

***Aircraft Performance An Engineering Approach 1st
edition Sadraey Solutions Manual***

SKU: 9781498776554-SOLUTIONS

Chapter 2. Equations of Motion

1. Determine the lift coefficient of an aircraft with a 180 ft^2 wing area and a mass of 3,200 kg in a cruising flight when flying at sea level with a speed of
- 80 knot
 - 130 knot.

$$S1 := 180 \cdot \text{ft}^2 \quad m1 := 3200 \cdot \text{kg} \quad g = 9.807 \text{m} \cdot \text{s}^{-2} \quad \rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3}$$

- 80 knot

$$V1 := 80 \cdot \text{knot} \quad V1 = 41.156 \frac{\text{m}}{\text{s}}$$

$$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 1.809 \quad (\text{Equ 2.4})$$

- 130 knot

$$V1 := 130 \cdot \text{knot} \quad V1 = 66.878 \frac{\text{m}}{\text{s}}$$

$$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 0.685 \quad (\text{Equ 2.4})$$

2. An aircraft with a mass of 1,200 kg and a wing area of 14 m² is cruising at 3,000 ft altitude. Determine its lift coefficient when the true air speed is 100 knot.

$$S1 := 14 \cdot \text{m}^2 \quad m1 := 1200 \cdot \text{kg} \quad g = 9.807 \frac{\text{m}}{\text{s}^2} \quad h := 3000 \cdot \text{ft}$$

$$\rho_3 := 0.002175 \cdot \frac{\text{slug}}{\text{ft}^3} \quad \rho_3 = 1.121 \frac{\text{kg}}{\text{m}^3}$$

$$V1 := 100 \cdot \text{knot} \quad V1 = 51.444 \frac{\text{m}}{\text{s}}$$

$$C_L := \frac{2 \cdot m1 \cdot g}{\rho_3 \cdot V1^2 \cdot S1} = 0.567 \quad (\text{Equ 2.4})$$

3. Assume that the aircraft in problem 2 has drag coefficient of 0.05. How much thrust the engine is producing?

$$S1 := 14 \text{m}^2 \quad m1 := 1200 \text{kg} \quad g = 9.807 \frac{\text{m}}{\text{s}^2} \quad h := 3000 \text{ft} \quad C_D := 0.05$$

$$\rho_3 := 0.002175 \frac{\text{slug}}{\text{ft}^3} \quad \rho_3 = 1.121 \frac{\text{kg}}{\text{m}^3}$$

$$V1 := 100 \text{knot} \quad V1 = 51.444 \frac{\text{m}}{\text{s}}$$

$$D := \frac{1}{2} \cdot \rho_3 \cdot V1^2 \cdot S1 \cdot C_D = 1038.3 \text{N} \quad (\text{Equ 2.5})$$

$$T1 := D = 1038.3 \text{N} \quad (\text{Equ 2.17})$$

4. Determine lift curve slope (in 1/rad) of a wing with aspect ratio of 12.5. Then, calculate the lift coefficient of this wing when its angle of attack is 5 degrees. Assume that the zero lift angle of attack is zero, and a_0 is 2π 1/rad.

$$AR := 12.5 \quad \alpha := 5 \cdot \text{deg} \quad a_0 := 2 \cdot \pi \quad \alpha_0 := 0$$

$$C_{L\alpha} := \frac{a_0}{1 + \frac{a_0}{\pi \cdot AR}} = 5.417 \quad (\text{Equ 2.13})$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_0)} \quad C_L := C_{L\alpha} \cdot (\alpha - \alpha_0) = 0.473 \quad (\text{Equ 2.12})$$

5. Calculate the true and equivalent stall speeds of the aircraft in problem 2, when the maximum lift coefficient is 1.6.

$$S1 := 14 \cdot \text{m}^2 \quad m1 := 1200 \cdot \text{kg} \quad g = 9.807 \cdot \frac{\text{m}}{\text{s}^2} \quad h := 3000 \cdot \text{ft} \quad C_{L\max} := 1.6$$

$$\rho_3 := 0.002175 \cdot \frac{\text{slug}}{\text{ft}^3} \quad \rho_3 = 1.121 \cdot \frac{\text{kg}}{\text{m}^3} \quad \rho_0 := 1.225 \cdot \frac{\text{kg}}{\text{m}^3}$$

$$V_s := \sqrt{\frac{2 \cdot m1 \cdot g}{(\rho_3 \cdot C_{L\max} \cdot S1)}} = 30.616 \frac{\text{m}}{\text{s}} \quad V_s = 59.513 \text{knot} \quad (\text{Equ 2.51}) \quad \text{True}$$

$$\sigma := \frac{\rho_3}{\rho_0} = 0.915 \quad (\text{Equ 1.25})$$

$$V_T = \frac{V_E}{\sqrt{\sigma}} \quad V_E := V_s \cdot \sqrt{\sigma} = 29.287 \frac{\text{m}}{\text{s}} \quad V_E = 56.929 \text{knot} \quad (\text{Equ 2.49}) \quad \text{Equivalent}$$

6. An aircraft is required to climb with a 10 degrees of climb angle. The aircraft has a mass of 30,000 kg and producing 50,000 N of drag. Assume zero angle of attack and zero thrust setting angle.

- a. How much lift this aircraft must generate?
b. How much thrust the aircraft engine must produce?

$$\gamma := 10 \text{ deg} \quad m_1 := 30000 \text{ kg} \quad D_1 := 50000 \text{ N} \quad \alpha := 0 \quad i_T := 0$$

$$W_1 := m_1 \cdot g = 294200 \text{ N}$$

$$L_1 := W_1 \cdot \cos(\gamma) = 289730 \text{ N} \quad (\text{Equ 2.33})$$

$$T_1 := D_1 + W_1 \cdot \sin(\gamma) = 101087 \text{ N} \quad (\text{Equ 2.32})$$

7. An aircraft which is initially at rest is accelerating on a runway with an acceleration of 10 m/sec². Consider a moment when other features of this aircraft are:

$$S = 30 \text{ m}^2, m = 6,000 \text{ kg}, C_L = 0.7, C_D = 0.1, V = 60 \text{ knot}$$

Calculate the engine thrust, assuming that the friction force is constant and equal to 2% of the aircraft weight.

$$S_1 := 30 \text{ m}^2 \quad m_1 := 6000 \text{ kg} \quad C_L := 0.7 \quad C_D := 0.1 \quad a_1 := 10 \frac{\text{m}}{\text{s}^2}$$

$$\rho := 1.225 \frac{\text{kg}}{\text{m}^3} \quad V_1 := 60 \text{ knot} \quad V_1 = 30.9 \frac{\text{m}}{\text{s}}$$

$$W_1 := m_1 \cdot g = 58839.9 \text{ N}$$

$$F_f := 0.02 \cdot W_1 = 1176.8 \text{ N}$$

$$D_1 := \frac{1}{2} \cdot \rho \cdot (V_1)^2 \cdot S_1 \cdot C_D = 1750.7 \text{ N} \quad (\text{Equ 2.5})$$

$$T_1 := m_1 \cdot a_1 + D_1 + F_f \quad (\text{Equ 2.34})$$

8. A cargo aircraft with a weight of 145,000 lb and wing area of 1,318 ft² has a maximum lift coefficient of 2.5. Is this aircraft able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS?

$$W1 := 145000 \text{ lbf} \quad S1 := 1318 \text{ ft}^2 \quad C_{L\max} := 2.5 \quad h := 25000 \text{ ft} \quad \text{ISA}+15$$

$$V1 := 150 \text{ knot} \quad L1 := 2 \cdot \frac{\text{K}}{1000 \text{ ft}} \quad \text{Chapter 1} \quad R1 := 287 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\text{Sea level:} \quad P_0 := 101325 \text{ Pa} \quad T_0 := (15 + 15 + 273) \cdot \text{K} = 303 \text{ K}$$

$$25000 \text{ ft:} \quad T_{25} := T_0 - L1 \cdot h = 253 \text{ K} \quad (\text{Equ 1.6})$$

$$\text{Appendix B:} \quad P_{25} := 786.3 \frac{\text{lbf}}{\text{ft}^2}$$

$$\rho_{25} := \frac{P_{25}}{R1 \cdot (T_{25})} = 0.518 \frac{\text{kg}}{\text{m}^3} \quad \rho_{25} = 0.00101 \frac{\text{slug}}{\text{ft}^3} \quad (\text{Equ 1.23})$$

$$V_{s15} := \sqrt{\frac{2 \cdot W1}{\rho_{25} \cdot S1 \cdot C_{L\max}}} = 90.153 \frac{\text{m}}{\text{s}} \quad V_{s15} = 175.243 \text{ knot} \quad (\text{Equ 2.51})$$

This aircraft is not able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS, since this speed is less than the stall speed at that altitude.

9. A hang glider (*Nimbus*) has a mass (structure plus pilot) of 138 kg and wing area of 16.2 m² and stall speed of 16 knot. What is the maximum lift coefficient?

$$S1 := 16.2 \cdot \text{m}^2 \quad m1 := 138 \cdot \text{kg} \quad g = 9.807 \cdot \frac{\text{m}}{\text{s}^2} \quad \rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3}$$

$$V_s := 16 \cdot \text{knot} \quad V_s = 8.231 \cdot \frac{\text{m}}{\text{s}}$$

$$C_{L\max} := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V_s^2 \cdot S1} = 2.013 \quad (\text{Equ 2.51})$$

10. Calculate the wing area of a hang glider *Volmer VJ-23*. The aircraft geometry and weight information may be taken from Table 2.2. If the pilot mass is 75 kg, what is the mass of the aircraft structure?

$$g = 9.807 \cdot \frac{\text{m}}{\text{s}^2} \quad \rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad m_p := 75 \cdot \text{kg}$$

$$\text{From Table 2.2, row 4:} \quad V_s := 13 \cdot \text{knot} \quad C_{L\max} := 2.9 \quad m1 := 136 \cdot \text{kg}$$

$$S1 := \frac{2 \cdot m1 \cdot g}{(\rho_o \cdot C_{L\max} \cdot V_s^2)} = 16.616 \text{m}^2 \quad (\text{Equ 2.51})$$

$$m_{st} := m1 - m_p = 61 \text{kg}$$

11. The sport aircraft *Butterworth* has the following characteristic

$$m = 635 \text{ kg}, S = 10.4 \text{ m}^2, V_s = 56 \text{ knot (at sea level)}$$

Assume that the maximum speed of this aircraft at every altitude is 126 knot (TAS). At what altitude maximum true airspeed and stall true airspeed will be the same?

$$m_1 := 635 \cdot \text{kg} \quad S_1 := 10.4 \cdot \text{m}^2 \quad V_s := 56 \cdot \text{knot} \quad \rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad V_{s_alt} := 126 \cdot \text{knot}$$

$$C_{Lmax} := \frac{(2 \cdot m_1 \cdot g)}{(\rho_o \cdot S_1 \cdot V_s^2)} = 1.178 \quad (\text{Equ 2.51})$$

Aircraft C_{Lmax} is constant.

$$\rho := \frac{2 \cdot m_1 \cdot g}{S_1 \cdot C_{Lmax} \cdot V_{s_alt}^2} = 0.242 \frac{\text{kg}}{\text{m}^3} \quad (\text{Equ 2.51})$$

From Appendix A, this air density belongs to an altitude of 13700 m or 45000 ft.

