8b)}{8a^2 b^2}$$


$$\tau_{\text{MAX}} = \frac{500[3(0.05) + 1.8(0.03)]}{8(0.05^2)(0.03^2)} = 5.667 \text{ MPa}$$

$$\approx 5.7 \text{ MPa AT A} \quad \text{ANS.}$$

$$(b) K = ab^3 \left[\frac{16}{3} - 3.36 \left(\frac{b}{a} \right) \left(1 - \frac{b^4}{12a^4} \right) \right]$$

$$K = (0.05)(0.03^3) \left[\frac{16}{3} - 3.36 \left(\frac{0.03}{0.05} \right) \left(1 - \frac{0.03^4}{12(0.05^4)} \right) \right]$$

$$= 4.507 \times 10^{-6}$$

$$\theta = \frac{TL}{KG} = \frac{500(1.20)}{4.507 \times 10^{-6}(30 \times 10^9)}$$

$$\theta = 0.00444 \text{ rad} \quad \text{ANS.}$$

2.104 REFER TO THE SKETCH IN PROB. 2.103.

$$a = 2.5 \text{ in.}; b = 1.5 \text{ in.}; L = 3 \text{ ft}$$

$$\tau_{\text{MAX}} = \frac{T(3a + 1.8b)}{8a^2 b^2}; T_1 = \frac{8a^2 b^2 \tau_{\text{MAX}}}{(3a + 1.8b)}$$

$$T_1 = \frac{8(2.5^2)(1.5^2)(15)}{3(2.5) + 1.8(1.5)} = 165.441 \text{ kip}\cdot\text{in.}$$

$$K = ab^3 \left[\frac{16}{3} - 3.36 \left(\frac{b}{a} \right) \left(1 - \frac{b^4}{12a^4} \right) \right]$$

$$K = (2.5)(1.5^3) \left[\frac{16}{3} - 3.36 \left(\frac{1.5}{2.5} \right) \left(1 - \frac{1.5^4}{12(2.5^4)} \right) \right]$$

$$= 28.174$$

$$\phi_{\text{MAX}} = \frac{T}{KG}; T_2 = KG \phi_{\text{MAX}}$$

$$T_2 = 28.174(4 \times 10^3) \left(\frac{0.02}{12} \right)$$

$$= 187.827 \text{ kip}\cdot\text{in.}$$

 $\therefore \tau_{\text{MAX}}$ CONTROLS AND

$$Q = T_1 = 165.441 \approx 165.4 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

2.105 $\tau = \frac{T(3a+1.8b)}{8a^2b^2} \Rightarrow$
 $\frac{a^2b^2}{3a+1.8b} = \frac{T}{8\tau}$
 $\frac{0.045b^2}{3(0.045)+1.8b} = \frac{15 \times 10^3}{8(35 \times 10^6)} = 5.35714 \times 10^{-5}$
 SIMPLIFYING, WE OBTAIN
 $b^2 - 2.143 \times 10^{-3}b - 1.607 \times 10^{-4} = 0$
 SOLVING THIS QUADRATIC EQ. LEADS TO
 $b = 0.01379 \text{ m} = 13.8 \text{ mm}$ ANS.

2.106 $T = 2V(2G\phi/kp/P) = 2(2.5)(20.5) = 102.5 \text{ kip}\cdot\text{in.}$
 $\tau = \frac{T(3a+1.8b)}{8a^2b^2} = \frac{102.5[3(2.5)+1.8(1.75)]}{8(2.5^2)(1.75^2)}$
 $= 7.129 \text{ ksi}$ ANS.

$\theta = \frac{TL}{KG}; K = ab^3 \left[\frac{16}{3} - 3.36 \left(\frac{b}{a} \right) \left(1 - \frac{b^4}{12a^4} \right) \right]$
 $K = (2.5)(1.75^3) \left[\frac{16}{3} - 3.36 \left(\frac{1.75}{2.5} \right) \left(1 - \frac{1.75^4}{12(2.5^4)} \right) \right]$
 $= 40.576 \text{ in.}^4$
 $\therefore \theta = \frac{7.129(5 \times 12)}{40.576(10 \times 10^3)} = 1.054 \times 10^{-3} \text{ rad}$ ANS.

2.107 $\tau = \frac{20T}{a^3} = \frac{20(7.5 \times 10^3)}{(0.08^3)}$
 $= 292.969 \text{ MPa}$ AT MIDPOINT OF THREE SIDES ANS.
 $\theta = \frac{TL}{KG}; K = \frac{a^4\sqrt{3}}{80} = \frac{(0.08^4)\sqrt{3}}{80} = 8.86 \times 10^{-7} \text{ m}^4$
 $\therefore \theta = \frac{7.5 \times 10^3(2.0)}{8.86 \times 10^{-7}(27 \times 10^9)} = 0.627 \text{ rad}$ ANS.

2.108 $\tau = \frac{20T}{a^3} \Rightarrow T_1 = \frac{\tau a^3}{20}$
 $T_1 = \frac{12(3^3)}{20} = 16.2 \text{ kip}\cdot\text{in.}$
 $\theta = \frac{TL}{KG} = T_2 = \frac{KG\theta}{L}$
 $K = \frac{a^4\sqrt{3}}{80} = \frac{3^4\sqrt{3}}{80} = 1.754 \text{ in.}^4$
 $\therefore T_2 = \frac{1.754(4 \times 10^3)(0.15)}{3(12)} = 29.233 \text{ kip}\cdot\text{in.}$
 $\therefore \tau$ CONTROLS AND
 $Q = T_1 = 16.2 \text{ kip}\cdot\text{in.}$ ANS.

2.109 $\tau = \frac{20T}{a^3}; a = \sqrt[3]{\frac{20T}{\tau}}$
 $a_1 = \sqrt[3]{\frac{20(10 \times 10^3)}{20 \times 10^6}} = 0.2154 \text{ m}$
 $\theta = \frac{TL}{KG}; K = \frac{TL}{\theta G} = \frac{10 \times 10^3(2.0)}{(0.2)(27 \times 10^9)} = 3.7037 \times 10^{-6} \text{ m}^4$
 $\therefore K = \frac{a^4\sqrt{3}}{80} = 3.7037 \times 10^{-6}$
 $a_2 = 0.1144 \text{ m}$
 $\therefore \tau$ CONTROLS AND
 $a = a_1 = 0.2154 \text{ m} = 215.4 \text{ mm}$ ANS.

2.110 CIRCLE: $R = 1.5 \text{ in.}$
 $\tau = \frac{TR}{J}; T_1 = \frac{\tau J}{R} = \frac{15(\pi/2)(1.5^4)}{1.5}$
 $T_1 = 79.522 \text{ kip}\cdot\text{in.}$
 $A = \pi R^2 = \pi(1.5^2) = 7.06858 \text{ in.}^2$
 ELLIPSE: $b = 1.25 \text{ in.}$
 $A = \pi ab = 7.06858$
 $a = \frac{7.06858}{\pi(1.25)} = 1.80 \text{ in.}$
 $\tau = \frac{2T}{\pi ab^2}; T_2 = \frac{\tau(\pi ab^2)}{2} = \frac{15\pi(1.80)(1.25^2)}{2}$
 $T_2 = 66.268 \text{ kip}\cdot\text{in.}$

$\therefore$ THE CIRCULAR SHAFT IS CHOSEN BECAUSE IT CARRIES ABOUT 20% MORE TORQUE.

2.111 SQUARE: $2a = 65 \text{ mm} = 0.065 \text{ m}$
 $\tau = \frac{T(3a+1.8b)}{8a^2b^2} = \frac{T(4.8a)}{8a^4}; T_1 = \frac{8a^3\tau}{4.8}$
 $T_1 = \frac{8(0.0325^3)(75 \times 10^6)}{4.8} = 4291 \text{ N}\cdot\text{m}$
 $A = 4a^2 = 4(0.0325^2) = 0.004225 \text{ m}^2$

EQUILATERAL TRIANGLE: b ON A SIDE
 $A = \frac{\sqrt{3}b^2}{4} = 0.004225$
 $b = 0.09878 \text{ m}$
 $\tau = \frac{20T}{b^3}; T_2 = \frac{\tau b^3}{20}$
 $T_2 = \frac{75 \times 10^6(0.09878^3)}{20} = 3614 \text{ N}\cdot\text{m}$

$\therefore$ THE SQUARE SHAFT IS CHOSEN BECAUSE IT CARRIES ABOUT 19% MORE TORQUE.

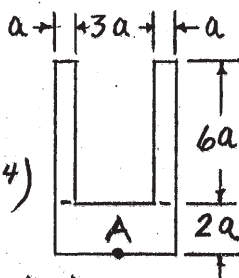
2.112

$$K = \frac{1}{3} \sum L t^3$$

$$= \frac{1}{3} [2(6a)a^3 + 5a(2a)^3]$$

$$= \frac{1}{3} (52a^4) = \frac{1}{3} (52)(1.5^4)$$

$$= 87.75 \text{ in}^4$$



$$(a) \tau_{\text{MAX}} = \frac{Tt}{K} = \frac{150(2)(1.5)}{87.75}$$

$$= 5.128 \text{ ksi AT A. } \underline{\text{ANS.}}$$

$$(b) \theta = \frac{TL}{KG} = \frac{150(4 \times 12)}{87.75(4 \times 10^3)}$$

$$= 20.5 \times 10^{-3} \text{ rad } \underline{\text{ANS.}}$$

2.113 AS IN PROB. 2.112, $K = \frac{1}{3}(52a^4)$

$$K = \frac{1}{3}(52)(0.05^4) = 1.083 \times 10^{-4} \text{ m}^4$$

$$\tau_{\text{MAX}} = \frac{Tt}{K}; T_1 = \frac{\tau K}{t} = \frac{40 \times 10^6 (1.083 \times 10^{-4})}{2(0.05)}$$

$$T_1 = 433.2 \times 10^2 \text{ N}\cdot\text{m}$$

$$\theta_{\text{MAX}} = \frac{TL}{KG}; T_2 = \frac{KG\theta}{L}$$

$$T_2 = \frac{1.083 \times 10^{-4} (25 \times 10^9) (6\pi/180)}{3.0} = 945.1 \times 10^2 \text{ N}\cdot\text{m}$$

 $\therefore \tau$ CONTROLS AND

$$Q = T_1 = 433.2 \times 10^2 \text{ N}\cdot\text{m} \approx 43.3 \text{ kN}\cdot\text{m} \underline{\text{ANS.}}$$

2.114 (a)

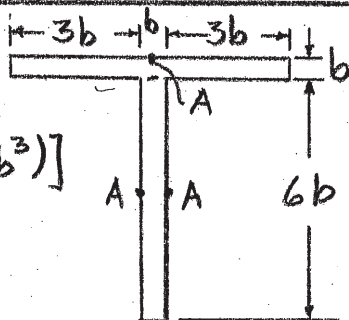
$$K = \frac{1}{3} \sum L t^3$$

$$= \frac{1}{3} [7b(b^3) + 6b(b^3)]$$

$$= \frac{1}{3} (13b^4)$$

$$= \frac{1}{3} (13)(1.25^4)$$

$$= 10.579 \text{ in}^4$$



$$\tau_{\text{MAX}} = \frac{Tt}{K} = \frac{100(1.25)}{10.579}$$

$$= 11.816 \text{ ksi AT POINTS A } \underline{\text{ANS.}}$$

(b)

$$\theta = \frac{TL}{KG} = \frac{100(3 \times 12)}{10.579(10 \times 10^3)}$$

$$= 3.403 \times 10^{-2} \text{ rad } \underline{\text{ANS.}}$$

2.115 AS IN PROB. 2.114, $K = \frac{1}{3}(13b^4)$

$$K = \frac{1}{3}(13)(0.04^4) = 11.093 \times 10^{-6} \text{ m}^4$$

$$\tau_{\text{MAX}} = \frac{Tt}{K}; T_1 = \frac{\tau K}{t} = \frac{11.093 \times 10^{-6} (50 \times 10^6)}{0.04}$$

$$T_1 = 13,866 \text{ N}\cdot\text{m}$$

$$\theta_{\text{MAX}} = \frac{TL}{KG}; T_2 = \frac{KG\theta}{L}$$

$$T_2 = \frac{11.093 \times 10^{-6} (27 \times 10^9) (5\pi/180)}{2} = 13,069 \text{ N}\cdot\text{m}$$

 $\therefore \theta$ CONTROLS AND

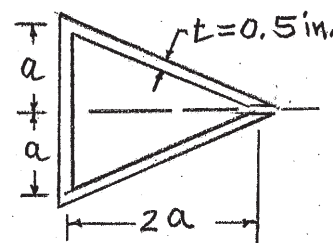
$$Q = T_2 = 13,069 \text{ N}\cdot\text{m} \approx 13.1 \text{ kN}\cdot\text{m} \underline{\text{ANS.}}$$

2.116 $K = \frac{1}{3} \sum L t^3$; $a =$

$$K = \frac{1}{3} [2(2.236a)t^3 + (2a)t^3]$$

$$= \frac{1}{3} (6.472at^3)$$

$$= \frac{1}{3} (6.472)(0.5^3)a$$



$$= 0.26967a$$

$$\tau = \frac{Tt}{K}; K = \frac{Tt}{\tau} = \frac{12(0.5)}{12} = 0.5 \text{ in}^4$$

$$\therefore 0.26967a = 0.5$$

$$a = 1.854 \text{ in. } \underline{\text{ANS.}}$$

2.117 AS IN PROB. 2.116, $K = \frac{1}{3}(6.472at^3)$

$$K = \frac{1}{3}(6.472)(0.1)(0.015^3) = 7.28 \times 10^{-7} \text{ m}^4$$

$$\tau_{\text{MAX}} = \frac{Tt}{K}; T_1 = \frac{\tau K}{t} = \frac{80 \times 10^6 (7.28 \times 10^{-7})}{0.015}$$

$$T_1 = 3,882.7 \text{ N}\cdot\text{m}$$

$$\theta_{\text{MAX}} = \frac{TL}{KG}; T_2 = \frac{KG\theta}{L} = \frac{7.28 \times 10^{-7} (77 \times 10^9) (0.10)}{1.5}$$

$$T_2 = 3,737.1 \text{ N}\cdot\text{m}$$

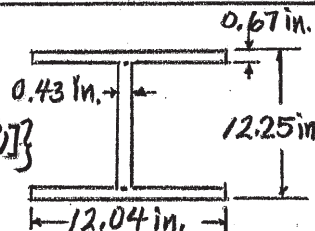
 $\therefore \theta$ CONTROLS AND

$$Q = T_2 = 3,737.1 \text{ N}\cdot\text{m} \approx 3.737 \text{ kN}\cdot\text{m} \underline{\text{ANS.}}$$

2.118 $K = \frac{1}{3} \sum L t^3$

$$K = \frac{1}{3} [2(12.04)(0.67^3) + [2(2.25 - 2(0.67))(0.43^3)]]$$

$$K = 2.70326 \text{ in}^4$$



$$(a) \tau_{\text{MAX}} = \frac{Tt}{K} = \frac{10 \times 12(0.67)}{2.70326} = 29.742 \text{ ksi } \underline{\text{ANS.}}$$

$$(b) \theta = \frac{TL}{KG} = \frac{10 \times 12(5 \times 12)}{2.70326(10 \times 10^3)} = 0.266 \text{ rad } \underline{\text{ANS.}}$$

2.119 $K = \frac{1}{3} \sum L t^3$

$$K = \frac{1}{3} \left\{ 2(2.527)(0.39^3) + [8.00 - 2(0.39)(0.487^3)] \right\}$$

$$= 0.37791 \text{ in.}^4$$

$$\tau_{\text{MAX}} = \frac{T t}{K}; T_1 = \frac{\tau K}{t}$$

$$T_1 = \frac{10(0.37791)}{0.487} = 7.760 \text{ kip} \cdot \text{in.}$$

$$\theta = \frac{T L}{K G}; T_2 = \frac{K G \theta}{L}$$

$$T_2 = \frac{0.37791(10 \times 10^3)(10\pi/180)}{4 \times 12} = 13.741 \text{ kip} \cdot \text{in.}$$

$\therefore \tau$ CONTROLS AND

$$\phi = T_1 = 7.760 \text{ kip} \cdot \text{in.}$$

ANS.

2.120 $a = 20 \text{ mm}$

$$A_m = \frac{1}{2} [10a(8.5a) + 5a(7a)]$$

$$A_m = 60a^2$$

$$= 60(0.02^2) = 0.024 \text{ m}^2$$

$$(a) \tau = \frac{T}{2A_m t} = \frac{7.5 \times 10^3}{2(0.024)(0.02)} = 7.813 \text{ MPa. ANS.}$$

$$(b) q = t \tau = (0.02)(7.813 \times 10^6) = 156.3 \text{ kN/m}$$

ANS.

$$(c) \theta = \frac{T L}{4 G A_m^2} \sum \left(\frac{s}{t} \right)$$

$$\theta = \frac{7.5 \times 10^3(1.0)}{4(27 \times 10^9)(0.024^2)} \left[\frac{10a}{2a} + \frac{6.5a}{2a} + \frac{7a}{1.5a} + \frac{6.5a}{a} \right]$$

$$= 120.563 \times 10^{-6} (19.417) = 2.34 \times 10^{-3} \text{ rad. ANS.}$$

2.121 AS IN PROB. 2.120, $A_m = 60a^2$

$$\tau = \frac{T}{2A_m t}; A_m = \frac{T}{2\tau t} = \frac{5}{2(8)(a)} = \frac{5}{16a}$$

$$\therefore \frac{5}{16a} = 60a^2 \Rightarrow a_1 = 0.173 \text{ in.}$$

$$\theta = \frac{T L}{4 G A_m^2} \sum \left(\frac{s}{t} \right) = \frac{5(5 \times 12)}{4(10 \times 10^3) A_m^2} \left[\frac{16.5a}{2a} + \frac{7a}{1.5a} + \frac{6.5a}{a} \right]$$

$$0.15 = \frac{7.5 \times 10^{-3}}{A_m^2} (19.41667); A_m^2 = 0.97083$$

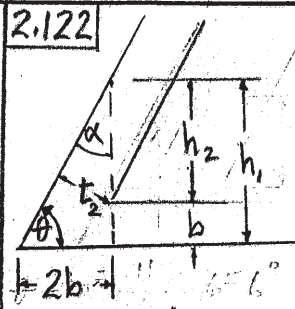
$$A_m = 0.9853 = 60a^2; a_2 = 0.128 \text{ in.}$$

$\therefore \tau$ CONTROLS AND

$$a = a_1 = 0.173 \text{ in.}$$

ANS.

