

Precalculus 12th edition Larson Solutions Manual

SKU: 9798214407050-SOLUTIONS

CONTENTS

Part I	Solutions to All Exercises	1
Chapter 1	Functions and Their Graphs.....	1
Chapter 2	Polynomial and Rational Functions	246
Chapter 3	Exponential and Logarithmic Functions	481
Chapter 4	Trigonometry	633
Chapter 5	Analytic Trigonometry.....	839
Chapter 6	Additional Topics in Trigonometry.....	981
Chapter 7	Systems of Equations and Inequalities.....	1137
Chapter 8	Matrices and Determinants	1336
Chapter 9	Sequences, Series, and Probability	1496
Chapter 10	Topics in Analytic Geometry.....	1634
Appendix A	Review of Fundamental Concepts of Algebra.....	1928
Appendix B	Concepts in Statistics	2019
Getting Ready Solutions		2050
	Solutions to Checkpoints	2071
Part II	Solutions to Chapter Tests	2350

Solution and Answer Guide

LARSON, PRECALCULUS, 12E, 2027, 9798214407050;

CHAPTER 1: FUNCTIONS AND THEIR GRAPHS

TABLE OF CONTENTS

Section 1.1	Rectangular Coordinates.....	2
Section 1.2	Graphs of Equations.....	13
Section 1.3	Linear Equations in Two Variables	27
Section 1.4	Functions	54
Section 1.5	Analyzing Graphs of Functions	71
Section 1.6	A Library of Parent Functions	97
Section 1.7	Transformations of Functions	112
Section 1.8	Combinations of Functions: Composite Functions	135
Section 1.9	Inverse Functions	154
Section 1.10	Mathematical Modeling and Variation.....	182
Review Exercises for Chapter 1		200
Problem Solving for Chapter 1.....		234

SECTION 1.1 RECTANGULAR COORDINATES

1.1.1.

Cartesian

1.1.2.

Midpoint Formula

1.1.3.

The x -axis is the horizontal real number line. Matches (c).

1.1.4.

The y -axis is the vertical real number line. Matches (f).

1.1.5.

The origin is the point of intersection of the vertical and horizontal axes. Matches (a).

1.1.6.

The quadrants are four regions of the coordinate plane. Matches (d).

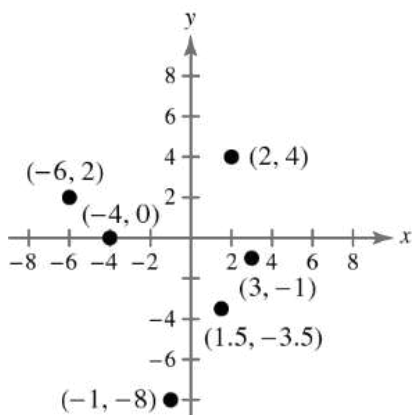
1.1.7.

An x -coordinate is the directed distance from the y -axis. Matches (e).

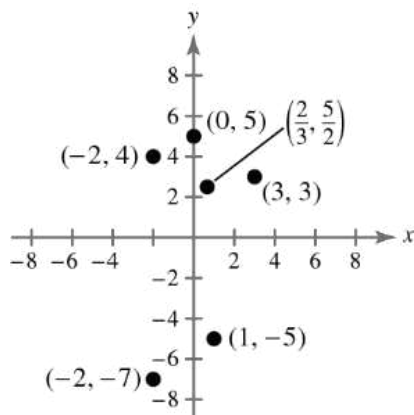
1.1.8.

A y -coordinate is the directed distance from the x -axis. Matches (b).

1.1.9.



1.1.10.



1.1.11.

$$(-3, 4)$$

1.1.12.

$$(-12, 0)$$

1.1.13.

$x > 0$ and $y < 0$ in Quadrant IV.

1.1.14.

$x < 0$ and $y < 0$ in Quadrant III.

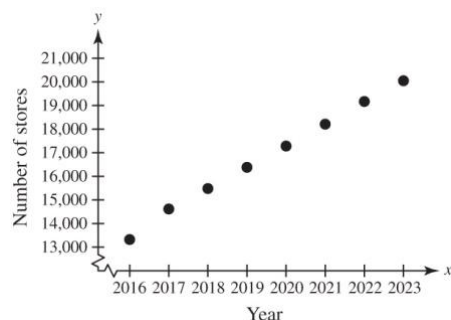
1.1.15.

$x + y = 0$, $x \neq 0$, $y \neq 0$ means $x = -y$ or $y = -x$. This occurs in Quadrant II or IV.

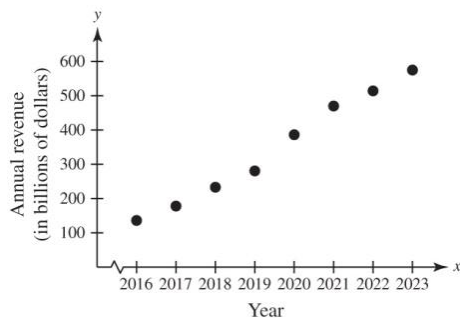
1.1.16.

(x, y) , $xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

1.1.17.



1.1.18.



1.1.19.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(3 - (-2))^2 + (-6 - 6)^2} \\
 &= \sqrt{(5)^2 + (-12)^2} \\
 &= \sqrt{25 + 144} \\
 &= 13 \text{ units}
 \end{aligned}$$

1.1.20.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(0 - 8)^2 + (20 - 5)^2} \\
 &= \sqrt{(-8)^2 + (15)^2} \\
 &= \sqrt{64 + 225} \\
 &= \sqrt{289} \\
 &= 17 \text{ units}
 \end{aligned}$$

1.1.21.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2} \\
 &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2} \\
 &= \sqrt{\frac{9}{4} + \frac{49}{9}} \\
 &= \sqrt{\frac{277}{36}} \\
 &= \frac{\sqrt{277}}{6} \text{ units}
 \end{aligned}$$

1.1.22.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2} \\
 &= \sqrt{(-13.4)^2 + (10.8)^2} \\
 &= \sqrt{179.56 + 116.64} \\
 &= \sqrt{296.2}
 \end{aligned}$$

1.1.23.

$$\begin{aligned}
 &\text{(a) } (1, 0), (13, 5) \\
 &\text{Distance} = \sqrt{(13 - 1)^2 + (5 - 0)^2} \\
 &\quad = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \\
 &(13, 5), (13, 0) \\
 &\text{Distance} = |5 - 0| = |5| = 5 \\
 &(1, 0), (13, 0) \\
 &\text{Distance} = |1 - 13| = |-12| = 12 \\
 &\text{(b) } 5^2 + 12^2 = 25 + 144 = 169 = 13^2
 \end{aligned}$$

1.1.24.

(a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

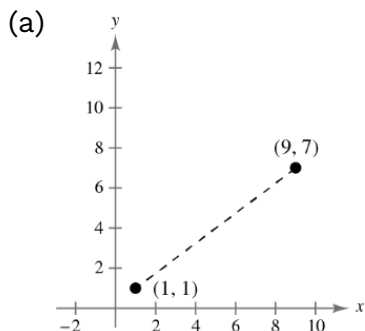
The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = (\sqrt{109})^2$

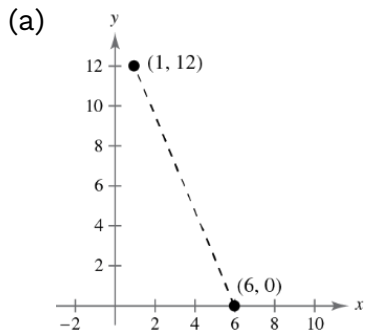
1.1.25.



(b) $d = \sqrt{(9 - 1)^2 + (7 - 1)^2} = \sqrt{64 + 36} = 10$

(c) $\left(\frac{9+1}{2}, \frac{7+1}{2}\right) = (5, 4)$

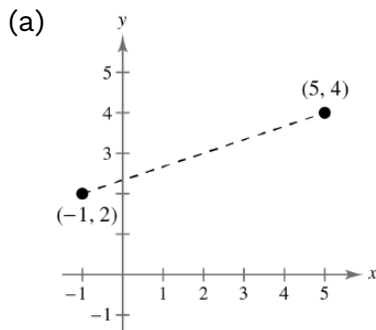
1.1.26.



(b) $d = \sqrt{(1 - 6)^2 + (12 - 0)^2} = \sqrt{25 + 144} = 13$

(c) $\left(\frac{1+6}{2}, \frac{12+0}{2}\right) = \left(\frac{7}{2}, 6\right)$

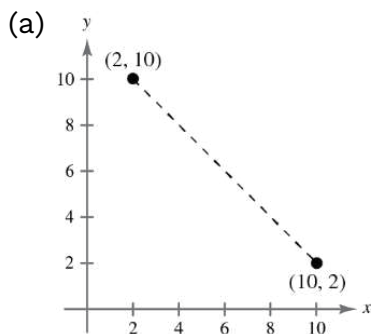
1.1.27.



$$(b) \ d = \sqrt{(5+1)^2 + (4-2)^2} = \sqrt{36+4} = 2\sqrt{10}$$

$$(c) \ \left(\frac{-1+5}{2}, \frac{2+4}{2} \right) = (2, 3)$$

1.1.28.



$$(b) \ d = \sqrt{(2-10)^2 + (10-2)^2} \\ = \sqrt{64+64} = 8\sqrt{2}$$

$$(c) \ \left(\frac{2+10}{2}, \frac{10+2}{2} \right) = (6, 6)$$

1.1.29.

$$\begin{aligned} d &= \sqrt{(42-18)^2 + (50-12)^2} \\ &= \sqrt{24^2 + 38^2} \\ &= \sqrt{2020} \\ &= 2\sqrt{505} \\ &\approx 45 \end{aligned}$$

The pass is about 45 yards.

1.1.30.

$$\begin{aligned} d &= \sqrt{120^2 + 150^2} \\ &= \sqrt{36,900} \\ &= 30\sqrt{41} \\ &\approx 192.09 \end{aligned}$$

The plane flies about 192 kilometers.

1.1.31.

$$\begin{aligned} \text{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2021 + 2023}{2}, \frac{572.8 + 648.1}{2} \right) \\ &= (2022, 610.45) \end{aligned}$$

In 2022, the sales were about \$610.45 billion.

1.1.32.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

In this problem, you have $\left(\frac{2022 + 2024}{2}, \frac{8.59 + y_2}{2} \right) = (2023, 14.87)$. So,

$$\frac{8.59 + y_2}{2} = 14.87$$

$$8.59 + y_2 = 29.74$$

$$y_2 = 21.15.$$

In 2024, the earnings per share were about \$21.15.

1.1.33.

$$(-2 + 2, -4 + 5) = (0, 1)$$

$$(2 + 2, -3 + 5) = (4, 2)$$

$$(-1 + 2, -1 + 5) = (1, 4)$$

1.1.34.

$$(-3 + 6, 6 - 3) = (3, 3)$$

$$(-5 + 6, 3 - 3) = (1, 0)$$

$$(-3 + 6, 0 - 3) = (3, -3)$$

$$(-1 + 6, 3 - 3) = (5, 0)$$

1.1.35.

True. Because $x < 0$ and $y > 0$, $2x < 0$ and $-3y < 0$, which is located in Quadrant III.

1.1.36.

False. The Midpoint Formula would be used 15 times.

1.1.37.

True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

1.1.38.

False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

1.1.39.

The y -coordinate of a point on the x -axis is 0. The x -coordinates of a point on the y -axis is 0.

1.1.40.

Because $x_m = \frac{x_1 + x_2}{2}$ and $y_m = \frac{y_1 + y_2}{2}$ we have:

$$\begin{aligned} 2x_m &= x_1 + x_2 & 2y_m &= y_1 + y_2 \\ 2x_m - x_1 &= x_2 & 2y_m - y_1 &= y_2 \end{aligned}$$

$$\text{So, } (x_2, y_2) = (2x_m - x_1, 2y_m - y_1).$$

1.1.41.

The midpoint of the given line segment is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is

$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right).$$

The midpoint between $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2} \right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4} \right).$$

So, the three points are $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4} \right)$, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$, and $\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4} \right)$.

1.1.42.

- (a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III.
Matches (ii).
- (b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I.
Matches (iii).
- (c) Because (x_0, y_0) lies in Quadrant II, $(x_0, \frac{1}{2}y_0)$ must lie in Quadrant II.
Matches (iv).
- (d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV.
Matches (i).

1.1.43.

(a)

First Set

$$d(A, B) = \sqrt{(2-2)^2 + (3-6)^2} = \sqrt{9} = 3$$

$$d(B, C) = \sqrt{(2-6)^2 + (6-3)^2} = \sqrt{16+9} = 5$$

$$d(A, C) = \sqrt{(2-6)^2 + (3-3)^2} = \sqrt{16} = 4$$

Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

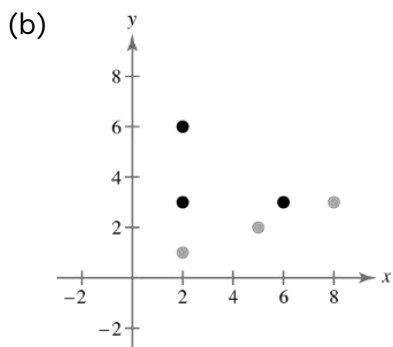
$$d(A, B) = \sqrt{(8-5)^2 + (3-2)^2} = \sqrt{10}$$

$$d(B, C) = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{10}$$

$$d(A, C) = \sqrt{(8-2)^2 + (3-1)^2} = \sqrt{40}$$

A, B, and C are the vertices of an isosceles triangle or are collinear:

$$\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}.$$



First set: Not collinear

Second set: Collinear

- (c) A set of three points is collinear when the sum of two distances among the points is exactly equal to the third distance.

1.1.44.

$$\begin{aligned} \text{(a) distance} &= \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(y_2 - y_1)^2} = |y_2 - y_1| \end{aligned}$$

$$\begin{aligned} \text{(b) distance} &= \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2} \\ &= \sqrt{(x_2 - x_1)^2} = |x_2 - x_1| \end{aligned}$$

Answers will vary.

1.1.45.

$$4x - 6$$

$$\text{(a) } 4(-1) - 6 = -4 - 6 = -10$$

$$\text{(b) } 4(0) - 6 = 0 - 6 = -6$$

1.1.46.

$$9 - 7x$$

$$\text{(a) } 9 - 7(-3) = 9 + 21 = 30$$

$$\text{(b) } 9 - 7(3) = 9 - 21 = -12$$

1.1.47.

$$\text{(a) When } x = -3, 2x^3 = 2(-3)^3 = 2(-27) = -54.$$

$$\text{(b) When } x = 0, 2x^3 = 2(0)^3 = 0.$$

1.1.48.

(a) When $x = 0$, $-3x^{-4} = \frac{-3}{x^4} = -\frac{3}{0}$. Division by 0 is undefined.

(b) When $x = -2$, $-3x^{-4} = \frac{-3}{x^4} = \frac{-3}{(-2)^4} = -\frac{3}{16}$.

1.1.49.

$$(-x)^2 - 2 = x^2 - 2$$

1.1.50.

$$(-x)^3 - (-x)^2 + 2 = -x^3 - x^2 + 2$$

1.1.51.

$$-x^2 + (-x)^2 = -x^2 + x^2 = 0$$

1.1.52.

$$\begin{aligned} -x^3 + (-x)^3 + (-x)^4 - x^4 &= -x^3 - x^3 + x^4 - x^4 \\ &= -2x^3 \end{aligned}$$

1.1.53.

$$\begin{aligned} \text{(a) } P &= R - C \\ &= 135x - (93x + 35,000) \\ &= 42x - 35,000 \\ \text{(b) } P &= 42(5000) - 35,000 = \$175,000 \end{aligned}$$

1.1.54.

(a) Time to copy one page: $\frac{1}{50}$ min

(b) Time to copy x pages: $\frac{x}{50}$ min

(c) Time to copy 120 pages: $\frac{120}{50} = \frac{12}{5} = 2.4$ min

(d) Time to copy 10,000 pages: $\frac{10,000}{50} = 200$ min

SECTION 1.2 GRAPHS OF EQUATIONS

1.2.1.

solution or solution point

1.2.2.

graph

1.2.3.

Three approaches to solve problems mathematically are algebraic, graphical, and numerical.

1.2.4.

Let (x, y) be any point on the circle. The distance between the center (h, k) and (x, y) is the radius r . So, $\sqrt{(x - h)^2 + (y - k)^2} = r$.

1.2.5.

$$\begin{aligned} \text{(a)} \quad (2, 0) : (2)^2 - 3(2) + 2 & \stackrel{?}{=} 0 \\ 4 - 6 + 2 & \stackrel{?}{=} 0 \\ 0 & = 0 \end{aligned}$$

Yes, the point *is* on the graph.

$$\begin{aligned} \text{(b)} \quad (-2, 8) : (-2)^2 - 3(-2) + 2 & \stackrel{?}{=} 8 \\ 4 + 6 + 2 & \stackrel{?}{=} 8 \\ 12 & \neq 8 \end{aligned}$$

No, the point *is not* on the graph.

1.2.6.

$$\begin{aligned} \text{(a)} \quad (0, 2) : 2 & \stackrel{?}{=} \sqrt{0 + 4} \\ 2 & = 2 \end{aligned}$$

Yes, the point *is* on the graph.

$$\begin{aligned} \text{(b)} \quad (5, 3) : 3 & \stackrel{?}{=} \sqrt{5 + 4} \\ 3 & \stackrel{?}{=} \sqrt{9} \\ 3 & = 3 \end{aligned}$$

Yes, the point *is* on the graph.

1.2.7.

$$(a) (1, 5): 5 \stackrel{?}{=} 4 - |1 - 2|$$

$$5 \stackrel{?}{=} 4 - 1$$

$$5 \neq 3$$

No, the point *is not* on the graph.

$$(b) (6, 0): 0 \stackrel{?}{=} 4 - |6 - 2|$$

$$0 \stackrel{?}{=} 4 - 4$$

$$0 = 0$$

Yes, the point *is* on the graph.

1.2.8.

$$(a) (6, 0): 2(6)^2 + 5(0)^2 \stackrel{?}{=} 8$$

$$2(36) + 5(0) \stackrel{?}{=} 8$$

$$72 + 0 \stackrel{?}{=} 8$$

$$72 \neq 8$$

No, the point *is not* on the graph.

$$(b) (0, 4): 2(0)^2 + 5(4)^2 \stackrel{?}{=} 8$$

$$2(0) + 5(16) \stackrel{?}{=} 8$$

$$0 + 80 \stackrel{?}{=} 8$$

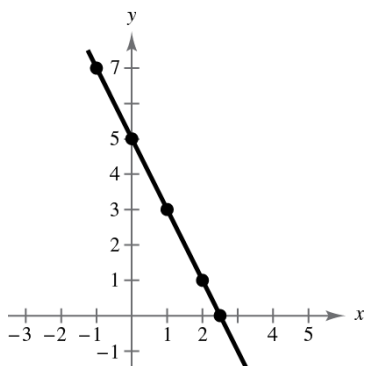
$$80 \neq 8$$

No, the point *is not* on the graph.

1.2.9.

$$y = -2x + 5$$

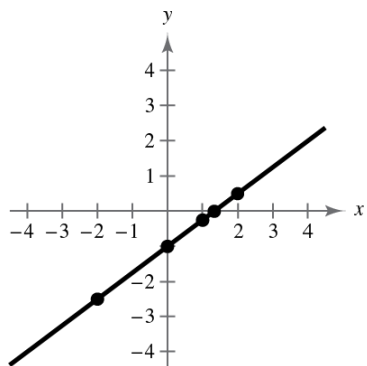
x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$(\frac{5}{2}, 0)$



1.2.10.

$$y = \frac{3}{4}x - 1$$

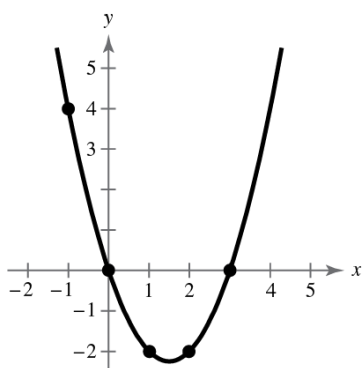
x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$(-2, -\frac{5}{2})$	$(0, -1)$	$(1, -\frac{1}{4})$	$(\frac{4}{3}, 0)$	$(2, \frac{1}{2})$



1.2.11.

$$y = x^2 - 3x$$

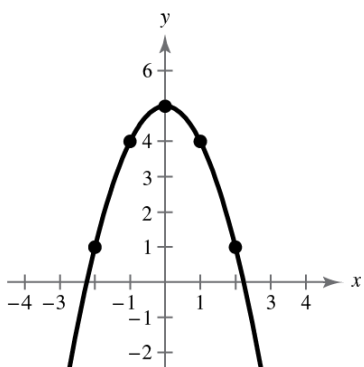
x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



1.2.12.

$$y = 5 - x^2$$

x	-2	-1	0	1	2
y	1	4	5	4	1
(x, y)	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



1.2.13.

x -intercept: $(-2, 0)$

y -intercept: $(0, 2)$

1.2.14.

x -intercept: $(4, 0)$

y -intercept: $(0, \pm 2)$

1.2.15.

x -intercept: $(3, 0)$

y -intercept: $(0, 9)$

1.2.16.

x -intercept: $(\pm 2, 0)$

y -intercept: $(0, 16)$

1.2.17.

$$x^2 - y = 0$$

$$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow y\text{-axis symmetry}$$

$$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \text{No origin symmetry}$$

1.2.18.

$$x - y^2 = 0$$

$$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow x\text{-axis symmetry}$$

$$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow \text{No origin symmetry}$$

1.2.19.

$$y = x^3$$

$$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = x^3 \Rightarrow y = -x^3 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow \text{Origin symmetry}$$

1.2.20.

$$y = x^4 - x^2 + 3$$

$$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow y\text{-axis symmetry}$$

$$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow \text{No origin symmetry}$$

1.2.21.

$$y = \frac{x}{x^2 + 1}$$

$$y = \frac{-x}{(-x)^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = \frac{x}{x^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = \frac{-x}{(-x)^2 + 1} \Rightarrow -y = \frac{-x}{x^2 + 1} \Rightarrow y = \frac{x}{x^2 + 1} \Rightarrow \text{Origin symmetry}$$

1.2.22.

$$y = \frac{1}{1 + x^2}$$

$$y = \frac{1}{1 + (-x)^2} \Rightarrow y = \frac{1}{1 + x^2} \Rightarrow y\text{-axis symmetry}$$

$$-y = \frac{1}{1 + x^2} \Rightarrow y = \frac{-1}{1 + x^2} \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = \frac{1}{1 + (-x)^2} \Rightarrow y = \frac{-1}{1 + x^2} \Rightarrow \text{No origin symmetry}$$

1.2.23.

$$xy^2 + 10 = 0$$

$$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow x\text{-axis symmetry}$$

$$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No origin symmetry}$$

1.2.24.

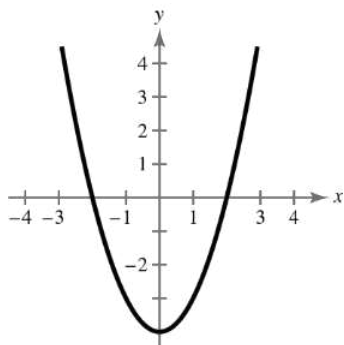
$$xy = 4$$

$$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow \text{No } y\text{-axis symmetry}$$

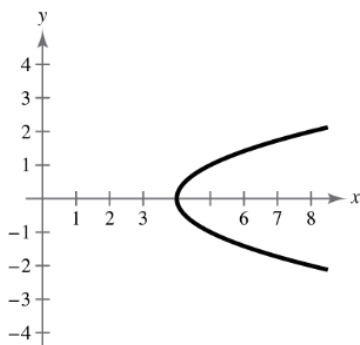
$$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow \text{Origin symmetry}$$

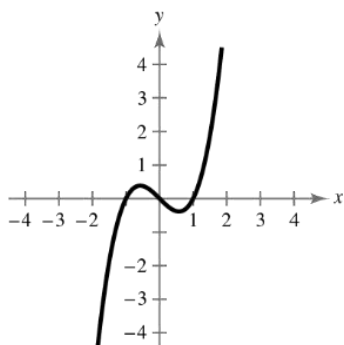
1.2.25.