12. The cargo aircraft *C-130* has an empty mass of 13,000 kg, a wing area of 85 m², and a stall speed of 94 knot (EAS). If maximum lift coefficient is 2.2, determine maximum the mass of payload (cargo and crew) plus fuel to satisfy this stall speed.

$$m_E := 13000 \cdot \text{kg} \quad S_1 := 85 \cdot \text{m}^2 \quad V_s := 94 \cdot \text{knot} \quad \rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad C_{Lmax} := 2.2$$

$$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{Lmax} \quad (\text{Equ 2.51})$$

$$m_{TO} := \frac{\rho_o \cdot V_s^2 \cdot S_1 \cdot C_{Lmax}}{2 \cdot g} = 27312 \text{ kg}$$

$$m_{pf} := m_{TO} - m_E = 14312 \text{ kg}$$

- 13.** The bomber *B-1B* has a maximum take-off mass of 216,367 kg, a 181 m² wing area, and a maximum velocity of Mach 2.2. Assume drag coefficient of this aircraft at cruise is 0.03, how much thrust the four engines are generating for this flight condition?

$$m1 := 216367 \cdot \text{kg} \quad S1 := 181 \cdot \text{m}^2 \quad g = 9.807 \cdot \frac{\text{m}}{\text{s}^2} \quad C_D := 0.03 \quad M1 := 2.2$$

$$\rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad a := 340 \cdot \frac{\text{m}}{\text{s}} \quad \text{speed of sound at sea level}$$

$$V1 := M1 \cdot a = 748 \frac{\text{m}}{\text{s}} \quad (\text{Equ 1.35})$$

$$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1860840.4 \text{ N} \quad (\text{Equ 2.5})$$

$$T1 := D = 1860.8 \text{ kN} \quad (\text{Equ 2.17})$$

14. The trainer aircraft *PC-7* with a mass of 2,700 kg and a wing area of 16.6 m² has a cruising speed of 330 km/hr.

- What is the lift coefficient when cruising at 5,000 m altitude, ISA condition?
- In a summer day, the temperature at sea level is 42 °C. How much lift coefficient must be increased when cruising at this day and at the same altitude?

$$m_1 := 2700 \text{ kg} \quad S_1 := 16.6 \text{ m}^2 \quad h := 5000 \text{ m} \quad V_C := 330 \frac{\text{km}}{\text{hr}} \quad V_C = 178.186 \text{ knot} \quad V_C = 91.667 \frac{\text{m}}{\text{s}}$$

a 5000 m, ISA $\rho_5 := 0.7364 \frac{\text{kg}}{\text{m}^3}$

$$C_{Lc1} := \frac{2 \cdot m_1 \cdot g}{\rho_5 \cdot V_C^2 \cdot S_1} = 0.516 \quad (\text{Equ 2.4})$$

b Non-ISA

$$L_1 := 6.5 \frac{\text{K}}{1000 \text{ m}} \quad \text{Chapter 1} \quad R_1 := 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

Sea level: $P_0 := 101325 \text{ Pa} \quad T_0 := (42 + 273) \cdot \text{K} = 315 \text{ K}$

5000 m: $T_5 := T_0 - L_1 \cdot h = 282.5 \text{ K} \quad (\text{Equ 1.6})$

Appendix B: $P_5 := 54048 \text{ Pa}$

$$\rho_5 := \frac{P_5}{R_1 \cdot T_5} = 0.667 \frac{\text{kg}}{\text{m}^3} \quad (\text{Equ 1.23})$$

$$C_{Lc2} := \frac{2 \cdot m_1 \cdot g}{\rho_5 \cdot V_C^2 \cdot S_1} = 0.57 \quad (\text{Equ 2.4})$$

$$\Delta C_L := C_{Lc2} - C_{Lc1} = 0.054$$

$$\% \Delta C_L := \frac{C_{Lc2} - C_{Lc1}}{C_{Lc1}} = 10.5\%$$

- 15.** A maneuverable aircraft has a mass of 6,800 kg, a wing area of 32 m² and a drag coefficient of 0.02. The aircraft is required to climb vertically with the speed of 100 knot. How much thrust the engine needs to produce?

In a vertical climb, $T = D + W$

$$m1 := 6800 \cdot \text{kg} \quad S1 := 32 \cdot \text{m}^2 \quad C_D := 0.02 \quad V1 := 100 \cdot \text{knot} \quad V1 = 51.444 \frac{\text{m}}{\text{s}}$$

$$\rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad a := 340 \cdot \frac{\text{m}}{\text{s}} \quad \text{speed of sound at sea level} \quad g = 9.807 \cdot \frac{\text{m}}{\text{s}^2}$$

$$W1 := m1 \cdot g = 66685.2 \text{ N}$$

$$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1037.4 \text{ N} \quad (\text{Equ 2.5})$$

$$T1 := D + W1 = 67.7 \cdot \text{kN} \quad (\text{Equ 2.32})$$

- 16.** Calculate the wing area of the aircraft *EMB-121A1*. The aircraft geometry and weight data may be taken from Table 2.2.

$$\text{From Table 2.2, row 6:} \quad V_s := 76 \cdot \text{knot} \quad C_{L_{\max}} := 2.16 \quad m1 := 5670 \cdot \text{kg}$$

$$\rho_o := 1.225 \cdot \frac{\text{kg}}{\text{m}^3} \quad g = 9.807 \cdot \frac{\text{m}}{\text{s}^2}$$

$$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{L_{\max}} \quad (\text{Equ 2.51})$$

$$S1 := \frac{2 \cdot m1 \cdot g}{(\rho_o \cdot C_{L_{\max}} \cdot V_s^2)} = 27.49 \text{ m}^2 \quad (\text{Equ 2.51})$$

17. Is fighter aircraft *F-14* able to fly vertically? The aircraft data may be taken from table 2.2. Assume drag coefficient to be 0.03.

From Table 2.2, row 21:

$$m_1 := 33724 \cdot \text{kg} \quad C_{L_{\max}} := 2.9 \quad V_s := 115 \cdot \text{knot} \quad V_s = 59.161 \frac{\text{m}}{\text{s}} \quad T_{\max} := 2 \cdot 93 \cdot \text{kN}$$

$$W_1 := m_1 \cdot g = 330.7 \text{ kN}$$

$$T_{\max} = 186 \text{ kN}$$

The maximum engine thrust is less than aircraft weight, so, it is unable, even with the lowest possible speed.

18. The dynamic pressure of an aircraft that is cruising at an altitude is 9,000 N/m².

- Determine the altitude, if the aircraft speed is 389 KTAS.
- Calculate aircraft equivalent airspeed in terms of KEAS.

- Altitude

$$q := 9000 \frac{\text{N}}{\text{m}^2} \quad V_T := 389 \text{ knot} \quad V_T = 200.119 \frac{\text{m}}{\text{s}} \quad \rho_o := 1.225 \frac{\text{kg}}{\text{m}^3}$$

$$q = \frac{1}{2} \cdot \rho \cdot V^2 \quad \rho := \frac{2 \cdot q}{V_T^2} = 0.449 \frac{\text{kg}}{\text{m}^3} \quad (\text{Equ 2.47})$$

The altitude for this air density is 9,300 m or 30,500 ft.

- Equivalent airspeed in terms of KEAS

$$V_E := V_T \cdot \sqrt{\frac{\rho}{\rho_o}} = 235.63 \text{ knot} \quad (\text{Equ 2.49})$$

19. A transport aircraft is cruising at 20,000 ft altitude with a speed of Mach 0.5. If a 50 m/sec head-wind is blowing, what is the ground speed and true airspeed in terms of knot?

- **Ground speed**

$$h := 20000\text{ft} \quad M1 := 0.5 \quad V_W := 50 \frac{\text{m}}{\text{s}} \quad V_W = 97.192\text{knot} \quad R1 := 287 \frac{\text{J}}{\text{kg} \cdot \text{K}} \quad \gamma := 1.4$$

From Appendix B: $T1 := 447.4\text{R} \quad T1 = 248.55\text{K}$

$$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 316.022 \frac{\text{m}}{\text{s}} \quad (\text{Equ 1.34})$$

$$V1 := M1 \cdot a1 \quad V1 = 158.011 \frac{\text{m}}{\text{s}} \quad (\text{Equ 1.35})$$

$$V_G := V1 - V_W = 108.011 \frac{\text{m}}{\text{s}} \quad V_G = 210\text{knot}$$

- **True airspeed**

$$V_T := V1 = 307.148\text{knot}$$

20. The aircraft *Voyager* is able to fly around the globe without refueling. In one mission the aircraft is flying at the equator at an altitude of 15,000 ft with the speed of 110 knot. Assume there is a 15 m/sec wind from West to East all the time.

a. How many days does it take to do this mission if cruising from West to East?

b. How many days does it take to do this mission if cruising from East to West?

Note: The Earth has a diameter of 12,800 km.