2.122



$$h_1 = 2b(\tan \theta)$$

$$= 2b\left(\frac{11}{5}\right) = 4.4b$$

$$\therefore h_2 = 3.4b; \alpha = \tan^{-1}\left(\frac{24}{44}\right) = 24.444^\circ$$

$$t_2 = 3.4b \sin \alpha = 1.407b.$$

$\therefore \tau_{\text{MAX}}$ IN RECTANGLE OF THICKNESS $t_1 = b.$

$$A_m = \frac{1}{2} \left[\frac{1}{2}(10b)(11b) + \frac{1}{2}(6b)(8b) \right] = 39.5b^2$$

$$= 39.5 \text{ in.}^2$$

$$(a) \tau_{\text{MAX}} = \frac{T}{2A_m t_1} = \frac{10(12)}{2(39.5)(1.0)} = 1.519 \text{ ksi. ANS.}$$

$$(b) \theta = \frac{T L}{4 G A_m^2} \sum \left(\frac{s}{t} \right) = \frac{10(12)(6 \times 12)}{4(6.4 \times 10^3)(39.5)^2} \left[\frac{8b}{b} + \frac{\sqrt{(9.5b)^2 + (4.0b)^2}}{(1.407b)} \right]$$

$$= 0.131 \text{ rad. ANS.}$$

2.123 AS IN PROB. 2.122, $A_m = 39.5b^2$

$$A_m = 39.5(0.02^2) = 0.0158 \text{ m}^2$$

$$\tau_{\text{MAX}} = \frac{T}{2A_m t_1}; T_1 = 2\tau_{\text{MAX}} A_m t_1$$

$$T_1 = 2(40 \times 10^6)(0.0158)(0.02) = 25.280 \text{ kN} \cdot \text{m}$$

$$\theta = \frac{T L}{4 G A_m^2} \sum \left(\frac{s}{t} \right); T_2 = \frac{4 G A_m^2}{L} \left[\frac{1}{\sum \left(\frac{s}{t} \right)} \right]$$

$$T_2 = \frac{4(\pi/180)(50 \times 10^9)(0.0158^2)}{1.5} \left[\frac{1}{15.326} \right] = 37.905 \text{ kN} \cdot \text{m}$$

NOTE THAT $\sum \left(\frac{s}{t} \right) = 15.326$ WAS OBTAINED IN PROB. 2.122.

$\therefore \tau_{\text{MAX}}$ CONTROLS AND

$$\phi = T_1 = 25.280 \text{ kN} \cdot \text{m}$$

ANS.

2.124

$$A_m = (10a)(10a) + \frac{1}{2}(10a)(5a)$$

$$= 125a^2$$

$$\tau_{MAX} = \frac{T}{2A_mt}; A_m = \frac{T}{2t\tau_{MAX}}$$

$$A_m = \frac{100}{2(a)(12)} = 125a^2$$

$$\therefore a = \sqrt[3]{\frac{100}{2(12)(125)}} = 0.322 \text{ in.}$$

$$\theta_{MAX} = \frac{TL}{4GA_m^2} \sum \left(\frac{s}{E} \right); A_m^2 = \frac{TL}{4G\theta} \sum \left(\frac{s}{E} \right)$$

$$A_m^2 = \frac{100(4)(12)}{4(4 \times 10^3)(10\pi/180)} \left[\frac{30\pi}{\pi} + 2 \left(\frac{\sqrt{5^2 + 5^2} \pi}{\pi} \right) \right]$$

$$44.142$$

$$A_m^2 = 75.875 = (125a^2)^2$$

$$a = 0.264 \text{ in.}$$

$$\therefore \tau \text{ CONTROLS AND } a = 0.322 \text{ in. ANS.}$$

$$2.125 \text{ AS IN PROB. 2.124, } A_m = 125a^2$$

$$A_m = 125(0.015^2) = 0.028125 \text{ m}^2$$

$$\text{ALSO, FROM PROB. 2.124, } \sum \left(\frac{s}{E} \right) = 44.142$$

$$\tau_{MAX} = \frac{T}{2A_mt}; T_1 = 2A_mt\tau_{MAX}$$

$$T_1 = 2(0.028125)(0.015)(25 \times 10^6)$$

$$= 21.094 \text{ kN}\cdot\text{m}$$

$$\theta_{MAX} = \frac{TL}{4GA_m^2} \sum \left(\frac{s}{E} \right); T_2 = \frac{4GA_m^2\theta}{L \sum \left(\frac{s}{E} \right)}$$

$$T_2 = \frac{4(27 \times 10^9)(0.028125^2)(0.05)}{2(44.142)}$$

$$= 48.388 \text{ kN}\cdot\text{m}$$

$$\therefore \tau_{MAX} \text{ CONTROLS AND}$$

$$Q = T_1 = 21.094 \text{ kN}\cdot\text{m}$$

ANS.

$$2.126 \quad A_m = \frac{1}{2} [3.5b(2b) + 2 \left(\frac{\pi}{2} \right) (1.75b)^2$$

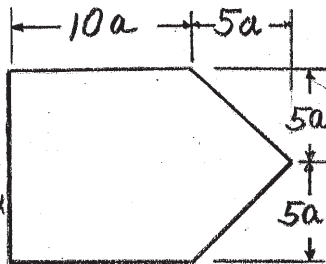
$$+ (3.0b)(2b) + 2 \left(\frac{\pi}{2} \right) (1.5b)^2]$$

$$A_m = 14.845b^2$$

$$\tau_{MAX} = \frac{T}{2A_mt}; A_m = \frac{T}{2t\tau_{MAX}}$$

$$A_m = \frac{15}{2(10)(0.25b)} = 14.845b^2$$

$$b_1 = 0.587 \text{ in.}$$



2.126 CONT'D

$$\theta = \frac{TL}{4GA_m^2} \sum \left(\frac{s}{E} \right); A_m^2 = \frac{TL}{4G\theta} \sum \left(\frac{s}{E} \right)$$

$$A_m^2 = \frac{15(3 \times 12)}{4(5.5 \times 10^3)(15\pi/180)} \left[2 \left(\frac{2b}{0.25b} \right) + 2 \left(\frac{\pi(1.625b)}{0.25b} \right) \right]$$

$$56.841$$

$$A_m = 2.309 \text{ in.}^2 = 14.845b^2$$

$$b_2 = 0.394 \text{ in.}$$

$$\therefore \tau_{MAX} \text{ CONTROLS AND}$$

$$b_{MIN} = b_1 = 0.587 \text{ in.}$$

ANS.

$$2.127 \text{ AS IN PROB. 2.126, } A_m = 14.845b^2$$

$$A_m = 14.845(0.03^2) = 1.336 \times 10^{-2} \text{ m}^2$$

$$\tau_{MAX} = \frac{T}{2A_mt}; T_1 = 2\tau_{MAX}A_mt$$

$$T_1 = 2(30 \times 10^6)(1.336 \times 10^{-2})(7.5 \times 10^{-3})$$

$$= 6.012 \text{ kN}\cdot\text{m}$$

$$\theta_{MAX} = \frac{TL}{4GA_m^2} \sum \left(\frac{s}{E} \right); T_2 = \frac{4\theta_{MAX}GA_m^2}{L \sum \left(\frac{s}{E} \right)}$$

$$\sum \left(\frac{s}{E} \right) = 56.841 \text{ FROM PROB. 2.126.}$$

$$T_2 = \frac{4(4\pi/180)(26 \times 10^9)(1.336 \times 10^{-2})^2}{1.5(56.841)}$$

$$= 15.199 \text{ kN}\cdot\text{m}$$

$$\therefore \tau_{MAX} \text{ CONTROLS AND}$$

$$Q = T_1 = 6.012 \text{ kN}\cdot\text{m}$$

ANS.

$$2.128 \quad \tau_y = 12.0 \text{ ksi}; \gamma_y = 2 \times 10^{-3}$$

$$(a) G = \frac{\tau_y}{\gamma_y} = \frac{12.0}{2 \times 10^{-3}} = 6 \times 10^3 \text{ ksi}$$

ANS.

$$(b) \tau_y = \frac{T_y R_o}{J}; T_y = \frac{\tau_y J}{R_o}$$

$$T_y = \frac{12.0(\pi/32)(2.5^4 - 1.5^4)}{2.5}$$

$$= 16.022 \text{ ksi}$$

ANS.

$$(c) \theta_y = \frac{T_y L}{JG} = \frac{16.022(2 \times 12)}{(\pi/32)(2.5^4 - 1.5^4)(6 \times 10^3)}$$

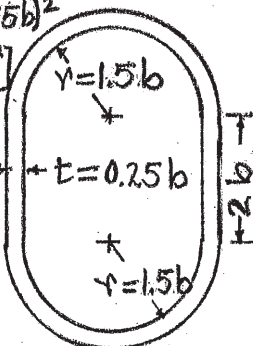
$$\theta_y = 19.200 \times 10^{-3} \text{ rad}$$

ANS.

$$(d) u_r = \frac{1}{2} \tau_y \gamma_y = \frac{1}{2} (12.0)(2 \times 10^{-3})$$

$$= 12.0 \times 10^{-3} \text{ kip}\cdot\text{in.}/\text{in.}^3$$

ANS.



2.129 $T_y = 30.0 \text{ kN}\cdot\text{m}$; $\tau_{xy} = 350 \text{ MPa}$

(a) $\tau_{xy} = \frac{T_y R}{J} = \frac{T_y (\frac{D}{2})}{(\pi/32) D^4} = \frac{16 T_y}{\pi D^3}$

$D^3 = \frac{16 T_y}{\pi \tau_{xy}} = \frac{16 (30.0 \times 10^3)}{\pi (350 \times 10^6)} = 0.436539 \times 10^3$

$D = 0.0759 \text{ m} = 75.9 \text{ mm}$ ANS.

(b) $\theta = \frac{TL}{JG}$; $G = \frac{TL}{J\theta} = \frac{30.0 \times 10^3 (1.5)}{(\pi/32)(0.0759^4)(\pi/180)}$

$G = 98.919 \times 10^9 \approx 98.9 \text{ GPa}$ ANS.

2.130 $T_y = \tau_{xy} J = \frac{17(\pi/32)(3.5^4)}{1} = 71.557 \text{ kip}\cdot\text{in}$

(a) $\frac{T}{T_y} = \left(\frac{R}{R_y}\right)^3 \left[1 - \left(\frac{1}{4}\right)\left(\frac{R}{R_y}\right)^4\right]$; $\frac{T}{T_y} = \frac{90}{71.557} = 1.258$

$1.258 = \left(\frac{R}{R_y}\right)^3 \left[1 - \left(\frac{1}{4}\right)\left(\frac{R}{R_y}\right)^4\right]$

$\left(\frac{R}{R_y}\right)^3 = 4 - 3\left(\frac{T}{T_y}\right) = 4 - 3(1.258) = 0.226$

$\frac{R}{R_y} = 0.609$; $R_y = 0.609 R = 0.609 (1.75)$

$R_y = 1.066 \text{ in.}$

$\therefore d = 1.75 - 1.066 = 0.684 \text{ in.}$ ANS.

(b) AS IN PART (a),

$\left(\frac{R}{R_y}\right)^3 = 4 - 3\left(\frac{T}{T_y}\right) = 4 - 3\left(\frac{80}{71.557}\right) = 0.646$

$\frac{R}{R_y} = 0.864$; $R_y = 0.864 R = 0.864 (1.75)$

$R_y = 1.512 \text{ in.}$

$\therefore d = 1.75 - 1.512 = 0.238 \text{ in.}$ ANS.

2.131 $T = \frac{\pi \tau_{xy}}{6} (4R^3 - R_y^3)$

(a) $R_y = 2.0 - 0.5 = 1.5 \text{ in.}$

$T = \frac{\pi (20)}{6} [4(2.0)^3 - (1.5)^3] = 299.760 \text{ kip}\cdot\text{in.}$ ANS.

(b) $R_y = 2.0 - 1.0 = 1.0 \text{ in.}$

$T = \frac{\pi (20)}{6} [4(2.0)^3 - (1.0)^3] = 324.631 \text{ kip}\cdot\text{in.}$ ANS.

(c) $R_y = 2.0 - 1.5 = 0.5 \text{ in.}$

$T = \frac{\pi (20)}{6} [4(2.0)^3 - (0.5)^3] = 333.794 \text{ kip}\cdot\text{in.}$ ANS.

(d) $R_y = 2.0 - 2.0 = 0$

$T = \frac{\pi (20)}{6} [4(2.0)^3 - 0] = 335.103 \text{ kip}\cdot\text{in.}$ ANS.

2.132 SEE DEVELOPMENT

OF EQ. (2.38).

$T = \left(\frac{2\pi \tau_{xy}}{R_y}\right) \int_{R_i}^{R_o} \rho^3 d\rho$

$+ (2\pi \tau_{xy}) \int_{R_y}^{R_o} \rho^2 d\rho$

$T = \left(\frac{2\pi \tau_{xy}}{R_y}\right) \left[\frac{\rho^4}{4}\right]_{R_i}^{R_o} + (2\pi \tau_{xy}) \left[\frac{\rho^3}{3}\right]_{R_y}^{R_o}$

$= \left(\frac{2\pi \tau_{xy}}{R_y}\right) \left(\frac{R_o^4 - R_i^4}{4}\right) + 2\pi \tau_{xy} \left(\frac{R_o^3 - R_y^3}{3}\right)$

$T = \frac{\pi \tau_{xy}}{2} \left[\frac{4R_o^3 - R_y^3}{3} - \frac{R_i^4}{R_y}\right]$ ANS.

2.133 FROM STATEMENT OF PROB. 2.132,

$T = \frac{\pi \tau_{xy}}{2} \left[\frac{4R_o^3 - R_y^3}{3} - \frac{R_i^4}{R_y}\right]$

$32 \times 10^3 = \pi (175 \times 10^6) \left[\frac{4(0.05^3 - R_y^3)}{6} - \frac{0.03^4}{2R_y}\right]$

SIMPLIFYING, WE OBTAIN

$R_y^4 - 15.078 \times 10^{-5} R_y + 0.243 \times 10^{-5} = 0$

A TRIAL-AND-ERROR SOLUTION YIELDS

$R_y = 0.0461 \text{ m}$

$d = 0.05 - 0.0461 = 0.0039 \text{ m}$

$= 3.9 \text{ mm}$ ANS.

2.134 $\frac{T}{T_y} = \frac{4}{3} \left[1 - \frac{1}{4(\theta/\theta_y)^3}\right]$

$T_y = \frac{\pi \tau_{xy} R^3}{2} = \frac{\pi (15)(3.5^3)}{2} = 1010.218 \text{ kip}\cdot\text{in.}$

$\theta_y = \frac{T_y L}{JG} = \frac{1010.218 (2.0 \times 12)}{(\pi/32)(3.5^4)(10.5 \times 10^3)} = 0.157 \text{ rad}$

$T = \left(\frac{4}{3}\right) T_y \left[1 - \frac{1}{4(\theta/\theta_y)^3}\right]$

(a) $\theta = 0.2 \text{ rad}$

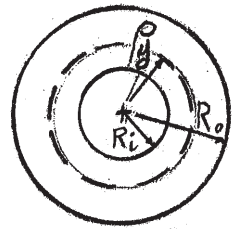
$T = \left(\frac{4}{3}\right) (1010.218) \left[1 - \frac{1}{4(0.2/0.157)^3}\right]$

$= 1084.064 \text{ kip}\cdot\text{in.}$ ANS.