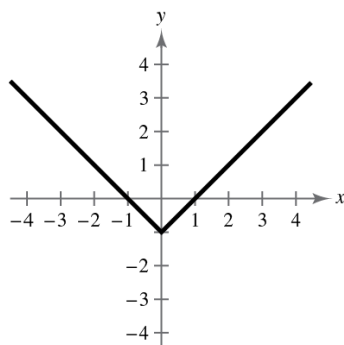
1.2.26.



1.2.27.



1.2.28.



1.2.29.

Center: $(0, 0)$; Radius: 3

$$\begin{aligned}(x-0)^2 + (y-0)^2 &= 3^2 \\ x^2 + y^2 &= 9\end{aligned}$$

1.2.30.

Center: $(0, 0)$; Radius: 7

$$\begin{aligned}(x-0)^2 + (y-0)^2 &= 7^2 \\ x^2 + y^2 &= 49\end{aligned}$$

1.2.31.

Center: $(-4, 5)$; Radius: 2

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ [x-(-4)]^2 + [y-5]^2 &= 2^2 \\ (x+4)^2 + (y-5)^2 &= 4\end{aligned}$$

1.2.32.

Center: $(1, -3)$; Radius: $\sqrt{11}$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-1)^2 + [y-(-3)]^2 &= \sqrt{11}^2 \\ (x-1)^2 + (y+3)^2 &= 11\end{aligned}$$

1.2.33.

Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned}r &= \sqrt{(x-h)^2 + (y-k)^2} \\ &= \sqrt{(-9-3)^2 + (13-8)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13\end{aligned}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-8)^2 = 13^2$$

$$(x-3)^2 + (y-8)^2 = 169$$

1.2.34.

Center: $(-2, -6)$; Solution point: $(1, -10)$

$$\begin{aligned} r &= \sqrt{(x-h)^2 + (y-k)^2} \\ &= \sqrt{[1-(-2)]^2 + [-10-(-6)]^2} \\ &= \sqrt{(3)^2 + (-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$[x-(-2)]^2 + [y-(-6)]^2 = 5^2$$

$$(x+2)^2 + (y+6)^2 = 25$$

1.2.35.

Endpoints of a diameter: $(3, 2)$, $(-9, -8)$

$$\begin{aligned} r &= \frac{1}{2} \sqrt{(-9-3)^2 + (-8-2)^2} \\ &= \frac{1}{2} \sqrt{(-12)^2 + (-10)^2} \\ &= \frac{1}{2} \sqrt{144+100} \\ &= \frac{1}{2} \sqrt{244} = \frac{1}{2} (2) \sqrt{61} = \sqrt{61} \end{aligned}$$

$$\begin{aligned} (h, k) &: \left(\frac{3+(-9)}{2}, \frac{2+(-8)}{2} \right) = \left(\frac{-6}{2}, \frac{-6}{2} \right) \\ &= (-3, -3) \end{aligned}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$[x-(-3)]^2 + [y-(-3)]^2 = (\sqrt{61})^2$$

$$(x+3)^2 + (y+3)^2 = 61$$

1.2.36.

Endpoints of a diameter: $(11, -5), (3, 15)$

$$\begin{aligned} r &= \frac{1}{2} \sqrt{(3-11)^2 + [15-(-5)]^2} \\ &= \frac{1}{2} \sqrt{(-8)^2 + (20)^2} \\ &= \frac{1}{2} \sqrt{64 + 400} \\ &= \frac{1}{2} \sqrt{464} = \frac{1}{2} (4) \sqrt{29} = 2\sqrt{29} \end{aligned}$$

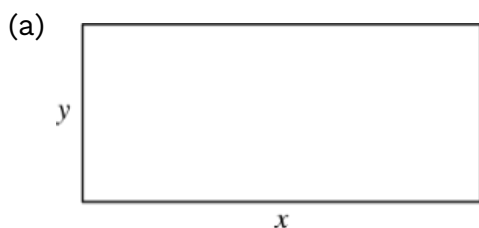
$$(h, k): \left(\frac{11+3}{2}, \frac{-5+15}{2} \right) = \left(\frac{14}{2}, \frac{10}{2} \right) = (7, 5)$$

$$(x-h)^2 + (y-k)^2 = r^2$$

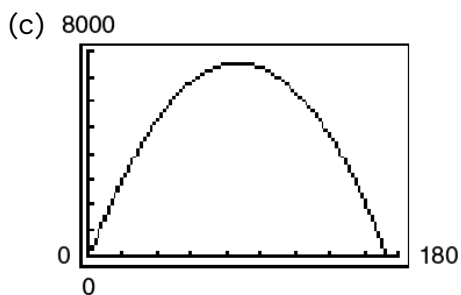
$$(x-7)^2 + (y-5)^2 = (2\sqrt{29})^2$$

$$(x-7)^2 + (y-5)^2 = 116$$

1.2.37.



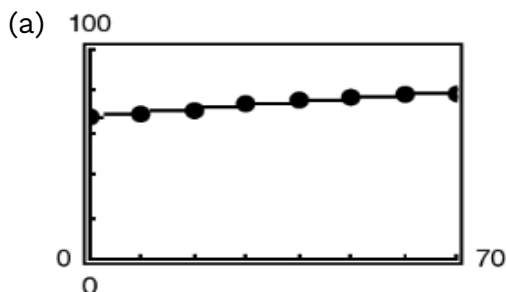
$$\begin{aligned} \text{(b)} \quad 2x + 2y &= \frac{1040}{3} \\ 2y &= \frac{1040}{3} - 2x \\ y &= \frac{520}{3} - x \\ A = xy &= x \left(\frac{520}{3} - x \right) \end{aligned}$$



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

- (e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

1.2.38.



The model fits the data well.

Each data value is close to the graph of the model.

- (b) Using the graph or the model (with $t = 40$), the life expectancy is approximately 75.2 years.
- (c) The model and the line $y = 70.1$ intersect at $t = 11.1$, corresponding to the year 1961.

Algebraically,
$$\frac{68.0 + 0.33(11.1)}{1 + 0.002(11.1)} \approx 70.1.$$

- (d) The y -intercept is $(0, 68.0)$. In 1950 ($t = 0$), the life expectancy of a child (at birth) was 68.0 years.

- (e) Answers will vary.

1.2.39.

False. The line $y = x$ is symmetric with respect to the origin.

1.2.40.

False. The line $y = 0$ has infinitely many x -intercepts.

1.2.41.

The test is for symmetry with respect to the x -axis. The statement should read: The graph of $x = 3y^2$ is symmetric with respect to the x -axis because replacing y with $-y$ yields an equivalent equation.

1.2.42.

x-axis symmetry: $x^2 + y^2 = 1$

$$x^2 + (-y)^2 = 1$$

$$x^2 + y^2 = 1$$

y-axis symmetry: $x^2 + y^2 = 1$

$$(-x)^2 + y^2 = 1$$

$$x^2 + y^2 = 1$$

Origin symmetry: $x^2 + y^2 = 1$

$$(-x)^2 + (-y)^2 = 1$$

$$x^2 + y^2 = 1$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

1.2.43.

$$y = ax^2 + bx^3$$

(a) $y = a(-x)^2 + b(-x)^3$

$$= ax^2 - bx^3$$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

Sample answer: $a = 1, b = 0$

(b) $-y = a(-x)^2 + b(-x)^3$

$$-y = ax^2 - bx^3$$

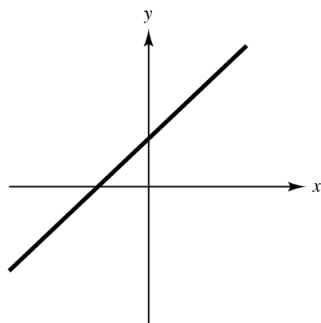
$$y = -ax^2 + bx^3$$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

Sample answer: $a = 0, b = 1$

1.2.44.

The line would rise from left to right, passing through Quadrants I, II, and III.



1.2.45.

$$3(7x + 1) = 3(7x) + 3(1) = 21x + 3$$

1.2.46.

$$5(x - 6) = 5(x) - 5(6) = 5x - 30$$

1.2.47.

$$6(x - 1) + 4 = 6(x) - 6(1) = 6x - 6 + 4 = 6x - 2$$

1.2.48.

$$\begin{aligned} 4(x + 2) - 12 &= 4(x) + 4(2) = 4x + 8 - 12 \\ &= 4x - 4 \end{aligned}$$

1.2.49.

The least common denominator is $3(4) = 12$.

1.2.50.

The least common denominator is 9.

1.2.51.

The least common denominator is $x - 4$.

1.2.52.

The least common denominator is $x^2 - 4 = (x + 2)(x - 2)$.

1.2.53.

$$\begin{aligned} 7\sqrt{72} - 5\sqrt{18} &= 7\sqrt{2 \cdot 36} - 5\sqrt{2 \cdot 9} \\ &= 7(6)\sqrt{2} - 5(3)\sqrt{2} \\ &= (42 - 15)\sqrt{2} \\ &= 27\sqrt{2} \end{aligned}$$

1.2.54.

$$\begin{aligned} -10\sqrt{25y} - \sqrt{y} &= (-10)(5)(\sqrt{y} - \sqrt{y}) \\ &= -50(\sqrt{y} - \sqrt{y}) \\ &= -51\sqrt{y} \end{aligned}$$

1.2.55.

$$7^{3/2} \cdot 7^{11/2} = 7^{3/2 + 11/2} = 7^7 = 823,543$$

1.2.56.

$$\frac{10^{17/4}}{10^{5/4}} = 10^{17/4 - 5/4} = 10^{12/4} = 10^3 = 1000$$

1.2.57.

$$(9x - 4) + (2x^2 - x + 15) = 2x^2 + 8x + 11$$

1.2.58.

$$\begin{aligned} 4x(11 - x + 3x^2) &= 44x - 4x^2 + 12x^3 \\ &= 12x^3 - 4x^2 + 44x \end{aligned}$$

1.2.59.

$$\begin{aligned} (2x + 9)(x - 7) &= 2x^2 + 9x - 14x - 63 \\ &= 2x^2 - 5x - 63 \end{aligned}$$

1.2.60.

$$\begin{aligned} (3x^2 - 5)(-x^2 + 1) &= -3x^4 + 5x^2 + 3x^2 - 5 \\ &= -3x^4 + 8x^2 - 5 \end{aligned}$$

SECTION 1.3 LINEAR EQUATIONS IN TWO VARIABLES

1.3.1.

linear

1.3.2.

slope

1.3.3.

point-slope

1.3.4.

parallel

1.3.5.

rate or rate of change

1.3.6.

linear extrapolation

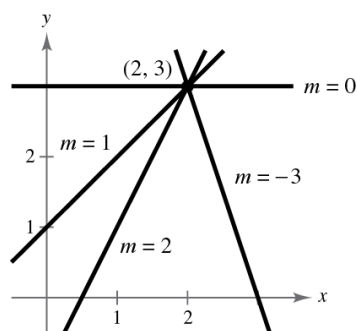
1.3.7.

They are perpendicular to each other.

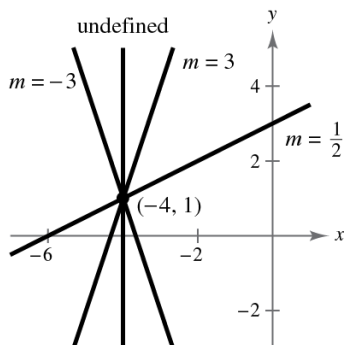
1.3.8.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - y_1 &= mx - mx_1 \\ mx_1 - y_1 &= mx - y \\ 0 &= mx - y - mx_1 + y_1 \\ mx - y - (mx_1 - y_1) &= 0 \end{aligned}$$

1.3.9.



1.3.10.



1.3.11.

Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{4 - 0} = \frac{6}{4} = \frac{3}{2}$$

1.3.12.

The line appears to go through $(0, 7)$ and $(7, 0)$.

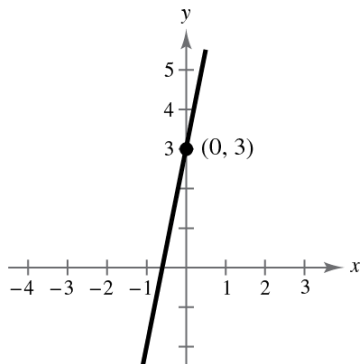
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

1.3.13.

$$y = 5x + 3$$

Slope: $m = 5$

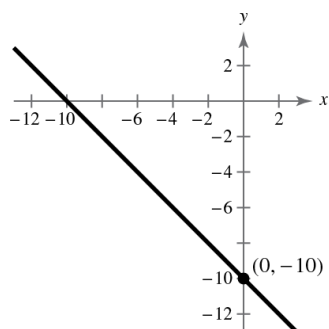
y-intercept: $(0, 3)$



1.3.14.

Slope: $m = -1$

y -intercept: $(0, -10)$

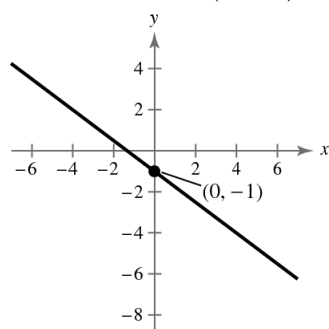


1.3.15.

$$y = -\frac{3}{4}x - 1$$

Slope: $m = -\frac{3}{4}$

y -intercept: $(0, -1)$

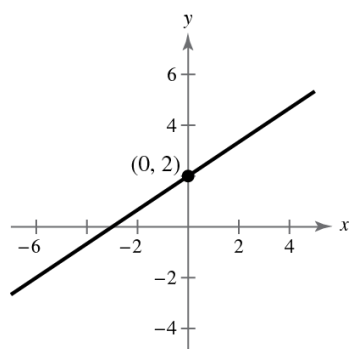


1.3.16.

$$y = \frac{2}{3}x + 2$$

Slope: $m = \frac{2}{3}$

y -intercept: $(0, 2)$



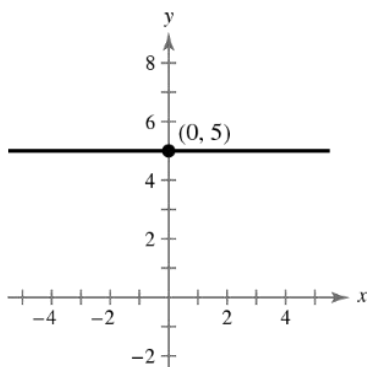
1.3.17.

$$y - 5 = 0$$

$$y = 5$$

Slope: $m = 0$

y -intercept: $(0, 5)$



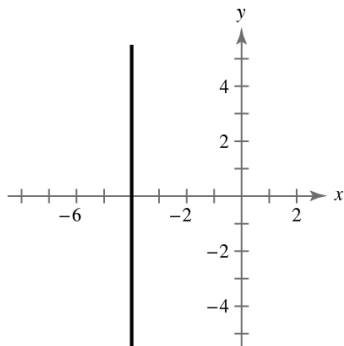
1.3.18.

$$x + 4 = 0$$

$$x = -4$$

Slope: undefined (vertical line)

y -intercept: none



1.3.19.

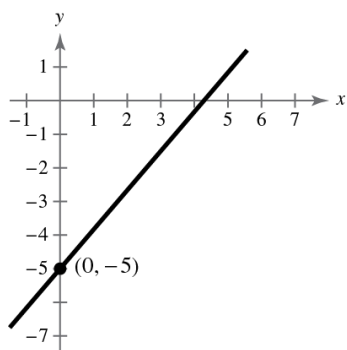
$$7x - 6y = 30$$

$$-6y = -7x + 30$$

$$y = \frac{7}{6}x - 5$$

Slope: $m = \frac{7}{6}$

y -intercept: $(0, -5)$



1.3.20.

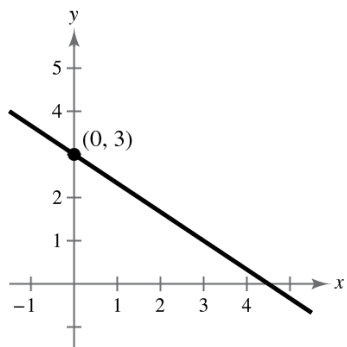
$$2x + 3y = 9$$

$$3y = -2x + 9$$

$$y = -\frac{2}{3}x + 3$$

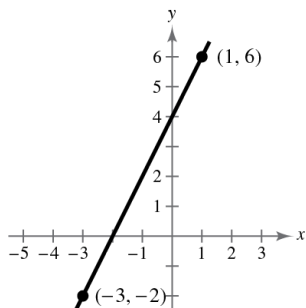
Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$



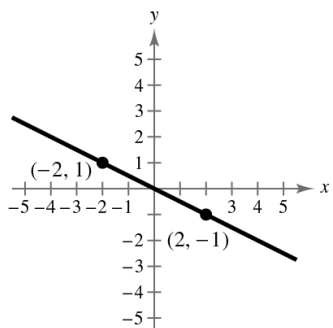
1.3.21.

$$m = \frac{6 - (-2)}{1 - (-3)} = \frac{8}{4} = 2$$



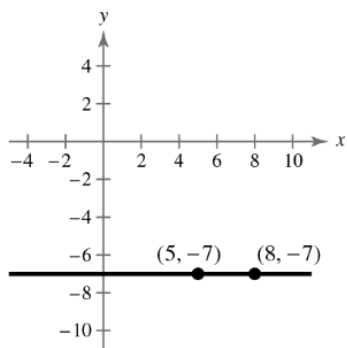
1.3.22.

$$m = \frac{1 - (-1)}{-2 - 2} = \frac{2}{-4} = -\frac{1}{2}$$



1.3.23.

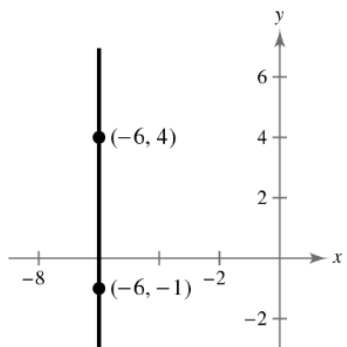
$$m = \frac{-7 - (-7)}{8 - 5} = \frac{0}{3} = 0$$



1.3.24.

$$m = \frac{4 - (-1)}{-6 - (-6)} = \frac{5}{0}$$

m is undefined.



1.3.25.

Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x . Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

1.3.26.

Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x . Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

1.3.27.

Point: $(-4, 3)$, Slope is undefined.

Because m is undefined, x does not change. Three points are $(-4, 0)$, $(-4, 5)$, and $(-4, 2)$.

1.3.28.

Point: $(2, 14)$, Slope is undefined.

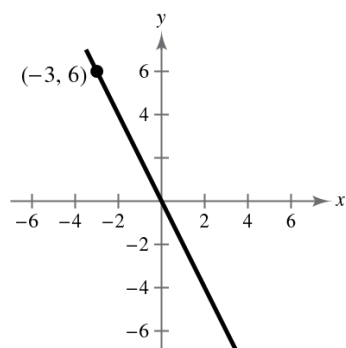
Because m is undefined, x does not change. Three other points are $(2, -3)$, $(2, 0)$, and $(2, 4)$.

1.3.29.

Point: $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$

$$y = -2x$$

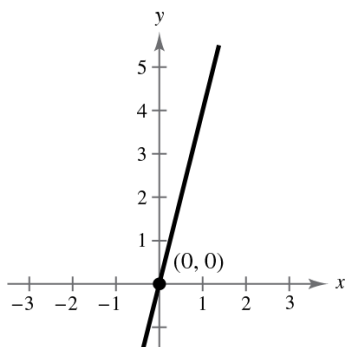


1.3.30.

Point: $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$

$$y = 4x$$

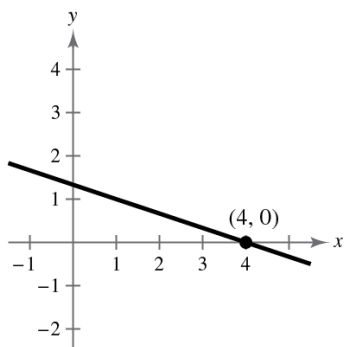


1.3.31.

Point: $(4, 0)$; $m = -\frac{1}{3}$

$$y - 0 = -\frac{1}{3}(x - 4)$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$



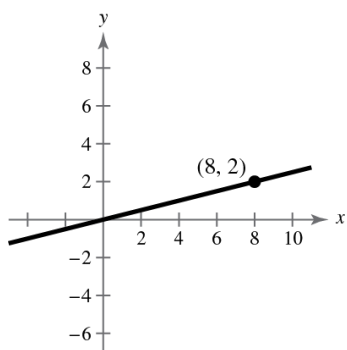
1.3.32.

Point: $(8, 2)$; $m = \frac{1}{4}$

$$y - 2 = \frac{1}{4}(x - 8)$$

$$y - 2 = \frac{1}{4}x - 2$$

$$y = \frac{1}{4}x$$



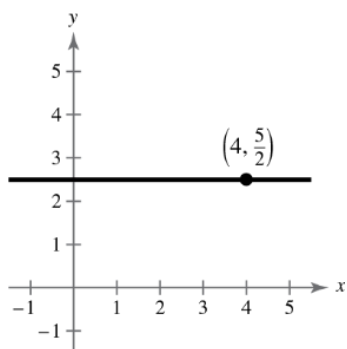
1.3.33.

Point: $(4, \frac{5}{2})$; $m = 0$

$$y - \frac{5}{2} = 0(x - 4)$$

$$y - \frac{5}{2} = 0$$

$$y = \frac{5}{2}$$



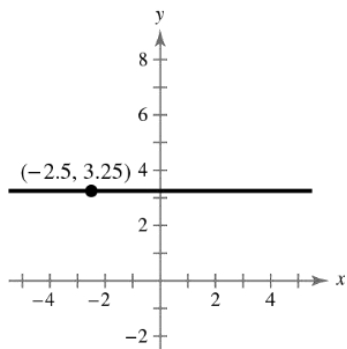
1.3.34.

Point: $(-2.5, 3.25)$; $m = 0$

$$y - 3.25 = 0(x - (-2.5))$$

$$y - 3.25 = 0$$

$$y = 3.25$$



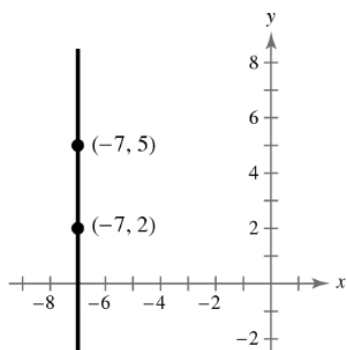
1.3.35.

$$(-7, 2), (-7, 5)$$

$$m = \frac{5 - 2}{-7 - (-7)} = \frac{3}{0}$$

m is undefined.

$$x = -7$$



1.3.36.

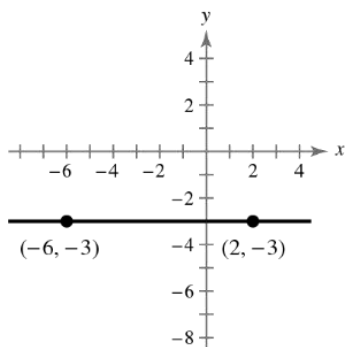
$$(-6, -3), (2, -3)$$

$$y - 4 = \frac{-3 - (-3)}{2 - (-6)}(x + 6)$$

$$y + 3 = 0(x + 6)$$

$$y + 3 = 0$$

$$y = -3$$



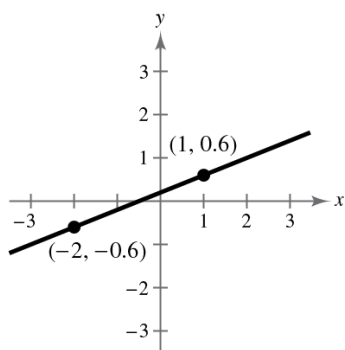
1.3.37.

$$(1, 0.6), (-2, -0.6)$$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$

$$y = 0.4(x - 1) + 0.6$$

$$y = 0.4x + 0.2$$



1.3.38.

$$(-8, 0.6), (2, -2.4)$$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

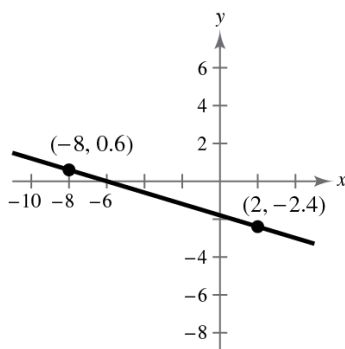
$$y - 0.6 = -\frac{3}{10}(x + 8)$$

$$10y - 6 = -3(x + 8)$$

$$10y - 6 = -3x - 24$$

$$10y = -3x - 18$$

$$y = -\frac{3}{10}x - \frac{9}{5} \text{ or } y = -0.3x - 1.8$$



1.3.39.

$$L_1: (0, -1), (5, 9)$$

$$m_1 = \frac{9 + 1}{5 - 0} = 2$$

$$L_2: (0, 3), (4, 1)$$

$$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$$

The slopes are negative reciprocals, so the lines are perpendicular.