- From West to East

$$h := 15000\text{ft} \quad h = 4572\text{m} \quad V_T := 110\text{knot} \quad V_T = 56.6 \frac{\text{m}}{\text{s}} \quad V_w := 15 \frac{\text{m}}{\text{sec}} \quad D_E := 12800\text{km}$$

$$\text{Total Distance traveled; Circumference:} \quad X := \pi \cdot (D_E + 2 \cdot h) = 40241.1\text{km}$$

$$V_{G1} := V_T + V_w = 71.589 \frac{\text{m}}{\text{s}} \quad V_{G1} = 139.158\text{knot}$$

$$V = \frac{X}{t} \quad t1 := \frac{X}{V_{G1}} = 6.506\text{day} \quad t1 = 5.621 \times 10^5 \text{ s} \quad t1 = 156.1\text{hr}$$

- From East to West

$$V_{G2} := V_T - V_w = 41.589 \frac{\text{m}}{\text{s}} \quad V_{G2} = 80.842\text{knot}$$

$$V = \frac{X}{t} \quad t2 := \frac{X}{V_{G2}} = 11.199\text{day} \quad t2 = 9.676 \times 10^5 \text{ s} \quad t2 = 268.8\text{hr}$$

21. The aircraft *Cessna Citation II* is climbing with a 3 degrees of angle of attack. The geometry and weight data of this aircraft may be taken from Table 2.2.

- If the drag coefficient is 0.035, determine its climb angle, when climbing with a speed of 160 knot.
- Determine the ratio between lift to weight at this climbing flight.

- Climb angle

From Table 2.2, row 18: $m_3 := 6033 \text{ kg}$ $V_s := 82 \text{ knot}$ $C_{L_{\max}} := 1.73$ $T_3 := 2 \cdot 11.1 \text{ kN}$

$V_{\text{climb}} := 160 \text{ knot}$ $C_D := 0.035$ $\alpha := 3 \text{ deg}$ $\rho_o := 1.225 \frac{\text{kg}}{\text{m}^3}$

$W_3 := m_3 g = 59163.5 \text{ N}$

$$S_3 := \frac{2 \cdot W_3}{\rho_o \cdot V_s^2 \cdot C_{L_{\max}}} = 31.376 \text{ m}^2 \quad (\text{Equ 2.51})$$

$$D_3 := \frac{1}{2} \cdot \rho_o \cdot V_{\text{climb}}^2 \cdot S_3 \cdot C_D = 4557.1 \text{ N} \quad (\text{Equ 2.5})$$

$$T \cdot \cos(\alpha) = D + W \cdot \sin(\gamma) \quad (\text{Equ 2.32})$$

$$\gamma := \arcsin\left(\frac{T_3 \cos(\alpha) - D_3}{W_3}\right) = 0.302 \text{ rad} \quad \gamma = 17.319 \text{ deg}$$

- Ratio between lift to weight

$$L = W \cdot \cos(\gamma) - T \cdot \sin(\alpha) \quad (\text{Equ 2.33})$$

$$L_3 := W_3 \cdot \cos(\gamma) - T_3 \cdot \sin(\alpha) = 55319.3 \text{ N}$$

$$\frac{L_3}{W_3} = 0.935$$

- 22.** A transport aircraft with a wing area of 200 m² is cruising with a speed of Mach 0.6 at 35,000 ft altitude, ISA condition.
- determine the mass of aircraft, if lift coefficient is 0.24.
 - determine the engine thrust, if drag coefficient is 0.035.

- Mass of aircraft

$$S1 := 200 \text{ m}^2 \quad M1 := 0.1 \quad h := 35000 \text{ ft} \quad C_L := 0.24 \quad R1 := 287 \frac{\text{J}}{\text{kg} \cdot \text{K}} \quad \gamma := 1.4 \quad C_D := 0.035$$

$$g = 9.807 \frac{\text{m}}{\text{s}^2}$$

$$\text{From Appendix B: } T1 := 394.1 \text{ R} \quad T1 = 218.94 \text{ K} \quad \rho_{35} := 0.000738 \frac{\text{slug}}{\text{ft}^3} \quad \rho_{35} = 0.38 \frac{\text{kg}}{\text{m}^3}$$

$$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 296.601 \frac{\text{m}}{\text{s}} \quad (\text{Equ 1.34})$$

$$V1 := M1 \cdot a1 \quad V1 = 177.96 \frac{\text{m}}{\text{s}} \quad V1 = 345.927 \text{ knot} \quad (\text{Equ 1.35})$$

$$m1 := \frac{\rho_{35} V1^2 \cdot S1}{2 \cdot g} = 122831.2 \text{ kg} \quad (\text{Equ 2.4})$$

- Engine thrust

$$D1 := \frac{1}{2} \cdot \rho_{35} V1^2 \cdot S1 \cdot C_D = 42159.7 \text{ N} \quad (\text{Equ 2.5})$$

$$T1 := D1 = 4.216 \times 10^4 \text{ N} \quad (\text{Equ 2.17})$$

- 23.** A transport aircraft with a wing area of 420 m^2 is cruising with a constant speed of 550 knot (KTAS) at 38,000 ft altitude. The aircraft has a mass of 390,000 kg at the beginning of a cruising flight and consumes 150,000 kg of fuel at the end of the cruise. Determine wing angle of attack at the beginning and at the end of the cruise. Also assume:

$$AR = 8.5; a_o = 2\pi \text{ 1/rad}; \alpha_o = -1 \text{ deg};$$

- **Wing angle of attack at the beginning of the cruise**

$$S1 := 420 \text{ m}^2 \quad V1 := 550 \text{ knot} \quad V1 = 282.944 \frac{\text{m}}{\text{s}} \quad h := 38000 \text{ ft} \quad m1 := 390000 \text{ kg}$$

$$m_f := 150000 \text{ kg} \quad AR := 8.5 \quad a_o := 2\pi \cdot \frac{1}{\text{rad}} \quad \alpha_o := -1 \cdot \text{deg}$$

$$\text{Appendix B:} \quad \rho_{38} := 6.462910^{-4} \cdot \frac{\text{slug}}{\text{ft}^3} \quad \rho_{38} = 0.333 \frac{\text{kg}}{\text{m}^3}$$

$$C_{L1} := \frac{2 \cdot m1 \cdot g}{\rho_{38} (V1)^2 \cdot S1} = 0.683 \quad (\text{Equ 2.4})$$

$$C_{L\alpha w} := \frac{a_o}{1 + \frac{a_o}{\pi \cdot AR}} = 5.086 \quad (\text{Equ 2.13})$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_o)} \quad \alpha_1 := \frac{C_{L1}}{C_{L\alpha w}} + \alpha_o = 0.117 \text{ rad} \quad \alpha_1 = 6.693 \text{ deg} \quad (\text{Equ 2.12})$$

- **Wing angle of attack at the end of the cruise**

$$\text{b.} \quad C_{L2} := \frac{2 \cdot [(m1 - m_f)g]}{\rho_{38} (V1)^2 \cdot S1} = 0.42$$

$$\alpha_2 := \frac{C_{L2}}{C_{L\alpha w}} + \alpha_o = 0.065 \text{ rad} \quad \alpha_2 = 3.734 \text{ deg}$$

24. The aircraft *Falco 900* is going to take-off from a runway in a winter day (ISA-20). It starts from rest and after a few seconds, when speed reaches $0.5 V_s$, friction force is 1% of aircraft weight, and drag coefficient is 0.1.

- Determine aircraft acceleration for this moment.
- How long does it take to come to this point? Assume the acceleration is constant during this period.

Note: Aircraft geometry and weight data may be taken from Table 2.2.

- Acceleration

From Table 2.2, row 20: $m_1 := 20640 \text{ kg}$ $V_s := 82 \text{ knot}$ $C_{L_{\max}} := 2.25$ $T_h := 3 \cdot 20 \text{ kN}$

$F_f := 0.01 \cdot W_1$ $C_D := 0.1$ $V_1 := 0.5 \cdot V_s$ $g = 9.807 \frac{\text{m}}{\text{s}^2}$ $R_1 := 287 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$ $\rho_o := 1.225 \frac{\text{kg}}{\text{m}^3}$

From Appendix B: $P_1 := 101325 \text{ Pa}$

$$T_1 := (15 - 20 + 273) \cdot \text{K} \quad (\text{Equ 2.5})$$

$$W_1 := m_1 \cdot g = 202409.5 \text{ N}$$

$$\rho := \frac{P_1}{R_1 \cdot (T_1)} = 1.317 \frac{\text{kg}}{\text{m}^3} \quad (\text{Equ 1.23})$$

$$S_1 := \frac{2 \cdot W_1}{\rho_o \cdot V_s^2 \cdot C_{L_{\max}}} = 82.535 \text{ m}^2 \quad (\text{Equ 2.51})$$

$$D_1 := \frac{1}{2} \cdot \rho \cdot (0.5 V_s)^2 \cdot S_1 \cdot C_D = 2418.5 \text{ N} \quad (\text{Equ 2.5})$$

$$F_f := 0.01 \cdot W_1 = 2024.1 \text{ N}$$

$$a_{TO} := \frac{T_h - D_1 - F_f}{m_1} = 2.692 \frac{\text{m}}{\text{s}^2} \quad (\text{Equ 2.34})$$

- Duration

$$V_{TO} := 0.5 V_s = 21.092 \frac{\text{m}}{\text{s}}$$

$$\text{Time} := \frac{V_{TO}}{a_{TO}} = 7.836 \text{ s}$$

25. Repeat problem 24, assuming the aircraft is taking off in a summer day (ISA+20).

- **Acceleration**

From Table 2.2, row 20: $m1 := 20640 \text{ kg}$ $V_s := 82 \text{ knot}$ $C_{L\max} := 2.25$ $Th := 3 \cdot 20 \text{ kN}$

$F_f = 0.01 \cdot W1$ $C_D := 0.1$ $V1 := 0.5 \cdot V_s$ $g = 9.807 \frac{\text{m}}{\text{s}^2}$ $R1 := 287 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$ $\rho_o := 1.225 \frac{\text{kg}}{\text{m}^3}$

From Appendix B: $P1 := 101325 \text{ Pa}$

$$T1 := (15 + 20 + 273) \cdot \text{K} = 308 \text{ K} \quad (\text{Equ 2.5})$$

$$W1 := m1 \cdot g = 202409.3 \text{ N}$$

$$\rho := \frac{P1}{R1 \cdot (T1)} = 1.146 \frac{\text{kg}}{\text{m}^3} \quad (\text{Equ 1.23})$$

$$S1 := \frac{2 \cdot W1}{\rho_o \cdot V_s^2 \cdot C_{L\max}} = 82.535 \text{ m}^2 \quad (\text{Equ 2.51})$$

$$D1 := \frac{1}{2} \cdot \rho \cdot (0.5 V_s)^2 \cdot S1 \cdot C_D = 2104.4 \text{ N} \quad (\text{Equ 2.5})$$

$$F_f := 0.01 \cdot W1 = 2024.1 \text{ N}$$

$$a_{TO} := \frac{Th - D1 - F_f}{m1} = 2.707 \frac{\text{m}}{\text{s}^2} \quad (\text{Equ 2.34})$$