(b) $\theta = 0.4 \text{ rad}$

$T = \left(\frac{4}{3}\right) (1010.218) \left[1 - \frac{1}{4(0.4/0.157)^3}\right]$

$= 1326.596 \text{ kip}\cdot\text{in.}$ ANS.



$$2.135 \quad \frac{T}{T_y} = \left(\frac{4}{3}\right) \left[1 - \left(\frac{1}{4}\right) \left(\frac{\rho_y}{R}\right)^3\right]$$

$$\rho_y = R \left[4 - 3 \left(\frac{T}{T_y}\right)^{1/3}\right]^{1/3}; R = 3/5 \text{ in.}$$

(a) FROM PROB. 2.134, $T = 1084.064 \text{ kip}\cdot\text{in.}$
 $T_y = 1010.218 \text{ kip}\cdot\text{in.}$ FOR $\theta = 0.20 \text{ rad.}$

$$\therefore \rho_y = 3.5 \left[4 - 3 \left(\frac{1084.064}{1010.218}\right)^{1/3}\right]^{1/3}$$

$$= 3.223 \text{ in.} \quad \text{ANS.}$$

(b) FROM PROB. 2.134, $T = 1326.596 \text{ kip}\cdot\text{in.}$
 $T_y = 1010.218 \text{ kip}\cdot\text{in.}$ FOR $\theta = 0.40 \text{ rad.}$

$$\therefore \rho_y = 3.5 \left[4 - 3 \left(\frac{1326.596}{1010.218}\right)^{1/3}\right]^{1/3}$$

$$= 1.374 \text{ in.} \quad \text{ANS.}$$

$$2.136 \quad T = \frac{\pi \tau_y}{6} (4R^3 - \rho_y^3)$$

$$\rho_y = \left[4R^3 - \frac{6T}{\pi \tau_y}\right]^{1/3} = \left[4(0.045^3) - \frac{6(25 \times 10^3)}{\pi(150 \times 10^6)}\right]^{1/3}$$

$$= 0.0359 \text{ m} = 35.9 \text{ mm} \quad \text{ANS.}$$

$$\frac{\rho_y}{R} = \frac{\theta_y}{\theta}; \theta = \left(\frac{R}{\rho_y}\right) \theta_y$$

$$\theta_y = \frac{T_y L}{J G} = \frac{(\tau_y T/R) L}{J G} = \frac{\tau_y L}{R G}$$

$$= \frac{150 \times 10^6 (0.75)}{0.045 (75 \times 10^9)} = 0.0333 \text{ rad.}$$

$$\therefore \theta = \left(\frac{0.75}{0.0359}\right) (0.0333)$$

$$= 0.696 \text{ rad} \quad \text{ANS.}$$

2.137 (a)

$$dF_y = (2\pi p dp) \tau_y$$

$$= (2\pi p dp) \left(\frac{\rho}{R} \tau_y\right)$$

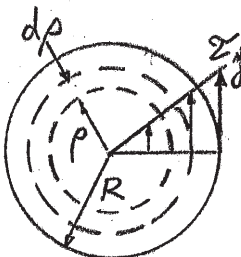
$$= (2\pi \tau_y \rho^2 dp) / R$$

$$dT_y = (2\pi \tau_y \rho^3 dp) / R$$

$$T_y = \frac{2\pi \tau_y}{R} \int \rho^3 dp = \frac{2\pi \tau_y}{R} \left[\frac{\rho^4}{4}\right]_0^R$$

$$T_y = \frac{\pi \tau_y R^3}{2} = \frac{\pi (25) (2^3)}{2}$$

$$= 314.159 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$



2.137 CONT'D

$$dF_y = (2\pi p dp) \tau_y$$

$$dT_y = (2\pi \tau_y \rho^2 dp)$$

$$T_y = 2\pi \tau_y \int \rho^2 dp$$

$$= 2\pi \tau_y \left[\frac{\rho^3}{3}\right]_0^R = \left(\frac{2}{3}\right) \pi R^3 \tau_y$$

$$T_y = \left(\frac{2}{3}\right) \pi (2^3) (25) = 418.879 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

(b)

$$T = 1.15 T_y = 1.15 (314.159) = 361.283 \text{ kip}\cdot\text{in.}$$

$$T = \frac{\pi \tau_y}{6} (4R^3 - \rho_y^3)$$

$$\rho_y = \left[4R^3 - \frac{6T}{\pi \tau_y}\right]^{1/3} = \left[4(2^3) - \frac{6(361.283)}{\pi(25)}\right]^{1/3}$$

$$\rho_y = 1.639 \text{ in.}$$

$$d = 2.0 - 1.639 = 0.361 \text{ in.} \quad \text{ANS.}$$

$$T = 1.25 T_y = 1.25 (314.159) = 392.699 \text{ kip}\cdot\text{in.}$$

$$\rho_y = \left[4(2^3) - \frac{6(392.699)}{\pi(25)}\right]^{1/3} = 1.260 \text{ in.}$$

$$d = 2.0 - 1.260 = 0.740 \text{ in.} \quad \text{ANS.}$$

2.138 (a)

$$\text{FROM PROB. 2.137, } T_y = \frac{\pi \tau_y R^3}{2}$$

$$T_y = \frac{\pi (160 \times 10^6) (0.04^3)}{2} = 16.085 \text{ kN}\cdot\text{m}$$

$$\text{FOR } d = 0.02 \text{ m; } \rho_y = 0.04 - 0.02 = 0.02 \text{ m}$$

$$T = \frac{\pi \tau_y}{6} (4R^3 - \rho_y^3) = \frac{\pi (160 \times 10^6)}{6} [4(0.04^3) - (0.02^3)]$$

$$T = 20.776 \text{ kN}\cdot\text{m}$$

$$\frac{T}{T_y} = \frac{20.776}{16.085} = 1.292 \quad \text{ANS.}$$

$$\text{FOR } d = 0.04 \text{ m; } \rho_y = 0.04 - 0.04 = 0$$

$$T = T_y = \left(\frac{2}{3}\right) \pi R^3 \tau_y \text{ (SEE PROB. 2.137)}$$

$$= \left(\frac{2}{3}\right) \pi (0.04^3) (160 \times 10^6) = 21.447 \text{ kN}\cdot\text{m}$$

$$\frac{T}{T_y} = \frac{21.447}{16.085} = 1.333 \quad \text{ANS.}$$

(b)

$$\frac{\rho_y}{R} = \frac{\theta_y}{\theta}; \theta = \left(\frac{R}{\rho_y}\right) \theta_y; \theta_y = \frac{T_y L}{J G}$$

2.138 CONT'D

$$\therefore \theta = \frac{RT_y L}{P_y J G}; \text{ FOR } d=0.02\text{m}; P_y=0.02\text{m}$$

$$\theta = \frac{(0.04)(16.085 \times 10^3)(0.80)}{(0.02)(\pi/32)(0.08^4)(70 \times 10^9)} = 0.0914 \text{ rad}$$

$$\theta = 0.0914 \text{ rad} \quad \text{ANS.}$$

$$\text{FOR } d=0.04\text{m}; P_y=0$$

$$\theta = \infty \quad \text{ANS.}$$

2.139 FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2} \left[\frac{4R_o^3 - P_y^3}{3} - \frac{R_i^4}{P_y} \right]$$

$$R_o = \left[\frac{P_y^3 + (6T/\pi \tau_y) + 3R_i^4/P_y}{4} \right]^{1/3}$$

$$= \left[\frac{(3.5)^3 + 6(250)/\pi(20) + 3(3.0)^4/3.5}{4} \right]^{1/3}$$

$$= 3.241 \text{ in.} \quad \text{ANS.}$$

2.140 FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2} \left[\frac{4R_o^3 - P_y^3}{3} - \frac{R_i^4}{P_y} \right]$$

$$R_i = \left\{ P_y \left[\frac{4R_o^3 - P_y^3}{3} - \left(\frac{2T}{\pi \tau_y} \right) \right] \right\}^{1/4}$$

$$= \left\{ 3.3 \left[\frac{4(3.5)^3 - (3.3)^3}{3} - \frac{2(200)}{\pi(10)} \right] \right\}^{1/4}$$

$$= 3.217 \text{ in.} \quad \text{ANS.}$$

2.141 FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2} \left[\frac{4R_o^3 - P_y^3}{3} - \frac{R_i^4}{P_y} \right]$$

$$(a) d=0.010\text{m}; P_y=0.05-0.01=0.04\text{m}$$

$$T = \frac{\pi(150 \times 10^6)}{2} \left[\frac{4(0.05)^3 - (0.04)^3}{3} - \frac{(0.02)^4}{0.04} \right]$$

$$= 33.301 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

$$\theta = \frac{R_o \theta_y}{P_y} = \frac{R_o (T_y L / J G)}{P_y}; T_y = \frac{\tau_y J}{R_o}$$

$$\therefore \theta = \frac{L \tau_y}{G P_y} = \frac{(0.6)(150 \times 10^6)}{75 \times 10^9 (0.04)}$$

$$= 0.030 \text{ rad} \quad \text{ANS.}$$

$$(b) d=0.020\text{m}; P_y=0.05-0.02=0.03\text{m}$$

$$T = \frac{\pi(150 \times 10^6)}{2} \left[\frac{4(0.05)^3 - (0.03)^3}{3} - \frac{(0.02)^4}{0.03} \right]$$

2.141 CONT'D

$$T = 35.893 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

$$\text{AS IN PART (a), } \theta = \frac{L \tau_y}{G P_y}$$

$$\theta = \frac{(0.6)(150 \times 10^6)}{75 \times 10^9 (0.03)} = 0.040 \text{ rad} \quad \text{ANS.}$$

2.142

$$(a) T_y = \frac{\tau_y J}{R_o} = \frac{(160 \times 10^6) \left(\frac{\pi}{32} \right) (0.07^4 - 0.04^4)}{0.07}$$

$$T_y = 77.014 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2} \left[\frac{4R_o^3 - P_y^3}{3} - \frac{R_i^4}{P_y} \right] \dots (a)$$

$$T = T_p \text{ WHEN } P_y = R_i$$

$$\therefore T_p = \frac{2\pi \tau_y}{3} (R_o^3 - R_i^3)$$

$$= \frac{2\pi(160 \times 10^6)}{3} [(0.07)^3 - (0.04)^3]$$

$$= 93.494 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

$$\theta_y = \frac{T_y L}{J G} = \frac{77.014 \times 10^3 (0.6)}{\frac{\pi}{32} (0.07^4 - 0.04^4) (70 \times 10^9)}$$

$$= 0.0196 \text{ rad} \quad \text{ANS.}$$

$$\theta_p = \infty \text{ (SEE PROB. 2.138)} \quad \text{ANS.}$$

$$(b) \frac{T}{T_y} = 1.20$$

$$T = 1.20 T_y = 1.20 (77.014) = 92.417 \text{ kN}\cdot\text{m}$$

FROM EQ. (a) ABOVE, WE CONCLUDE THAT

$$P_y^4 + \left[\frac{6T}{\pi \tau_y} - 4R_o^3 \right] P_y + 3R_i^4 = 0$$

SUBSTITUTING NUMERICAL VALUES WE OBTAIN

$$P_y^4 - 2.689 \times 10^{-4} P_y + 7.68 \times 10^{-6} = 0$$

A TRIAL-AND-ERROR SOLUTION YIELDS

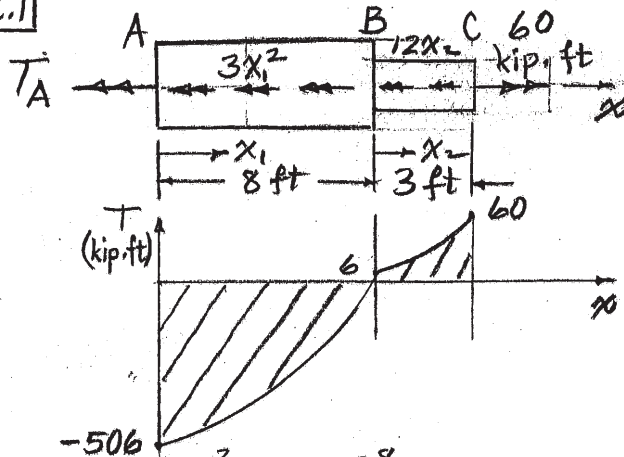
$$P_y = 0.0475 \text{ m}; \frac{P_y}{R_o} = \frac{\theta_y}{\theta}$$

$$\therefore \theta = \frac{R_o \theta_y}{P_y}$$

$$= \frac{(0.07)(0.0196)}{0.0475}$$

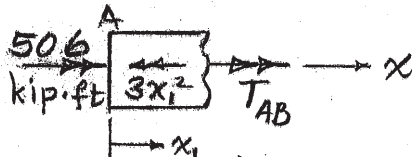
$$= 0.02888 \text{ rad} \quad \text{ANS.}$$

R2.1



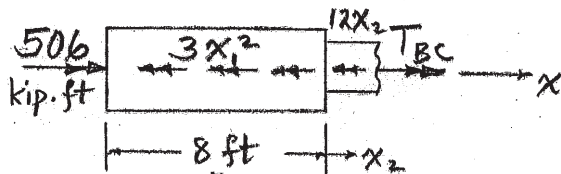
$$\sum T_x = 0: 60 - \int_0^3 12x_2 dx_2 - \int_0^8 3x_1^2 dx_1 - T_A = 0$$

$$T_A = 60 - 54 - 512 = -506 \text{ kip}\cdot\text{ft}$$



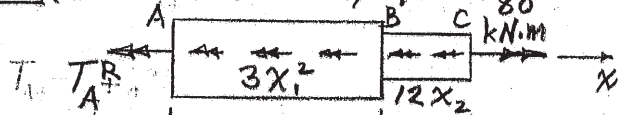
$$\sum T_x = 0: 506 - \int_0^{x_1} 3x_1^2 dx_1 + T_{AB} = 0$$

$$T_{AB} = x_1^3 - 506 \quad \text{ANS.}$$



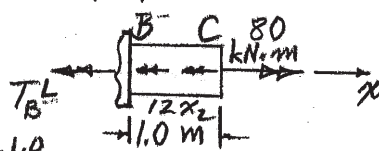
$$\sum T_x = 0: 506 - \int_0^8 3x_1^2 dx_1 - \int_0^{x_2} 12x_2 dx_2 + T_{BC} = 0$$

$$T_{BC} = 6x_2^2 + 6 \quad \text{ANS.}$$

R2.2 (a) JUST RIGHT OF A, $x_1 \approx 0$.

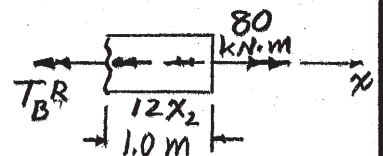
$$\sum T_x = 0: 80 - \int_0^{1.0} 12x_2 dx_2 - \int_0^{3.0} 3x_1^2 dx_1 - T_A^R = 0$$

$$T_A^R = 47 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

(b) JUST LEFT OF B, $x_1 \approx 3.0 \text{ m}$ 

$$\sum T_x = 0: 80 - \int_0^{1.0} 12x_2 dx_2 - T_B^L = 0$$

$$T_B^L = 74 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

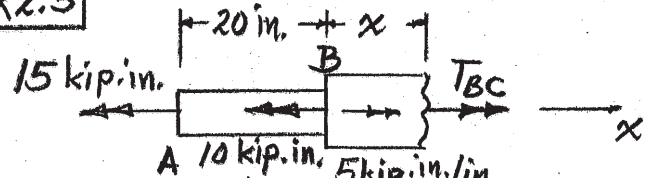
R2.2 CONT'D (c) JUST RIGHT OF B, $x_2 \approx 1.0 \text{ m}$ 

$$\sum T_x = 0:$$

$$80 - \int_0^{1.0} 12x_2 dx_2 - T_B^R = 0$$

$$T_B^R = 74 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

R2.3



$$\sum T_x = 0: T_{BC} + \int_0^{x_1} 5 dx - 10 - 15 = 0$$

$$T_{BC} = 25 - 5x \Big|_0^{x_1} = 25 - 5x$$

$$(a) \theta_A = \theta_{AB} + \theta_{BC}$$

$$\theta_{AB} = \left(\frac{TL}{JG_{AB}} \right) = \frac{15(20)}{(\pi/32)(3^4)(4 \times 10^3)} = 0.015915 \text{ rad}$$

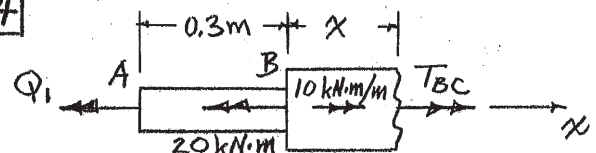
$$\theta_{BC} = \left(\frac{TL}{JG_{BC}} \right) = \frac{1}{JG_{BC}} \int_0^{30} (25 - 5x) dx = \frac{1}{(\pi/32)(3^4)(4 \times 10^3)} [25x - 2.5x^2]_0^{30} = -0.047157 \text{ rad}$$

$$\theta_A = -0.0312 \text{ rad WHEN VIEWED FROM A TO C.} \quad \text{ANS.}$$

$$(b) (T_{BC})_{x=10''} = 25 - 5(10) = -25 \text{ kip}\cdot\text{in.}$$

$$\left| \tau_{x=10''} \right| = \left| \frac{TR}{J} \right| = \frac{25(1.5)}{(\pi/32)(3^4)} = 4.716 \text{ ksi} \quad \text{ANS.}$$

R2.4



$$T_{AB} = Q_1$$

$$\tau_{AB} = \left(\frac{TR}{J} \right)_{AB}; 120 \times 10^6 = \frac{Q_1(0.03)}{(\pi/32)(0.06^4)}$$

$$Q_1 = 5.089 \text{ kN}\cdot\text{m}$$

$$\sum T_x = 0: T_{BC} + \int_0^{x_1} 10 dx - 20 - Q_1 = 0$$

$$T_{BC} = 20 + Q_1 - 10x; (T_{BC})_{\text{MAX}} = 20 + Q_1 \text{ JUST RIGHT OF B.}$$

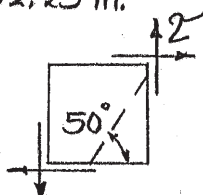
$$\tau_{BC} = \left(\frac{TR}{J} \right)_{BC}; 70 \times 10^6 = \frac{(20 + Q_1)(0.06)}{(\pi/32)(0.120^4)}$$

$$Q_1 = 3.750 \text{ kN}\cdot\text{m}; \therefore (Q_1)_{\text{MAX}} = 3.750 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

R2.5 $T = Q$; $R = 12 + 0.25 = 12.25$ in.

$$\tau = \frac{TR}{J} = \frac{Q(12.25)}{(\pi/32)(24.5^4)}$$

$$= 3.463 \times 10^{-4} Q$$



$$\tau_n = \tau \sin 2\theta$$

$$20 = (3.463 \times 10^{-4} Q) \sin(100^\circ)$$

$$Q_1 = 58,644.3 \text{ kip}\cdot\text{in.}$$

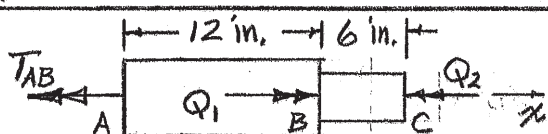
$$\tau_{nt} = \tau \cos 2\theta$$

$$14 = (3.463 \times 10^{-4} Q) \cos(100^\circ)$$

$$|Q_2| = 232,812.0 \text{ kip}\cdot\text{in.}$$

$$\therefore Q_{MAX} = Q_1 = 58,644.3 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

R2.6



$$\sum T_x = 0: Q_1 - Q_2 - T_{AB} = 0$$

$$T_{AB} = Q_1 - Q_2$$

$$\tau_{MAX} = \frac{TR}{J}; \theta = \frac{TL}{JG}$$

$$AB: 15 = \frac{(Q_1 - Q_2)(2)}{(\pi/32)(4^4)}; Q_1 - Q_2 = 188,496 \text{ kip}\cdot\text{in.} \quad \text{--- (1)}$$

$$\theta_C = \theta_{C/B} + \theta_{B/A}$$

$$0.03 = \frac{Q_2(6)}{(\pi/32)(2^4)(4 \times 10^3)} + \frac{(Q_1 - Q_2)(12)}{(\pi/32)(4^4)(4 \times 10^3)}$$

$$0.03 = 0.000955 Q_2 + 0.000119 (Q_1 - Q_2)$$

$$\therefore 300 = 8.36 Q_2 + 1.19 Q_1 \quad \text{--- (2)}$$

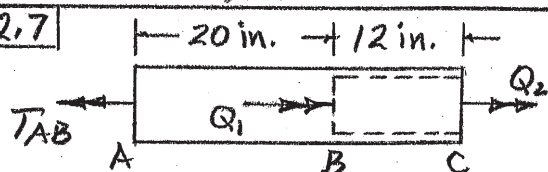
SOLVE (1) AND (2) TO GET

$$Q_1 = 196.4 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

$$Q_2 = 7.9 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

$$BC: \tau = \frac{7.9(1)}{(\pi/32)(2^4)} = 5.0 \text{ ksi} < \tau_{ALL} \text{ OK}$$