1.3.40.

$$L_1 : (-2, -1), (1, 5)$$

$$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$$

$$L_2 : (1, 3), (5, -5)$$

$$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

1.3.41.

$$L_1 : (-6, -3), (2, -3)$$

$$m_1 = \frac{-3 - (-3)}{2 - (-6)} = \frac{0}{8} = 0$$

$$L_2 : (3, -\frac{1}{2}), (6, -\frac{1}{2})$$

$$m_2 = \frac{-\frac{1}{2} - (-\frac{1}{2})}{6 - 3} = \frac{0}{3} = 0$$

L_1 and L_2 are both horizontal lines, so they are parallel.

1.3.42.

$$L_1 : (4, 8), (-4, 2)$$

$$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$$

$$L_2 : (3, -5), (-1, \frac{1}{3})$$

$$m_2 = \frac{\frac{1}{3} - (-5)}{-1 - 3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The slopes are negative reciprocals, so the lines are perpendicular.

1.3.43.

$$4x - 2y = 3$$

$$y = 2x - \frac{3}{2}$$

$$\text{Slope: } m = 2$$

$$(a) (2, 1), m = 2$$

$$y - 1 = 2(x - 2)$$

$$y = 2x - 3$$

$$\begin{aligned} \text{(b)} \quad (2, 1), m &= -\frac{1}{2} \\ y - 1 &= -\frac{1}{2}(x - 2) \\ y &= -\frac{1}{2}x + 2 \end{aligned}$$

1.3.44.

$$\begin{aligned} x + y &= 7 \\ y &= -x + 7 \\ \text{Slope: } m &= -1 \\ \text{(a)} \quad m &= -1, (-3, 2) \\ y - 2 &= -1(x + 3) \\ y - 2 &= -x - 3 \\ y &= -x - 1 \\ \text{(b)} \quad m &= 1, (-3, 2) \\ y - 2 &= 1(x + 3) \\ y &= x + 5 \end{aligned}$$

1.3.45.

$$\begin{aligned} 3x + 4y &= 7 \\ y &= -\frac{3}{4}x + \frac{7}{4} \\ \text{Slope: } m &= -\frac{3}{4} \\ \text{(a)} \quad \left(-\frac{2}{3}, \frac{7}{8}\right), m &= -\frac{3}{4} \\ y - \frac{7}{8} &= -\frac{3}{4}\left(x - \left(-\frac{2}{3}\right)\right) \\ y &= -\frac{3}{4}x + \frac{3}{8} \\ \text{(b)} \quad \left(-\frac{2}{3}, \frac{7}{8}\right), m &= \frac{4}{3} \\ y - \frac{7}{8} &= \frac{4}{3}\left(x - \left(-\frac{2}{3}\right)\right) \\ y &= \frac{4}{3}x + \frac{127}{72} \end{aligned}$$

1.3.46.

$$\begin{aligned} 5x + 3y &= 0 \\ 3y &= -5x \\ y &= -\frac{5}{3}x \\ \text{Slope: } m &= -\frac{5}{3} \\ \text{(a)} \quad m &= -\frac{5}{3}, \left(\frac{7}{8}, \frac{3}{4}\right) \end{aligned}$$

$$\begin{aligned}y - \frac{3}{4} &= -\frac{5}{3}\left(x - \frac{7}{8}\right) \\24y - 18 &= -40\left(x - \frac{7}{8}\right) \\24y - 18 &= -40x + 35 \\24y &= -40x + 53 \\y &= -\frac{5}{3}x + \frac{53}{24}\end{aligned}$$

$$\begin{aligned}\text{(b) } m &= \frac{3}{5}, \left(\frac{7}{8}, \frac{3}{4}\right) \\y - \frac{3}{4} &= \frac{3}{5}\left(x - \frac{7}{8}\right) \\40y - 30 &= 24\left(x - \frac{7}{8}\right) \\40y - 30 &= 24x - 21 \\40y &= 24x + 9 \\y &= \frac{3}{5}x + \frac{9}{40}\end{aligned}$$

1.3.47.

$$\begin{aligned}y + 5 &= 0 \\y &= -5 \\ \text{Slope: } m &= 0 \\ \text{(a) } (-2, 4), m &= 0 \\y &= 4 \\ \text{(b) } (-2, 4), m &\text{ is undefined.} \\x &= -2\end{aligned}$$

1.3.48.

$$\begin{aligned}x - 4 &= 0 \\x &= 4 \\ \text{Slope: } m &\text{ is undefined.} \\ \text{(a) } (3, -2), m &\text{ is undefined.} \\x &= 3 \\ \text{(b) } (3, -2), m &= 0 \\y &= -2\end{aligned}$$

1.3.49.

$$\begin{aligned}x - y &= 4 \\y &= x - 4 \\ \text{Slope: } m &= 1 \\ \text{(a) } (2.5, 6.8), m &= 1\end{aligned}$$

$$y - 6.8 = 1(x - 2.5)$$

$$y = x + 4.3$$

(b) $(2.5, 6.8)$, $m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$

$$y = -x + 9.3$$

1.3.50.

$$6x + 2y = 9$$

$$2y = -6x + 9$$

$$y = -3x + \frac{9}{2}$$

Slope: $m = -3$

(a) $(-3.9, -1.4)$, $m = -3$

$$y - (-1.4) = -3(x - (-3.9))$$

$$y + 1.4 = -3x - 11.7$$

$$y = -3x - 13.1$$

(b) $(-3.9, -1.4)$, $m = \frac{1}{3}$

$$y - (-1.4) = \frac{1}{3}(x - (-3.9))$$

$$y + 1.4 = \frac{1}{3}x + 1.3$$

$$y = \frac{1}{3}x - 0.1$$

1.3.51.

$(3, 0)$, $(0, 5)$

$$\frac{x}{3} + \frac{y}{5} = 1$$

$$(15)\left(\frac{x}{3} + \frac{y}{5}\right) = 1(15)$$

$$5x + 3y - 15 = 0$$

1.3.52.

$(-3, 0)$, $(0, 4)$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$$

$$4x - 3y + 12 = 0$$

1.3.53.

$$\left(-\frac{1}{6}, 0\right), \left(0, -\frac{2}{3}\right)$$

$$\frac{x}{-1/6} + \frac{y}{-2/3} = 1$$

$$6x + \frac{3}{2}y = -1$$

$$12x + 3y + 2 = 0$$

1.3.54.

$$\left(\frac{2}{3}, 0\right), (0, -2)$$

$$\frac{x}{2/3} + \frac{y}{-2} = 1$$

$$\frac{3x}{2} - \frac{y}{2} = 1$$

$$3x - y - 2 = 0$$

1.3.55.

- (a) $m = 135$. The sales are increasing 135 units per year.
- (b) $m = 0$. There is no change in sales during the year.
- (c) $m = -40$. The sales are decreasing 40 units per year.

1.3.56.

- (a) The slopes of each line segment are:

$$\frac{265.60 - 229.23}{18 - 17} = 36.37$$

$$\frac{274.52 - 260.17}{20 - 19} = 14.35$$

$$\frac{394.33 - 365.82}{22 - 21} = 28.51$$

$$\frac{260.17 - 265.60}{19 - 18} = -5.43$$

$$\frac{365.82 - 274.52}{21 - 20} = 91.3$$

$$\frac{383.29 - 394.33}{23 - 22} = -11.04$$

The sales showed the greatest increase from 2020 to 2021. The sales showed the greatest decrease from 2022 to 2023.

$$(b) m = \frac{383.29 - 229.23}{23 - 17} = \frac{154.06}{6} \approx 25.68$$

The slope of the line is about 25.68.

- (c) Sales increased at an average rate of about \$25.68 billion per year from 2017 to 2023.

1.3.57.

Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

Since the point $(0, 32)$ is the F -intercept, $b = 32$, the equation is

$$F = 1.8C + 32 \text{ or } C = \frac{5}{9}F - \frac{160}{9}.$$

1.3.58.

(a) Using the points $(1, 970)$ and $(3, 1270)$, you have $m = \frac{1270 - 970}{3 - 1} = \frac{300}{2} = 150$.

Using the point-slope form with $m = 150$ and the point $(1, 970)$, you have

$$\begin{aligned} y - y_1 &= m(t - t_1) \\ y - 970 &= 150(t - 1) \\ y - 970 &= 150t - 150 \\ y &= 150t + 820. \end{aligned}$$

(b) The slope is $m = 150$. The slope tells you the amount of increase in the weight of an average male child's brain each year.

(c) Let $t = 2$:

$$\begin{aligned} y &= 150(2) + 820 \\ y &= 300 + 820 \\ y &= 1120 \end{aligned}$$

The average brain weight at age 2 is 1120 grams.

(d) Answers will vary.

(e) Answers will vary. *Sample answer:* No. The brain stops growing after reaching a certain age.

1.3.59.

Using the points $(0, 1100)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V , you have

$$V = \frac{0 - 1100}{5 - 0} t + 1100 = -220t + 1100, 0 \leq t \leq 5.$$

1.3.60.

Using the points $(0, 30,000)$ and $(10, 2500)$, where the first coordinate represents the year t and the second coordinate represents the value V , you have

$$m = \frac{2500 - 30,000}{10 - 0} = \frac{-27,500}{10} = -2750.$$

The point $(0, 30,000)$ is the V -intercept, so $b = 30,000$. The equation is $V = -2750t + 30,000, 0 \leq t \leq 10$.

1.3.61.

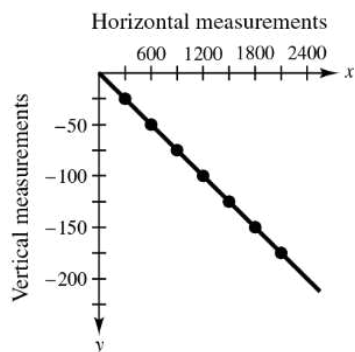
$$y = \frac{6}{100}x$$

$$y = \frac{6}{100}(200) = 12 \text{ feet}$$

1.3.62.

(a) and (b)

x	300	600	900	1200	1500	1800	2100
y	-25	-50	-75	-100	-125	-150	-175



$$(c) \ m = \frac{-50 - (-25)}{600 - 300} = \frac{-25}{300} = -\frac{1}{12}$$

$$y - (-50) = -\frac{1}{12}(x - 600)$$

$$y + 50 = -\frac{1}{12}x + 50$$

$$y = -\frac{1}{12}x$$

(d) Because $m = -\frac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\frac{1}{12} \approx 0.083 = 8.3\%$ grade

1.3.63.

(a) Total Cost = cost for fuel and maintenance + cost for operator + purchase cost

$$C = 9.5t + 11.5t + 42,000$$

$$C = 21t + 42,000$$

(b) Revenue = Rate per hour · Hours

$$R = 45t$$

(c) $P = R - C$

$$P = 45t - (21t + 42,000)$$

$$P = 24t - 42,000$$

(d) Let $P = 0$, and solve for t .

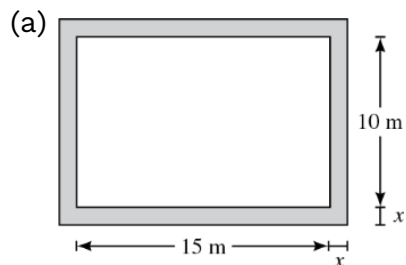
$$0 = 24t - 42,000$$

$$42,000 = 24t$$

$$1750 = t$$

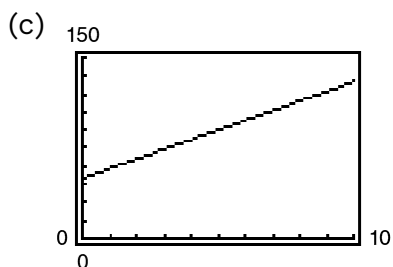
The equipment must be used 1750 hours to yield a profit of 0 dollars.

1.3.64.



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.



1.3.65.

False. The slope with the greatest magnitude corresponds to the steepest line.

1.3.66.

$$(-8, 2) \text{ and } (-1, 4): m_1 = \frac{4-2}{-1-(-8)} = \frac{2}{7}$$

$$(0, -4) \text{ and } (-7, 7): m_2 = \frac{7-(-4)}{-7-0} = \frac{11}{-7}$$

False. The lines are not parallel.

1.3.67.

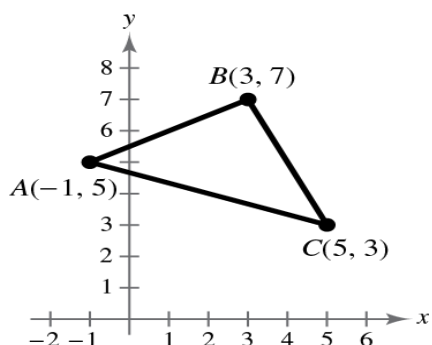
$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & d_2 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-0)^2 + (m_1-0)^2} & &= \sqrt{(1-0)^2 + (m_2-0)^2} \\ &= \sqrt{1+(m_1)^2} & &= \sqrt{1+(m_2)^2} \end{aligned}$$

Using the Pythagorean Theorem:

$$\begin{aligned} (d_1)^2 + (d_2)^2 &= (\text{distance between } (1, m_1), \text{ and } (1, m_2))^2 \\ \left(\sqrt{1+(m_1)^2}\right)^2 + \left(\sqrt{1+(m_2)^2}\right)^2 &= \left(\sqrt{(1-1)^2 + (m_2 - m_1)^2}\right)^2 \\ 1+(m_1)^2 + 1+(m_2)^2 &= (m_2 - m_1)^2 \\ (m_1)^2 + (m_2)^2 + 2 &= (m_2)^2 - 2m_1m_2 + (m_1)^2 \\ 2 &= -2m_1m_2 \\ -\frac{1}{m_2} &= m_1 \end{aligned}$$

1.3.68.

Find the slope of the line segments between the points A and B , and B and C .



$$m_{AB} = \frac{7-5}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

1.3.69.

(a) The slope is $\frac{0-(-1)}{2-0} = \frac{1}{2}$, not 2.

(b) The y -intercept is $(0, -3)$, not $(0, 4)$.

1.3.70.

(a) Matches graph (ii).

The slope is -20 , which represents the decrease in the amount of the loan each week. The y -intercept is $(0, 200)$, which represents the original amount of the loan.

(b) Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y -intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

(c) Matches graph (i).

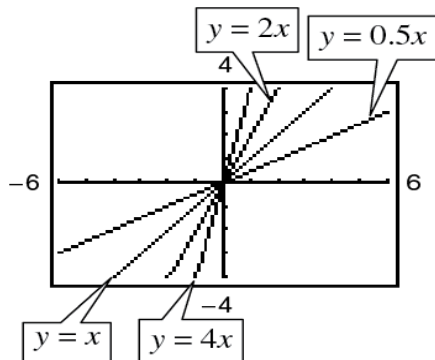
The slope is 0.32, which represents the increase in travel cost for each mile driven. The y -intercept is $(0, 32)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

(d) Matches graph (iv).

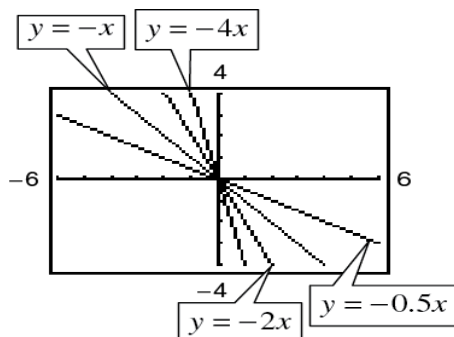
The slope is -100 , which represents the amount by which the computer depreciates each year. The y -intercept is $(0, 750)$, which represents the original purchase price.

1.3.71.

The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

1.3.72.

Because $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of -4 . The slope with the greatest magnitude corresponds to the steepest line.

1.3.73.

$$y = x^2 - x$$

$$\text{When } x = -2: y = (-2)^2 - (-2) = 4 + 2 = 6$$

$$\text{When } x = 0: y = 0^2 - 0 = 0$$

$$\text{When } x = 3: y = 3^2 - 3 = 9 - 3 = 6$$

$$\text{When } x = 6: y = 6^2 - 6 = 36 - 6 = 30$$

1.3.74.

$$y = x + 5y \Rightarrow 4y = -x \Rightarrow y = \frac{-x}{4}$$

$$\text{When } x = -2: y = \frac{-(-2)}{4} = \frac{1}{2}$$

$$\text{When } x = 0: y = \frac{-0}{4} = 0$$

$$\text{When } x = 3: y = \frac{-3}{4}$$

$$\text{When } x = 6: y = \frac{-6}{4} = \frac{-3}{2}$$

1.3.75.

$$2f = \frac{(7 - x^3)}{5} \Rightarrow f = \frac{(7 - x^3)}{10}$$

$$\text{When } x = -2: f = \frac{(7 - (-2)^3)}{10} = \frac{(7 + 8)}{10} = \frac{15}{10} = \frac{3}{2}$$

$$\text{When } x = 0: f = \frac{(7 - 0^3)}{10} = \frac{7}{10}$$

$$\text{When } x = 3: f = \frac{(7 - 3^3)}{10} = \frac{(7 - 27)}{10} = \frac{-20}{10} = -2$$

$$\text{When } x = 6: f = \frac{(7 - 6^3)}{10} = \frac{(7 - 216)}{10} = \frac{-209}{10}$$

1.3.76.

$$\frac{g}{4} = \frac{\sqrt{2+x}}{3} \Rightarrow g = \frac{4\sqrt{2+x}}{3}$$

$$\text{When } x = -2: g = \frac{4\sqrt{2-2}}{3} = \frac{4\sqrt{0}}{3} = 0$$

$$\text{When } x = 0: g = \frac{4\sqrt{2+0}}{3} = \frac{4\sqrt{2}}{3}$$

$$\text{When } x = 3: g = \frac{4\sqrt{2+3}}{3} = \frac{4\sqrt{5}}{3}$$

$$\text{When } x = 6: g = \frac{4\sqrt{2+6}}{3} = \frac{4\sqrt{8}}{3} = \frac{8\sqrt{2}}{3}$$

1.3.77.

$$\begin{aligned} 2x^3 - 5x^2 - 2x + 5 &= 0 \\ x^2(2x - 5) - (2x - 5) &= 0 \\ (x^2 - 1)(2x - 5) &= 0 \\ x &= \pm 1, \frac{5}{2} \end{aligned}$$

1.3.78.

$$\begin{aligned} x^4 - 3x^3 - x - 2 &= 1 - 2x \\ x^4 - 3x^3 + x - 3 &= 0 \\ x^3(x - 3) + (x - 3) &= 0 \\ (x^3 + 1)(x - 3) &= 0 \\ (x + 1)(x^2 - x + 1)(x - 3) &= 0 \\ x &= -1, 3 \end{aligned}$$

1.3.79.

$$\begin{aligned} x^6 + 4x^3 - 5 &= 0 \\ (x^3 + 5)(x^3 - 1) &= 0 \\ x &= -\sqrt[3]{5}, 1 \end{aligned}$$

1.3.80.

$$\begin{aligned} x^4 - 5x^2 + 4 &= 0 \\ (x^2 - 4)(x^2 - 1) &= 0 \\ (x + 2)(x - 2)(x + 1)(x - 1) &= 0 \\ x &= \pm 2, \pm 1 \end{aligned}$$

1.3.81.

$$\begin{aligned} 3\sqrt{x} - 5\sqrt{18} &= 0 \\ 3\sqrt{x} &= 5(3)\sqrt{2} \\ \sqrt{x} &= 5\sqrt{2} \\ x &= 25(2) = 50 \end{aligned}$$

1.3.82.

$$\begin{aligned}(x+1)^{3/5} - 8 &= 0 \\ (x+1)^{3/5} &= 8 \\ (x+1) &= 8^{5/3} = 2^5 = 32 \\ x &= 31\end{aligned}$$

1.3.83.

$$\begin{aligned}x &= \frac{2}{x} + 1 \\ x^2 &= 2 + x \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, -1\end{aligned}$$

1.3.84.

$$\begin{aligned}\frac{3}{x} - \frac{1}{2} &= \frac{x}{4} \\ 3 - \frac{1}{2}x &= \frac{x^2}{4} \\ 12 - 2x &= x^2 \\ x^2 + 2x - 12 &= 0 \\ x &= \frac{-2 \pm \sqrt{2^2 - 4(-12)}}{2} \\ &= \frac{-2 \pm \sqrt{52}}{2} \\ &= -1 \pm \sqrt{13}\end{aligned}$$

1.3.85.

$$\begin{aligned}x + |x-9| &= 7 \\ |x-9| &= 7-x\end{aligned}$$

If $x-9 \geq 0$, then $x-9 = 7-x \Rightarrow 2x = 16 \Rightarrow x = 8$, which is extraneous.

If $x-9 < 0$, then $-(x-9) = 7-x$, which is extraneous.

No real solution.

1.3.86.

$$|3x - 6| = x^2 - 4$$

If $3x - 6 \geq 0$, then $3x - 6 = x^2 - 4 \Rightarrow x^2 - 3x - 2 = 0$

$$\Rightarrow (x - 2)(x - 1) = 0 \Rightarrow x = 1, 2$$

($x = 1$ is extraneous.)

If $3x - 6 < 0$, then $6 - 3x = x^2 - 4 \Rightarrow x^2 + 3x - 10 = 0$

$$\Rightarrow (x + 5)(x - 2) = 0 \Rightarrow x = 2, -5$$

Solutions: $x = 2, -5$

1.3.87.

$$\begin{aligned} \frac{[(x-1)^2 + 1] - (x^2 + 1)}{x} &= \frac{(x^2 - 2x + 1 + 1) - (x^2 + 1)}{x} \\ &= \frac{-2x + 1}{x} \end{aligned}$$

1.3.88.

$$\begin{aligned} \frac{[(t+1)^2 - (t+1) - 2] - (t^2 - t - 2)}{t} &= \frac{(t^2 + 2t + 1 - t - 1 - 2) - (t^2 - t - 2)}{t} \\ &= \frac{t^2 + t - 2 - t^2 + t + 2}{t} \\ &= \frac{2t}{t} = 2, t \neq 0 \end{aligned}$$

1.3.89.

$$\begin{aligned} \frac{4 - x^2 - 3}{x - 1} + x^2 + x + 4 &= \frac{4 - x^2 - 3 + (x - 1)(x^2 + x + 4)}{x - 1} \\ &= \frac{4 - x^2 - 3 + (x^3 + x^2 + 4x - x^2 - x - 4)}{x - 1} \\ &= \frac{4 - x^2 - 3 + x^3 + 3x - 4}{x - 1} \\ &= \frac{x^3 - x^2 + 3x - 3}{x - 1} \\ &= \frac{x^2(x - 1) + 3(x - 1)}{x - 1} \\ &= \frac{(x - 1)(x^2 + 3)}{x - 1} \\ &= x^2 + 3, \quad x \neq 1 \end{aligned}$$

Alternate solution:

$$\begin{aligned}\frac{4-x^2-3}{x-1} + x^2 + x + 4 &= \frac{(1-x)(1+x)}{x-1} + x^2 + x + 4 \\ &= -1 - x + x^2 + x + 4, \quad x \neq 1 \\ &= x^2 + 3, \quad x \neq 1\end{aligned}$$

1.3.90.

$$\begin{aligned}\frac{x^3+x}{x^2-4} - \frac{5}{2(x+2)} + \frac{5}{2(x-2)} + 2 &= \frac{2(x^3+x) - 5(x-2) + 5(x+2) + 4(x^2-4)}{2(x^2-4)} \\ &= \frac{2x^3 + 2x - 5x + 10 + 5x + 10 + 4x^2 - 16}{2(x^2-4)} \\ &= \frac{2x^3 + 4x^2 + 2x + 4}{2(x^2-4)} \\ &= \frac{x^3 + 2x^2 + x + 2}{x^2-4} \\ &= \frac{x^2(x+2) + (x+2)}{x^2-4} \\ &= \frac{(x+2)(x^2+1)}{(x+2)(x-2)} \\ &= \frac{x^2+1}{x-2}, \quad x \neq -2\end{aligned}$$

SECTION 1.4 FUNCTIONS

1.4.1.

independent; dependent

1.4.2.

difference quotient

1.4.3.