- **Duration**

$$V_{TO} := 0.5 V_s = 21.092 \frac{\text{m}}{\text{s}}$$

$$\text{Time} := \frac{V_{TO}}{a_{TO}} = 7.792 \text{ s}$$

- 26.** An aircraft that is initially at rest is accelerating on a runway for a take-off operation. When the aircraft speed is 35 KTAS, the acceleration is 10 m/sec². Other features of this aircraft at this time are:

$$S = 35 \text{ m}^2, m = 6400 \text{ kg}, C_L = 0.8, C_D = 0.037.$$

If the friction coefficient is 0.02, calculate the engine thrust. Assume sea-level ISA condition.

$$S1 := 35 \text{ m}^2 \quad m1 := 6400 \text{ kg} \quad C_L := 0.8 \quad C_D := 0.037 \quad V_{TO} := 35 \text{ knot} \quad V_{TO} = 18 \frac{\text{m}}{\text{s}}$$

$$a := 10 \frac{\text{m}}{\text{s}^2} \quad \mu := 0.02 \quad g = 9.807 \frac{\text{m}}{\text{s}^2} \quad \rho_o := 1.225 \frac{\text{kg}}{\text{m}^3}$$

$$W1 := m1 \cdot g = 62762.6 \text{ N}$$

$$L1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_L = 5560 \text{ N} \quad (\text{Equ 2.4})$$

$$D1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_D = 257.2 \text{ N} \quad (\text{Equ 2.5})$$

$$\text{Normal force:} \quad N1 := W1 - L1 = 57202.6 \text{ N}$$

$$F_f := \mu \cdot N1 = 1144.1 \text{ N}$$

$$T1 := m1 \cdot a + D1 + F_f = 65.4 \text{ kN} \quad (\text{Equ 2.34})$$

27. The following aircraft (Figure 2.24) is descending with a constant airspeed and a descent angle of 20 degrees. Aircraft has an angle of attack of 5 degrees, and engines have 3 degrees of setting angle. Draw forces (weight, aerodynamic forces, and engine thrust) on the aircraft and derive the governing equations for this flight phase.

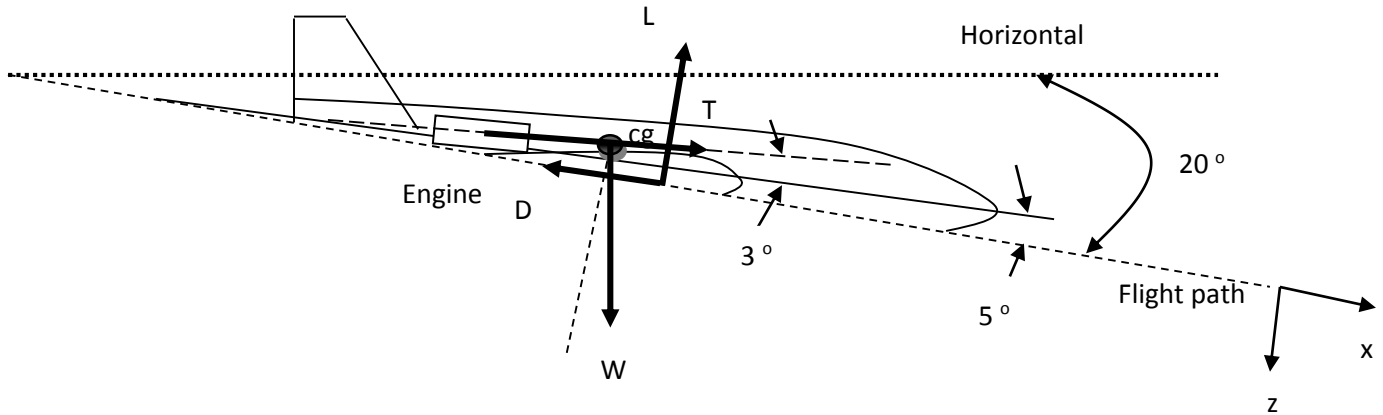


Figure 2.1. An aircraft is descending flight

Governing equations:

$$\sum F_x = 0 \Rightarrow T \cdot \cos(i_t + \alpha) + W \cdot \sin(\gamma) = D$$

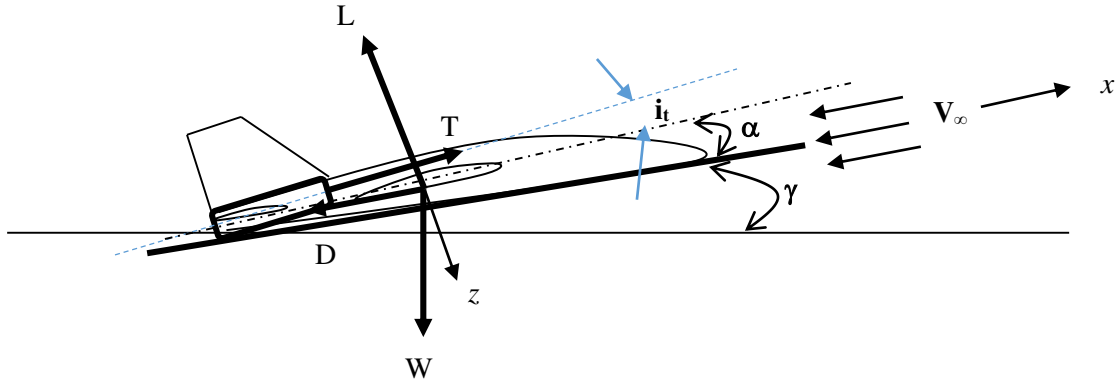
$$\sum F_z = 0 \Rightarrow L + T \cdot \sin(i_t + \alpha) = W \cdot \cos(\gamma)$$

Thus:

$$\sum F_x = 0 \Rightarrow T \cdot \cos(3 + 5) + W \cdot \sin(20) = D$$

$$\sum F_z = 0 \Rightarrow L + T \cdot \sin(3 + 5) = W \cdot \cos(20)$$

28. A fighter aircraft is climbing with an arbitrary climb angle. The aircraft has two turbofan engines; both engines have a positive i_t degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.

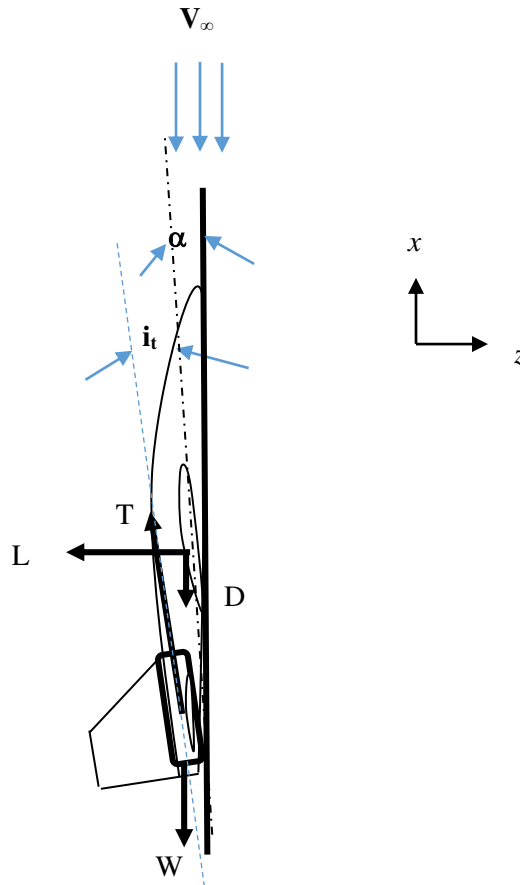


Governing equations:

$$\sum F_x = 0 \Rightarrow T \cdot \cos(i_t + \alpha) = D + W \cdot \sin(\gamma)$$

$$\sum F_z = 0 \Rightarrow L + T \cdot \sin(i_t + \alpha) = W \cdot \cos(\gamma)$$

29. A non-VTOL fighter aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; both engines have a positive i , degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.

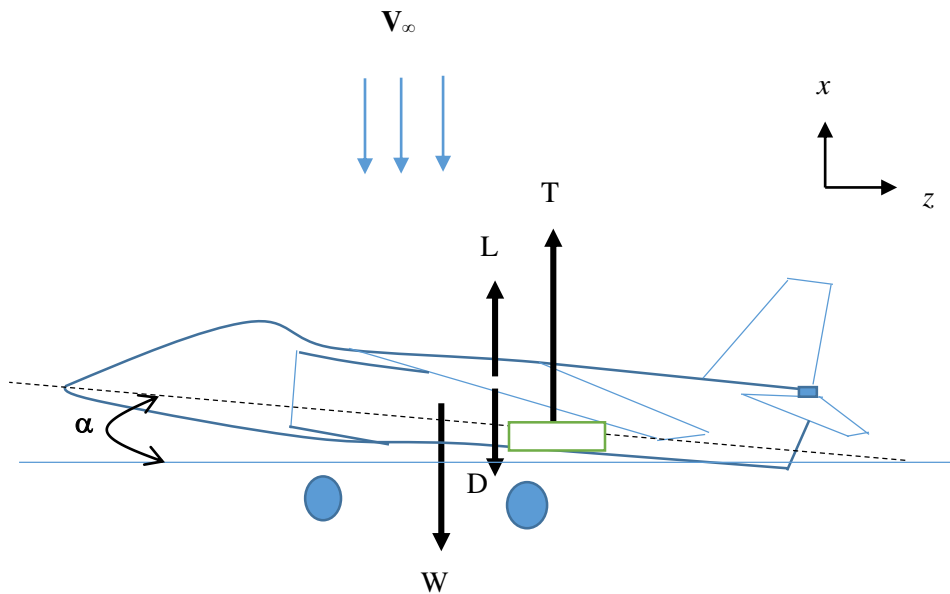


Governing equations:

$$\sum F_x = 0 \Rightarrow T \cdot \cos(i_t + \alpha) = D + W$$

$$\sum F_z = 0 \Rightarrow L + T \cdot \sin(i_t + \alpha) = 0$$

30. A VTOL aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; where during take-off they are arranged such that they produce a thrust which is upward. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.