R2.7



$$\tau_{BC} = \frac{Q_2(2)}{(\pi/32)(4.0^4 - 3.5^4)} = 0.19230 Q_2$$

$$\tau_{AB} = \frac{(Q_1 + Q_2)(2)}{(\pi/32)(4.0^4)} = 0.07958 (Q_1 + Q_2)$$

$$\therefore 0.19230 Q_2 = 0.07958 (Q_1 + Q_2)$$

R2.7 CONT'D

$$\therefore 0.11272 Q_2 = 0.07958 Q_1$$

$$Q_1/Q_2 = 1.416 \quad \text{ANS.}$$

$$\theta_{C/B} = \frac{Q_2(12)}{(\pi/32)(4.0^4 - 3.5^4)G} = \frac{1.1538 Q_2}{G}$$

$$\theta_{B/A} = \frac{(Q_1 + Q_2)(20)}{(\pi/32)(4^4)G} = \frac{0.7958 (Q_1 + Q_2)}{G}$$

$$\frac{\theta_{B/A}}{\theta_{C/B}} = \frac{0.7958 (Q_1 + Q_2)}{1.1538 Q_2} = \frac{0.7958 (\frac{Q_1}{Q_2} + 1)}{1.1538}$$

$$= \frac{0.7958 (1.416 + 1)}{1.1538} = 1.666 \quad \text{ANS.}$$

R2.8 REFER TO THE SKETCH IN PROB. R2.7.

$$(a) \tau_{AB} = \left(\frac{TR}{J} \right)_{AB} = \frac{(25+15)(10^3)(0.06)}{(\pi/32)(0.120^4)}$$

$$\tau_{AB} = 117,893 \text{ MPa} \quad \text{ANS.}$$

$$(b) \tau_{BC} = \left(\frac{TR}{J} \right)_{BC}$$

$$= \frac{15(10^3)(0.06)}{(\pi/32)(0.120^4 - 0.100^4)}$$

$$= 85,389 \text{ MPa} \quad \text{ANS.}$$

$$\tau_n = \tau \sin 2\theta$$

$$= 85,389 \sin 60^\circ$$

$$= 73,949 \text{ MPa} \quad \text{ANS. SHAFT AXIS}$$

$$\tau_n = \tau \sin 2(\theta + 90^\circ)$$

$$= 85,389 \sin 240^\circ$$

$$= -73,949 \text{ MPa} \quad \text{ANS.}$$

$$\tau_{nt} = \tau \cos 2\theta = 85,389 \cos 60^\circ = 42,695 \text{ MPa}$$

R2.9 $\sum T_{CD} = 0$

$$5 - T_{CD} - 3F = 0 \quad \text{--- (1)}$$

$$\sum T_{AB} = 0: T_{AB} - 7F = 0$$

$$F = (1/7) T_{AB} \quad \text{--- (2)}$$

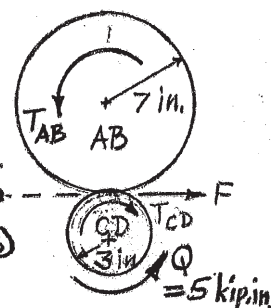
(2) INTO (1) YIELDS

$$5 - T_{CD} - (3/7) T_{AB} = 0 \quad \text{--- (3)}$$

$$3\theta_{C/D} = 7\theta_{B/A}$$

$$3 \left[\frac{T_{CD}(24)}{(\pi/32)d^4 G} \right] = 7 \left[\frac{T_{AB}(20)}{(\pi/32)d^4 G} \right]$$

$$T_{CD} = (140/72) T_{AB} \quad \text{--- (4)}$$



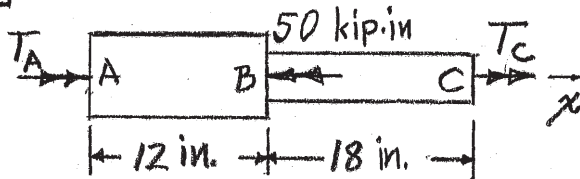
R2.9 CONT'D SOLVE ③ AND ④ TO GET

$$T_{AB} = 2.107 \text{ kip}\cdot\text{in.}; T_{CD} = 4.097 \text{ kip}\cdot\text{in.}$$

$$\therefore \tau_{\text{MAX}} = 20 \text{ ksi IN CD.}$$

$$20 = \frac{4.097(d/2)}{(\pi/32)d^4}; d = 1.014 \text{ in. ANS.}$$

R2.10



$$\sum T_x = 0: T_A + T_C = 50 \quad \text{--- ①}$$

$$\theta_{B/A} = \theta_{B/C}; \left(\frac{TL}{JG}\right)_{AB} = \left(\frac{TL}{JG}\right)_{BC}$$

$$\frac{T_A(12)}{(\pi/32)d^4(4 \times 10^3)} = \frac{T_C(18)}{(\pi/32)(3.0^4)(6.4 \times 10^3)}$$

$$86.400 T_A = d^4 T_C \quad \text{--- ②}$$

ASSUME BRASS CONTROLS.

$$\therefore 18 = \frac{T_C(1.5)}{(\pi/32)(3.0^4)}; T_C = 42.412 \text{ kip}\cdot\text{in.}$$

$$\text{①} \Rightarrow T_A = 50 - 42.412 = 7.588 \text{ kip}\cdot\text{in.}$$

$$\text{②} \Rightarrow 86.400(7.588) = d^4(42.412)$$

$$d = 1.983 \text{ in.}$$

$$\tau_{\text{AL}} = \frac{7.588(1.983/2)}{(\pi/32)(1.983^4)} = 4.956 \text{ ksi} < \tau_{\text{ALL}}$$

$$\therefore d_{\text{MIN}} = 1.983 \text{ in. ANS.}$$

R2.11 $\sum T_{AB} = 0$

$$T_{AB} + 0.18F - Q = 0 \quad \text{--- ①}$$

$$\sum T_{CD} = 0$$

$$0.09F - T_{CD} = 0$$

$$F = 11.111 T_{CD} \quad \text{--- ②}$$

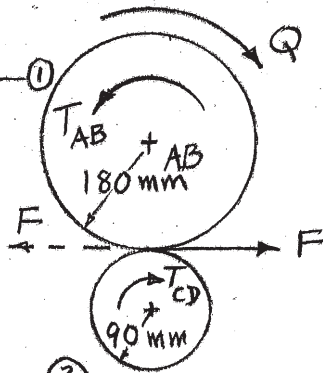
② INTO ① $\Rightarrow$

$$T_{AB} + 2T_{CD} = Q \quad \text{--- ③}$$

$$\text{NOW } 0.18\theta_{AB} = 0.09\theta_{CD}; \theta_{CD} = 2\theta_{AB}$$

$$\therefore \frac{T_{CD}(0.3)}{(\pi/32)(0.06^4)(25 \times 10^9)} = 2 \left[\frac{T_{AB}(0.3)}{(\pi/32)(0.03^4)(70 \times 10^9)} \right]$$

$$T_{CD} = 11.429 T_{AB} \quad \text{--- ④}$$



R2.11 CONT'D

ASSUME ALUMINUM CONTROLS. THUS,

$$80 \times 10^6 = \frac{T_{CD}(0.03)}{(\pi/32)(0.06^4)}; T_{CD} = 3.393 \text{ kN}\cdot\text{m}$$

$$\text{④} \Rightarrow T_{AB} = 0.297 \text{ kN}\cdot\text{m}$$

$$\tau_{AB} = \frac{(0.297)(10^3)(0.015)}{(\pi/32)(0.03^4)} = 56.0 \text{ MPa} < \tau_{\text{ALL}}$$

$$\text{③} \Rightarrow Q = 0.297 + 2(3.393) = 7.083 \text{ kN}\cdot\text{m ANS.}$$

R2.12 $P = T\omega; T = \frac{P}{\omega}$

$$T = \frac{500(6600)}{(400)(2\pi/60)} = 78,781.697 \text{ lb}\cdot\text{in.}$$

$$\tau = \frac{78,781.697(3.0)}{(\pi/32)(6.0^4 - d_i^4)} = 10 \times 10^3$$

$$d_i = 5.6995 \approx 5.7 \text{ in.}$$

$$\theta = \frac{78,781.697(12)}{(\pi/32)(6.0^4 - d_i^4)(4 \times 10^6)} = 0.015$$

$$d_i = 5.8049 \approx 5.8 \text{ in}$$

$$\therefore (d_i)_{\text{MAX}} = 5.7 \text{ in. ANS.}$$

R2.13 $U = \frac{1}{2} F \Delta$

$$500(0.05 + \Delta) = \frac{1}{2} F \Delta \quad \text{--- ①}$$

$$\Delta = \theta b = \left(\frac{TL}{JG}\right)b = \frac{Fb^2L}{JG} = \frac{F(0.4^2)(0.6)}{(\pi/32)(0.11^4 - d_i^4)(25 \times 10^9)}$$

$$\Delta = \frac{3.91139 \times 10^{-11} F}{X} \text{ WHERE } X = (0.11^4 - d_i^4)$$

SUBSTITUTING INTO ① AND SIMPLIFYING YIELDS

$$F^2 - 1 \times 10^3 F - 12.78317 \times 10^{-11} X = 0 \quad \text{--- ②}$$

$$\tau = \frac{TR}{J}; 100 \times 10^6 = \frac{0.4F(0.055)}{(\pi/32)X}$$

$$X = 2.241 \times 10^{-9} F \quad \text{--- ③}$$

③ INTO ② LEADS TO

$$F^2 - 3.8647 \times 10^3 F = 0; F = 3,865 \text{ N}$$

$$\text{③} \Rightarrow X = 2.241 \times 10^{-9} (3,865) = 8.661 \times 10^{-8}$$

$$\therefore 0.11^4 - d_i^4 = 8.661 \times 10^{-8}; d_i = 0.108 \text{ m}$$

$$\theta = \frac{TL}{JG}; 0.05 = \frac{0.4F(0.6)}{(\pi/32)X(25 \times 10^9)}$$

$$X = 1.956 \times 10^{-9} F \quad \text{--- ④}$$

④ INTO ② YIELDS

$$F^2 - 26 \times 10^3 F = 0; F = 3,500 \text{ N}$$

$$\text{④} \Rightarrow X = 6.825 \times 10^{-6}$$

$$\therefore 0.11^4 - d_i^4 = 6.825 \times 10^{-6}; d_i = 0.109 \text{ m}$$

$$(d_i)_{\text{MAX}} = 0.108 \text{ m} = 108 \text{ mm ANS.}$$

$$R2.14 \quad \sigma_n = \frac{75 \times 10^9}{(1-0.33^2)} [500 + 0.33(-300)] \times 10^{-6}$$

$$= 33.750 \text{ MPa}$$

$$\sigma_t = \frac{75 \times 10^9}{(1-0.33^2)} [-300 + 0.33(500)] \times 10^{-6}$$

$$= -11.362 \text{ MPa}$$

FROM EXAMPLE 2.12, WE HAVE

$$\sigma_m = \frac{1}{2} \sigma_x (1 + \cos 2\theta) + \tau_{xy} \sin 2\theta$$

$$33.750 = \frac{1}{2} \sigma_x (1 + \cos 100^\circ) + \tau_{xy} \sin 100^\circ$$

$$33.750 = 0.413 \sigma_x + 0.985 \tau_{xy} \quad \text{--- ①}$$

$$\sigma_t = \frac{1}{2} \sigma_x (1 + \cos 2(\theta + 90^\circ)) + \tau_{xy} \sin 2(\theta + 90^\circ)$$

$$-11.362 = \frac{1}{2} \sigma_x (1 + \cos 280^\circ) + \tau_{xy} \sin 280^\circ$$

$$-11.362 = 0.587 \sigma_x - 0.985 \tau_{xy} \quad \text{--- ②}$$

SOLVE ① & ② TO GET

$$\sigma_x = 22.388 \text{ MPa}; \tau_{xy} = 24.883 \text{ MPa}$$

$$\sigma_x = \frac{P}{A}; P = \sigma_x A = 22.388 \times 10^6 \left(\frac{\pi}{4}\right) (0.1^2)$$

$$P \approx 175.8 \text{ kN} \quad \text{ANS.}$$

$$\tau_{xy} = \frac{T R}{J}; T = \frac{\tau_{xy} J}{R} = \frac{24.883 \times 10^6 (\pi/32) (0.1^4)}{0.05}$$

$$Q = T = 4.886 \text{ kN}\cdot\text{m} \quad \text{ANS.}$$

R2.15

$$A_o = 2 \left[\left(\frac{1}{2} \right) (8.75) (2.526) \right]$$

$$+ 5.052 (8.75)$$

$$A_o = 66.3075 \text{ in}^2$$

$$A_i = 2 \left[\left(\frac{1}{2} \right) (7.25) (2.093) \right]$$

$$+ 4.186 (7.25)$$

$$A_i = 45.5228 \text{ in}^2$$

$$A_m = \frac{1}{2} (A_o + A_i) = 55.915 \text{ in}^2$$

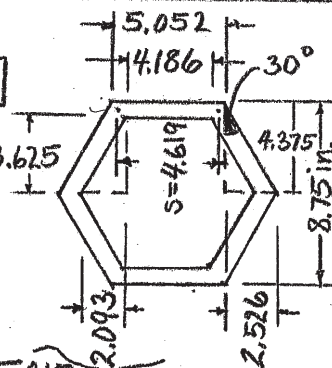
$$\tau = \frac{T}{2t A_m}; T = 2t A_m \tau$$

$$T = 2 (0.75) (55.915) (12) = 1006.5 \text{ kip}\cdot\text{in.}$$

$$\theta = \frac{TL}{4GA_m^2} \sum \left(\frac{s}{t} \right); T = \frac{4GA_m^2 \theta}{L \sum (s/t)}$$

$$T = \frac{4(5.0 \times 10^3) (55.915^2) (2\pi/180)}{4(12) [6(4.619/0.75)]} = 1230.6 \text{ kip}\cdot\text{in.}$$

$$Q_{\text{MAX}} = 1006.5 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$



R2.16

$$K = \frac{1}{3} \sum L t^3 = \frac{1}{3} [6(4.619)(0.75)^3] = 3897 \text{ in}^4$$

$$\tau = \frac{T t}{K}; T = \frac{\tau K}{t} = \frac{12(3897)}{0.75} = 62.352 \text{ kip}\cdot\text{in.}$$

$$\theta = \frac{TL}{KG}; T = \frac{KG\theta}{L} = \frac{3897(5 \times 10^3)(2\pi/180)}{4(12)}$$

$$= 14.170 \text{ kip}\cdot\text{in.}$$

$$\therefore Q_{\text{MAX}} = 14.170 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

R2.17

FROM PROBLEM 2.132 WE HAVE

$$T = \frac{\pi \tau_y}{2} \left[\frac{4R_o^3}{3} - \frac{p_y^3}{3} - \frac{R_i^4}{p_y} \right]$$

REWRITING THIS EQ. WE OBTAIN THE FOLLOWING POLYNOMIAL:

$$p_y^4 + \left(\frac{6T}{\pi \tau_y} - 4R_o^3 \right) p_y + 3R_i^4 = 0$$

SUBSTITUTING GIVEN NUMERICAL VALUES, THE ABOVE EQ. REDUCES TO

$$p_y^4 - 5.313 \times 10^{-4} p_y + 7.68 \times 10^{-6} = 0$$

A TRIAL-AND-ERROR SOLUTION YIELDS

$$p_y = 0.07545 \text{ m}$$

$$d = R_o - p_y = 0.08 - 0.07545$$

$$= 0.00455 \text{ m} = 4.55 \text{ mm} \quad \text{ANS.}$$

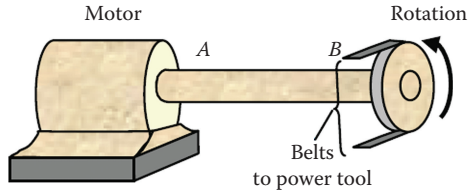


FIGURE 2.1 Example of a motor transmitting power, through shaft AB , to a power tool.

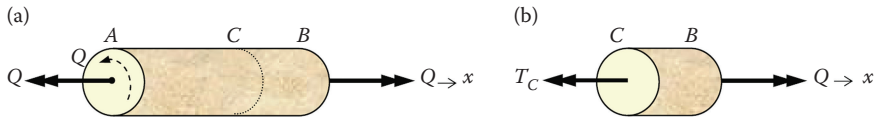


FIGURE 2.3 Representation of a concentrated torque Q at the center point in a circular shaft.

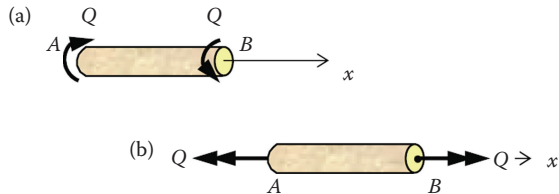


FIGURE 2.2 Representation of a three-dimensional torque Q by a double-headed arrow. Interpretation is made by the right-hand rule.