A relation is any set of ordered pairs. A function is a relation in which no two ordered pairs have the same first component and different second components.

1.4.4.

For $g(x) = 4x - 5$, to find $g(x + 2)$, substitute $x + 2$ for x , and simplify:

$$\begin{aligned} g(x + 2) &= 4(x + 2) - 5 \\ &= 4x + 3. \end{aligned}$$

1.4.5.

The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

1.4.6.

The domain of $y = \sqrt{4 - x^2}$ is determined by solving the inequality $4 - x^2 \geq 0$.

1.4.7.

Yes, the relationship is a function. Each domain value is matched with exactly one range value.

1.4.8.

No, the relationship is not a function. The domain value of -1 is matched with two output values.

1.4.9.

No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

1.4.10.

Yes, the table does represent a function. Each input value is matched with exactly one output value.

Input, x	-2	0	2	4	6
Output, y	1	1	1	1	1

1.4.11.

$$x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$$

No, y is not a function of x .

1.4.12.

$$x^2 - y = 9 \Rightarrow y = x^2 - 9$$

Yes, y is a function of x .

1.4.13.

$$y = \sqrt{16 - x^2}$$

Yes, y is a function of x .

1.4.14.

$$y = \sqrt{x + 5}$$

Yes, y is a function of x .

1.4.15.

$$y = 4 - |x|$$

Yes, y is a function of x .

1.4.16.

$$|y| = 4 - x \Rightarrow y = 4 - x \text{ or } y = -(4 - x)$$

No, y is not a function of x .

1.4.17.

$$y = -75 \text{ or } y = -75 + 0x$$

Yes, y is a function of x .

1.4.18.

$$x - 1 = 0$$

$$x = 1$$

No, this is not a function of x .

1.4.19.

$$g(t) = 4t^2 - 3t + 5$$

$$(a) \ g(2) = 4(2)^2 - 3(2) + 5 = 16 - 6 + 5 = 15$$

$$(b) \ g(-1) = 4(-1)^2 - 3(-1) + 5 = 4 + 3 + 5 = 12$$

$$\begin{aligned} \text{(c)} \quad g(t+2) &= 4(t+2)^2 - 3(t+2) + 5 \\ &= 4(t^2 + 4t + 4) - 3t - 6 + 5 \\ &= 4t^2 + 13t + 15 \end{aligned}$$

1.4.20.

$$V(r) = \frac{4}{3} \pi r^3$$

$$\text{(a)} \quad V(3) = \frac{4}{3} \pi (3)^3 = \frac{4}{3} \pi (27) = 36\pi$$

$$\text{(b)} \quad V\left(\frac{3}{2}\right) = \frac{4}{3} \pi \left(\frac{3}{2}\right)^3 = \frac{4}{3} \pi \left(\frac{27}{8}\right) = \frac{9}{2} \pi$$

$$\text{(c)} \quad V(2r) = \frac{4}{3} \pi (2r)^3 = \frac{4}{3} \pi (8r^3) = \frac{32}{3} \pi r^3$$

1.4.21.

$$f(y) = 3 - \sqrt{y}$$

$$\text{(a)} \quad f(4) = 3 - \sqrt{4} = 1$$

$$\text{(b)} \quad f(0.25) = 3 - \sqrt{0.25} = 2.5$$

$$\text{(c)} \quad f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$$

1.4.22.

$$f(x) = \sqrt{x+8} + 2$$

$$\text{(a)} \quad f(-8) = \sqrt{(-8)+8} + 2 = 2$$

$$\text{(b)} \quad f(1) = \sqrt{(1)+8} + 2 = 5$$

$$\text{(c)} \quad f(x-8) = \sqrt{(x-8)+8} + 2 = \sqrt{x} + 2$$

1.4.23.

$$q(x) = \frac{1}{x^2 - 9}$$

$$\text{(a)} \quad q(0) = \frac{1}{0^2 - 9} = -\frac{1}{9}$$

(b) $q(3) = \frac{1}{3^2 - 9}$ is undefined.

(c) $q(y+3) = \frac{1}{(y+3)^2 - 9} = \frac{1}{y^2 + 6y}$

1.4.24.

$$q(t) = \frac{2t^2 + 3}{t^2}$$

(a) $q(2) = \frac{2(2)^2 + 3}{(2)^2} = \frac{8 + 3}{4} = \frac{11}{4}$

(b) $q(0) = \frac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \frac{2(-x)^2 + 3}{(-x)^2} = \frac{2x^2 + 3}{x^2}$

1.4.25.

$$f(x) = \frac{|x|}{x}$$

(a) $f(2) = \frac{|2|}{2} = 1$

(b) $f(-2) = \frac{|-2|}{-2} = -1$

(c) $f(x-1) = \frac{|x-1|}{x-1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

1.4.26.

$$f(x) = |x| + 4$$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

1.4.27.

$$\begin{aligned}f(x) &= 15 - 3x \\15 - 3x &= 0 \\3x &= 15 \\x &= 5\end{aligned}$$

1.4.28.

$$\begin{aligned}f(x) &= 4x + 6 \\4x + 6 &= 0 \\4x &= -6 \\x &= -\frac{3}{2}\end{aligned}$$

1.4.29.

$$\begin{aligned}f(x) &= \frac{3x - 4}{5} \\\frac{3x - 4}{5} &= 0 \\3x - 4 &= 0 \\x &= \frac{4}{3}\end{aligned}$$

1.4.30.

$$\begin{aligned}f(x) &= \frac{12 - x^2}{8} \\\frac{12 - x^2}{8} &= 0 \\x^2 &= 12 \\x &= \pm\sqrt{12} = \pm 2\sqrt{3}\end{aligned}$$

1.4.31.

$$\begin{aligned}f(x) &= x^2 - 81 \\x^2 - 81 &= 0 \\x^2 &= 81 \\x &= \pm 9\end{aligned}$$

1.4.32.

$$\begin{aligned} f(x) &= x^2 - 6x - 16 \\ x^2 - 6x - 16 &= 0 \\ (x - 8)(x + 2) &= 0 \\ x - 8 = 0 &\Rightarrow x = 8 \\ x + 2 = 0 &\Rightarrow x = -2 \end{aligned}$$

1.4.33.

$$\begin{aligned} f(x) &= x^3 - x \\ x^3 - x &= 0 \\ x(x^2 - 1) &= 0 \\ x(x + 1)(x - 1) &= 0 \\ x = 0, x = -1, \text{ or } x = 1 \end{aligned}$$

1.4.34.

$$\begin{aligned} f(x) &= x^3 - x^2 - 3x + 3 \\ x^3 - x^2 - 3x + 3 &= 0 \\ x^2(x - 1) - 3(x - 1) &= 0 \\ (x - 1)(x^2 - 3) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x^2 - 3 = 0 &\Rightarrow x = \pm\sqrt{3} \end{aligned}$$

1.4.35.

$$\begin{aligned} f(x) &= g(x) \\ x^2 &= x + 2 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \\ x - 2 = 0 \quad x + 1 = 0 \\ x = 2 \quad x &= -1 \end{aligned}$$

1.4.36.

$$\begin{aligned} f(x) &= g(x) \\ x^2 + 2x + 1 &= 5x + 19 \\ x^2 - 3x - 18 &= 0 \\ (x - 6)(x + 3) &= 0 \\ x - 6 = 0 \quad x + 3 = 0 \\ x = 6 \quad x &= -3 \end{aligned}$$

1.4.37.

$$\begin{aligned} f(x) &= g(x) \\ x^4 - 2x^2 &= 2x^2 \\ x^4 - 4x^2 &= 0 \\ x^2(x^2 - 4) &= 0 \\ x^2(x + 2)(x - 2) &= 0 \\ x^2 = 0 &\Rightarrow x = 0 \\ x + 2 = 0 &\Rightarrow x = -2 \\ x - 2 = 0 &\Rightarrow x = 2 \end{aligned}$$

1.4.38.

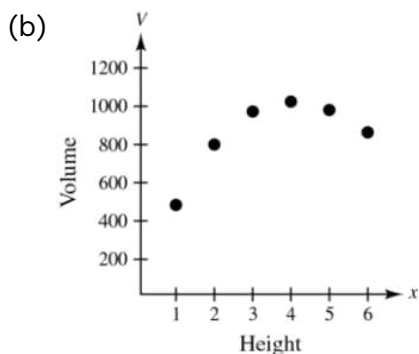
$$\begin{aligned} f(x) &= g(x) \\ \sqrt{x} - 4 &= 2 - x \\ x + \sqrt{x} - 6 &= 0 \\ (\sqrt{x} + 3)(\sqrt{x} - 2) &= 0 \\ \sqrt{x} + 3 = 0 &\Rightarrow \sqrt{x} = -3, \text{ which is a contradiction, since } \sqrt{x} \text{ represents the} \\ &\text{principal square root.} \\ \sqrt{x} - 2 = 0 &\Rightarrow \sqrt{x} = 2 \Rightarrow x = 4 \end{aligned}$$

1.4.39.

(a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.



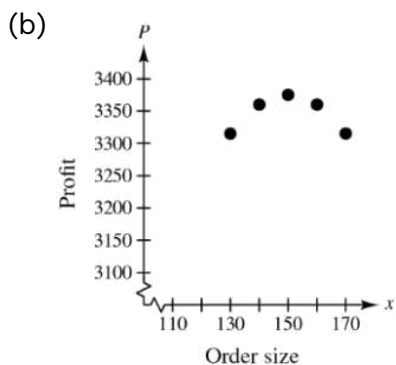
V is a function of x .

$$V = x(24 - 2x)^2$$

Domain: $0 < x < 12$

1.4.40.

(a) The maximum profit is \$3375.



Yes, P is a function of x .

Profit = Revenue - Cost

$$\begin{aligned}
 &= \left(\begin{array}{c} \text{price} \\ \text{per unit} \end{array} \right) \left(\begin{array}{c} \text{number} \\ \text{of units} \end{array} \right) - \left(\begin{array}{c} \text{cost} \end{array} \right) \left(\begin{array}{c} \text{number} \\ \text{of units} \end{array} \right) \\
 &= [90 - (x - 100)(0.15)]x - 60x, \quad x > 100 \\
 &= (90 - 0.15x + 15)x - 60x \\
 &= (105 - 0.15x)x - 60x \\
 &= 105x - 0.15x^2 - 60x \\
 &= 45x - 0.15x^2, \quad x > 100
 \end{aligned}$$

1.4.41.

$$y = -\frac{1}{10}x^2 + 3x + 6$$

$$y(25) = -\frac{1}{10}(25)^2 + 3(25) + 6 = 18.5 \text{ feet}$$

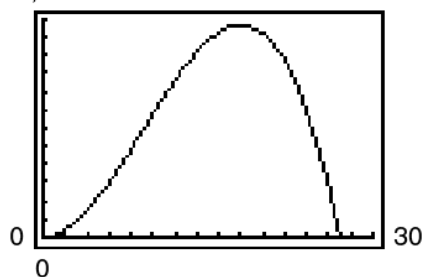
If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 18.5 feet.

1.4.42.

(a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2y$ where $4x + y = 108$. So, $y = 108 - 4x$ and
 $V = x^2(108 - 4x) = 108x^2 - 4x^3$.

Domain: $0 < x < 27$

(b) 12,000

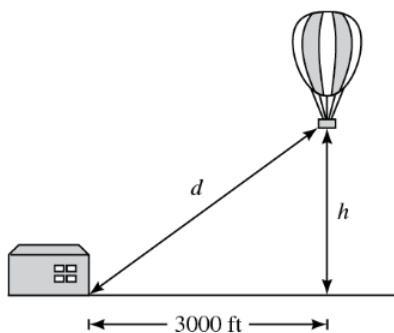


(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y , use the equation $y = 108 - 4x$.

$$y = 108 - 4x = 108 - 4(18) - 72 = 36$$

1.4.43.

(a)



(b) $(3000)^2 + h^2 = d^2$

$$h = \sqrt{d^2 - (3000)^2}$$

Domain: $d \geq 3000$ (because both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

1.4.44.

$$p(t) = \begin{cases} 0.38t + 81.2, & 17 \leq t \leq 21 \\ 1.35t + 57.5, & 21 \leq t \leq 23 \end{cases}$$

For 2017 to 2020, use the first equation.

$$2017: p(17) = 0.38(17) + 81.2 = 87.66\%$$

$$2018: p(18) = 0.38(18) + 81.2 = 88.04\%$$

$$2019: p(19) = 0.38(19) + 81.2 = 88.42\%$$

$$2020: p(20) = 0.38(20) + 81.2 = 88.80\%$$

For 2021 through 2023, use the second equation.

$$2021: p(21) = 1.35(21) + 57.5 = 85.85\%$$

$$2022: p(22) = 1.35(22) + 57.5 = 87.20\%$$

$$2023: p(23) = 1.35(23) + 57.5 = 88.55\%$$

1.4.45.

$$p(t) = \begin{cases} 4.011t^2 - 76.89t + 586.7, & 7 \leq t < 12 \\ 14.94t + 70.0, & 12 \leq t < 17 \\ 8.352t^2 - 304.97t - 3098.5, & 17 \leq t < 22 \\ 0.234t^2 - 16.93t - 692.5, & 22 \leq t \leq 24 \end{cases}$$

For 2007 through 2011, use the first equation.

$$2007: p(7) = 4.011(7)^2 - 76.89(7) + 586.7 = 245.009 \Rightarrow \$245,009$$

$$2008: p(8) = 4.011(8)^2 - 76.89(8) + 586.7 = 228.284 \Rightarrow \$228,284$$

$$2009: p(9) = 4.011(9)^2 - 76.89(9) + 586.7 = 219.581 \Rightarrow \$219,581$$

$$2010: p(10) = 4.011(10)^2 - 76.89(10) + 586.7 = 218.900 \Rightarrow \$218,900$$

$$2011: p(11) = 4.011(11)^2 - 76.89(11) + 586.7 = 226.241 \Rightarrow \$226,241$$

For 2012 through 2016, use the second equation.

$$2012: p(12) = 14.94(12) + 70.0 = 249.28 \Rightarrow \$249,280$$

$$2013: p(13) = 14.94(13) + 70.0 = 264.22 \Rightarrow \$264,220$$

$$2014: p(14) = 14.94(14) + 70.0 = 279.16 \Rightarrow \$279,160$$

$$2015: p(15) = 14.94(15) + 70.0 = 294.1 \Rightarrow \$294,100$$

$$2016: p(16) = 14.94(16) + 70.0 = 309.04 \Rightarrow \$309,040$$

For 2017 through 2021, use the third equation.

$$2017 : p(17) = 8.352(17)^2 - 304.97(17) + 3098.5 = 327.738 \Rightarrow \$327,738$$

$$2018 : p(18) = 8.352(18)^2 - 304.97(18) + 3098.5 = 315.088 \Rightarrow \$315,088$$

$$2019 : p(19) = 8.352(19)^2 - 304.97(19) + 3098.5 = 319.142 \Rightarrow \$319,142$$

$$2020 : p(20) = 8.352(20)^2 - 304.97(20) + 3098.5 = 339.9 \Rightarrow \$339,900$$

$$2021 : p(21) = 8.352(21)^2 - 304.97(21) + 3098.5 = 377.362 \Rightarrow \$377,362$$

For 2022 through 2024, use the fourth equation.

$$2022 : p(22) = 0.234(22)^2 + 16.93(22) + 292.5 = 433.296 \Rightarrow \$433,296$$

$$2023 : p(23) = 0.234(23)^2 + 16.93(23) + 692.5 = 426.896 \Rightarrow \$426,896$$

$$2024 : p(24) = 0.234(24)^2 + 16.93(24) + 692.5 = 420.964 \Rightarrow \$420,964$$

1.4.46.

$$F(y) = 149.76\sqrt{10}y^{5/2}$$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

$$(c) \quad 1,000,000 = 149.76\sqrt{10}y^{5/2}$$

$$\frac{1,000,000}{149.76\sqrt{10}} = y^{5/2}$$

$$2111.56 \approx y^{5/2}$$

$$21.37 \text{ feet} \approx y$$

1.4.47.

$$(a) \quad R = n(\text{rate}) = n[8.00 - 0.05(n - 80)], n \geq 80$$

$$R = 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20}, n \geq 80$$

(b)

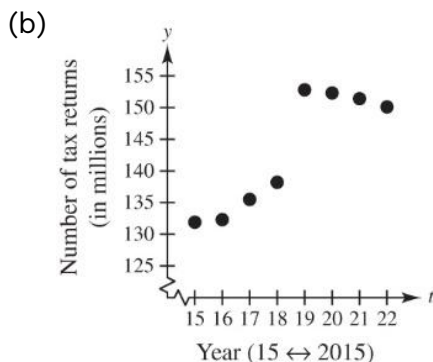
n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

1.4.48.

$$(a) \frac{f(2022) - f(2015)}{2022 - 2015} = \frac{150.1 - 131.9}{7} = \frac{18.2}{7} = 2.6$$

The number of e-filed tax returns increased at an average rate of 2,600,000 per year from 2015 to 2022.



(c) *Sample answer:* $N = 2.6t + 92.9$, $15 \leq t \leq 22$

(d) *Sample answer:*

t	15	16	17	18	19	20	21	22
N	131.9	134.5	137.1	139.7	142.3	144.9	147.5	150.1

(e) The model does not appear to be a good fit for the actual data.

(f) Using technology, $N = 3.43t + 79.7$. The models are similar.

1.4.49.

$$f(x) = x^2 - 2x + 4$$

$$\begin{aligned} f(2+h) &= (2+h)^2 - 2(2+h) + 4 \\ &= 4 + 4h + h^2 - 4 - 2h + 4 \\ &= h^2 + 2h + 4 \end{aligned}$$

$$f(2) = (2)^2 - 2(2) + 4 = 4$$

$$f(2+h) - f(2) = h^2 + 2h$$

$$\frac{f(2+h) - f(2)}{h} = \frac{h^2 + 2h}{h} = h + 2, h \neq 0$$

1.4.50.

$$\begin{aligned}
 f(x) &= 5x - x^2 \\
 f(5+h) &= 5(5+h) - (5+h)^2 \\
 &= 25 + 5h - (25 + 10h + h^2) \\
 &= 25 + 5h - 25 - 10h - h^2 \\
 &= -h^2 - 5h \\
 f(5) &= 5(5) - (5)^2 \\
 &= 25 - 25 = 0 \\
 \frac{f(5+h) - f(5)}{h} &= \frac{-h^2 - 5h}{h} \\
 &= \frac{-h(h+5)}{h} = -(h+5), h \neq 0
 \end{aligned}$$

1.4.51.

$$\begin{aligned}
 f(x) &= x^3 + 3x \\
 f(x+h) &= (x+h)^3 + 3(x+h) \\
 &= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h \\
 \frac{f(x+h) - f(x)}{h} &= \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h} \\
 &= \frac{h(3x^2 + 3xh + h^2 + 3)}{h} \\
 &= 3x^2 + 3xh + h^2 + 3, h \neq 0
 \end{aligned}$$

1.4.52.

$$\begin{aligned}
 f(x) &= 4x^3 - 2x \\
 f(x+h) &= 4(x+h)^3 - 2(x+h) \\
 &= 4(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h \\
 &= 4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h \\
 \frac{f(x+h) - f(x)}{h} &= \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 2x - 2h) - (4x^3 - 2x)}{h} \\
 &= \frac{12x^2h + 12xh^2 + 4h^3 - 2h}{h} \\
 &= \frac{h(12x^2 + 12xh + 4h^2 - 2)}{h} \\
 &= 12x^2 + 12xh + 4h^2 - 2, h \neq 0
 \end{aligned}$$

1.4.53.

$$\begin{aligned}
 g(x) &= \frac{1}{x^2} \\
 \frac{g(x) - g(3)}{x - 3} &= \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3} \\
 &= \frac{9 - x^2}{9x^2(x - 3)} \\
 &= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)} \\
 &= -\frac{x + 3}{9x^2}, x \neq 3
 \end{aligned}$$

1.4.54.

$$\begin{aligned}
 f(t) &= \frac{1}{t - 2} \\
 f(1) &= \frac{1}{1 - 2} = -1 \\
 \frac{f(t) - f(1)}{t - 1} &= \frac{\frac{1}{t - 2} - (-1)}{t - 1} \\
 &= \frac{1 + (t - 2)}{(t - 2)(t - 1)} \\
 &= \frac{(t - 1)}{(t - 2)(t - 1)} \\
 &= \frac{1}{t - 2}, t \neq 1
 \end{aligned}$$

1.4.55.

$$\begin{aligned}
 f(x) &= \sqrt{5x} \\
 \frac{f(x) - f(5)}{x - 5} &= \frac{\sqrt{5x} - 5}{x - 5}, x \neq 5
 \end{aligned}$$

1.4.56.

$$\begin{aligned}
 f(x) &= x^{2/3} + 1 \\
 f(8) &= 8^{2/3} + 1 = 5 \\
 \frac{f(x) - f(8)}{x - 8} &= \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8}, x \neq 8
 \end{aligned}$$

1.4.57.

False. The equation $y^2 = x^2 + 4$ is a relation between x and y . However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

1.4.58.

True. A function is a relation by definition.

1.4.59.

False. The range is $[-1, \infty)$.

1.4.60.

True. The set represents a function. Each x -value is mapped to exactly one y -value.

1.4.61.

By plotting the points, you have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

1.4.62.

By plotting the data, you can see that they represent a line, or $f(x) = cx$.

Because $(0, 0)$ and $(1, \frac{1}{4})$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

1.4.63.

Because the function is undefined at 0, you have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

1.4.64.

By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y -values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

1.4.65.

The domain of $f(x) = \sqrt{x-1}$ includes $x = 1$, $x \geq 1$ and the domain of

$g(x) = \frac{1}{\sqrt{x-1}}$ does not include $x = 1$ because you cannot divide by 0. The

domain of $g(x) = \frac{1}{\sqrt{x-1}}$ is $x > 1$. So, the functions do not have the same domain.

1.4.66.

- (a) The height h is a function of t because for each value of t there is a corresponding value of h for $0 \leq t \leq 2.6$.
- (b) Using the graph when $t = 0.5$, $h \approx 20$ feet and when $t = 1.25$, $h \approx 28$ feet.
- (c) The domain of h is approximately $0 \leq t \leq 2.6$.
- (d) No, the time t is not a function of the height h because some values of h correspond to more than one value of t .

1.4.67.

No; x is the independent variable, f is the name of the function.

1.4.68.

$$\begin{aligned} f(x) &= x^{2/3} + 1 \\ f(8) &= 8^{2/3} + 1 = 5 \\ \frac{f(x) - f(8)}{x - 8} &= \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8}, \quad x \neq 8 \end{aligned}$$

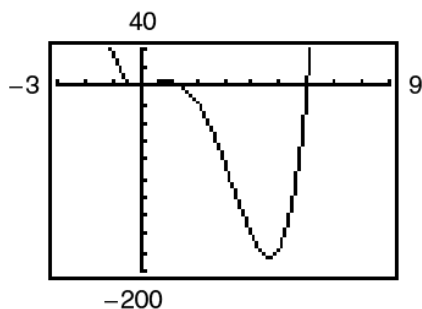
1.4.69.

x -intercepts: $(-2, 0), (1, 0)$
 y -intercept: $(0, -2)$

1.4.70.