Governing equations:

$$\sum F_x = 0 \Rightarrow T + L = D + W$$

$$\sum F_z = 0$$

- 31.** A pilot is planning to fly east to west at 5,000 m altitude such that he can watch the sunset for a couple of hours. What must be the flight speed (in Mach number) in order to achieve such objective? Assume there is a 30 knot headwind during this flight. The radius of Earth at sea level is 6400 km.

If the aircraft can fly with the speed of air (i.e., earth) at 5,000 m, the pilot can watch the sunset as long as he/she has fuel to fly.

$$h := 5000\text{m} \quad V_W := 30\text{-knot} \quad V_W = 15.433 \frac{\text{m}}{\text{s}} \quad R_E := 6400\text{km} \quad \gamma := 1.4 \quad R1 := 287 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

The Earth is revolving around itself in each 24 hours.

$$t := 24\text{hr} \quad t = 8.64 \times 10^4 \text{s}$$

If the aircraft is flying at sea level, the aircraft velocity should be the same as the earth linear speed at the sea level. For the 5,000 m altitude:

$$\text{Total Distance traveled; Circumference:} \quad X := 2\pi \cdot (R_E + h) = 40243.8\text{km}$$

$$V_A := \frac{X}{t} = 465.785 \frac{\text{m}}{\text{s}}$$

$$V_G := V_A + V_W = 481.218 \frac{\text{m}}{\text{s}}$$

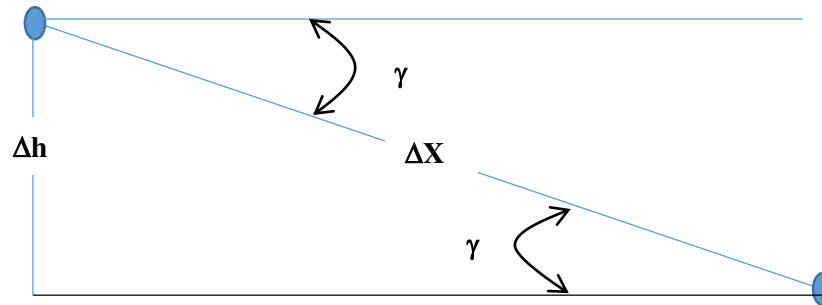
$$\text{From Appendix A:} \quad T_5 := 255.69\text{K}$$

$$a1 := \sqrt{\gamma \cdot R1 \cdot T_5} = 320.525 \frac{\text{m}}{\text{s}} \quad (\text{Equ 1.34})$$

$$M1 := \frac{V_G}{a1} = 1.501 \quad (\text{Equ 1.35})$$

32. A transport aircraft Boeing 777 (Figure 7.21) is descending with a velocity of 318 mph at 8,200 ft.

- If at After one minute, the altitude is 4,000 ft, and the airspeed is 234 mph, determine the average descend angle.
- If at the touchdown (sea level altitude), the airspeed is 180 mph, determine the average descend angle and deceleration. This phase takes one minute.



- a.....

$$V1 := 318 \frac{\text{mile}}{\text{hr}} \quad V2 := 234 \frac{\text{mile}}{\text{hr}} \quad h1 := 8200 \text{ft} \quad h2 := 4000 \text{ft} \quad t := 1 \cdot \text{min}$$

$$\Delta h := h1 - h2 = 4200 \text{ft} \quad V_{\text{avg}} := \frac{V1 + V2}{2} = 276 \frac{\text{mile}}{\text{hr}}$$

$$\text{Distance traveled} \quad \Delta X := V_{\text{avg}} \cdot t = 24288 \text{ft} \quad \Delta X = 7403 \text{m}$$

$$\sin(\gamma) = \frac{\Delta h}{\Delta X} \quad \gamma_{\text{avg}} := \text{asin}\left(\frac{\Delta h}{\Delta X}\right) \quad \gamma_{\text{avg}} = 9.958 \text{deg}$$

- b.....

$$V1 := 234 \frac{\text{mile}}{\text{hr}} \quad V2 := 180 \frac{\text{mile}}{\text{hr}} \quad h1 := 8200 \text{ft} \quad h2 := 4000 \text{ft} \quad t := 1 \cdot \text{min}$$

$$\Delta h := h1 - h2 = 4200 \text{ft} \quad V_{\text{avg}} := \frac{V1 + V2}{2} = 207 \frac{\text{mile}}{\text{hr}}$$

$$\text{Distance traveled} \quad \Delta X := V_{\text{avg}} \cdot t = 18216 \text{ft}$$

$$\sin(\gamma) = \frac{\Delta h}{\Delta X} \quad \gamma_{\text{avg}} := \text{asin}\left(\frac{\Delta h}{\Delta X}\right) \quad \gamma_{\text{avg}} = 13.33 \text{deg}$$

$$a := \frac{V1 - V2}{t} = 1.32 \frac{\text{ft}}{\text{s}^2} \quad a = 0.402 \frac{\text{m}}{\text{s}^2}$$

- 33.** The earth is moving in a circular orbit about the sun, with a radius of 147×10^9 m. The duration of one turn is one year. Determine the velocity of earth in terms of speed of sound (i.e., Mach number) at sea level.

$$X_{ES} := 147 \cdot 10^9 \cdot \text{m} \quad X_{ES} = 1.47 \times 10^8 \text{ km}$$

$$t := 1 \cdot \text{yr} \quad t = 3.156 \times 10^7 \text{ s} \quad t = 365.242 \text{ day} \quad t = 8.766 \times 10^3 \text{ hr}$$

$$\text{Total Distance traveled; Circumference:} \quad X := 2\pi \cdot (X_{ES}) = 9.2 \times 10^8 \text{ km}$$

$$V_{\text{Earth}} := \frac{X}{t} = 2.927 \times 10^4 \frac{\text{m}}{\text{s}} \quad V_{\text{Earth}} = 29.269 \frac{\text{km}}{\text{s}}$$

$$\text{Speed of sound:} \quad a := 340 \frac{\text{m}}{\text{s}}$$

$$M1 := \frac{V_{\text{Earth}}}{a} = 86.084$$

- 34.** In 24 Mar 1960, the maximum speed of a Tu-114 – World's fastest propeller-driven aircraft on a 1,000 km closed circuit with payloads of 0 to 25,000 kg was recorded to be 871.38 km/hr. Determine this velocity in terms of Mach number. Assume sea level.

$$V_A := 871.38 \frac{\text{km}}{\text{hr}} \quad V_A = 242.05 \frac{\text{m}}{\text{s}}$$

$$\text{Speed of sound:} \quad a := 340 \frac{\text{m}}{\text{s}}$$

$$M1 := \frac{V_A}{a} = 0.712 \quad (\text{Equ 1.35})$$

CHAPTER 2

Equations of Motion

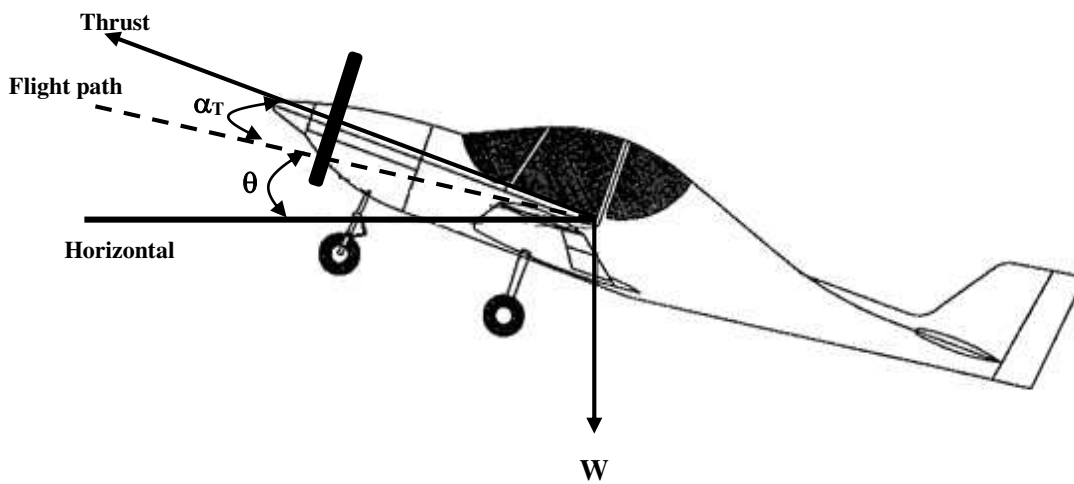


Figure 2.1. An aircraft with two non-aerodynamic forces (weight and thrust)

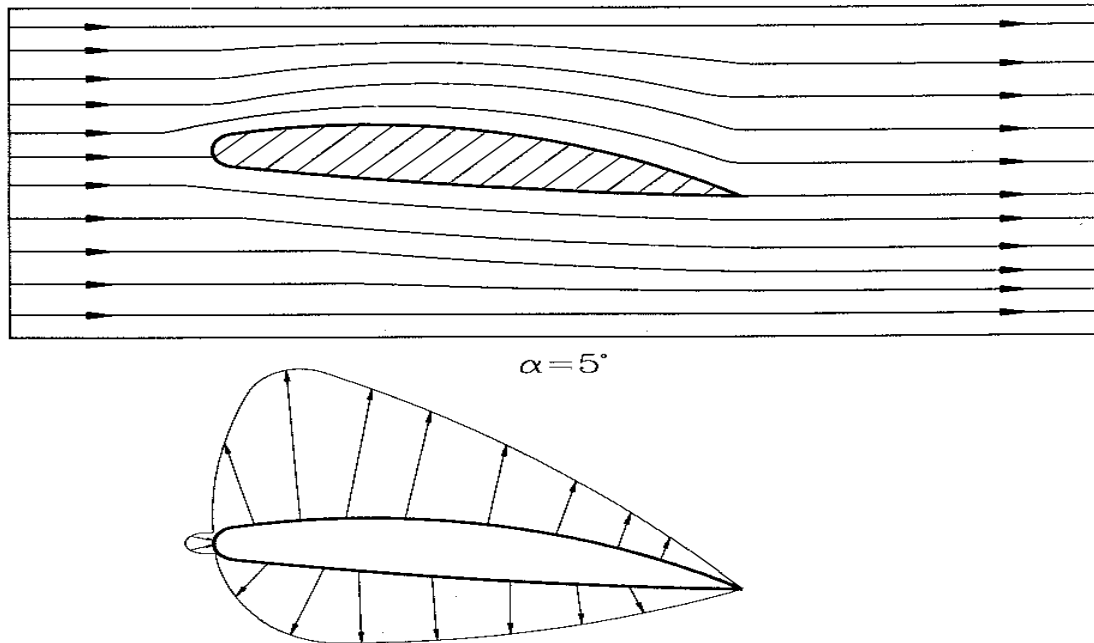


Figure 2.2. A typical pressure distribution over an airfoil with 5 degrees of angle of attack