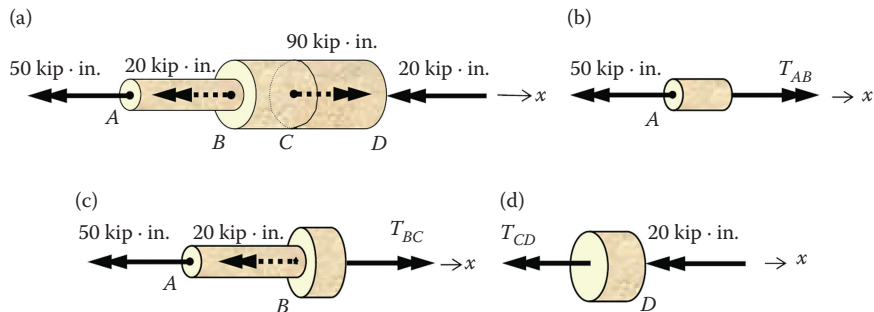


FIGURE 2.4 Shaft subjected to concentrated torques at a number of positions along its length, and determination of internal torques in segments AB, BC, and CD using equilibrium.

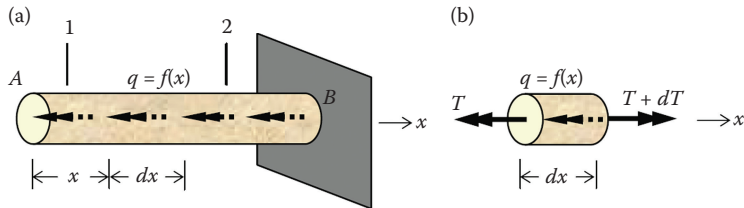


FIGURE 2.6 Example of a variably distributed torque $q = f(x)$ over the length of shaft AB and determination, using equilibrium, of internally distributed torque at any position in the shaft.

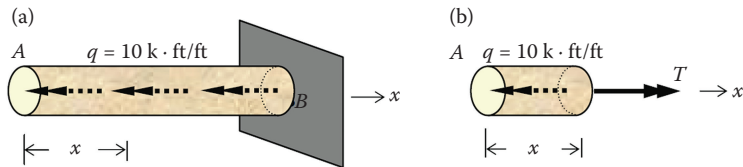


FIGURE 2.5 Example of a constantly distributed torque $q = 10 \text{ k} \cdot \text{ft/ft}$ over the entire length of the shaft AB and determination, by equilibrium, of internally distributed torque at any position a distance x from end A .

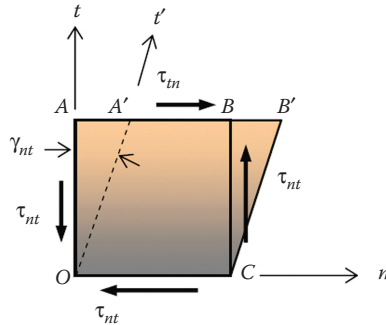


FIGURE 2.7 Definition of shearing strain γ_{nt} as being approximately equal to the tangent of the shearing angle γ_{nt} .

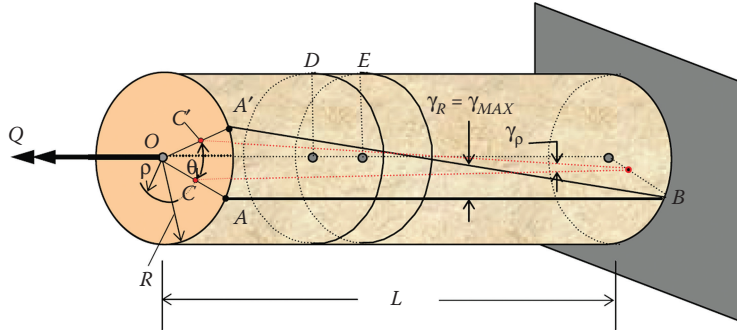


FIGURE 2.8 Shaft showing distortion under the influence of a torque Q and the definition of the angle of twist θ .

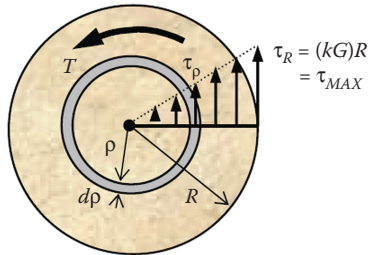


FIGURE 2.9 Circular cross section of a circular shaft showing the stress distribution due to a torque T .

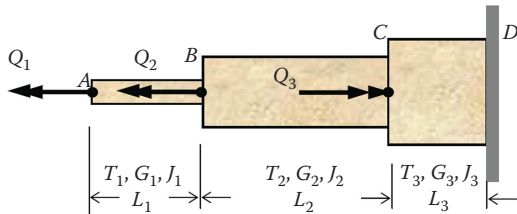


FIGURE 2.10 Shaft consisting of three component parts, each having its own properties T , G , and J , showing that the total angle of twist is the sum of three angles of twist.

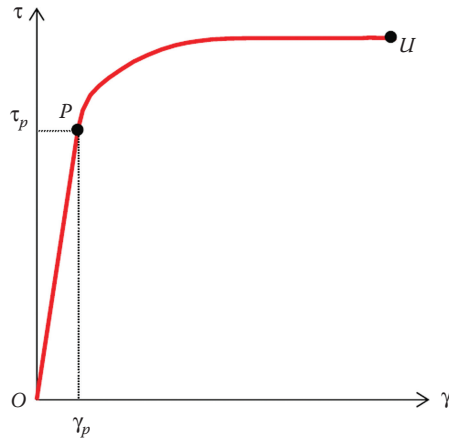


FIGURE 2.11 Shearing stress–strain diagram for a given ductile material, defining the properties: modulus of rigidity G and the modulus of rupture τ_U .

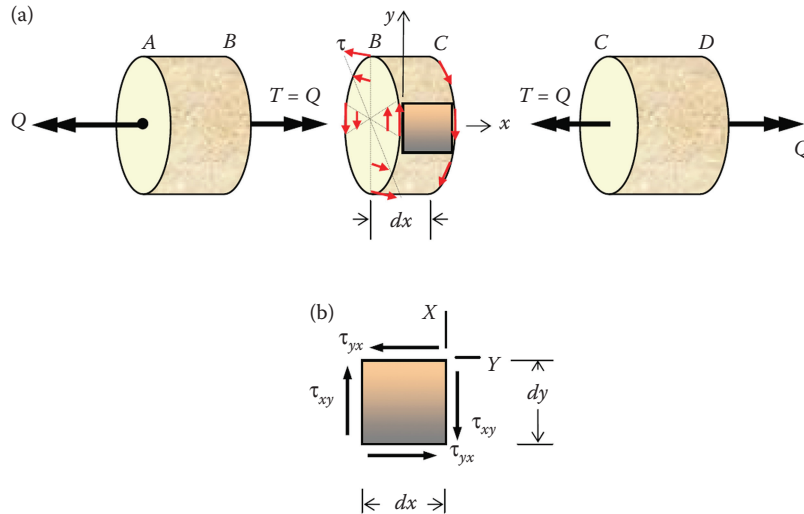


FIGURE 2.12 Diagram showing the state of pure shear in a shaft subjected to a torque Q .

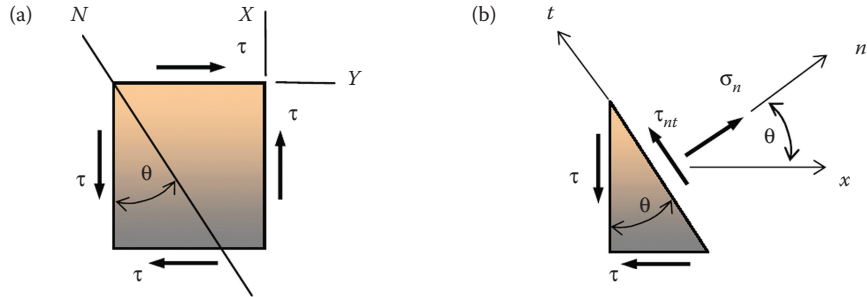


FIGURE 2.13 (a) The state of pure shear and (b) determination of normal and shearing stresses on inclined planes.

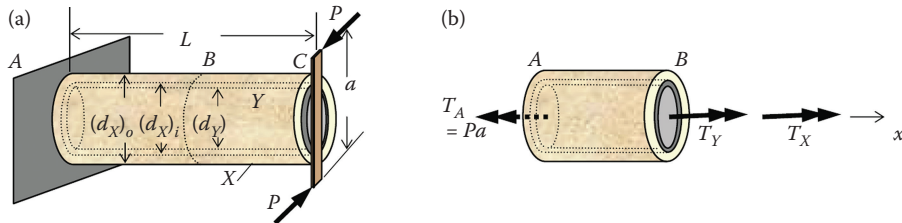


FIGURE 2.14 (a) A two-material shaft subjected to a torque Pa and (b) the free-body diagram of segment AB .

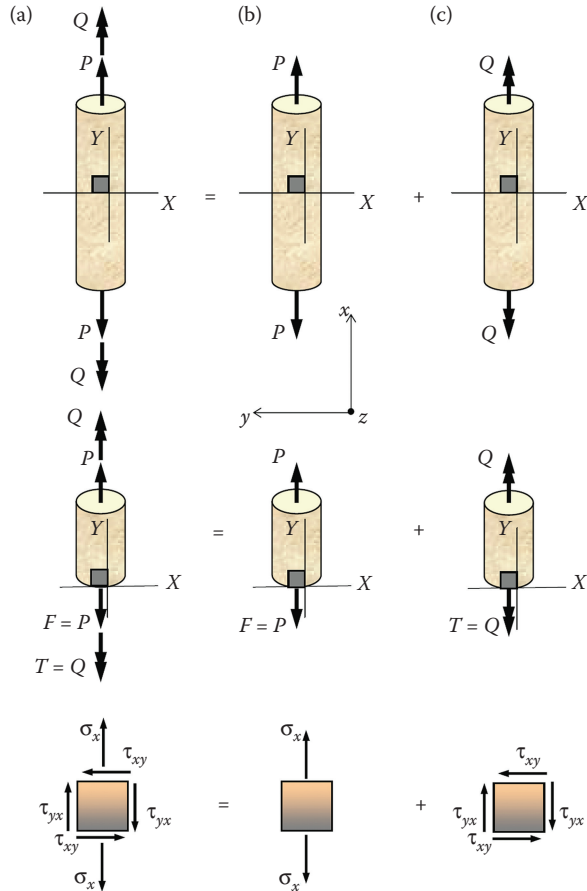


FIGURE 2.15 Analysis of stresses in a shaft subjected to the combined loads, axial force P and a torque Q .

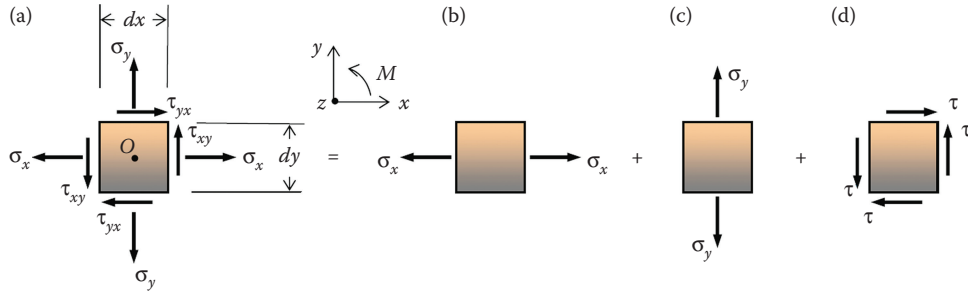


FIGURE 2.16 A general plane-stress condition broken down into three simple components, two uniaxial σ_x and σ_y , and one pure shear τ .

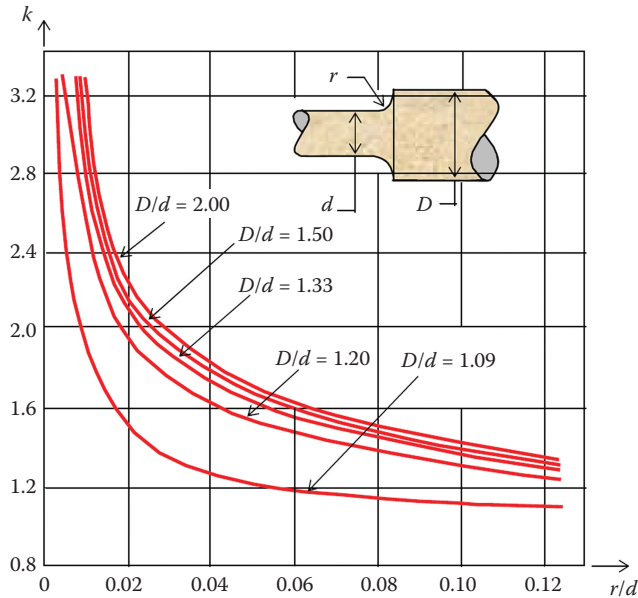


FIGURE 2.17 Stress-concentration factors for torsional loads. (Adapted from the work of L.S. Jacobsen, Torsional stress concentrations in shafts of circular cross section and variable diameter, *Trans ASME*, 47, 619–641, 1925.)

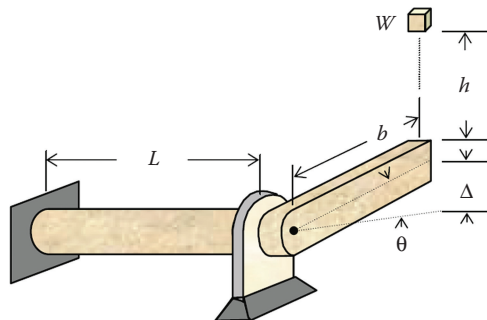


FIGURE 2.18 Diagram showing a shaft of length L subjected to impact loading due to a weight W dropping through a height h .

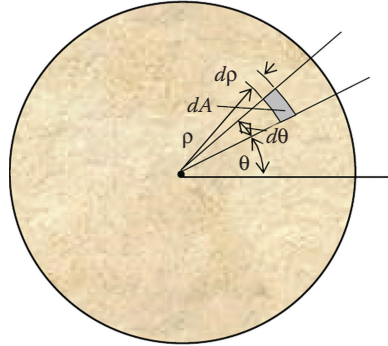


FIGURE 2.20 Cross section of a shaft subjected to a torque T and used to determine the total strain energy U .

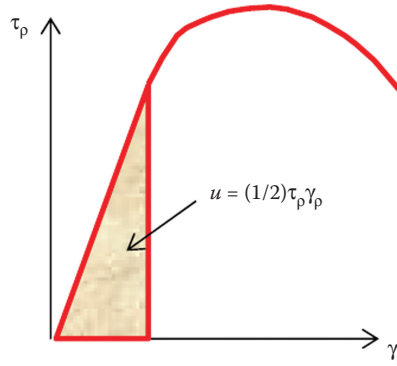


FIGURE 2.19 A shearing stress–strain diagram showing determination of the elastic strain energy u .

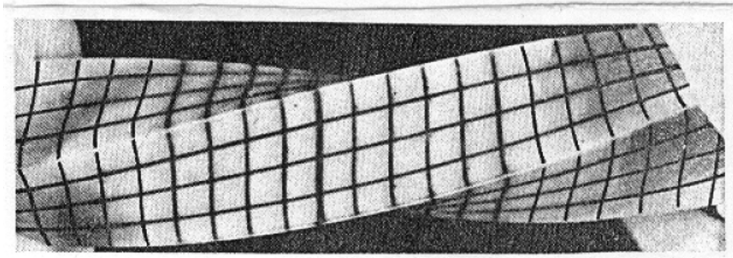


FIGURE 2.22 Diagram showing that, for a shaft of a noncircular cross section, plane sections before twisting do not remain plane after twisting.

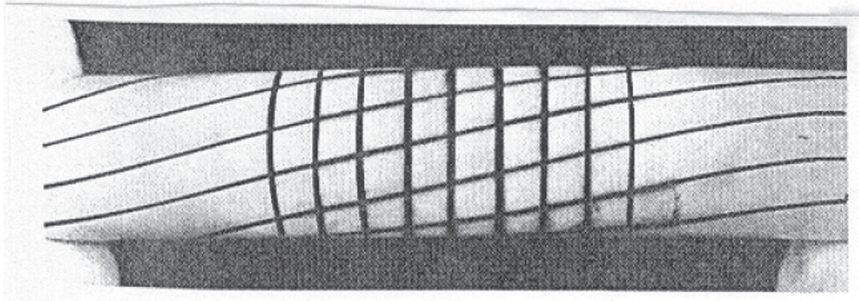


FIGURE 2.21 Diagram showing that, for a shaft of circular cross section, plane sections before twisting remain plane after twisting.

$$\tau_{MAX} = \frac{T(3a + 1.8b)}{8a^2b^2}; \quad \text{occurs at } A$$

$$\theta = \frac{TL}{KG}; \quad K = ab^3 \left[\left(\frac{16}{3} \right) - 3.36 \left(\frac{b}{a} \right) \left(1 - \frac{b^4}{12a^4} \right) \right]$$

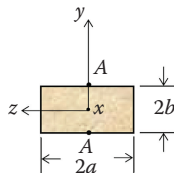


FIGURE 2.24 Mathematically obtained solutions for a shaft of a rectangular cross section.

$$\tau_{MAX} = \frac{2T}{\pi ab^2}; \quad \text{occurs at } A$$

$$\theta = \frac{TL}{KG}; \quad K = \frac{\pi a^3 b^3}{a^2 + b^2}$$

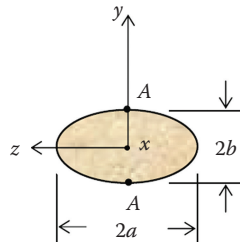


FIGURE 2.23 Mathematically obtained solutions for a shaft of an elliptical cross section.

$$\tau_{MAX} = \frac{20T}{a^3}; \quad \text{occurs at } A$$

$$\theta = \frac{TL}{KG}; \quad K = \frac{(\sqrt{3})a^4}{80}$$

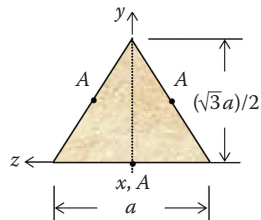


FIGURE 2.25 Mathematically obtained solutions for a shaft of an equilateral triangular cross section.