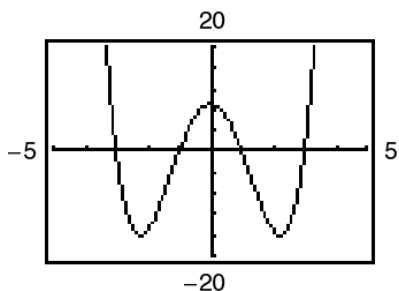
x -intercepts: $(-1, 0), (3, 0)$
 y -intercept: $(0, 3)$

1.4.71.



x -intercepts: $(0, 0), (\frac{3}{2}, 0), (6, 0)$

1.4.72.



x-intercepts: $(\pm 1, 0)$, $(\pm 3, 0)$

1.4.73.

$$y = x^2 - 3.61x + 2.86$$

Using technology, the x-intercepts are $(1.17, 0)$ and $(2.44, 0)$.

1.4.74.

$$y = x^3 + 1.27x^2 + 5.49$$

Using technology, the x-intercept is $(-2.30, 0)$.

1.4.75.

$$\frac{(3^2 + 4) - (1^2 + 4)}{3 - 1} = \frac{13 - 5}{2} = \frac{8}{2} = 4$$

1.4.76.

$$\frac{9^{3/2} - 4^{3/2}}{9 - 4} = \frac{27 - 8}{5} = \frac{19}{5}$$

1.4.77.

$$\frac{\frac{1}{3} - \frac{1}{2}}{6 - 4} = \frac{-\frac{1}{6}}{2} = -\frac{1}{12}$$

1.4.78.

$$\frac{-\sqrt{\frac{1}{9}} + \sqrt{\frac{1}{4}}}{9 - 4} = \frac{-\frac{1}{3} + \frac{1}{2}}{5} = \frac{\frac{1}{6}}{5} = \frac{1}{30}$$

SECTION 1.5 ANALYZING GRAPHS OF FUNCTIONS

1.5.1.

zeros

1.5.2.

decreasing

1.5.3.

average rate of change; secant

1.5.4.

odd

1.5.5.

No. If a vertical line intersects the graph more than once, then it does not represent y as a function of x .

1.5.6.

If $f(2) \geq f(x)$ for all x in $(0, 3)$, the $(2, f(2))$ is a relative maximum of f .

1.5.7.

Domain: $[-2, 2]$; Range: $[-1, 8]$

(a) $f(-1) = -1$

(b) $f(0) = 0$

(c) $f(1) = -1$

(d) $f(2) = 8$

1.5.8.

Domain: $[-1, \infty)$; Range: $(-\infty, 7]$

(a) $f(-1) = 4$

(b) $f(0) = 3$

(c) $f(1) = 6$

(d) $f(3) = 0$

1.5.9.

Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

1.5.10.

Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

(a) $f(-2) = -3$

(b) $f(1) = 0$

(c) $f(0) = 1$

(d) $f(2) = -3$

1.5.11.

A vertical line intersects the graph at most once, so y is a function of x .

1.5.12.

y is not a function of x . Some vertical lines intersect the graph twice.

1.5.13.

A vertical line intersects the graph more than once, so y is not a function of x .

1.5.14.

A vertical line intersects the graph at most once, so y is a function of x .

1.5.15.

$$f(x) = 2x^2 - 7x - 30$$

$$2x^2 - 7x - 30 = 0$$

$$(2x + 5)(x - 6) = 0$$

$$2x + 5 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = -\frac{5}{2} \quad x = 6$$

1.5.16.

$$f(x) = 3x^2 + 22x - 16$$

$$3x^2 + 22x - 16 = 0$$

$$(3x - 2)(x + 8) = 0$$

$$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$$

$$x + 8 = 0 \Rightarrow x = -8$$

1.5.17.

$$f(x) = \frac{1}{3}x^3 - 2x$$

$$\frac{1}{3}x^3 - 2x = 0$$

$$(3)\left(\frac{1}{3}x^3 - 2x\right) = 0(3)$$

$$x^3 - 6x = 0$$

$$x(x^2 - 6) = 0$$

$$x = 0 \quad \text{or} \quad x^2 - 6 = 0$$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$

1.5.18.

$$f(x) = -25x^4 + 9x^2$$

$$-x^2(25x^2 - 9) = 0$$

$$-x^2 = 0 \quad \text{or} \quad 25x^2 - 9 = 0$$

$$x = 0 \quad 25x^2 = 9$$

$$x^2 = \frac{9}{25}$$

$$x = \pm\frac{3}{5}$$

1.5.19.

$$f(x) = \sqrt{2x} - 1$$

$$\sqrt{2x} - 1 = 0$$

$$\sqrt{2x} = 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

1.5.20.

$$\begin{aligned} f(x) &= \sqrt{3x+2} \\ \sqrt{3x+2} &= 0 \\ 3x+2 &= 0 \\ -\frac{2}{3} &= x \end{aligned}$$

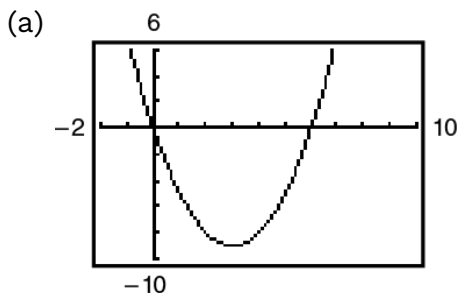
1.5.21.

$$\begin{aligned} f(x) &= \frac{x+3}{2x^2-6} \\ \frac{x+3}{2x^2-6} &= 0 \\ x+3 &= 0 \\ x &= -3 \end{aligned}$$

1.5.22.

$$\begin{aligned} f(x) &= \frac{x^2-9x+14}{4x} \\ \frac{x^2-9x+14}{4x} &= 0 \\ (x-7)(x-2) &= 0 \\ x-7 &= 0 \Rightarrow x=7 \\ x-2 &= 0 \Rightarrow x=2 \end{aligned}$$

1.5.23.

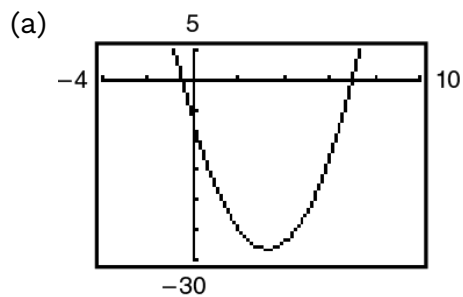


Zeros: $x = 0, 6$

(b)

$$\begin{aligned} f(x) &= x^2 - 6x \\ x^2 - 6x &= 0 \\ x(x-6) &= 0 \\ x &= 0 \Rightarrow x=0 \\ x-6 &= 0 \Rightarrow x=6 \end{aligned}$$

1.5.24.



Zeros: $x = -0.5, 7$

(b) $f(x) = 2x^2 - 13x - 7$

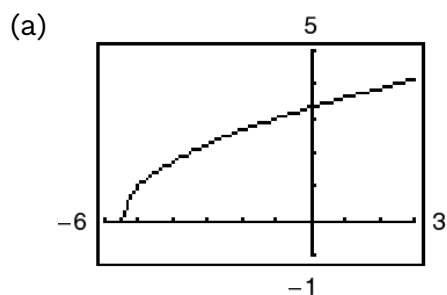
$$2x^2 - 13x - 7 = 0$$

$$(2x + 1)(x - 7) = 0$$

$$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

$$x - 7 = 0 \Rightarrow x = 7$$

1.5.25.



Zero: $x = -5.5$

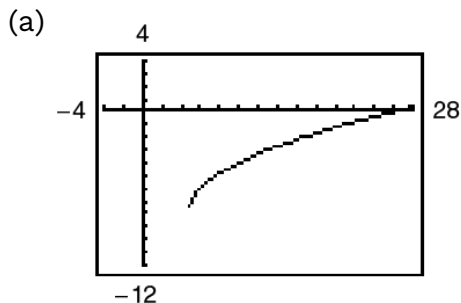
(b) $f(x) = \sqrt{2x + 11}$

$$\sqrt{2x + 11} = 0$$

$$2x + 11 = 0$$

$$x = -\frac{11}{2}$$

1.5.26.



Zero: $x = 26$

(b) $f(x) = \sqrt{3x - 14} - 8$

$$\begin{aligned}\sqrt{3x - 14} - 8 &= 0 \\ \sqrt{3x - 14} &= 8 \\ 3x - 14 &= 64 \\ x &= 26\end{aligned}$$

1.5.27.

$$f(x) = -\frac{1}{2}x^3$$

The function is decreasing on $(-\infty, \infty)$.

1.5.28.

$$f(x) = x^2 - 4x$$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

1.5.29.

$$f(x) = \sqrt{x^2 - 1}$$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

1.5.30.

$$f(x) = x^3 - 3x^2 + 2$$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

1.5.31.

$$f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$$

The function is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

1.5.32.

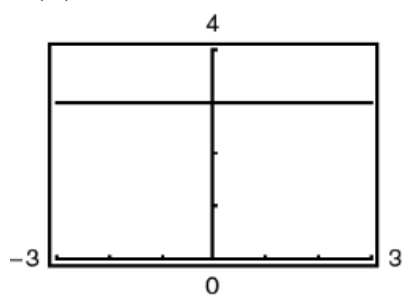
$$f(x) = \begin{cases} x+3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x+1, & x > 2 \end{cases}$$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$.

The function is constant on $(0, 2)$.

1.5.33.

$$f(x) = 3$$

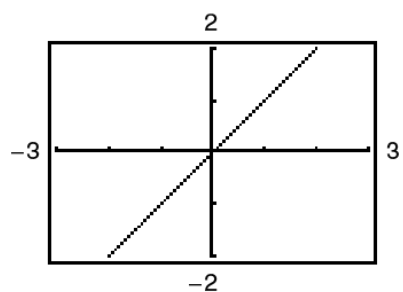


Constant on $(-\infty, \infty)$

x	-2	-1	0	1	2
$f(x)$	3	3	3	3	3

1.5.34.

$$g(x) = x$$

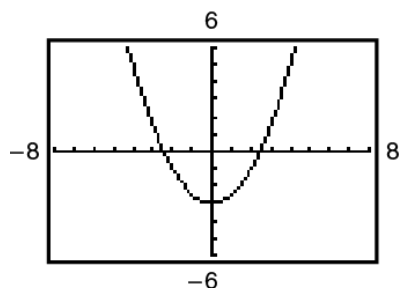


Increasing on $(-\infty, \infty)$

x	-2	-1	0	1	2
$g(x)$	-2	-1	0	1	2

1.5.35.

$$g(x) = \frac{1}{2}x^2 - 3$$



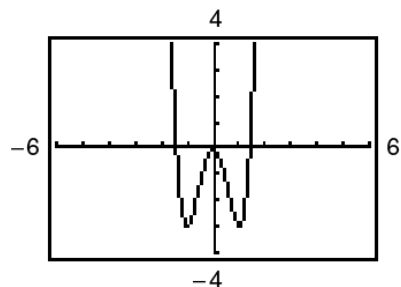
Decreasing on $(-\infty, 0)$.

Increasing on $(0, \infty)$.

x	-2	-1	0	1	2
$g(x)$	-1	$-\frac{5}{2}$	-3	$-\frac{5}{2}$	-1

1.5.36.

$$f(x) = 3x^4 - 6x^2$$

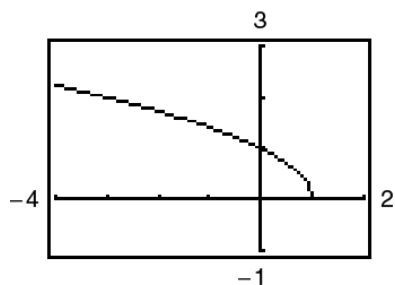


Increasing on $(-1, 0)$, $(1, \infty)$; Decreasing on $(-\infty, -1)$, $(0, 1)$

x	-2	-1	0	1	2
$f(x)$	24	-3	0	-3	24

1.5.37.

$$f(x) = \sqrt{1-x}$$

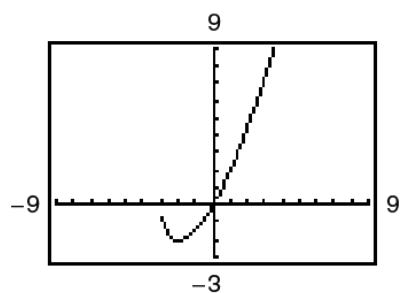


Decreasing on $(-\infty, 1)$

x	-3	-2	-1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

1.5.38.

$$f(x) = x\sqrt{x+3}$$

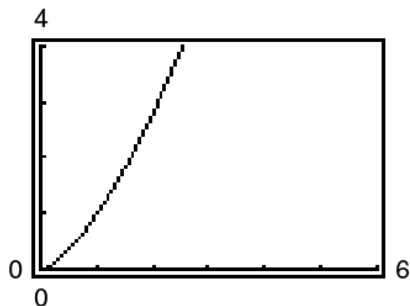


Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

x	-3	-2	-1	0	1
$f(x)$	0	-2	-1.414	0	2

1.5.39.

$$f(x) = x^{3/2}$$

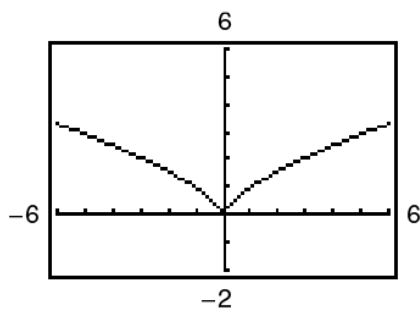


Increasing on $(0, \infty)$

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

1.5.40.

$$f(x) = x^{2/3}$$

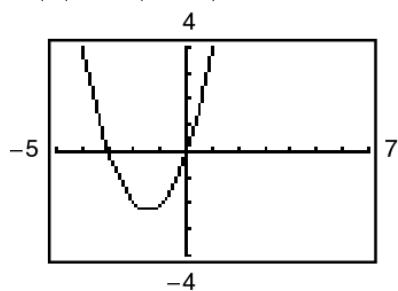


Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

x	-2	-1	0	1	2
$f(x)$	1.59	1	0	1	1.59

1.5.41.

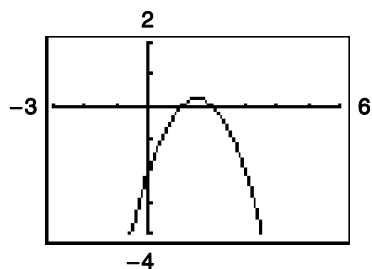
$$f(x) = x(x + 3)$$



Relative minimum: $(-1.5, -2.25)$

1.5.42.

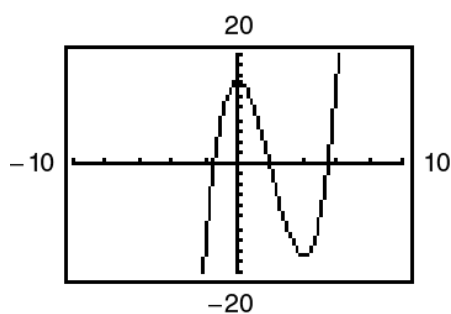
$$f(x) = -x^2 + 3x - 2$$



Relative maximum: $(1.5, 0.25)$

1.5.43.

$$h(x) = x^3 - 6x^2 + 15$$

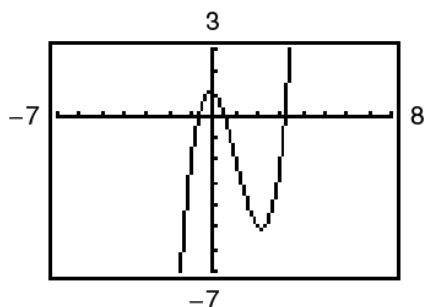


Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

1.5.44.

$$f(x) = x^3 - 3x^2 - x + 1$$

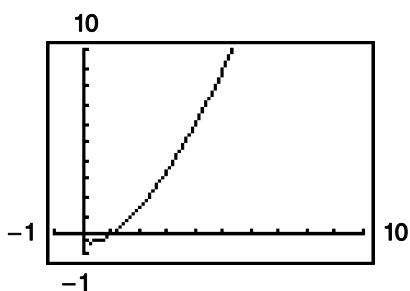


Relative maximum: $(-0.15, 1.08)$

Relative minimum: $(2.15, -5.08)$

1.5.45.

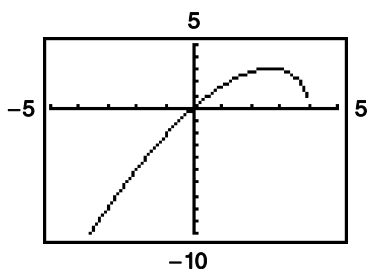
$$h(x) = (x-1)\sqrt{x}$$



Relative minimum: $(0.33, -0.38)$

1.5.46.

$$g(x) = x\sqrt{4-x}$$

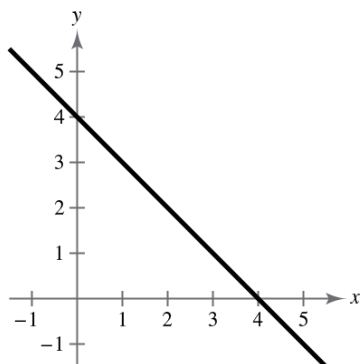


Relative maximum: $(2.67, 3.08)$

1.5.47.

$$f(x) = 4 - x$$

$$f(x) \geq 0 \text{ on } (-\infty, 4]$$



1.5.48.

$$f(x) = 4x + 2$$

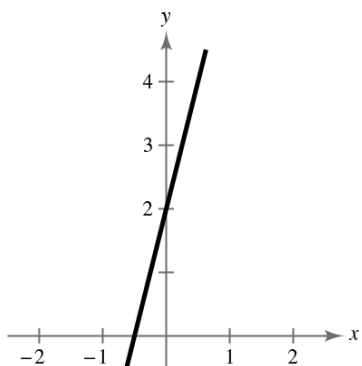
$$f(x) \geq 0 \text{ on } \left[-\frac{1}{2}, \infty\right)$$

$$4x + 2 \geq 0$$

$$4x \geq -2$$

$$x \geq -\frac{1}{2}$$

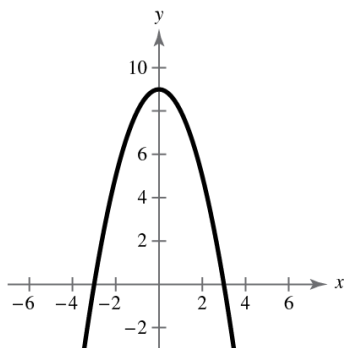
$$\left[-\frac{1}{2}, \infty\right)$$



1.5.49.

$$f(x) = 9 - x^2$$

$$f(x) \geq 0 \text{ on } [-3, 3]$$



1.5.50.

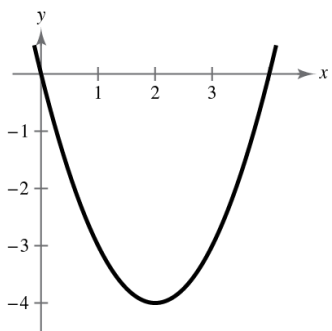
$$f(x) = x^2 - 4x$$

$$f(x) \geq 0 \text{ on } (-\infty, 0] \text{ and } [4, \infty)$$

$$x^2 - 4x \geq 0$$

$$x(x - 4) \geq 0$$

$$(-\infty, 0], [4, \infty)$$



1.5.51.

$$f(x) = \sqrt{x - 1}$$

$$f(x) \geq 0 \text{ on } [1, \infty)$$

$$\sqrt{x - 1} \geq 0$$

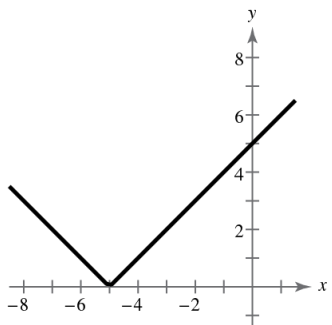
$$x - 1 \geq 0$$

$$x \geq 1$$

$$[1, \infty)$$

1.5.52.

$$f(x) = |x + 5|$$



$f(x)$ is always greater than or equal to 0. $f(x) \geq 0$ for all x .

$$(-\infty, \infty)$$

1.5.53.

$$f(x) = -2x + 15$$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2 .

1.5.54.

$$f(x) = x^2 - 2x + 8$$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4 .

1.5.55.

$$f(x) = x^3 - 3x^2 - x$$

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{-6 - (-3)}{3} = \frac{-3}{3} = -1$$

The average rate of change from $x_1 = -1$ to $x_2 = 2$ is -1 .

1.5.56.

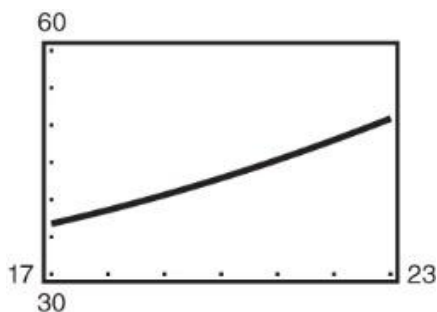
$$f(x) = -x^3 + 6x^2 + x$$

$$\frac{f(6) - f(1)}{6 - 1} = \frac{6 - 6}{5} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

1.5.57.

(a) $y = 0.1053t^2 - 1.861t + 37.96, 17 \leq t \leq 23$



(b) $\frac{y(23) - y(17)}{23 - 17} \approx \frac{50.8607 - 36.7547}{6} \approx 2.35$

The amount the U.S. federal government spent on applied research increased by an average rate of about \$2.35 billion per year from 2017 to 2023.

1.5.58.

$$\begin{aligned} \text{Average rate of change} &= \frac{s(t_2) - s(t_1)}{t_2 - t_1} \\ &= \frac{s(9) - s(0)}{9 - 0} \\ &= \frac{540 - 0}{9 - 0} \\ &= 60 \text{ feet per second.} \end{aligned}$$

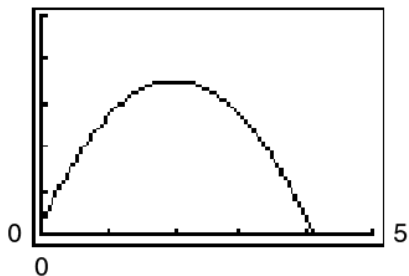
As the time traveled increases, the distance increases rapidly, causing the average speed to increase with each time increment. From $t = 0$ to $t = 4$, the average speed is less than from $t = 4$ to $t = 9$. Therefore, the overall average from $t = 0$ to $t = 9$ falls below the average found in part (b).

1.5.59.

$$s_0 = 6, v_0 = 64$$

(a) $s = -16t^2 + 64t + 6$

(b) 100



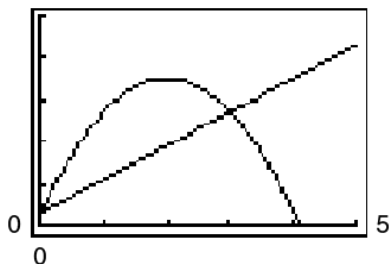
(c) $\frac{s(3) - s(0)}{3 - 0} = \frac{54 - 6}{3} = 16$

(d) The slope of the secant line is positive.

(e) $s(0) = 6, m = 16$

$$\begin{aligned} \text{Secant line: } y - 6 &= 16(t - 0) \\ y &= 16t + 6 \end{aligned}$$

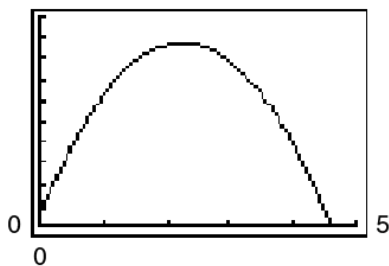
(f) 100



1.5.60.

(a) $s = -16t^2 + 72t + 6.5$

(b) 100

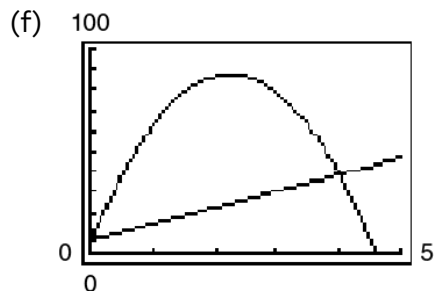


(c) The average rate of change from $t = 0$ to $t = 4$:

$$\frac{s(4) - s(0)}{4 - 0} = \frac{38.5 - 6.5}{4} = \frac{32}{4} = 8 \text{ feet per second}$$

(d) The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive.