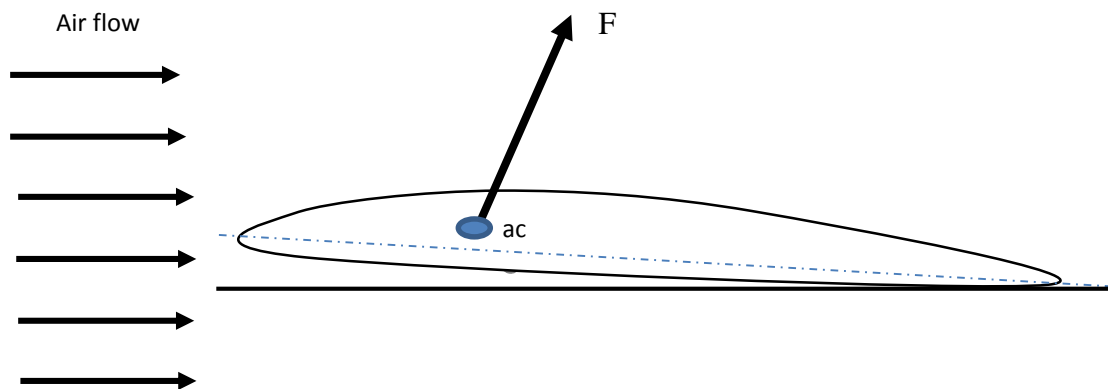
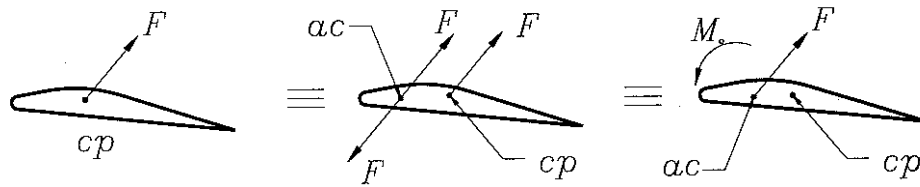


Figure 2.3. The resultant force out of integration of pressure distribution



a. original force b. adding two equal and opposite forces c. resultant force

Figure 2.4. The movement of resultant force to aerodynamic center

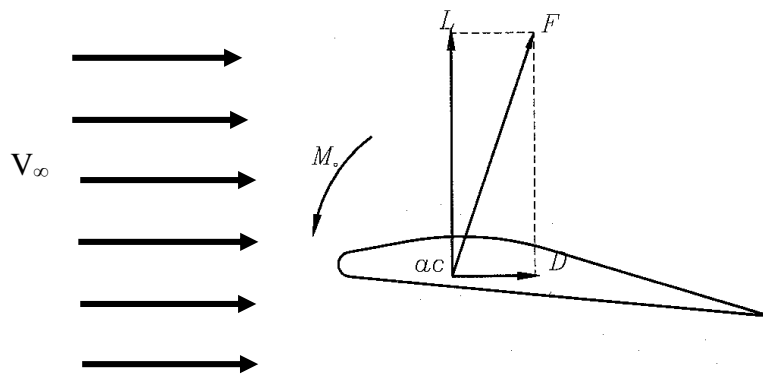


Figure 2.5. The definitions of lift, drag, and pitching moment

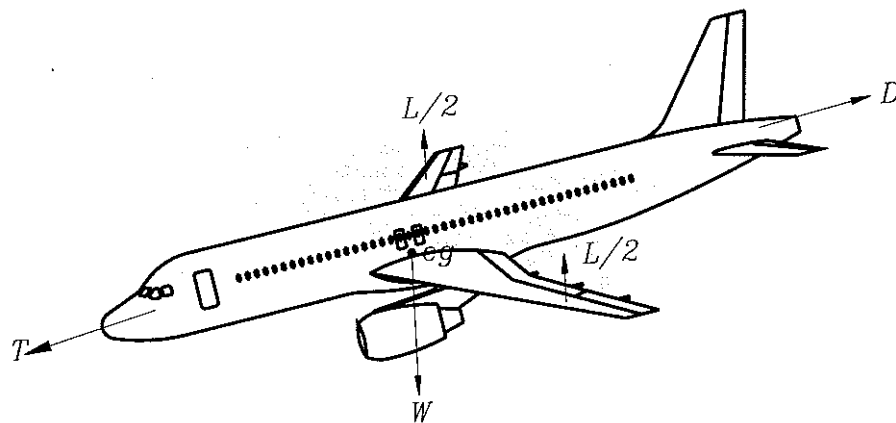


Figure 2.6. The major forces on an airplane

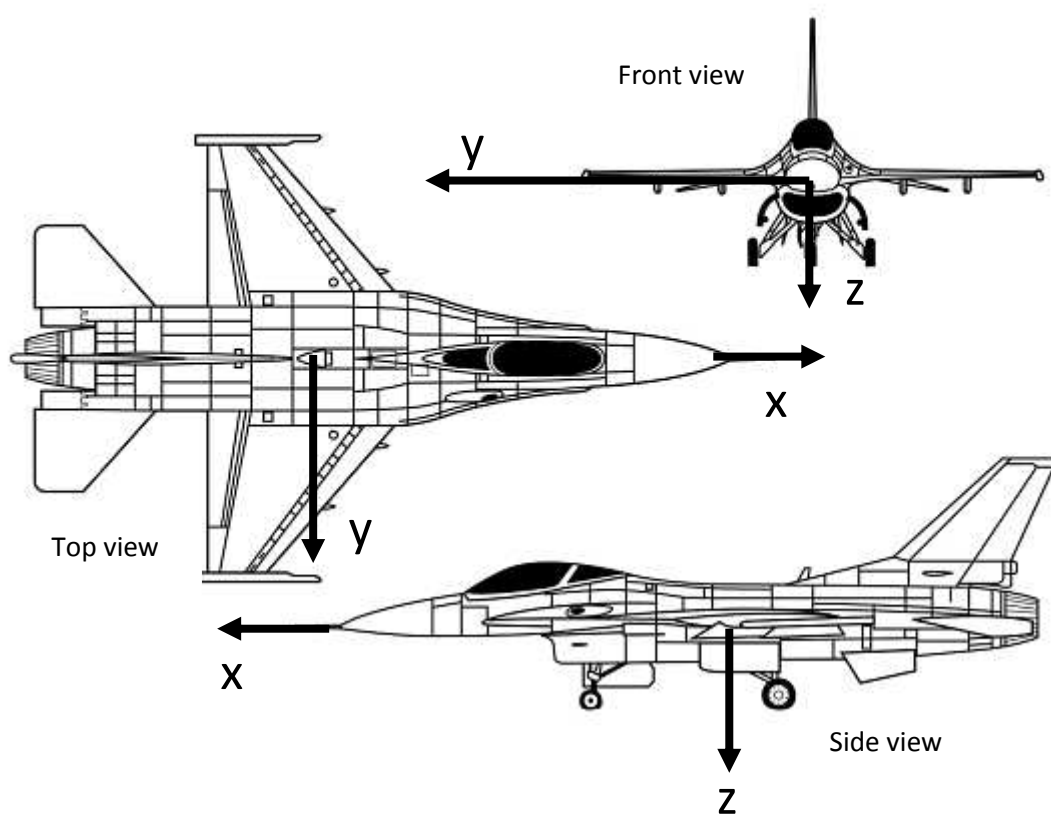


Figure 2.7. Aircraft body-fixed coordinate system (F-16)

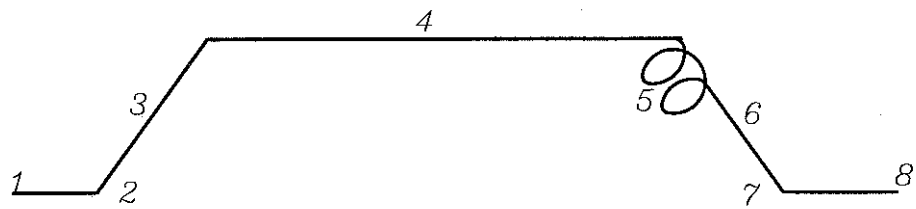


Figure 2.8. A basic and typical flight operation of an aircraft

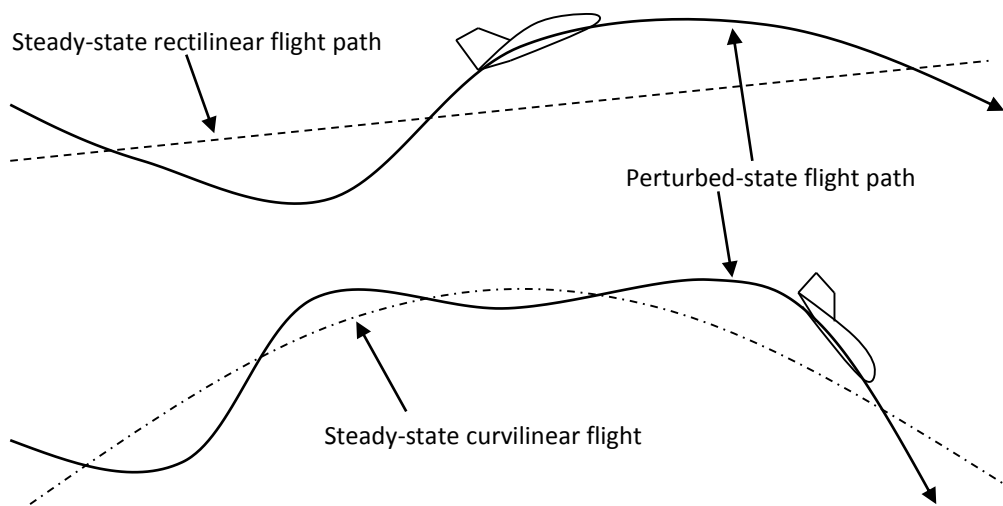
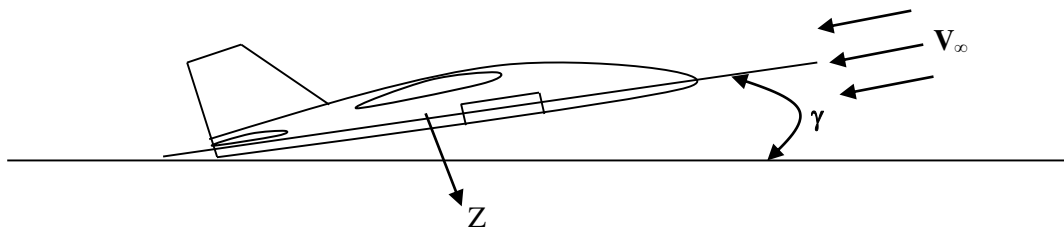
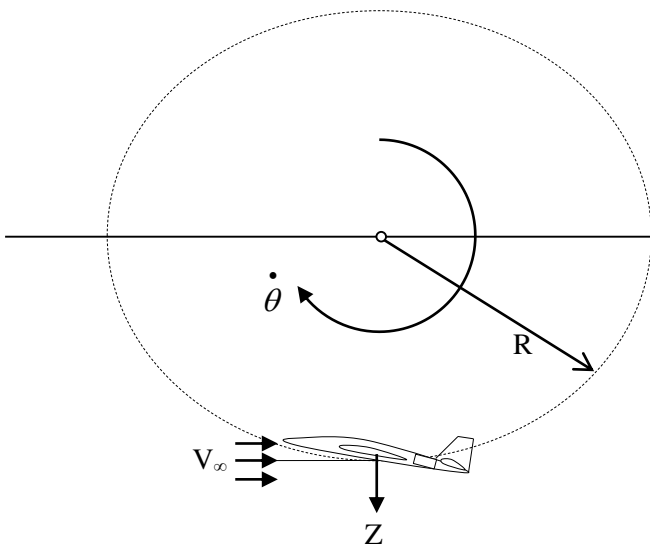