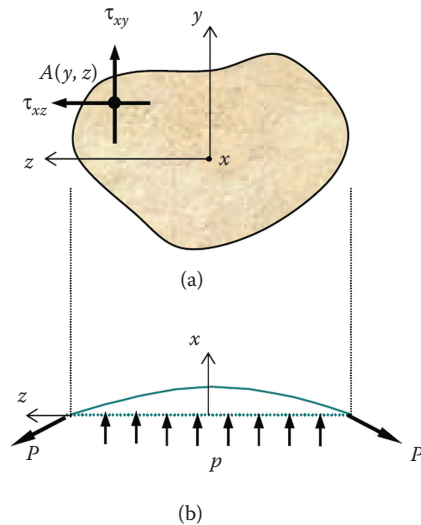


FIGURE 2.26 (a,b) The two views of a distended thin membrane subjected to a pressure p used to experimentally solve the torsion problem of noncircular shafts.

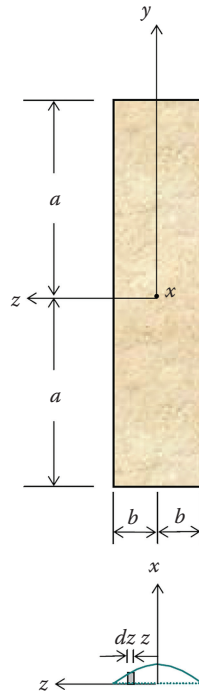


FIGURE 2.27 Experimental solution of the torsion problem of a shaft having a long and thin rectangular cross section.

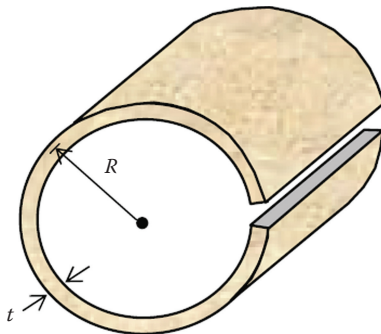


FIGURE 2.28 Solution of the torsion problem of a shaft with a narrow circular section having a thin slit.

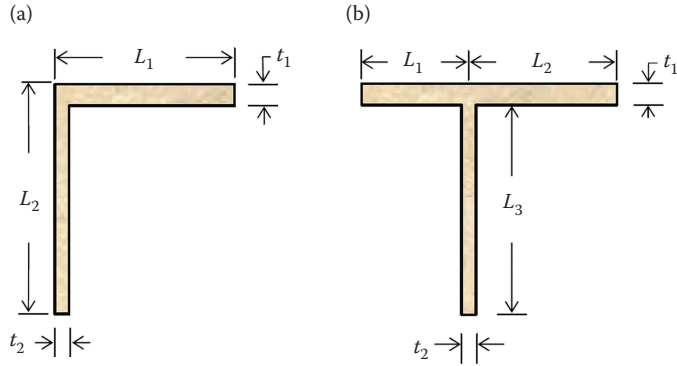


FIGURE 2.29 Diagrams showing cross sections composed of narrow rectangles for which solutions can be obtained.

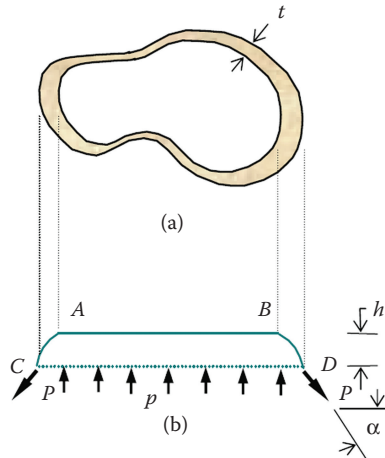


FIGURE 2.30 (a,b) The two views of a distended thin membrane subjected to a pressure p used to experimentally solve the torsion problem of thin-walled tubes of noncircular shafts.

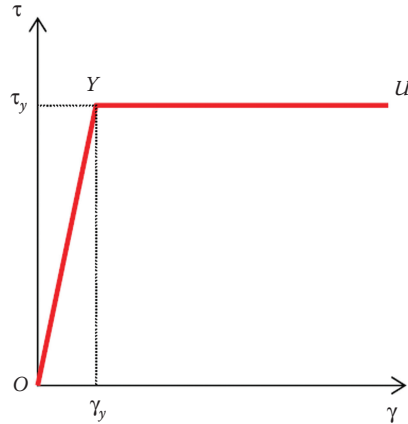


FIGURE 2.31 Stress–strain diagram for an elastoplastic material in which τ_y is the yield stress.

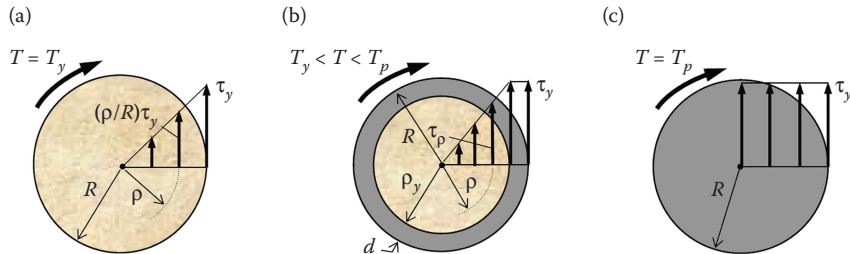


FIGURE 2.32 (a) The shearing stress distribution in a circular shaft when it reaches the yield stress τ_y . (b) The stress distribution when plastic action has reached the outer layers of the circular shaft. (c) The stress distribution when the entire shaft is under plastic action.

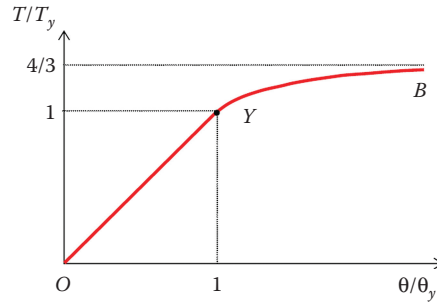


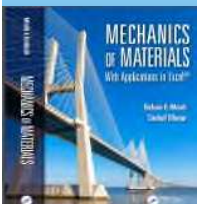
FIGURE 2.33 Diagram showing a plot of Equation 2.65.

MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL CHAPTER 2: TORSIONAL LOADS

LECTURE 7

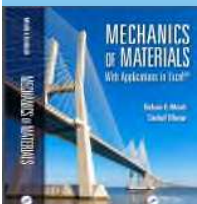
2.1 INTRODUCTION

2.2 INTERNAL TORQUE



Lecture Outline

- Introduction
- Internal Torque
- Examples



Lecture 7

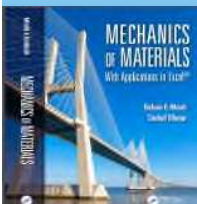
Chapter 2. Torsional Loads

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Introduction

- This topic deals with members subjected to torsional loads, namely, moments (couples) about the centroidal axes of such members.
- These types of moments or couples are generally referred to as torques and the members subjected to torques are generally known as shafts.
- Examples of these types of members include propeller shafts of ships and aircraft and drive shafts of automobiles, power tools, and other equipment.

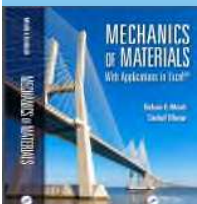
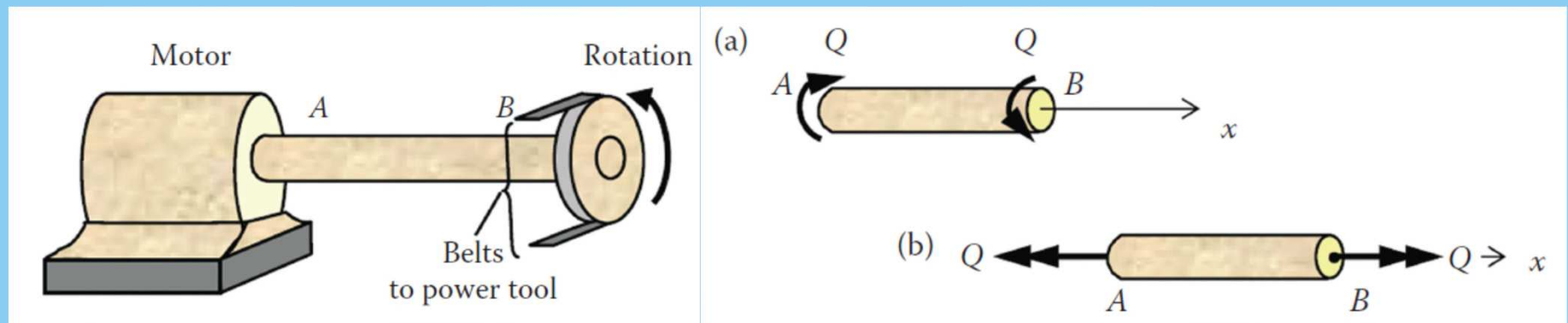


Lecture 7

Chapter 2. Torsional Loads

Concentrated Torque

- A concentrated torque may be defined as one that is applied over a very small area (assumed to be a point) of the member on which it acts.

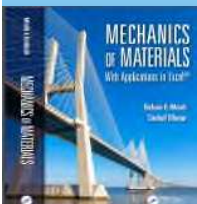
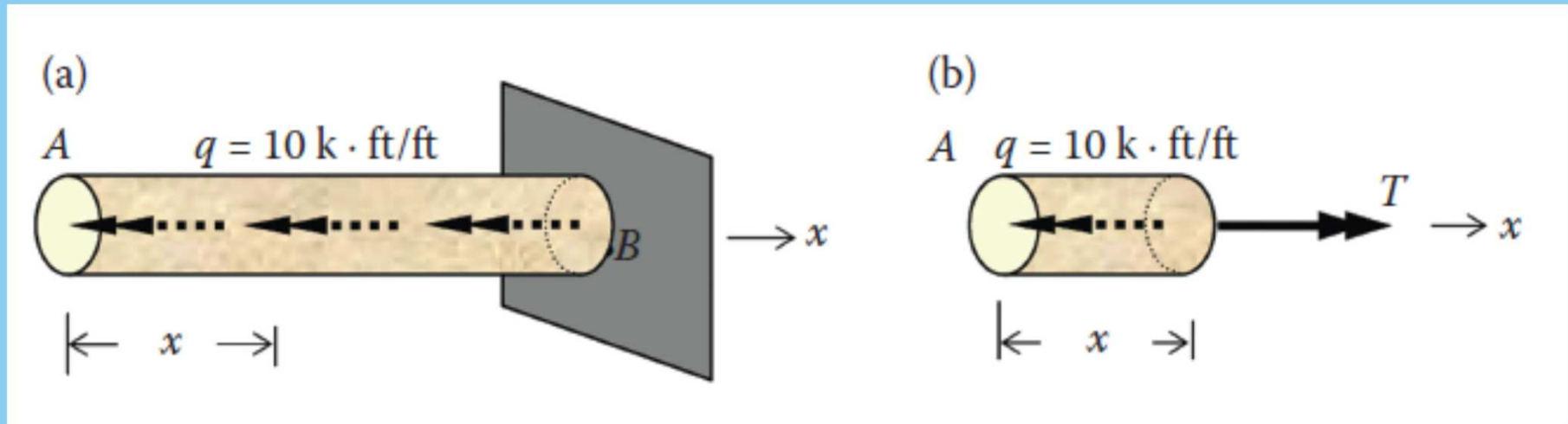


Lecture 7

Chapter 2. Torsional Loads

Distributed Torque

- A distributed torque is applied over the length of a member.

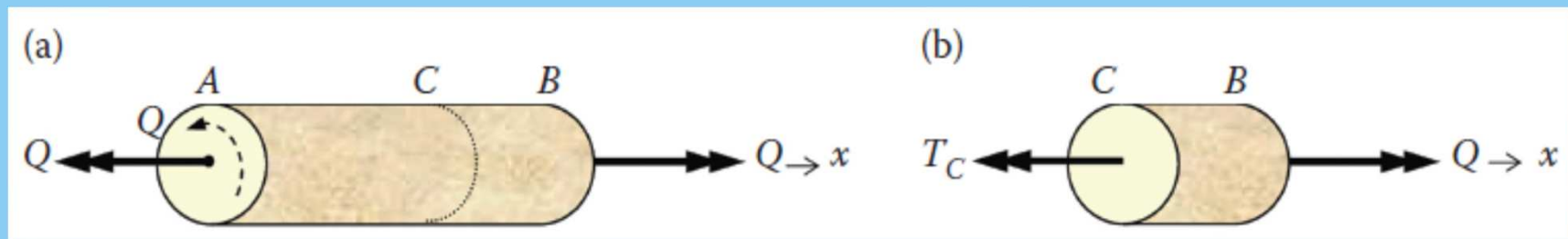


Lecture 7

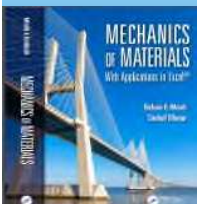
Chapter 2. Torsional Loads

Internal Torque

- Internal torque at any location along the member is obtained by introducing a section at the location and applying equilibrium. The figure below illustrates the internal torque at C, T_C .

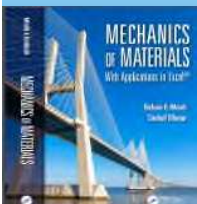
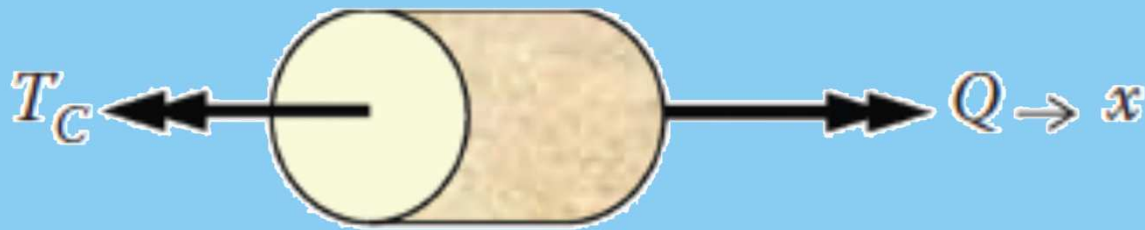


$$\sum T_x = 0 \Rightarrow Q - T_C = 0; \quad T_C = Q$$



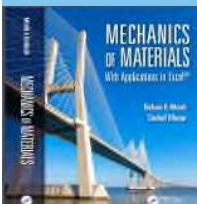
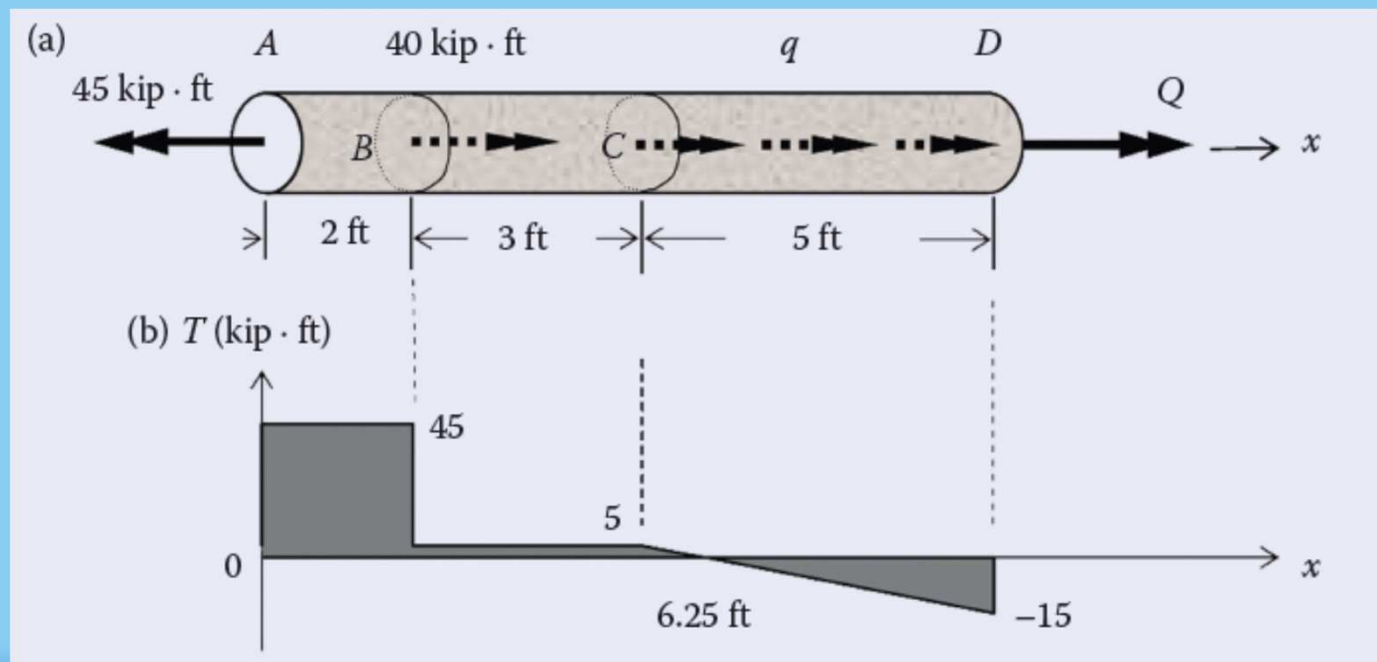
Sign Convention

- Torque is positive, if the two-headed vector representing it points away from the surface on which it acts, and negative, if it points into this surface.



Internal Torque Diagram

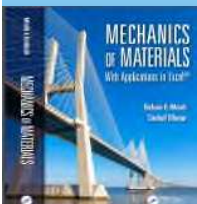
- An internal torque diagram is a diagram that shows, at a glance, how the internal torque changes from point to point along the length of the shaft



Lecture 7

Chapter 2. Torsional Loads

Examples



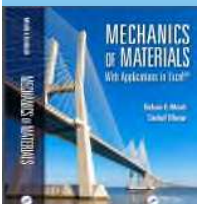
Lecture 7

Chapter 2. Torsional Loads

MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL CHAPTER 2: TORSIONAL LOADS

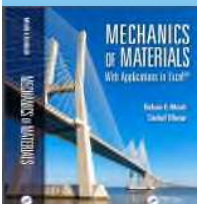
LECTURE 8

2.3 STRESSES AND DEFORMATIONS IN CIRCULAR SHAFTS



Lecture Outline

- Introduction
- Stresses and Deformation in Circular Shafts
- Material Properties in Shear
- Stress Element
- Examples



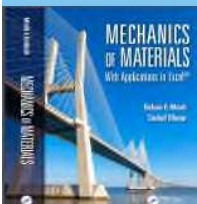
Lecture 8

Chapter 2. Torsional Loads

2

Introduction

- The torsion of circular shafts represents the simplest of all torsion problems.
- Pure torsion causes shear strain and shear stress.