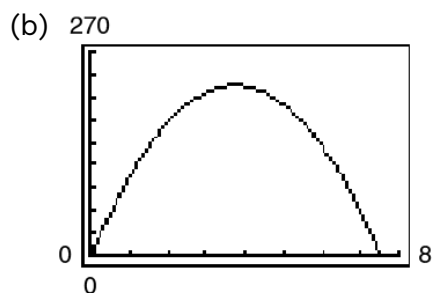
(e) The equation of the secant line: $m = 8$, $y = 8t + 6.5$



1.5.61.

$$v_0 = 120, s_0 = 0$$

(a) $s = -16t^2 + 120t$



(c) The average rate of change from $t = 3$ to $t = 5$:

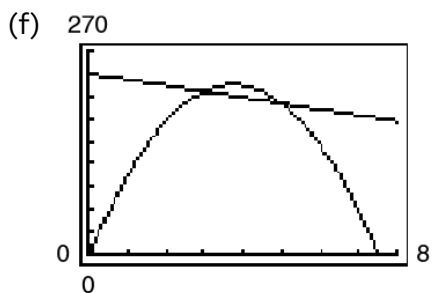
$$\frac{s(5) - s(3)}{5 - 3} = \frac{200 - 216}{2} = -\frac{16}{2} = -8 \text{ feet per second}$$

(d) The slope of the secant line through $(3, s(3))$ and $(5, s(5))$ is negative.

(e) The equation of the secant line: $m = -8$

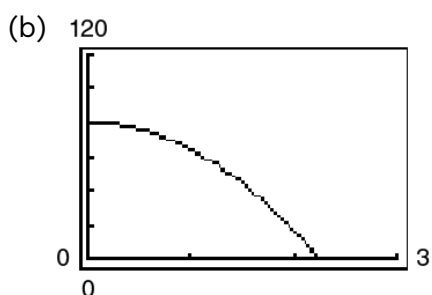
Using $(5, s(5)) = (5, 200)$ we have

$$\begin{aligned} y - 200 &= -8(t - 5) \\ y &= -8t + 240. \end{aligned}$$



1.5.62.

(a) $s = -16t^2 + 80$



(c) The average rate of change from $t = 1$ to $t = 2$:

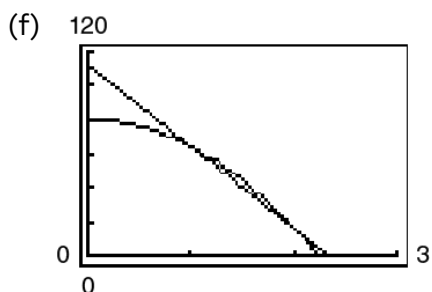
$$\frac{s(2) - s(1)}{2 - 1} = \frac{16 - 64}{1} = -\frac{48}{1} = -48 \text{ feet per second}$$

(d) The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative.

(e) The equation of the secant line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$$\begin{aligned} y - 64 &= -48(t - 1) \\ y &= -48t + 112. \end{aligned}$$



1.5.63.

$$\begin{aligned}f(x) &= x^6 - 2x^2 + 3 \\f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\&= x^6 - 2x^2 + 3 \\&= f(x)\end{aligned}$$

The function is even. y -axis symmetry.

1.5.64.

$$\begin{aligned}g(x) &= x^3 - 5x \\g(-x) &= (-x)^3 - 5(-x) \\&= -x^3 + 5x \\&= -g(x)\end{aligned}$$

The function is odd. Origin symmetry.

1.5.65.

$$\begin{aligned}h(x) &= x\sqrt{x+5} \\h(-x) &= (-x)\sqrt{-x+5} \\&= -x\sqrt{5-x} \\&\neq h(x) \\&\neq -h(x)\end{aligned}$$

The function is neither odd nor even. No symmetry.

1.5.66.

$$\begin{aligned}f(x) &= x\sqrt{1-x^2} \\f(-x) &= -x\sqrt{1-(-x)^2} \\&= -x\sqrt{1-x^2} \\&= -f(x)\end{aligned}$$

The function is odd. Origin symmetry.

1.5.67.

$$\begin{aligned}f(s) &= 4s^{3/2} \\&= 4(-s)^{3/2} \\&\neq f(s) \\&\neq -f(s)\end{aligned}$$

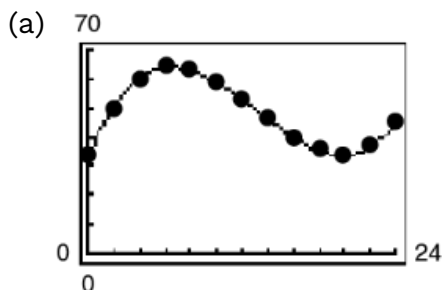
The function is neither odd nor even. No symmetry.

1.5.68.

$$\begin{aligned} g(s) &= 4s^{2/3} \\ g(-s) &= 4(-s)^{2/3} \\ &= 4s^{2/3} \\ &= g(s) \end{aligned}$$

The function is even. y -axis symmetry.

1.5.69.



(b) The model is an excellent fit.

(c) The temperature was increasing from 6 A.M. until noon ($x = 0$ to $x = 6$). Then it decreases until 2 A.M. ($x = 6$ to $x = 20$). Then the temperature increases until 6 A.M. ($x = 20$ to $x = 24$).

(d) The maximum temperature according to the model is about 63.93°F . According to the data, it is 64°F . The minimum temperature according to the model is about 33.98°F . According to the data, it is 34°F .

(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

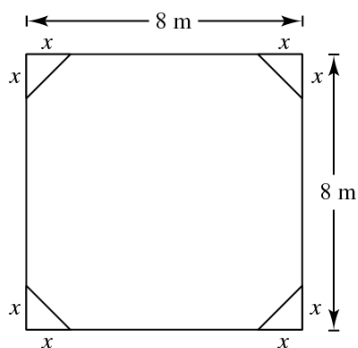
1.5.70.

$$\begin{aligned} L &= \text{right} - \text{left} \\ &= 2 - \sqrt[3]{2y} \end{aligned}$$

1.5.71.

$$\begin{aligned} L &= \text{right} - \text{left} \\ &= \frac{2}{y} - 0 \\ &= \frac{2}{y} \end{aligned}$$

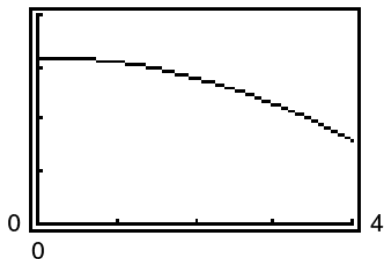
1.5.72.



$$(a) A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x) = 64 - 2x^2$$

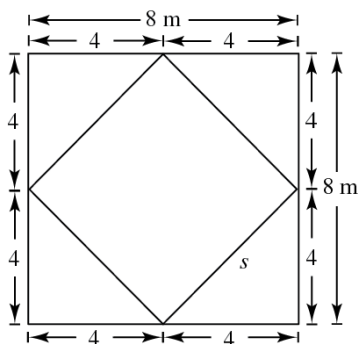
Domain: $0 \leq x \leq 4$

(b) 80



Range: $32 \leq A \leq 64$

(c) When $x = 4$, the resulting figure is a square.



By the Pythagorean Theorem, $4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

1.5.73.

False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

1.5.74.

False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

1.5.75.

The error is that $-2x^3 - 5 \neq -(2x^3 - 5)$. The correct process is as follows.

$$f(x) = 2x^3 - 5$$

$$\begin{aligned} f(-x) &= 2(-x)^3 - 5 \\ &= -2x^3 - 5 \\ &= -(2x^3 + 5) \end{aligned}$$

$f(-x) \neq -f(x)$ and $f(-x) \neq f(x)$, so the function $f(x) = 2x^3 - 5$ is neither odd nor even.

1.5.76.

(a) Domain: $[-4, 5]$; Range: $[0, 9]$

(b) $(3, 0)$

(c) Increasing: $(-4, 0) \cup (3, 5)$; Decreasing: $(0, 3)$

(d) Relative minimum: $(3, 0)$

Relative maximum: $(0, 9)$

(e) Neither

1.5.77.

$$\left(-\frac{5}{3}, -7\right)$$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

1.5.78.

$$(2a, 2c)$$

(a) $(-2a, 2c)$

(b) $(-2a, -2c)$

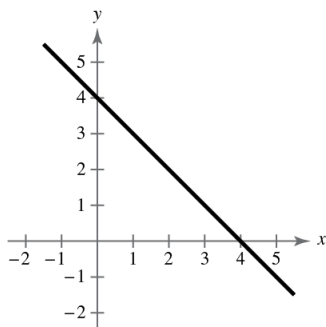
1.5.79.

$$(1, 3), (4, 0)$$

$$\text{Slope: } m = \frac{3-0}{1-4} = -1$$

$$y - 0 = -1(x - 4)$$

$$y = -x + 4$$



1.5.80.

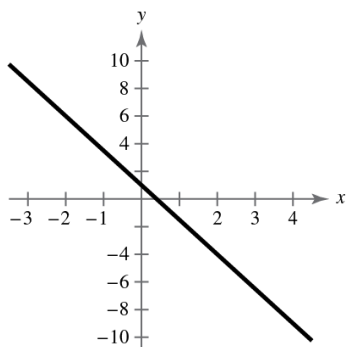
$$(-2, 6), (4, -9)$$

$$\text{Slope: } m = \frac{6 - (-9)}{-2 - 4} = \frac{15}{-6} = -\frac{5}{2}$$

$$y - 6 = -\frac{5}{2}(x + 2)$$

$$y = -\frac{5}{2}x - 5 + 6$$

$$y = -\frac{5}{2}x + 1$$

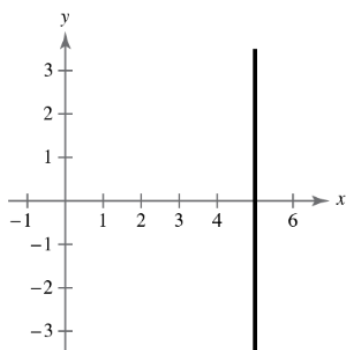


1.5.81.

$$(5, 0), (5, 1)$$

$$\text{Slope: } \frac{0-1}{5-5} = \frac{-1}{0} \Rightarrow m \text{ is undefined.}$$

$$x = 5$$

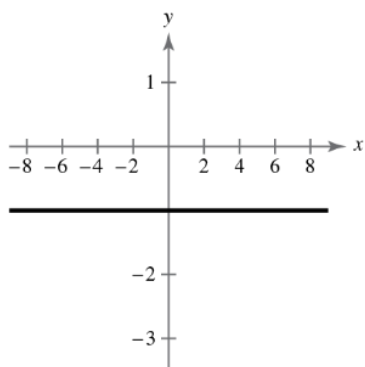


1.5.82.

$$(6, -1), (-6, -1)$$

$$\text{Slope: } m = \frac{-1 - (-1)}{6 - (-6)} = 0$$

$$y = -1$$



1.5.83.

$$f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$$

$$(a) f(0) = 2(0) + 3 = 3$$

$$(b) f(1) = 2(1) + 3 = 5$$

$$(c) f(2) = -2 + 4 = 2$$

1.5.84.

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \leq -4 \\ x + 5, & x > -4 \end{cases}$$

$$(a) f(-8) = -\frac{1}{2}(-8) - 6 = 4 - 6 = -2$$

$$(b) f(-4) = -\frac{1}{2}(-4) - 6 = 2 - 6 = -4$$

$$(c) f(0) = 0 + 5 = 5$$

1.5.85.

Verbal Model: (Sum) = (first number) + (second number)

Labels: Sum = S , first number = n , second number = $n + 1$

Equation: $S = n + (n + 1) = 2n + 1$

1.5.86.

Verbal Model: Product = (first number) · (second number)

Labels: Product = P , first number = n , second number = $n + 1$

Equation: $P = n(n + 1) = n^2 + n$

SECTION 1.6 A LIBRARY OF PARENT FUNCTIONS

1.6.1.

Greatest integer function

1.6.2.

Identity function

1.6.3.

Reciprocal function

1.6.4.

Squaring function

1.6.5.

Square root function

1.6.6.

Constant function

1.6.7.

Absolute value function

1.6.8.

Cubic function

1.6.9.

Linear function

1.6.10.

linear

1.6.11.

(a) $f(1) = 4, f(0) = 6$

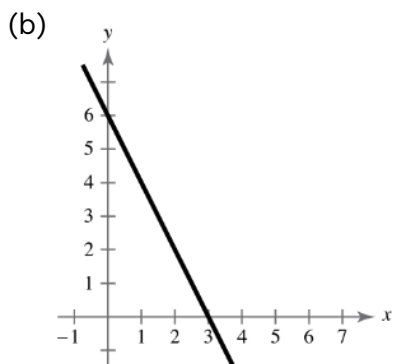
$(1, 4), (0, 6)$

$$m = \frac{6-4}{0-1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$



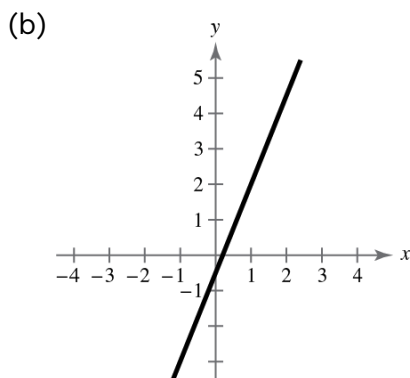
1.6.12.

(a) $f(-3) = -8, f(1) = 2$
 $(-3, -8), (1, 2)$

$$m = \frac{2 - (-8)}{1 - (-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x - 1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$



1.6.13.

(a) $f\left(\frac{1}{2}\right) = -\frac{5}{3}, f(6) = 2$
 $\left(\frac{1}{2}, -\frac{5}{3}\right), (6, 2)$

$$m = \frac{2 - \left(-\frac{5}{3}\right)}{6 - \left(\frac{1}{2}\right)}$$

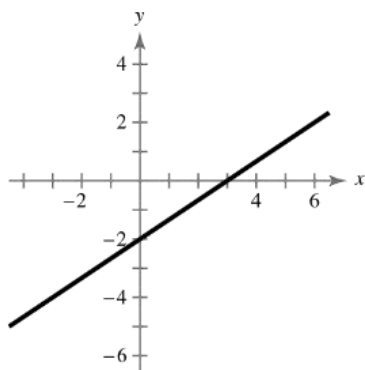
$$= \frac{\frac{11}{3}}{\frac{11}{2}} = \left(\frac{11}{3}\right) \cdot \left(\frac{2}{11}\right) = \frac{2}{3}$$

$$f(x) - 2 = \frac{2}{3}(x - 6)$$

$$f(x) - 2 = \frac{2}{3}x - 4$$

$$f(x) = \frac{2}{3}x - 2$$

(b)



1.6.14.

(a) $f\left(\frac{3}{5}\right) = \frac{1}{2}$, $f(4) = 9$

$$\left(\frac{3}{5}, \frac{1}{2}\right), (4, 9)$$

$$m = \frac{9 - \left(\frac{1}{2}\right)}{4 - \left(\frac{3}{5}\right)}$$

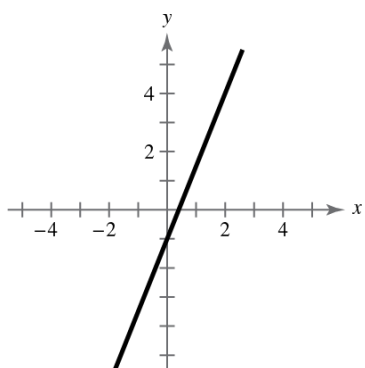
$$= \frac{\frac{17}{2}}{\frac{17}{5}} = \left(\frac{17}{2}\right) \cdot \left(\frac{5}{17}\right) = \frac{5}{2}$$

$$f(x) - 9 = \frac{5}{2}(x - 4)$$

$$f(x) - 9 = \frac{5}{2}x - 10$$

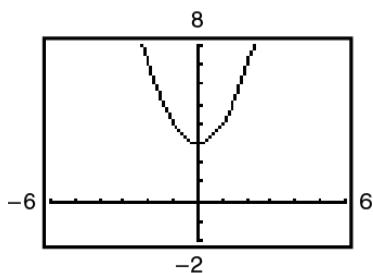
$$f(x) = \frac{5}{2}x - 1$$

(b)



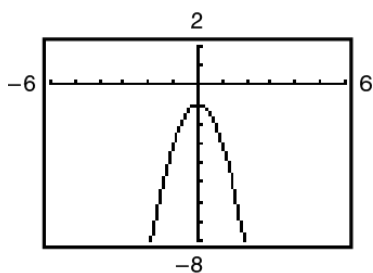
1.6.15.

$$g(x) = x^2 + 3$$



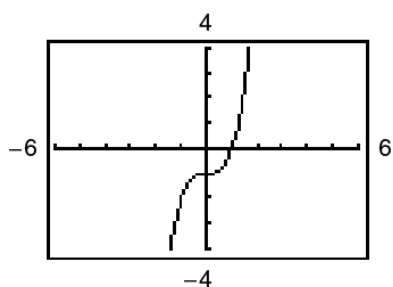
1.6.16.

$$g(x) = -2x^2 - 1$$



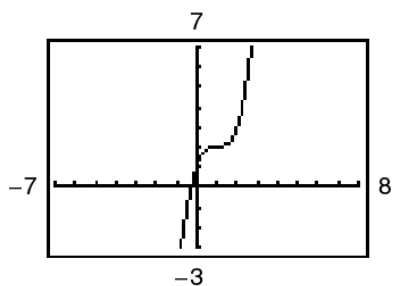
1.6.17.

$$f(x) = x^3 - 1$$



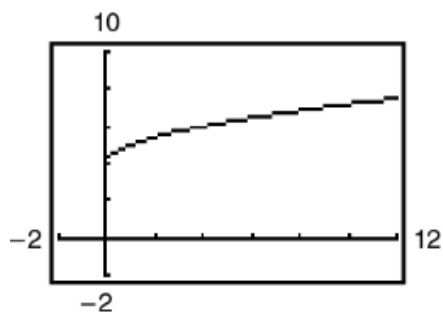
1.6.18.

$$f(x) = (x - 1)^3 + 2$$



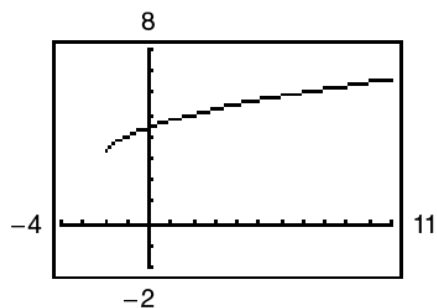
1.6.19.

$$f(x) = \sqrt{x} + 4$$



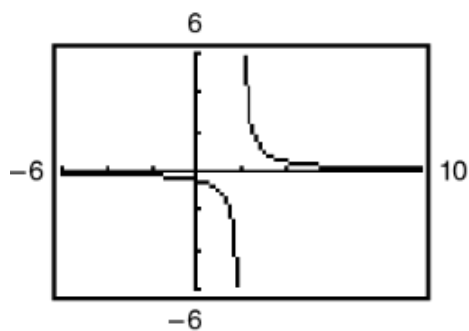
1.6.20.

$$h(x) = \sqrt{x+2} + 3$$



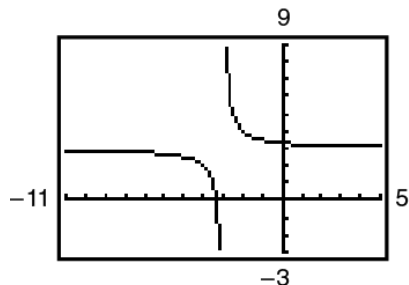
1.6.21.

$$f(x) = \frac{1}{x-2}$$



1.6.22.

$$k(x) = 3 + \frac{1}{x+3}$$



1.6.23.

$$f(x) = \llbracket x \rrbracket$$

(a) $f(2.1) = 2$

(b) $f(2.9) = 2$

(c) $f(-3.1) = -4$

(d) $f\left(\frac{7}{2}\right) = 3$

1.6.24.

$$h(x) = \llbracket x + 3 \rrbracket$$

(a) $h(-2) = \llbracket 1 \rrbracket = 1$

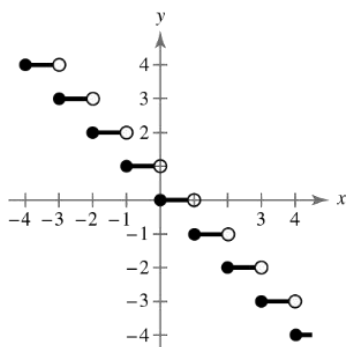
(b) $h\left(\frac{1}{2}\right) = \llbracket 3.5 \rrbracket = 3$

(c) $h(4.2) = \llbracket 7.2 \rrbracket = 7$

(d) $h(-21.6) = \llbracket -18.6 \rrbracket = -19$

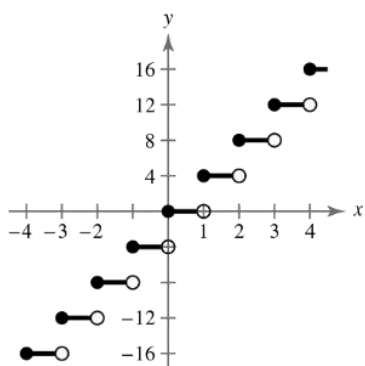
1.6.25.

$$g(x) = -\lceil x \rceil$$



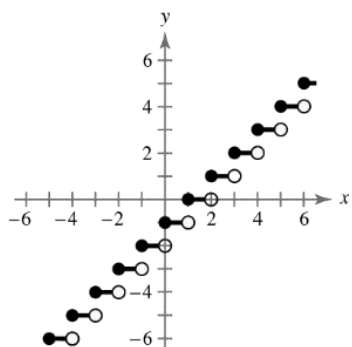
1.6.26.

$$g(x) = 4\lceil x \rceil$$



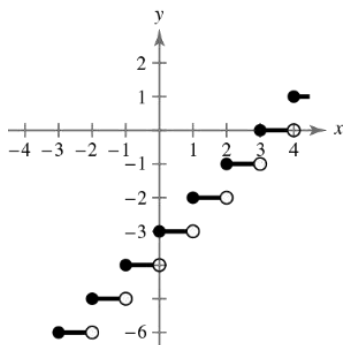
1.6.27.

$$g(x) = \lceil x \rceil - 1$$



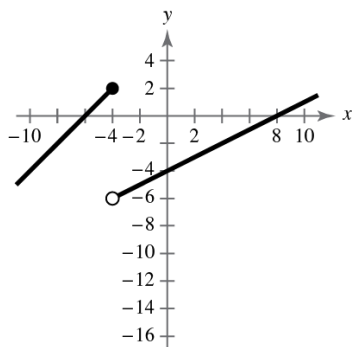
1.6.28.

$$g(x) = \lfloor x - 3 \rfloor$$



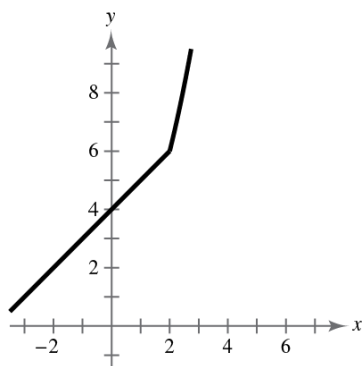
1.6.29.

$$g(x) = \begin{cases} x + 6, & x \leq -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$$



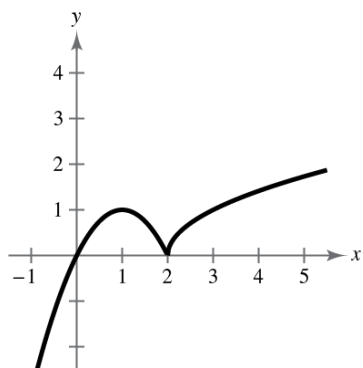
1.6.30.

$$f(x) = \begin{cases} 4 + x, & x \leq 2 \\ x^2 + 2, & x > 2 \end{cases}$$



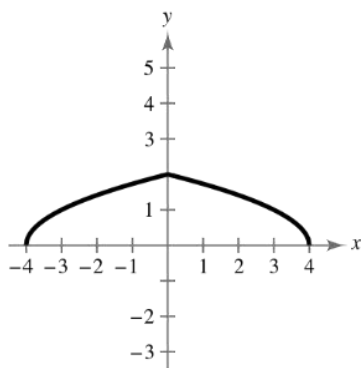
1.6.31.

$$f(x) = \begin{cases} 1 - (x-1)^2, & x \leq 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$$



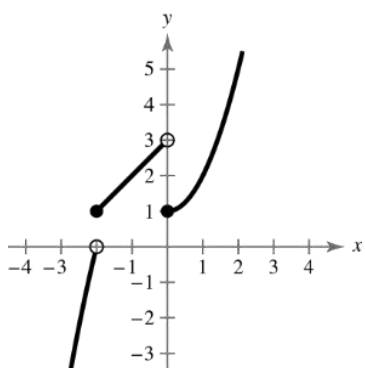
1.6.32.

$$f(x) = \begin{cases} \sqrt{4+x}, & x < 0 \\ \sqrt{4-x}, & x \geq 0 \end{cases}$$



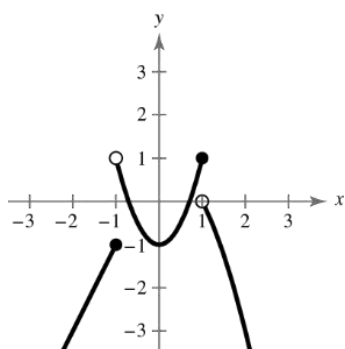
1.6.33.

$$h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$$



1.6.34.

$$k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$$



1.6.35.

$$\begin{aligned} \text{(a)} \quad W(30) &= 14(30) = 420 \\ W(40) &= 14(40) = 560 \\ W(45) &= 21(45 - 40) + 560 = 665 \\ W(50) &= 21(50 - 40) + 560 = 770 \end{aligned}$$

$$\text{(b)} \quad W(h) = \begin{cases} 14h, & 0 < h \leq 36 \\ 21(h - 36) + 504, & h > 36 \end{cases}$$

$$\text{(c)} \quad W(h) = \begin{cases} 16h, & 0 < h \leq 40 \\ 24(h - 40) + 640, & h > 40 \end{cases}$$

1.6.36.