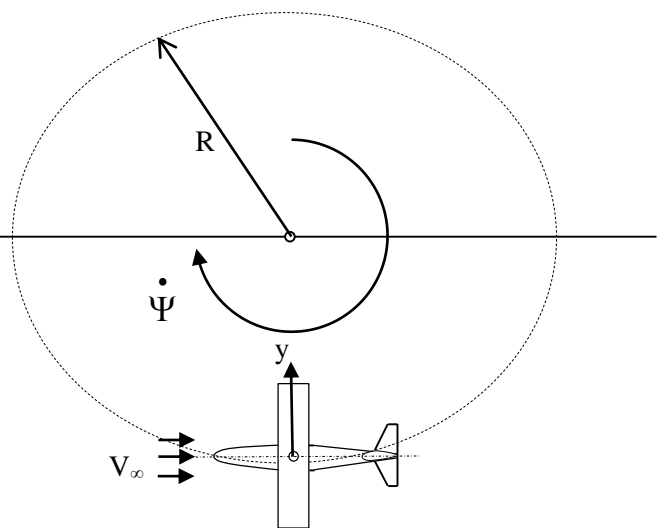
Figure 2.9. Examples of perturbed-state flight paths



a. Rectilinear flight (side-view)



b. Symmetrical pull-up (side-view)



c. Level turn (Top-view)

Figure 2.10. Examples of steady state flight conditions



Figure 2.11. NASA Global Hawk in a cruising flight

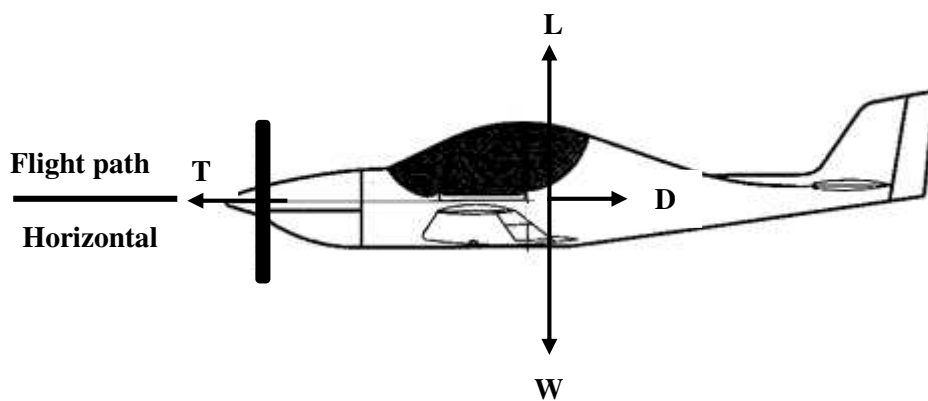


Figure 2.12. Straight line flight

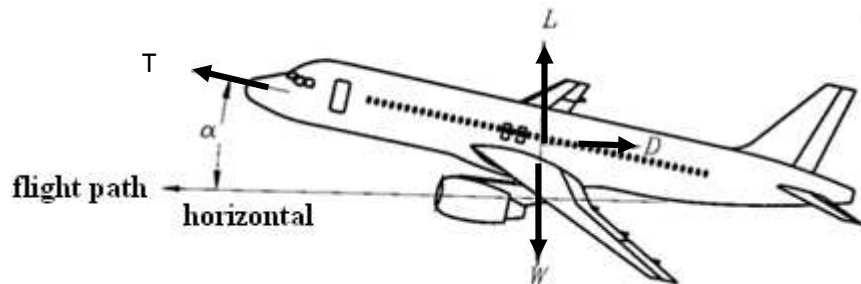


Figure 2.13. An aircraft with an angle of attack in a straight line horizontal flight

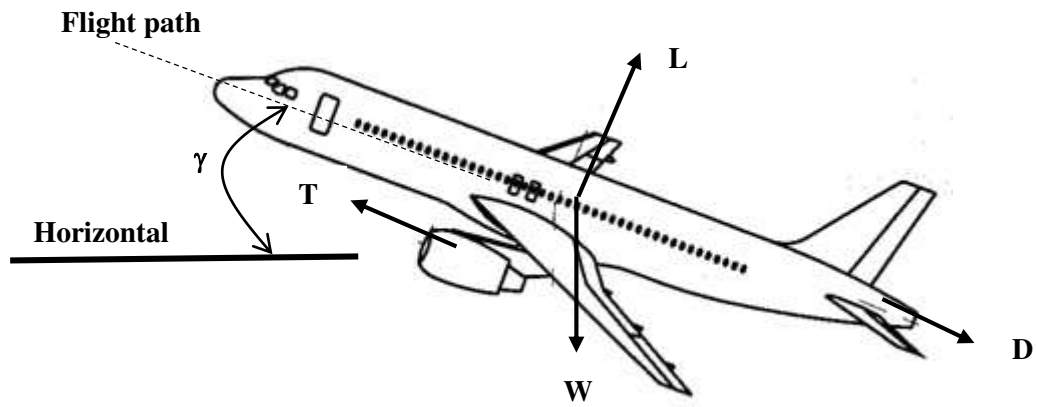


Figure 2.14. An aircraft in climb (assuming zero angle of attack)