Lecture 8

Chapter 2. Torsional Loads

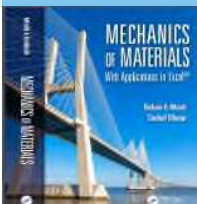
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Stresses and Deformations in Circular Shafts

- Shear stress-strain relations are used to study the behavior of torsional shafts. Stress-Strain relations are governed by Hooke's Law:

$$\tau = G\gamma$$



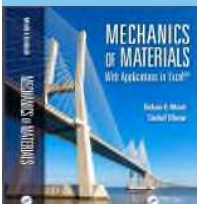
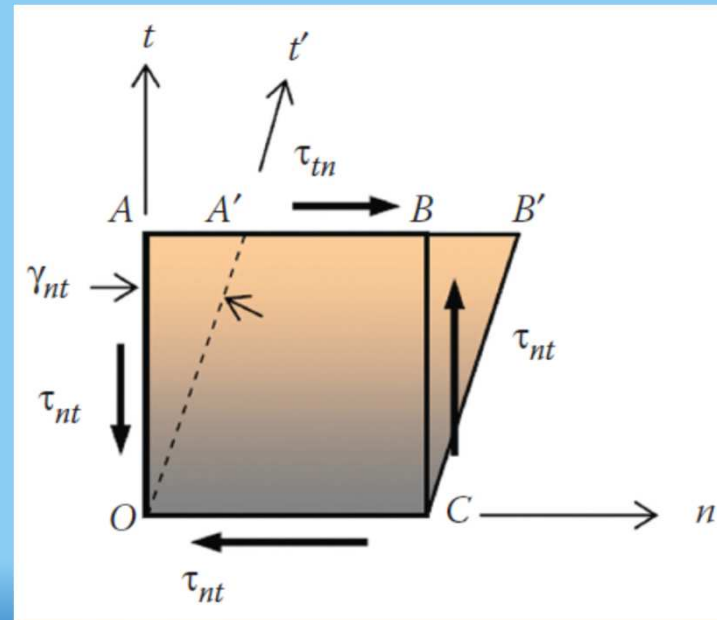
Lecture 8

Chapter 2. Torsional Loads

Shearing Strain

- Shear strain is a distortion – it represents a change in the initial shape of the body.
- Distortion can best be represented by the change in the angle between two initially perpendicular lines

$$\gamma_{nt} \approx \tan \gamma_{nt} = \frac{AA'}{OA}$$

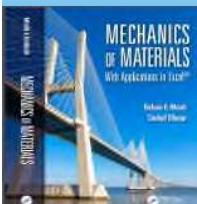
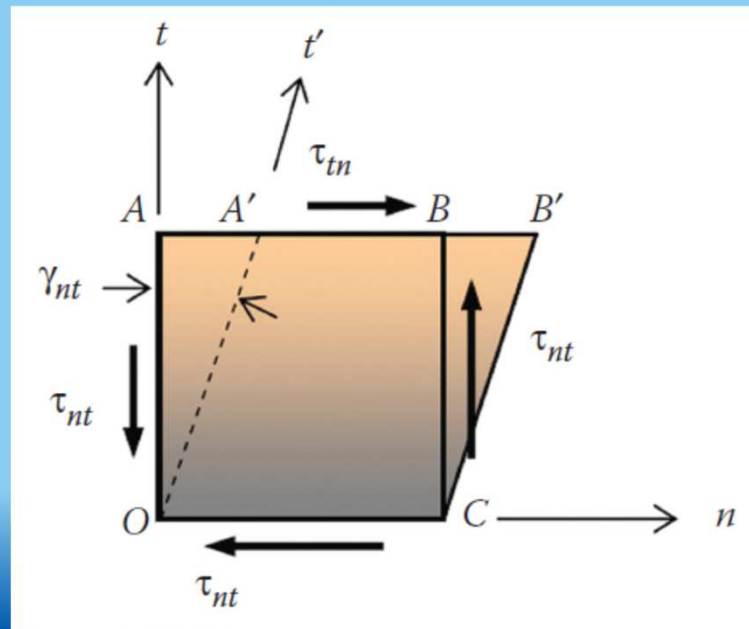


Lecture 8

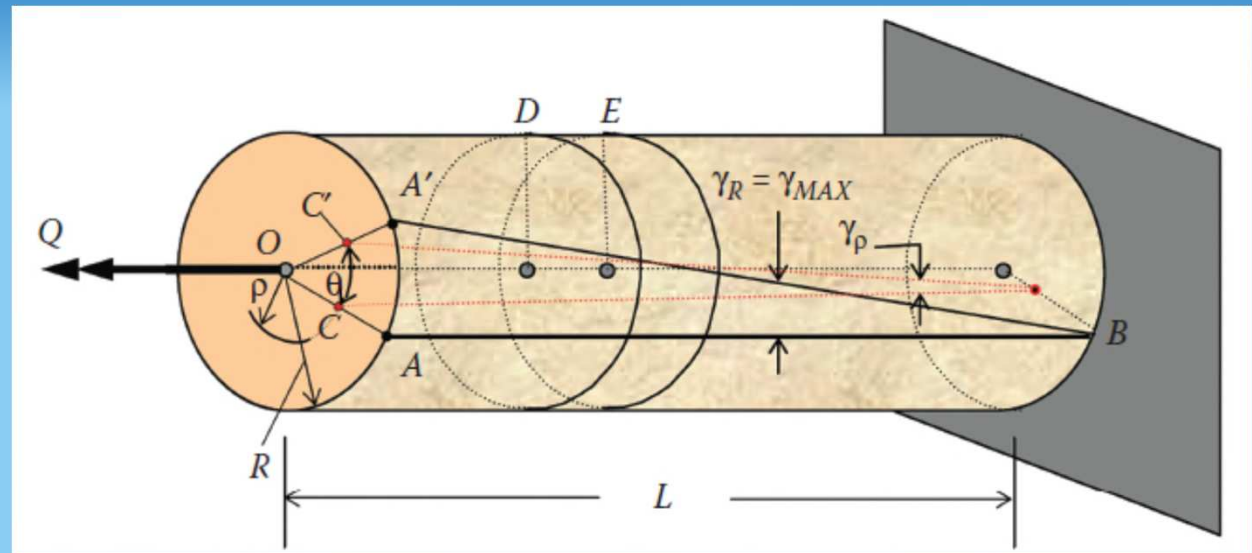
Chapter 2. Torsional Loads

Shearing Strain – Sign Convention

- A positive shearing strain represents a decrease in the 90° angle and a negative shearing strain represents an increase in this angle.
- The shearing strain shown below is positive since it represents a decrease in the 90° angle between OA and OC.

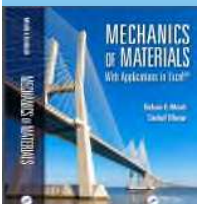


Shearing Stress and Shearing Deformation



Assumptions:

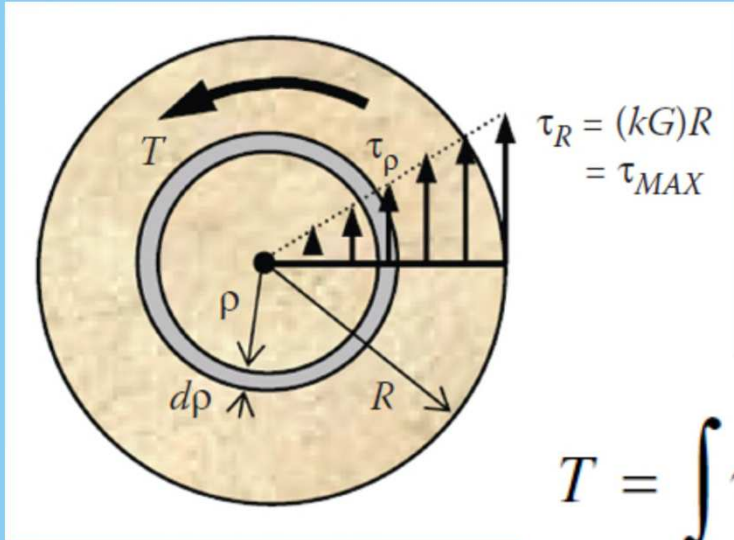
1. Cross sections such as those at D and E are plane prior to twisting and remain plane after twisting of the circular shaft.
2. Straight line elements on the surface of the shaft such as line AB are assumed to remain straight after twisting occurs. Point A moves to A' and line $A'B$ is assumed to be straight for small angles θ even though its true shape is helical.
3. The material is assumed to behave elastically in order to be able to use Hooke's law.



Lecture 8

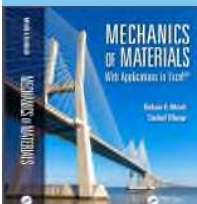
Chapter 2. Torsional Loads

Torque to Shearing Stress Relationship



$$T = \int \tau_\rho \rho dA = 2\pi \int_0^R \tau_\rho \rho^2 d\rho = 2\pi \int_0^R \left(\frac{\tau_\rho}{\rho} \right) \rho^3 d\rho$$

$$T = 2\pi \left(\frac{\tau_\rho}{\rho} \right) \int_0^R \rho^3 d\rho = \left(\frac{\tau_\rho}{\rho} \right) \left(\frac{\pi R^4}{2} \right) = \left(\frac{\tau_\rho}{\rho} \right) J$$



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Maximum Shearing Stress and Angle of Twist

- Maximum Shearing Stress:

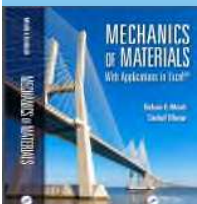
$$\tau_{MAX} = \tau_R = \frac{TR}{J}$$

- Angle of Twist:

$$\theta = \frac{TL}{JG}$$

For a shaft with multiple segments: $\theta = \sum \frac{TL}{JG}$

J= Polar Moment of Inertia of the Cross-Section



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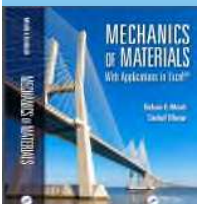
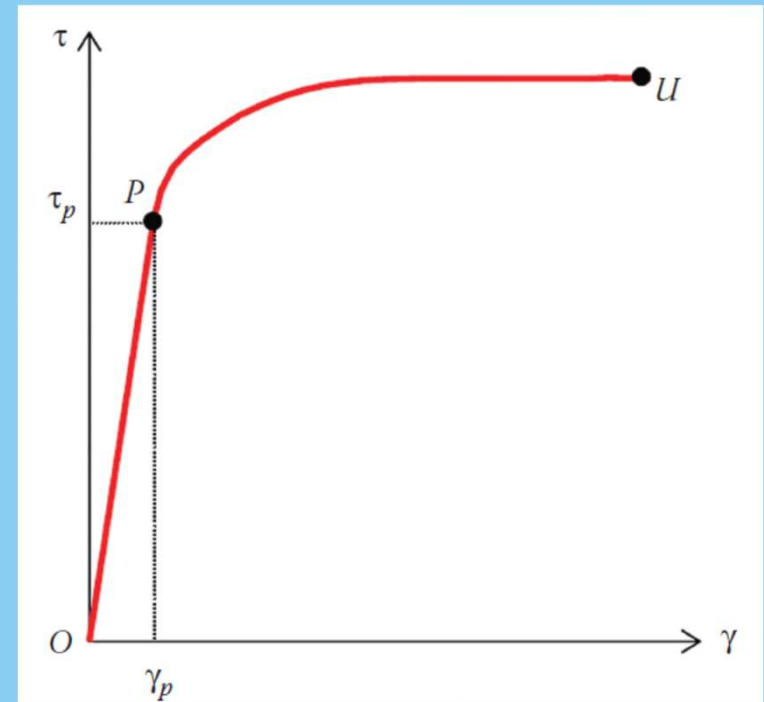
Material Properties in Shear

Shear Stress-Strain Diagram:

Hooke's Law applies in the linear-elastic region O-P:

$$\tau = G\gamma$$

G: Shear Modulus of Elasticity



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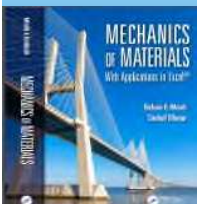
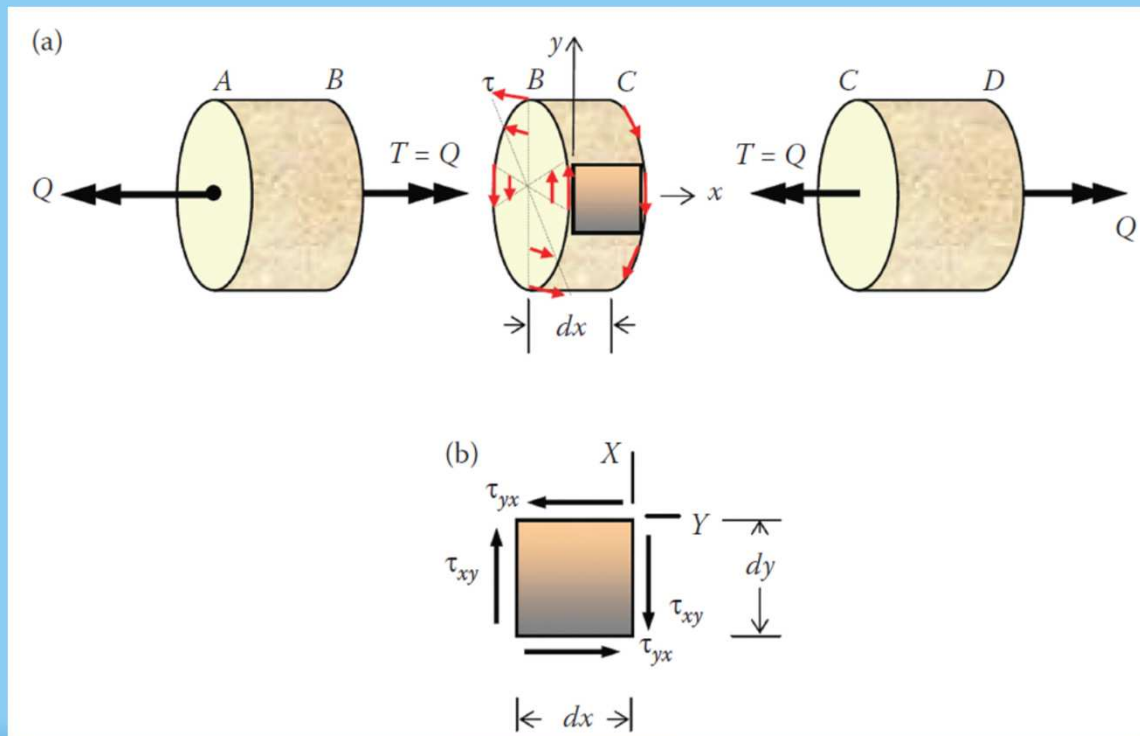
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Stress Element

- State of pure shear in a shaft subjected to a torque Q

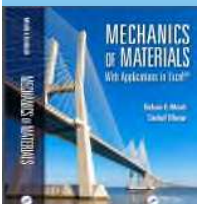
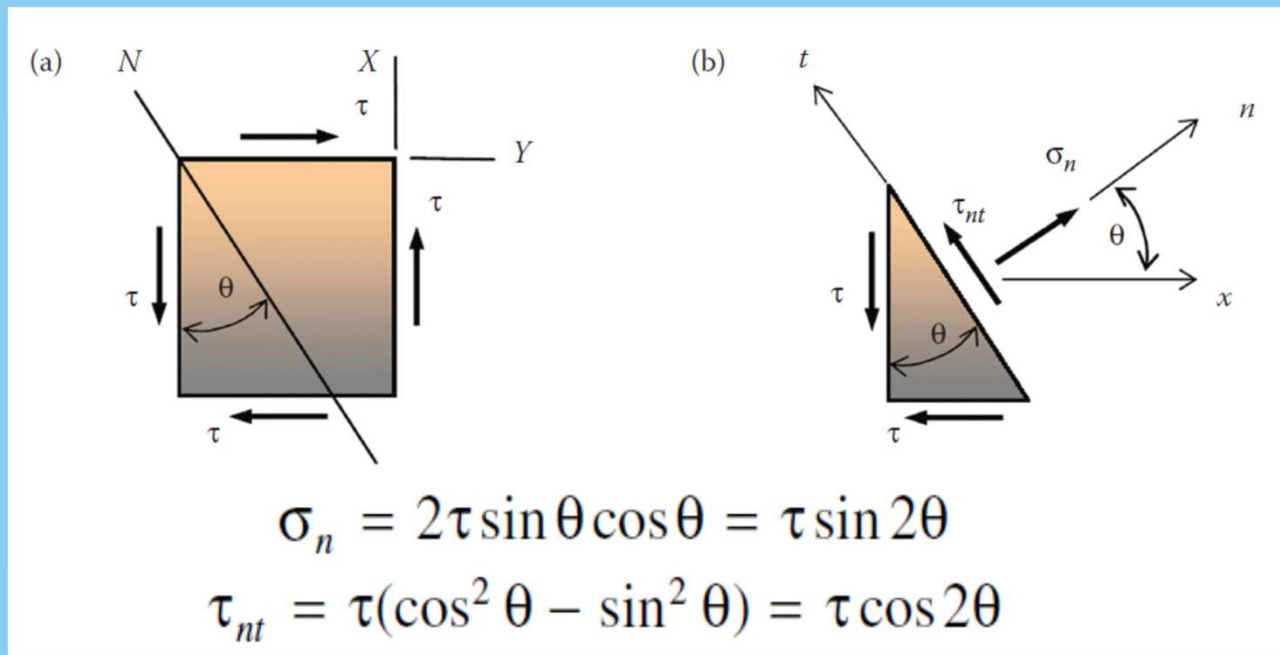


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Stresses on Inclined Planes

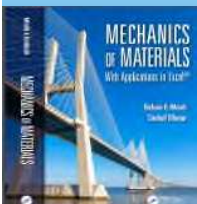
- From a state of pure shear to the determination of normal and shearing stresses on an inclined plane



Lecture 8

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Examples



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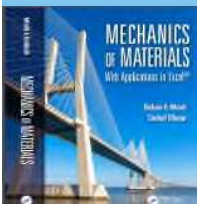
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MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL CHAPTER 2: TORSIONAL LOADS

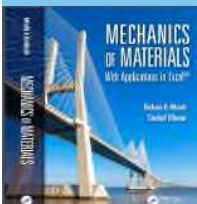
LECTURE 9

2.4 STATICALLY INDETERMINATE SHAFTS



Lecture Outline

- Definition
- Solution Procedure
- Examples



Lecture 9

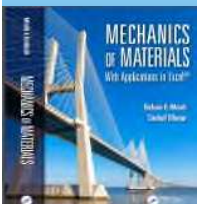
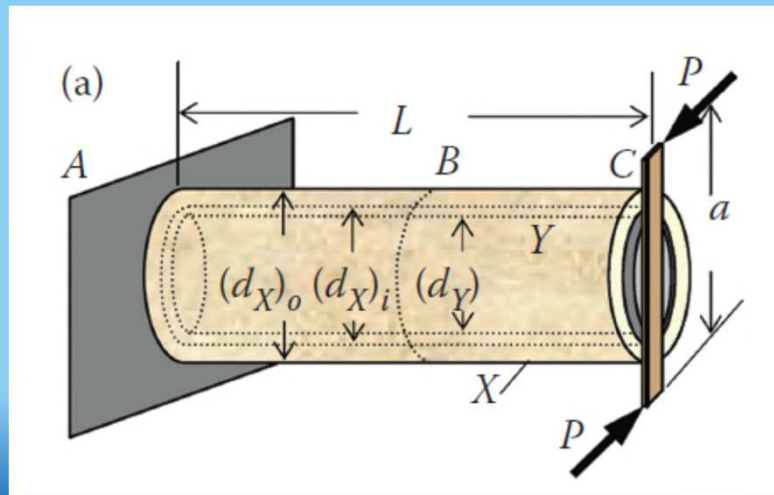
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Definition

- statically indeterminate shaft is one in which the number of unknown quantities exceeds the number of available equilibrium equations.
- Situation may happen when more than one material are involved or there are too many torsional reactions.