For the first two hours, the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.

$$f(t) = \begin{cases} t, & 0 \leq t \leq 2 \\ 2t - 2, & 2 < t \leq 8 \\ \frac{1}{2}t + 10, & 8 < t \leq 9 \end{cases}$$

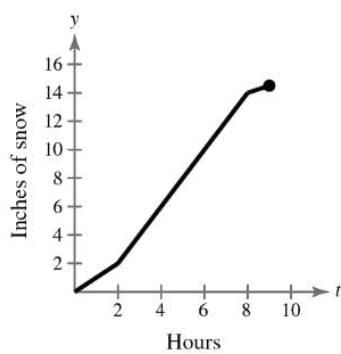
To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$$y - 2 = 2(t - 2) \Rightarrow y = 2t - 2$$

To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$$y - 14 = \frac{1}{2}(t - 8) \Rightarrow y = \frac{1}{2}t + 10$$

Total accumulation = 14.5 inches

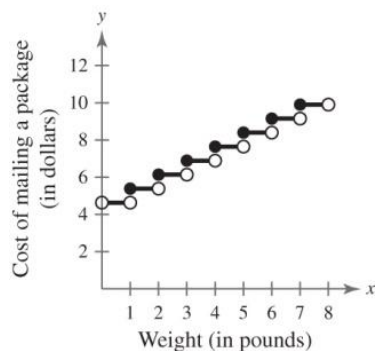


1.6.37.

$$(a) C = 0.75\lceil x \rceil + 4.63$$

Note that $C(0.9) = 4.63$ and $C(1) = 0.75 + 4.63$.

(b)



1.6.38.

$$f(x) = x^2$$

(a) Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$$f(x) = x^3$$

(a) Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

(d) Even; the graph has y-axis symmetry.

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(-\infty, \infty)$

(d) Odd; the graph has origin symmetry.

1.6.39.

False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

1.6.40.

False. The vertical line $x = 2$ has an x-intercept at the point $(2, 0)$ but does not have a y-intercept. The horizontal line $y = 3$ has a y-intercept at the point $(0, 3)$ but does not have an x-intercept.

1.6.41.

According to the graph, the domains should be $x \leq 3$ and $x > 3$.

1.6.42.

When $x = \frac{-4}{3}$, $\left\lceil \frac{-4}{3} \right\rceil = -2$, not -1 . So, $f\left(\frac{-4}{3}\right) = -2 + 5 = 3$.

1.6.43.

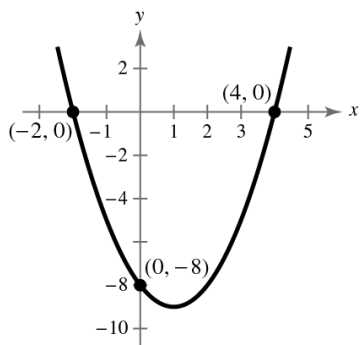
- (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.
- (b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

1.6.44.

- (a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).
- (b) Yes. The greater the height from which the acorn is dropped, the greater the speed with which the acorn will strike the ground.

1.6.45.

$$f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$$

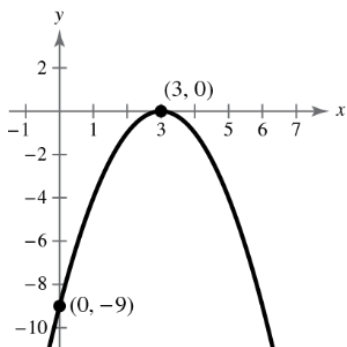


x-intercepts: $(4, 0)$, $(-2, 0)$

y-intercept: $(0, -8)$

1.6.46.

$$g(x) = -x^2 + 6x - 9 = -(x^2 - 6x + 9) = -(x - 3)^2$$

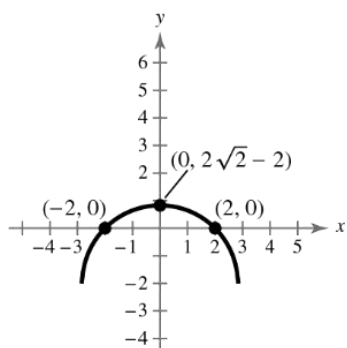


x-intercept: $(3, 0)$

y-intercept: $(0, -9)$

1.6.47.

$$p(x) = \sqrt{8 - x^2} - 2$$

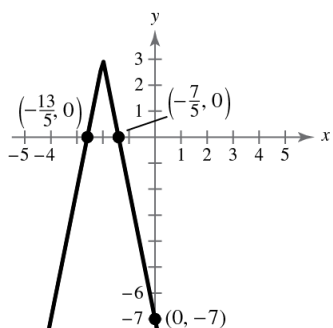


x -intercepts: $(2, 0), (-2, 0)$

y -intercept: $(0, 2\sqrt{2} - 2)$

1.6.48.

$$g(x) = -5|x + 2| + 3$$



x -intercepts: $(-\frac{7}{5}, 0), (-\frac{13}{5}, 0)$

y -intercept: $(0, -7)$

1.6.49.

$$\begin{aligned} \frac{\sqrt[3]{(-x)^2} + \sqrt[3]{-x}}{-x} &= -\frac{\sqrt[3]{x^2} + \sqrt[3]{x}}{x} \\ &= -\frac{(x^{1/3} - 1)}{x^{2/3}} \end{aligned}$$

1.6.50.

$$\begin{aligned} -\frac{\sqrt{(2-x)^3} + \sqrt{2-x}}{x-2} &= \frac{\sqrt{2-x}(2-x+1)}{2-x} \\ &= \frac{3-x}{\sqrt{2-x}} \\ &= \frac{(3-x)\sqrt{2-x}}{(2-x)} \end{aligned}$$

1.6.51.

$$\begin{aligned} (x-3)^2 - 3(x-3) + 2 &= x^2 - 6x + 9 - 3x + 9 + 2 \\ &= x^2 - 9x + 20 \\ &= (x-5)(x-4) \end{aligned}$$

1.6.52.

$$\begin{aligned} 3(x-1)^3 - 2(x-1)^2 - 3(x-1) + 2 &= (x-1)^2 [3(x-1) - 2] - [3(x-1) - 2] \\ &= [(x-1)^2 - 1] [3(x-1) - 2] \\ &= (x^2 - 2x)(3x - 5) \\ &= 3x^3 - 11x^2 + 10x \\ &= x(3x^2 - 11x + 10) \\ &= x(3x - 5)(x - 2) \end{aligned}$$

SECTION 1.7 TRANSFORMATIONS OF FUNCTIONS

1.7.1.

$$-f(x); f(-x)$$

1.7.2.

vertical stretch; vertical shrink

1.7.3.

The three types of rigid transformations are horizontal shifts, vertical shifts, and reflections.

1.7.4.

(a) iv

(b) ii

(c) iii

(d) i

1.7.5.

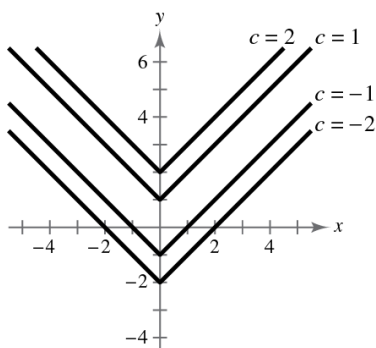
(a) $f(x) = |x| + c$ Vertical shifts

$c = -2: f(x) = |x| - 2$ 2 units down

$c = -1: f(x) = |x| - 1$ 1 unit down

$c = 1: f(x) = |x| + 1$ 1 unit up

$c = 2: f(x) = |x| + 2$ 2 units up

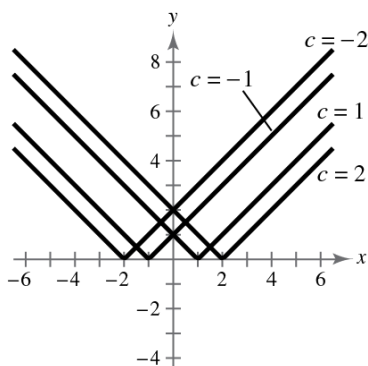


(b) $f(x) = |x - c|$

Horizontal shifts

$c = -2: f(x) = |x - (-2)| = |x + 2|$ 2 units left

$$\begin{aligned} c = -1: f(x) &= |x - (-1)| = |x + 1| && 1 \text{ unit left} \\ c = 1: f(x) &= |x - (1)| = |x - 1| && 1 \text{ unit right} \\ c = 2: f(x) &= |x - (2)| = |x - 2| && 2 \text{ units right} \end{aligned}$$



1.7.6.

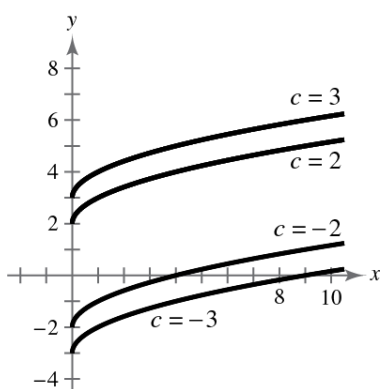
(a) $f(x) = \sqrt{x} + c$ Vertical shifts

$$c = -3: f(x) = \sqrt{x} - 3 \quad 3 \text{ units down}$$

$$c = -2: f(x) = \sqrt{x} - 2 \quad 2 \text{ units down}$$

$$c = 2: f(x) = \sqrt{x} + 2 \quad 2 \text{ units up}$$

$$c = 3: f(x) = \sqrt{x} + 3 \quad 3 \text{ units up}$$



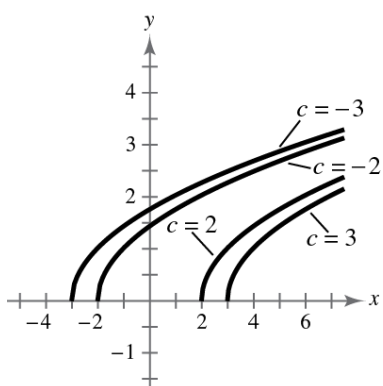
(b) $f(x) = \sqrt{x - c}$ Horizontal shifts

$$c = -3: f(x) = \sqrt{x - (-3)} = \sqrt{x + 3} \quad 3 \text{ units left}$$

$$c = -2: f(x) = \sqrt{x - (-2)} = \sqrt{x + 2} \quad 2 \text{ units left}$$

$$c = 2: f(x) = \sqrt{x-2} \quad 2 \text{ units right}$$

$$c = 3: f(x) = \sqrt{x-3} \quad 3 \text{ units right}$$



1.7.7.

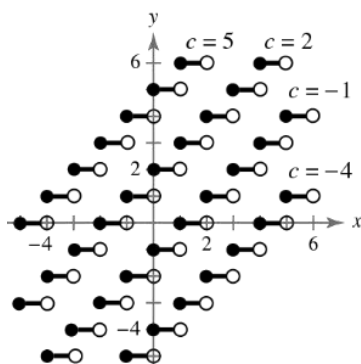
(a) $f(x) = \llbracket x \rrbracket + c$ Vertical shifts

$$c = -4: f(x) = \llbracket x \rrbracket - 4 \quad 4 \text{ units down}$$

$$c = -1: f(x) = \llbracket x \rrbracket - 1 \quad 1 \text{ unit down}$$

$$c = 2: f(x) = \llbracket x \rrbracket + 2 \quad 2 \text{ units up}$$

$$c = 5: f(x) = \llbracket x \rrbracket + 5 \quad 5 \text{ units up}$$



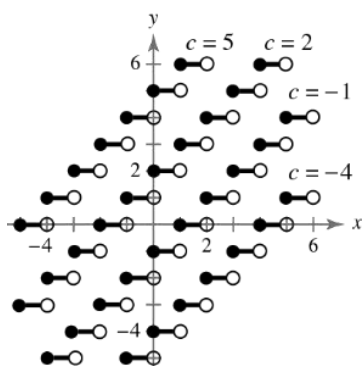
(b) $f(x) = \llbracket x + c \rrbracket$ Horizontal shifts

$$c = -4: f(x) = \llbracket x - (-4) \rrbracket = \llbracket x + 4 \rrbracket \quad 4 \text{ units left}$$

$$c = -1: f(x) = \llbracket x - (-1) \rrbracket = \llbracket x + 1 \rrbracket \quad 1 \text{ unit left}$$

$$c = 2: f(x) = \llbracket x - (2) \rrbracket = \llbracket x - 2 \rrbracket \quad 2 \text{ units right}$$

$$c = 5: f(x) = \llbracket x - (5) \rrbracket = \llbracket x - 5 \rrbracket \quad 5 \text{ units right}$$



1.7.8.

$$(a) f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \geq 0 \end{cases}$$

Vertical shifts

$$c = -3: f(x) = \begin{cases} x^2 - 3, & x < 0 \\ -x^2 - 3, & x \geq 0 \end{cases}$$

3 units down

$$c = -2: f(x) = \begin{cases} x^2 - 2, & x < 0 \\ -x^2 - 2, & x \geq 0 \end{cases}$$

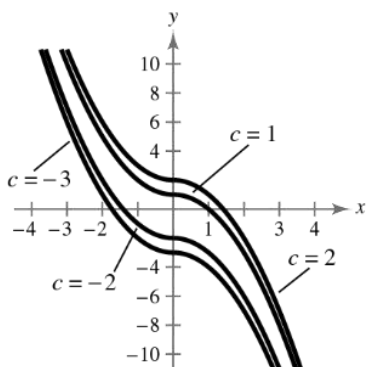
2 units down

$$c = 1: f(x) = \begin{cases} x^2 + 1, & x < 0 \\ -x^2 + 1, & x \geq 0 \end{cases}$$

1 unit up

$$c = 2: f(x) = \begin{cases} x^2 + 2, & x < 0 \\ -x^2 + 2, & x \geq 0 \end{cases}$$

2 units up



$$(b) f(x) = \begin{cases} (x+c)^2, & x < 0 \\ -(x+c)^2, & x \geq 0 \end{cases}$$

Horizontal shifts

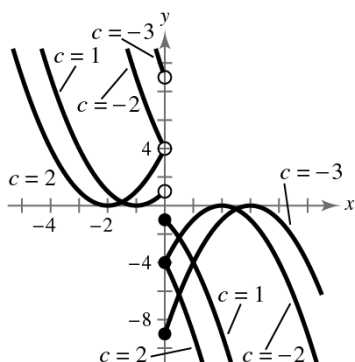
$$c = -3: f(x) = \begin{cases} (x-3)^2, & x < 0 \\ -(x-3)^2, & x \geq 0 \end{cases}$$

3 units right

$$c = -2: f(x) = \begin{cases} (x-2)^2, & x < 0 \\ -(x-2)^2, & x \geq 0 \end{cases} \quad \text{2 units right}$$

$$c = 1: f(x) = \begin{cases} (x+1)^2, & x < 0 \\ -(x+1)^2, & x \geq 0 \end{cases} \quad \text{1 unit left}$$

$$c = 2: f(x) = \begin{cases} (x+2)^2, & x < 0 \\ -(x+2)^2, & x \geq 0 \end{cases} \quad \text{2 units left}$$



1.7.9.

Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$$g(x) = x^2 - 1$$

(b) Reflection in the x -axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$$g(x) = -(x+1)^2 + 1$$

1.7.10.

Parent function: $f(x) = x^3$

(a) Shifted upward 1 unit

$$g(x) = x^3 + 1$$

(b) Reflection in the x -axis, shifted to the left 3 units and down 1 unit

$$g(x) = -(x+3)^3 - 1$$

1.7.11.

Parent function: $f(x) = |x|$

(a) Reflection in the x -axis and a horizontal shift 3 units to the left

$$g(x) = -|x + 3|$$

(b) Horizontal shift 2 units to the right and a vertical shift 4 units downward

$$g(x) = |x - 2| - 4$$

1.7.12.

Parent function: $f(x) = \sqrt{x}$

(a) A vertical shift 7 units downward and a horizontal shift 1 unit to the left

$$g(x) = \sqrt{x + 1} - 7$$

(d) Reflection in the x - and y -axis and shifted to the right 3 units and downward 4 units

$$g(x) = -\sqrt{-x + 3} - 4$$

1.7.13.

Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$$y = (x - 2)^3$$

1.7.14.

Parent function: $y = \llbracket x \rrbracket$

Vertical shift 4 units upward

$$y = \llbracket x \rrbracket + 4$$

1.7.15.

Parent function: $f(x) = \sqrt{x}$

Reflection in the x -axis and a vertical shift 1 unit upward

$$y = -\sqrt{x} + 1$$

1.7.16.

Parent function: $y = |x|$

Horizontal shift 2 units to the left

$$y = |x + 2|$$

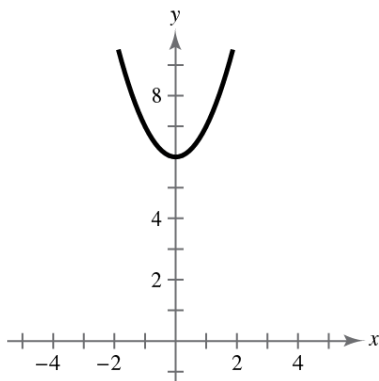
1.7.17.

$$g(x) = x^2 + 6$$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 6 units upward

(c)



(d) $g(x) = f(x) + 6$

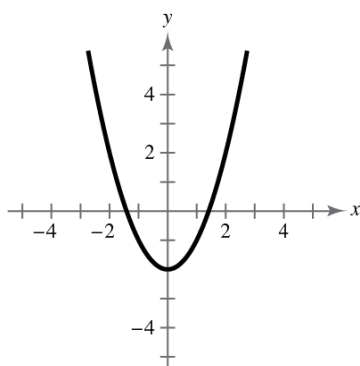
1.7.18.

$$g(x) = x^2 - 2$$

(a) Parent function: $f(x) = x^2$

(b) A vertical shift 2 units downward

(c)



(d) $g(x) = f(x) - 2$

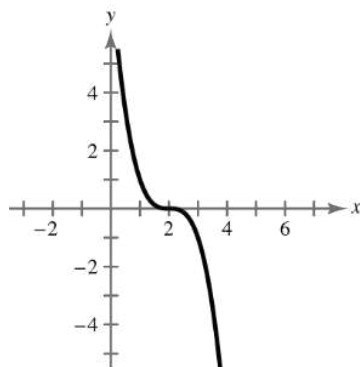
1.7.19.

$$g(x) = -(x - 2)^3$$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 2 units to the right and a reflection in the x-axis

(c)



(d) $g(x) = -f(x-2)$

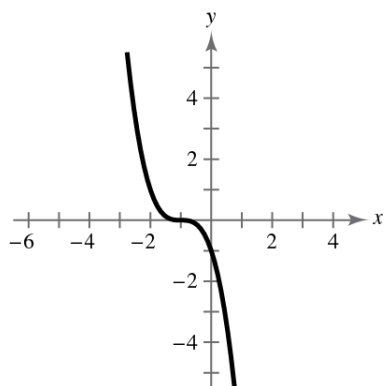
1.7.20.

$$g(x) = -(x+1)^3$$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the left and a reflection in the x -axis

(c)



(d) $g(x) = -f(x+1)$

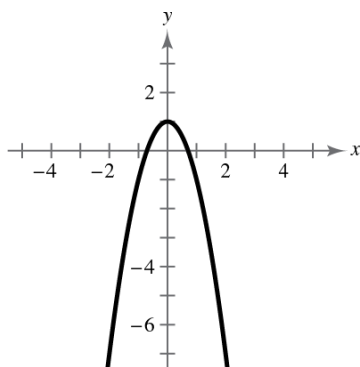
1.7.21.

$$g(x) = -2x^2 + 1$$

(a) Parent function: $f(x) = x^2$

(b) A vertical stretch, reflection in the x -axis and a vertical shift 1 unit upward

(c)



(d) $g(x) = -2f(x) + 1$

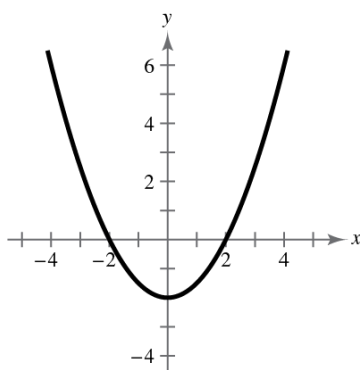
1.7.22.

$$g(x) = \frac{1}{2}x^2 - 2$$

(a) Parent function: $f(x) = x^2$

(b) A vertical shrink and a vertical shift 2 units downward

(c)



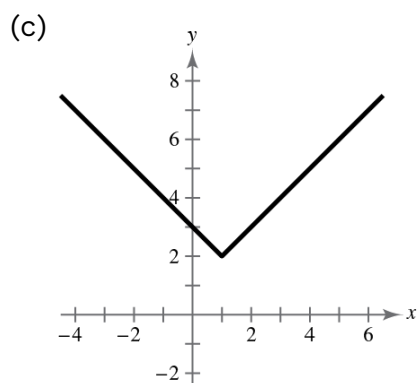
(d) $g(x) = \frac{1}{2}f(x) - 2$

1.7.23.

$$g(x) = |x - 1| + 2$$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 1 unit right and a vertical shift 2 units upward



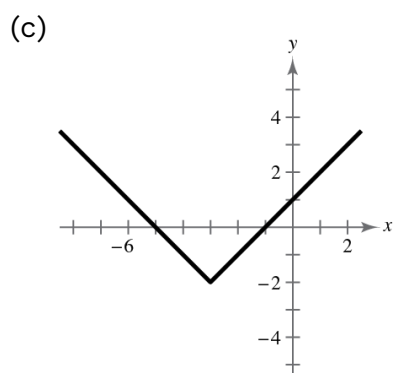
(d) $g(x) = f(x - 1) + 2$

1.7.24.