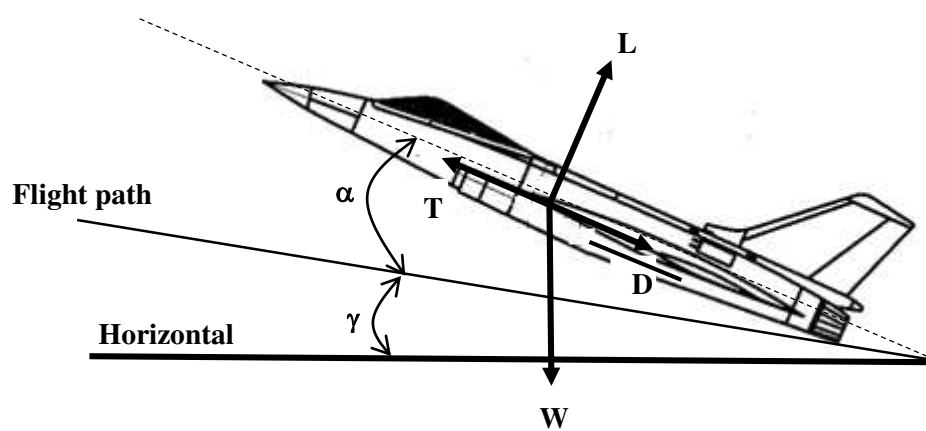


Figure 2.15. An aircraft in a climbing flight with an angle of attack, α

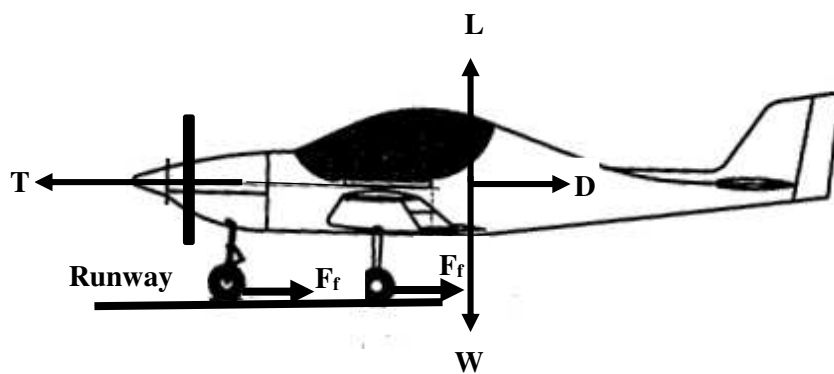


Figure 2.16. An aircraft in take off

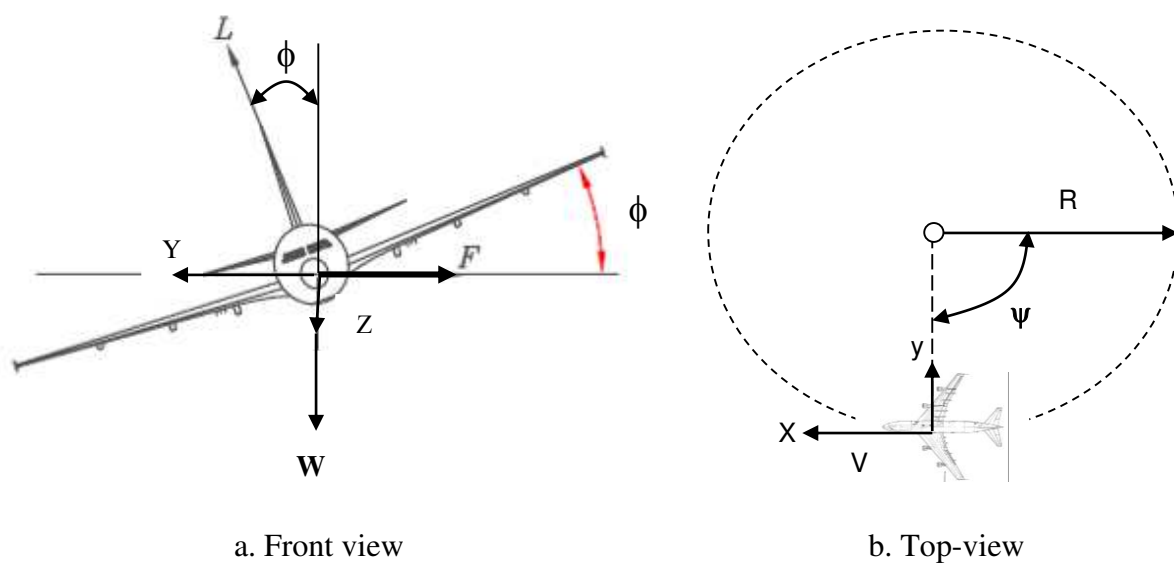


Figure 2.17. An aircraft in a coordinated turning flight

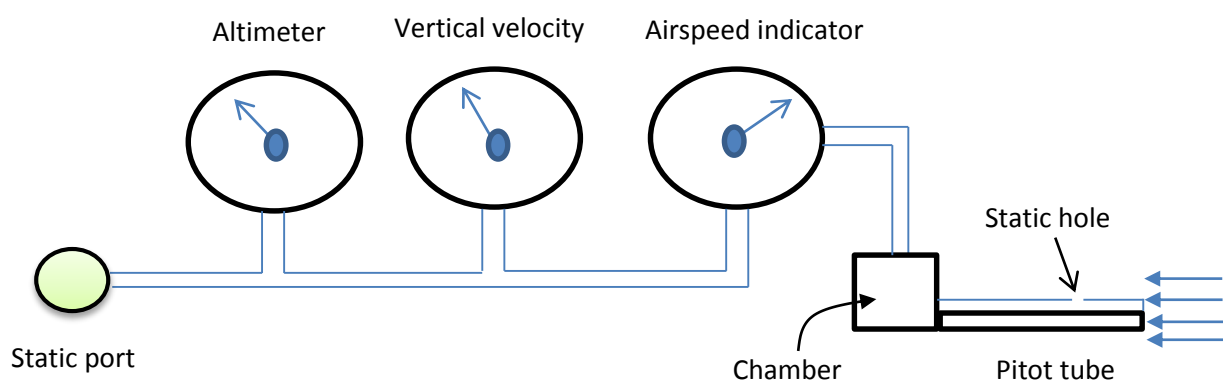


Figure 2.18. Pitot-static measurement device

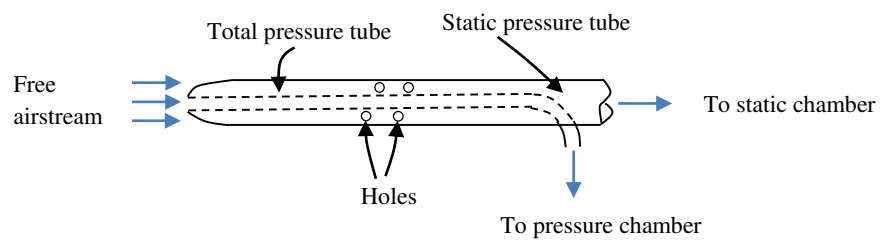
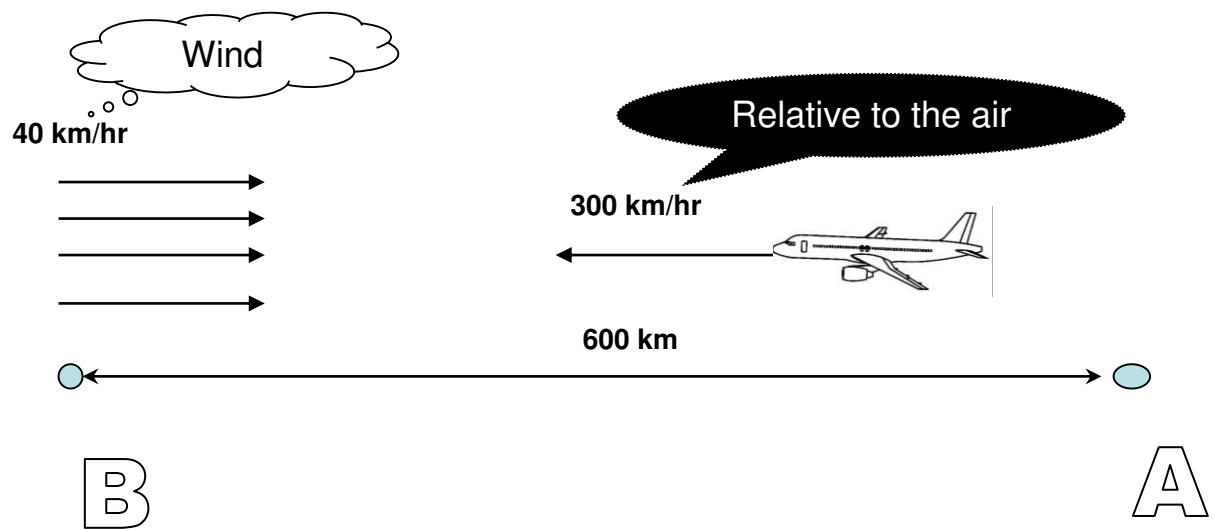
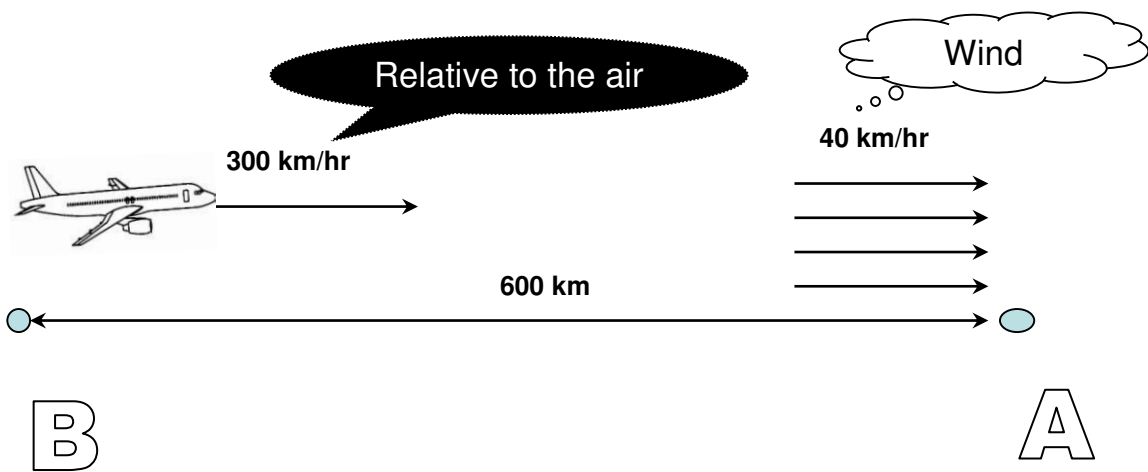


Figure 2.19. A pitot-static tube



a. Headwind



b. Tailwind

Figure 2.20. Airspeed and wind speed

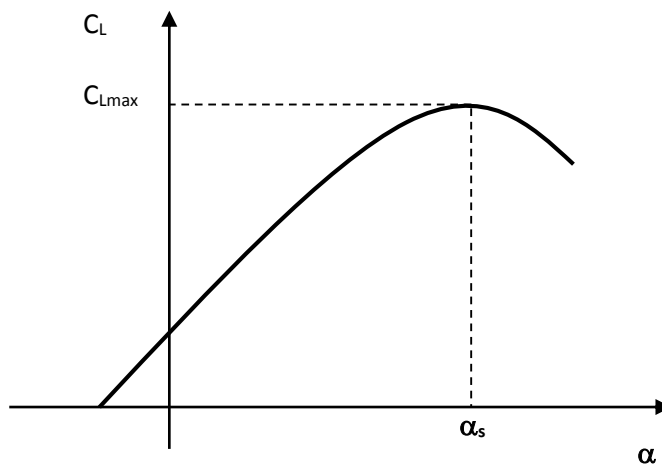


Figure 2.21. Wing lift curve slope



Figure 2.22. Aircraft Raytheon Hawker 800XP with a $C_{L_{max}}$ of 2.28 (Courtesy of Gustavo Corujo)



Figure 2.23. Eurofighter EF-2000 Typhoon, a single-seat fighter in an angle beyond stall angle (Courtesy of Fabrizio Capenti)

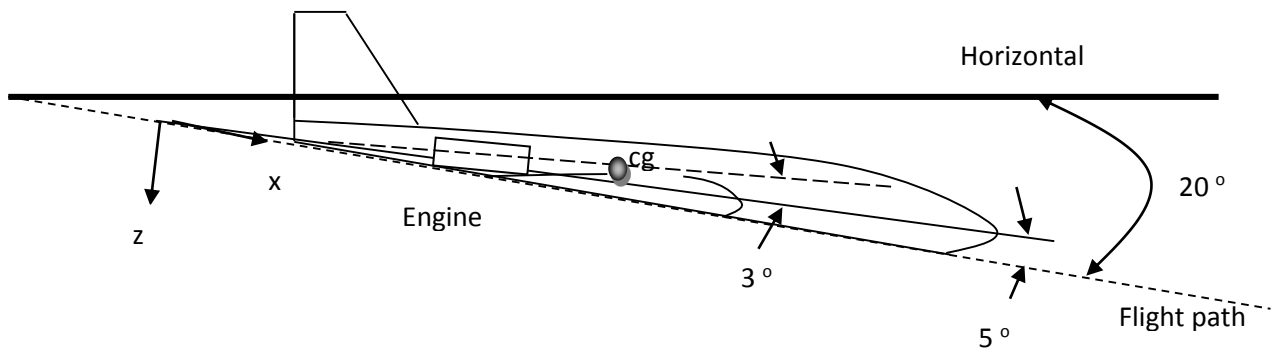


Figure 2.24. An aircraft is a descending flight