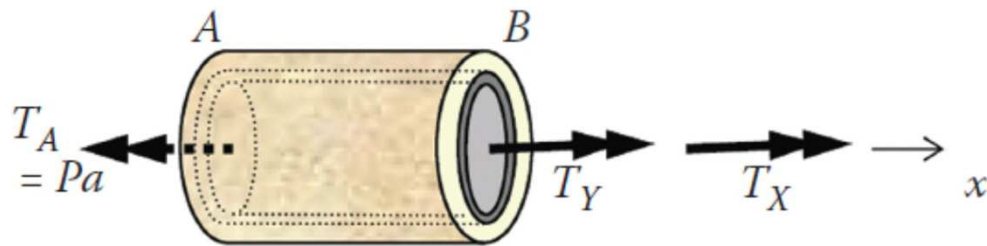


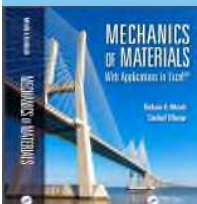
Lecture 9

Chapter 2. Torsional Loads

Solution Procedure

- Supplement the conditions of equilibrium with relations arising from the deformation characteristics of the shaft under consideration

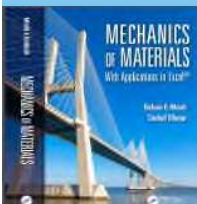
(b) 	$\sum T_x = 0 \Rightarrow T_X + T_Y - Pa = 0$ $\theta_X = \theta_Y \Rightarrow \left(\frac{TL}{JG} \right)_X = \left(\frac{TL}{JG} \right)_Y$
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Lecture 9

Chapter 2. Torsional Loads

Examples



Lecture 9

Chapter 2. Torsional Loads

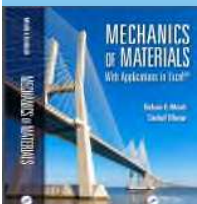
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MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL CHAPTER 2: TORSIONAL LOADS

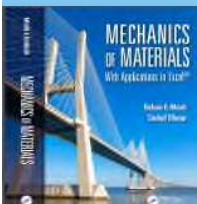
LECTURE 10

2.5 DESIGN OF POWER-TRANSMISSION SHAFTS



Lecture Outline

- Definitions
- Units of Power
- Examples



Lecture 10

Chapter 2. Torsional Loads

2

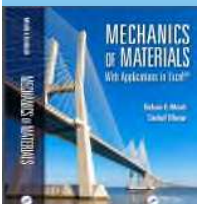
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Definitions

- Shafts are primarily used to transmit power from one location to another:

$$P = T\omega$$

- P: Transmitted power; i.e. the rate at which work is performed by the transmission shaft
- T: Internal torque in the transmission shaft
- ω : Angular speed



Lecture 10

Chapter 2. Torsional Loads

Units of Power

- SI System of Units:

$$\text{m}\cdot\text{N}/\text{s} = \text{joules}/\text{s} = \text{watt (W)}$$

or

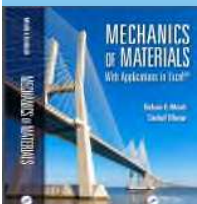
$$\text{m}\cdot\text{kN}/\text{s} = \text{kilowatt (kW)}$$

- US Customary System:

$$\text{in}\cdot\text{lb}/\text{s} \text{ or } \text{ft}\cdot\text{lb}/\text{s}$$

or

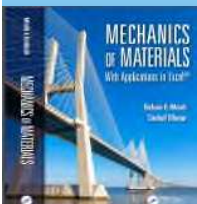
$$\text{horsepower (hp) where } 1 \text{ hp} = 550 \text{ ft}\cdot\text{lb}/\text{s} = 6600 \text{ in}\cdot\text{lb}/\text{s}$$



Lecture 10

Chapter 2. Torsional Loads

Examples



Lecture 10

Chapter 2. Torsional Loads

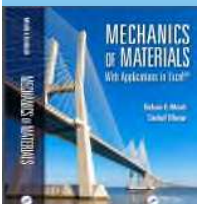
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MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL CHAPTER 2: TORSIONAL LOADS

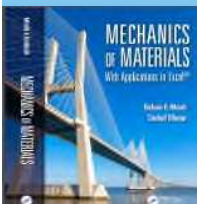
LECTURE 11

2.6 STRESSES UNDER COMBINED LOADS



Lecture Outline

- Superposition
- Hooke's Law in two dimensions
- Examples



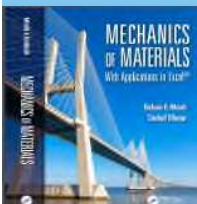
Lecture 11

Chapter 2. Torsional Loads

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Superposition

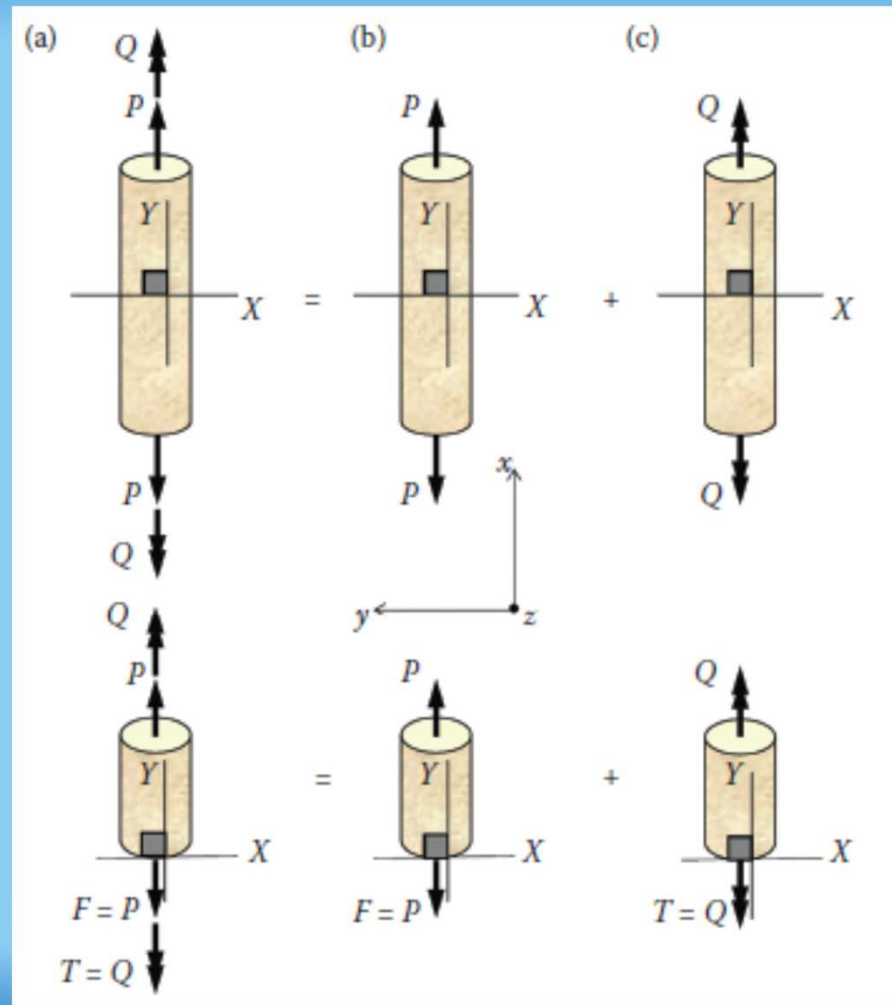
- Analysis of combined axial and torsional loads is most conveniently accomplished by using the principle of superposition.
- The method of superposition is equally applicable to any cross-sectional form as long as the material is not stressed beyond the elastic limit.



Lecture 11

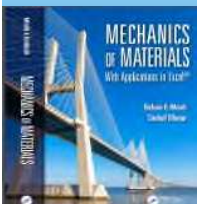
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Combined Axial Force and Torque



Applied Loads

Internal Loads



Lecture 11

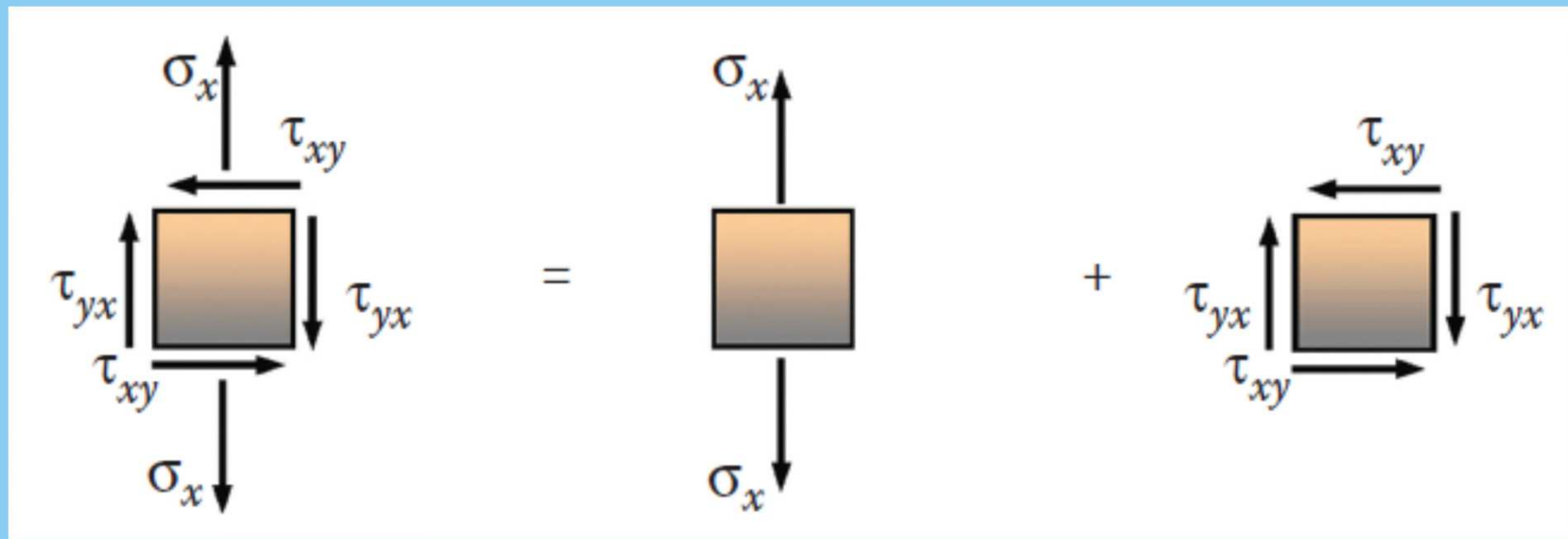
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Combined Axial Force and Torque - Stresses

From Both Loads

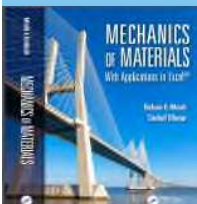
From Axial Load

From Torque



Normal Stress

Shear Stress



Lecture 11

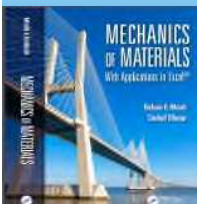
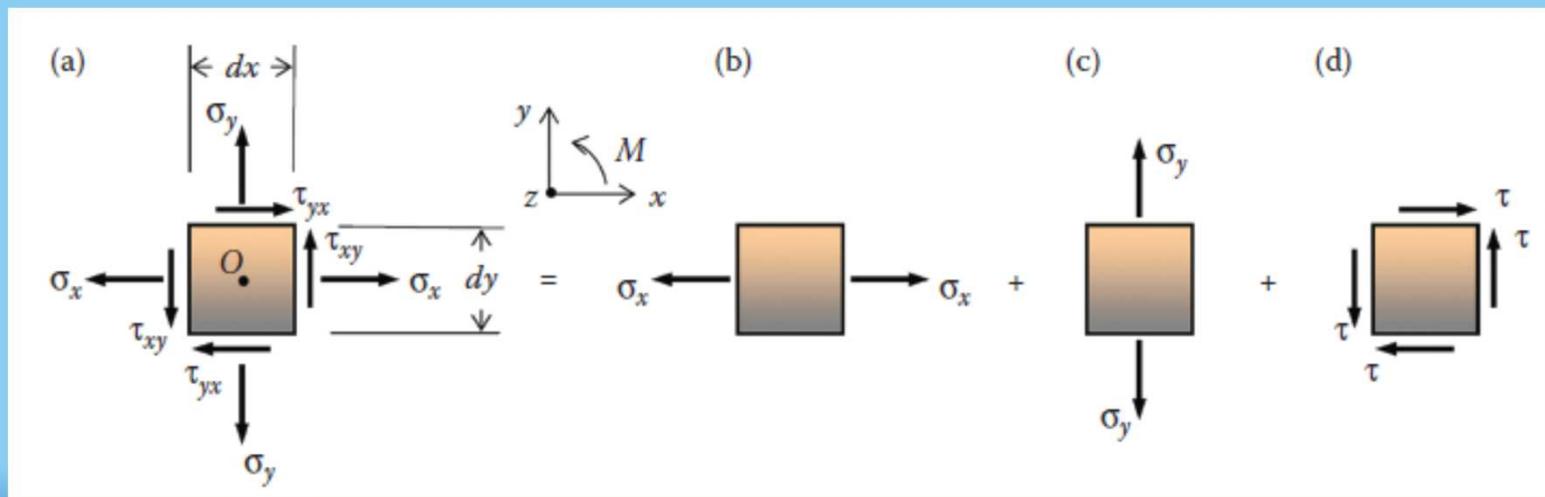
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Hooke's Law in Two Dimensions

- Condition is referred to as a state of plane stress
- Although stresses exist on one plane only (two-dimensions), strain components will be in three-dimensions because of Poisson's ratio



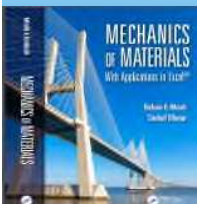
Lecture 11

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Hooke's Law in Two Dimensions

- Applying Hooke's Law and the definition of Poisson's ratio:

$$\left. \begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \mu \left(\frac{\sigma_y}{E} \right) = \frac{1}{E} (\sigma_x - \mu \sigma_y) \\ \varepsilon_y &= \frac{\sigma_y}{E} - \mu \left(\frac{\sigma_x}{E} \right) = \frac{1}{E} (\sigma_y - \mu \sigma_x) \\ \varepsilon_z &= -\frac{\mu}{E} (\sigma_x + \sigma_y); \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \end{aligned} \right\} \quad (2.22)$$



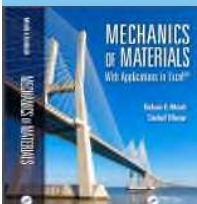
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Chapter 2. Torsional Loads

Hooke's Law in Two Dimensions

- Expressing stresses in terms of strains:

$$\left. \begin{aligned} \sigma_x &= \frac{E}{1-\mu^2}(\epsilon_x + \mu\epsilon_y) \\ \sigma_y &= \frac{E}{1-\mu^2}(\epsilon_y + \mu\epsilon_x) \\ \sigma_z &= 0 \end{aligned} \right\} \quad (2.23)$$



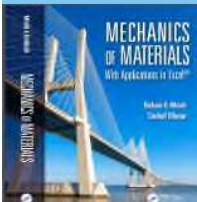
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Hooke's Law in Two Dimensions

- Relationship between the Modulus of Elasticity and the Shear Modulus:

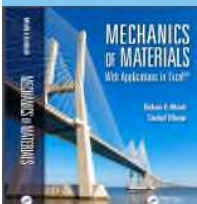
$$G = \frac{E}{2(1 + \mu)} \quad (2.24)$$



Lecture 11

Chapter 2. Torsional Loads

Examples



Lecture 11

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