$g(x) = |x + 3| - 2$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift 3 units left and a vertical shift 2 units downward



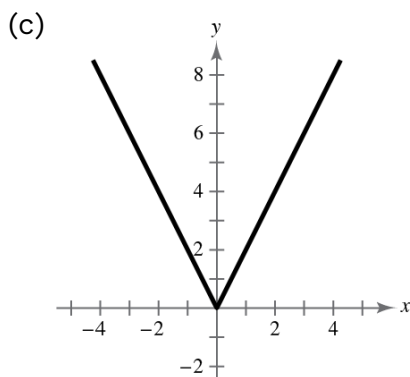
(d) $g(x) = f(x + 3) - 2$

1.7.25.

$g(x) = |2x|$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shrink



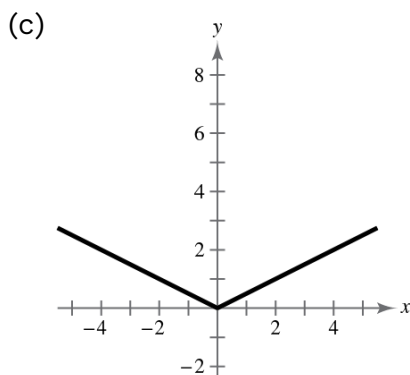
(d) $g(x) = f(2x)$

1.7.26.

$$g(x) = \left| \frac{1}{2}x \right|$$

(a) Parent function: $f(x) = |x|$

(b) A horizontal stretch



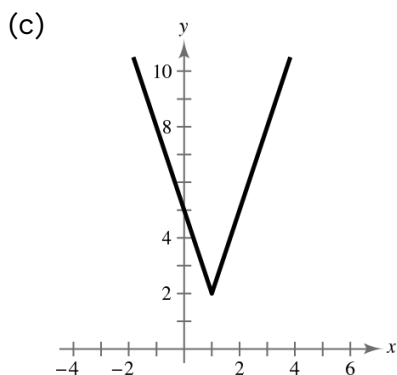
(d) $g(x) = f\left(\frac{1}{2}x\right)$

1.7.27.

$$g(x) = 3|x - 1| + 2$$

(a) Parent function: $f(x) = |x|$

(b) A horizontal shift of 1 unit to the right, a vertical stretch, and a vertical shift 2 units upward



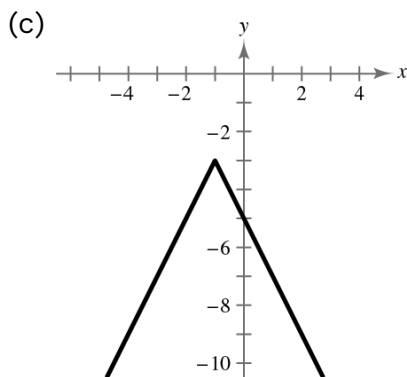
(d) $g(x) = 3f(x - 1) + 2$

1.7.28.

$$g(x) = -2|x + 1| - 3$$

(a) Parent function: $f(x) = |x|$

(b) A reflection in the x -axis, a vertical stretch, a horizontal shift 1 unit to the left, and a vertical shift 3 units downward



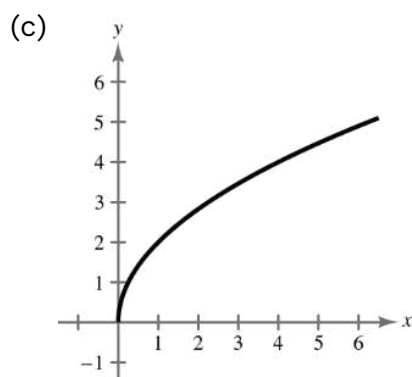
(d) $g(x) = -2f(x + 1) - 3$

1.7.29.

$$g(x) = 2\sqrt{x}$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical stretch (each y value is multiplied by 2)



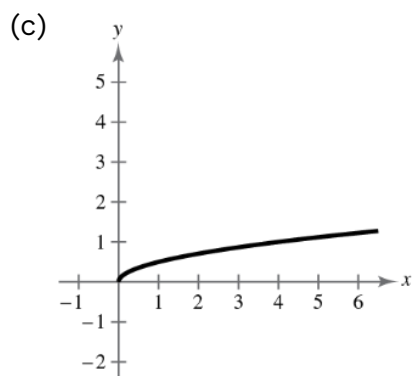
(d) $g(x) = 2f(x)$

1.7.30.

$$g(x) = \frac{1}{2}\sqrt{x}$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) A vertical shrink (each y value is multiplied by $\frac{1}{2}$)



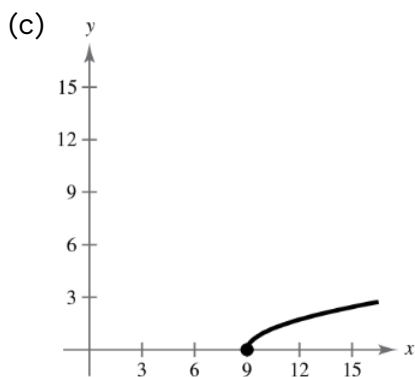
(d) $g(x) = \frac{1}{2}f(x)$

1.7.31.

$$g(x) = \sqrt{x-9}$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) horizontal shift 9 units to the right



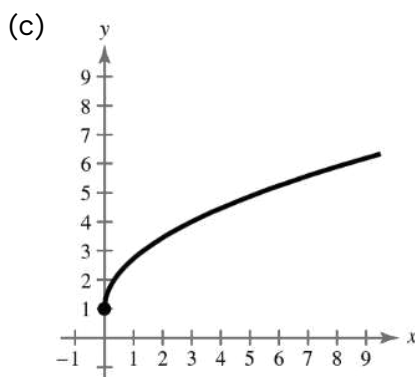
(d) $g(x) = f(x - 9)$

1.7.32.

$$g(x) = \sqrt{3x} + 1$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink (each x -value is multiplied by $\frac{1}{3}$), vertical shift of 1 unit upward



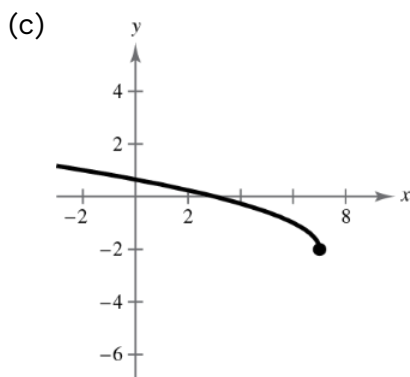
(d) $g(x) = f(3x) + 1$

1.7.33.

$$g(x) = \sqrt{7-x} - 2 \text{ or } g(x) = \sqrt{-(x-7)} - 2$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the y -axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward



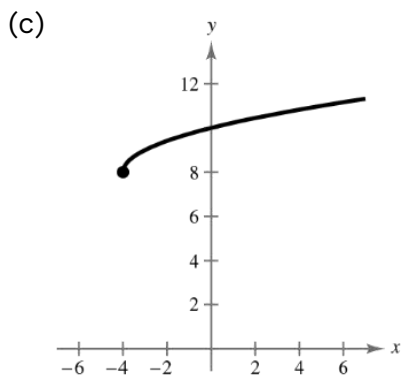
(d) $g(x) = f(7 - x) - 2$

1.7.34.

$$g(x) = \sqrt{x + 4} + 8$$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift of 4 units to the left, vertical shift of 8 units upward



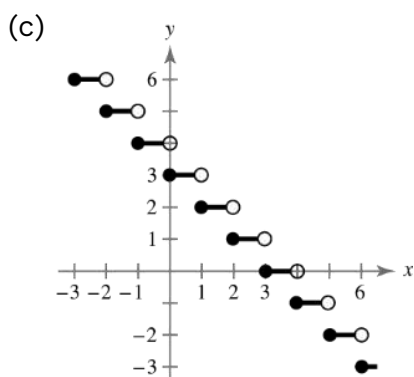
(d) $g(x) = f(x + 4) + 8$

1.7.35.

$$g(x) = 3 - \lceil x \rceil$$

(a) Parent function: $f(x) = \lceil x \rceil$

(b) Reflection in the x-axis and a vertical shift 3 units upward



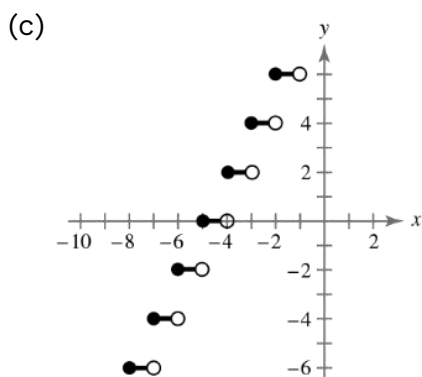
(d) $g(x) = 3 - f(x)$

1.7.36.

$$g(x) = 2 \llbracket x + 5 \rrbracket$$

(a) Parent function: $f(x) = \llbracket x \rrbracket$

(b) Horizontal shift 5 units to the left, vertical stretch (each y -value is multiplied by 2)



(d) $g(x) = 2f(x + 5)$

1.7.37.

$f(x) = x^2$ but shifted three units to the right and seven units down

$$g(x) = (x - 3)^2 - 7$$

1.7.38.

$f(x) = x^2$ but shifted two units to the left, nine units up, and then reflected in the x -axis

$$g(x) = -(x + 2)^2 + 9$$

1.7.39.

$f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x - and y -axes

$$g(x) = -\sqrt{-x+6}$$

1.7.40.

$f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x -axis and the y -axis

$$\begin{aligned} g(x) &= -(\sqrt{-x} - 9) \\ &= -\sqrt{-x} + 9 \end{aligned}$$

1.7.41.

$$f(x) = x^2$$

(a) Reflection in the x -axis and a vertical stretch (each y -value is multiplied by 3)

$$g(x) = -3x^2$$

(b) Vertical shift 3 units upward and a vertical stretch (each y -value is multiplied by 4)

$$g(x) = 4x^2 + 3$$

1.7.42.

$$f(x) = x^3$$

(a) Vertical shrink (each y -value is multiplied by $\frac{1}{4}$)

$$g(x) = \frac{1}{4}x^3$$

(b) Reflection in the x -axis and a vertical stretch (each y -value is multiplied by 2)

$$g(x) = -2x^3$$

1.7.43.

$$f(x) = |x|$$

(a) Reflection in the x -axis and a vertical shrink (each y -value is multiplied by $\frac{1}{2}$)

$$g(x) = -\frac{1}{2}|x|$$

- (b) Vertical stretch (each y -value is multiplied by 3) and a vertical shift 3 units downward

$$g(x) = 3|x| - 3$$

1.7.44.

$$f(x) = \sqrt{x}$$

- (a) Vertical stretch (each y -value is multiplied by 8)

$$g(x) = 8\sqrt{x}$$

- (b) Reflection in the x -axis and a vertical shrink
(each y -value is multiplied by $\frac{1}{4}$)

$$g(x) = -\frac{1}{4}\sqrt{x}$$

1.7.45.

Parent function: $f(x) = x^3$

- Vertical stretch (each y -value is multiplied by 2)

$$g(x) = 2x^3$$

1.7.46.

Parent function: $y = \llbracket x \rrbracket$

- Horizontal stretch (each x -value is multiplied by 2)

$$g(x) = \llbracket \frac{1}{2}x \rrbracket$$

1.7.47.

Parent function: $f(x) = \sqrt{x}$

- Reflection in the y -axis, vertical shrink (each y -value is multiplied by $\frac{1}{2}$)

$$g(x) = \frac{1}{2}\sqrt{-x}$$

1.7.48.

Parent function: $f(x) = |x|$

- Reflection in the x -axis, vertical shift of 2 units downward, vertical stretch
(each y -value is multiplied by 2)

$$g(x) = -2|x| - 2$$

1.7.49.

Parent function: $f(x) = x^3$

Reflection in the x -axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$$g(x) = -(x - 2)^3 + 2$$

1.7.50.

Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$$g(x) = |x + 4| - 2$$

1.7.51.

Parent function: $f(x) = \sqrt{x}$

Reflection in the x -axis and a vertical shift 3 units downward

$$g(x) = -\sqrt{x} - 3$$

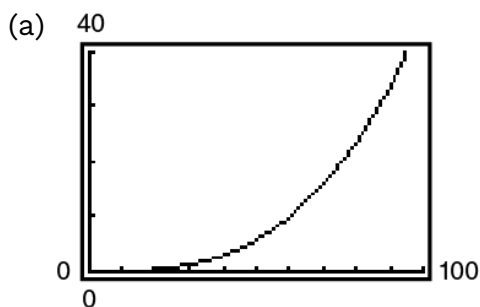
1.7.52.

Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward

$$g(x) = (x - 2)^2 + 4$$

1.7.53.



(b) $H(x) = 0.00004636x^3$

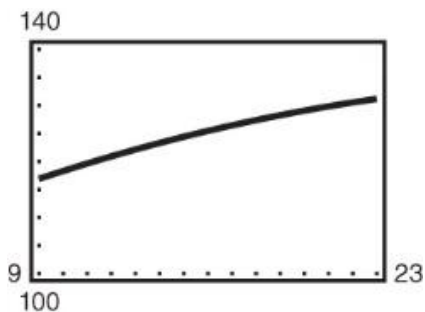
$$\begin{aligned} H\left(\frac{x}{1.6}\right) &= 0.00004636\left(\frac{x}{1.6}\right)^3 \\ &= 0.00004636\left(\frac{x^3}{4.096}\right) \end{aligned}$$

$$= 0.0000113184x^3 = 0.00001132x^3$$

The graph of $H\left(\frac{x}{1.6}\right)$ is a horizontal stretch of the graph of $H(x)$.

1.7.54.

- (a) The transformation consists of a reflection in the x -axis, vertical shrink (each y -value is multiplied by 0.02399), horizontal shift to the right 33.643 units, and a vertical shift 134.533 units upward.



(b) $\frac{N(23) - N(9)}{23 - 9} = \frac{131.25 - 116.93}{14} \approx 1.023$

The number of households in the United States increased by an average rate of about 1,023,000 households per year from 2009 to 2023.

(c) $N(19) = -0.02899(19 - 33.643)^2 + 134.533 \approx 128.3$

There were about 128.3 million households in 2019.

1.7.55.

False. $y = f(-x)$ is a reflection in the y -axis.

1.7.56.

False. $y = -f(x)$ is a reflection in the x -axis.

1.7.57.

True. Because $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

1.7.58.

False. The point $(-2, -61)$ lies on the transformation.

1.7.59.

$$y = f(x + 2) - 1$$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward

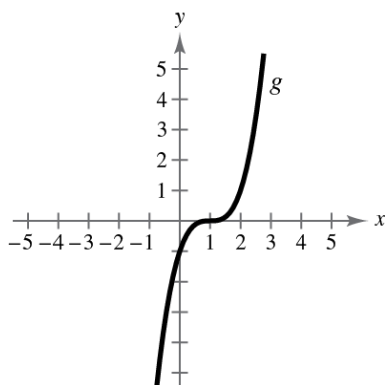
$$\begin{aligned}(0, 1) &\rightarrow (0 - 2, 1 - 1) = (-2, 0) \\ (1, 2) &\rightarrow (1 - 2, 2 - 1) = (-1, 1) \\ (2, 3) &\rightarrow (2 - 2, 3 - 1) = (0, 2)\end{aligned}$$

1.7.60.

- (a) Answers will vary. *Sample answer:* To graph $f(x) = 3x^2 - 4x + 1$ use the point-plotting method since it is not written in a form that is easily identified by a sequence of translations of the parent function $y = x^2$.
- (b) Answers will vary. *Sample answer:* To graph $f(x) = 2(x - 1)^2 - 6$ use the method of translating the parent function $y = x^2$ since it is written in a form such that a sequence of translations is easily identified.

1.7.61.

Since the graph of $g(x)$ is a horizontal shift one unit to the right of $f(x) = x^3$, the equation should be $g(x) = (x - 1)^3$ and not $g(x) = (x + 1)^3$.



1.7.62.

- (a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$
- (b) Increasing on the interval $(-1, 2)$ and decreasing on the intervals $(-\infty, -1)$ and $(2, \infty)$
- (c) Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$ and decreasing on the interval $(-1, 2)$

(d) Increasing on the interval $(0, 3)$ and decreasing on the intervals $(-\infty, 0)$ and $(3, \infty)$

(e) Increasing on the intervals $(-\infty, 1)$ and $(4, \infty)$ and decreasing on the interval $(1, 4)$

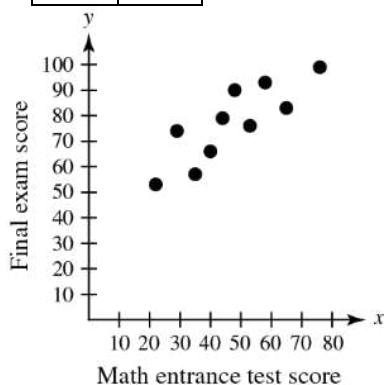
1.7.63.

No. $g(x) = -x^4 - 2$. Yes. $h(x) = -(x - 3)^4$

1.7.64.

(a)

x	y
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that will affect the final exam score.

1.7.65.

$$(2x + 1) + (x^2 + 2x - 1) = x^2 + 4x$$

1.7.66.

$$\begin{aligned}(2x + 1) - (x^2 + 2x - 1) &= 2x + 1 - x^2 - 2x + 1 \\ &= -x^2 + 2\end{aligned}$$

1.7.67.

$$\begin{aligned}(3x^2 + x - 1) - (1 - x) &= 3x^2 + x - 1 - 1 + x \\ &= 3x^2 + 2x - 2\end{aligned}$$

1.7.68.

$$(3x^2 + x - 1) + (1 - x) = 3x^2$$

1.7.69.

$$x^2(x - 3) = x^3 - 3x^2$$

1.7.70.

$$x^2(1 - x) = x^2 - x^3 = -x^3 + x^2$$

1.7.71.

$$\begin{aligned}-2x(0.1x + 17) &= (-2x)(0.1x) + (-2x)(17) \\ &= -0.2x^2 - 34x\end{aligned}$$

1.7.72.

$$\begin{aligned}6y\left(5 - \frac{3}{8}y\right) &= (6y)(5) - (6y)\left(\frac{3}{8}y\right) \\ &= 30y - \frac{9}{4}y^2 \\ &= -\frac{9}{4}y^2 + 30y\end{aligned}$$

1.7.73.

$$\begin{aligned}(3x + 5) \div (6x^2 + 10x) &= \frac{3x + 5}{6x^2 + 10x} = \frac{3x + 5}{2x(3x + 5)} \\ &= \frac{1}{2x}, \quad x \neq \frac{-5}{3}\end{aligned}$$

1.7.74.

$$(20x^2 - 43x) \div x = \frac{20x^2 - 43x}{x} = 20x - 43, \quad x \neq 0$$

SECTION 1.8 COMBINATIONS OF FUNCTIONS: COMPOSITE FUNCTIONS

1.8.1.

addition; subtraction; multiplication; division

1.8.2.

composition

1.8.3.

Since $(fg)(x) = 2x(x^2 + 1)$ and $f(x) = x^2 + 1$, $g(x) = 2x$, and $(fg)(x) = (gf)(x) = (2x)f(x)$.

1.8.4.

Since $(f \circ g)(x) = f(g(x))$ and $(f \circ g)(x) = f(x^2 + 1)$, $g(x)$ must equal $x^2 + 1$.

1.8.5.

$$f(x) = x + 2, \quad g(x) = x - 2$$

$$\begin{aligned} \text{(a)} \quad (f + g)(x) &= f(x) + g(x) \\ &= (x + 2) + (x - 2) \\ &= 2x \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (f - g)(x) &= f(x) - g(x) \\ &= (x + 2) - (x - 2) \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (fg)(x) &= f(x) \cdot g(x) \\ &= (x + 2)(x - 2) \\ &= x^2 - 4 \end{aligned}$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + 2}{x - 2}$$

Domain: all real numbers x except $x = 2$

1.8.6.

$$f(x) = 2x - 5, \quad g(x) = 2 - x$$

$$\text{(a)} \quad (f + g)(x) = 2x - 5 + 2 - x = x - 3$$

$$\begin{aligned} \text{(b)} \quad (f - g)(x) &= 2x - 5 - (2 - x) \\ &= 2x - 5 - 2 + x \\ &= 3x - 7 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (fg)(x) &= (2x - 5)(2 - x) \\ &= 4x - 2x^2 - 10 + 5x \\ &= -2x^2 + 9x - 10 \end{aligned}$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)(x) = \frac{2x - 5}{2 - x}$$

Domain: all real numbers x except $x = 2$

1.8.7.

$$f(x) = x^2, \quad g(x) = 4x - 5$$

$$\begin{aligned} \text{(a)} \quad (f + g)(x) &= f(x) + g(x) \\ &= x^2 + (4x - 5) \\ &= x^2 + 4x - 5 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (f - g)(x) &= f(x) - g(x) \\ &= x^2 - (4x - 5) \\ &= x^2 - 4x + 5 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (fg)(x) &= f(x) \cdot g(x) \\ &= x^2(4x - 5) \\ &= 4x^3 - 5x^2 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^2}{4x - 5} \end{aligned}$$

Domain: all real numbers x except $x = \frac{5}{4}$

1.8.8.

$$f(x) = 3x + 1, \quad g(x) = x^2 - 16$$

$$\begin{aligned} \text{(a)} \quad (f + g)(x) &= f(x) + g(x) \\ &= 3x + 1 + x^2 - 16 \\ &= x^2 + 3x - 15 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (f - g)(x) &= f(x) - g(x) \\ &= 3x + 1 - (x^2 - 16) \\ &= -x^2 + 3x + 17 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (fg)(x) &= f(x) \cdot g(x) \\ &= (3x + 1)(x^2 - 16) \\ &= 3x^3 + x^2 - 48x - 16 \end{aligned}$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x + 1}{x^2 - 16}$$

Domain: all real numbers x , except $x \neq \pm 4$

1.8.9.

$$f(x) = x^2 + 6, \quad g(x) = \sqrt{1 - x}$$

$$\text{(a)} \quad (f + g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1 - x}$$

$$\text{(b)} \quad (f - g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1 - x}$$

$$\text{(c)} \quad (fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 6}{\sqrt{1 - x}} = \frac{(x^2 + 6)\sqrt{1 - x}}{1 - x}$$

Domain: $x < 1$

1.8.10.

$$f(x) = \sqrt{x^2 - 4}, \quad g(x) = \sqrt{x + 2}$$

$$\text{(a)} \quad (f + g)(x) = \sqrt{x^2 - 4} + \sqrt{x + 2}$$

$$\text{(b)} \quad (f - g)(x) = \sqrt{x^2 - 4} - \sqrt{x + 2}$$

$$\text{(c)} \quad (fg)(x) = \sqrt{x^2 - 4} \sqrt{x + 2} = (x + 2)\sqrt{x - 2}$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x^2 - 4}}{\sqrt{x + 2}} = \sqrt{x - 2}$$

Domain of $\sqrt{x^2 - 4}$: All real numbers x such that $x \leq -2$ and $x \geq 2$

Domain of $\sqrt{x + 2}$: All real numbers x such that $x \geq -2$

Domain of $\frac{f}{g}$: All real numbers x such that $x \geq 2$

1.8.11.

$$f(x) = \frac{x}{x+1}, \quad g(x) = x^3$$

$$(a) \quad (f+g)(x) = \frac{x}{x+1} + x^3 = \frac{x + x^4 + x^3}{x+1}$$

$$(b) \quad (f-g)(x) = \frac{x}{x+1} - x^3 = \frac{x - x^4 - x^3}{x+1}$$

$$(c) \quad (fg)(x) = \frac{x}{x+1} \cdot x^3 = \frac{x^4}{x+1}$$

$$(d) \quad \left(\frac{f}{g}\right)(x) = \frac{x}{x+1} \div x^3 = \frac{x}{x+1} \cdot \frac{1}{x^3} \\ = \frac{1}{x^2(x+1)}$$

Domain: all real numbers x except $x = 0$ and $x = -1$

1.8.12.

$$f(x) = \frac{2}{x}, \quad g(x) = \frac{1}{x^2 - 1}$$

$$(a) \quad (f+g)(x) = \frac{2}{x} + \frac{1}{x^2 - 1} = \frac{2(x^2 - 1) + x}{x(x^2 - 1)} = \frac{2x^2 + x - 2}{x(x^2 - 1)}$$

$$(b) \quad (f-g)(x) = \frac{2}{x} - \frac{1}{x^2 - 1} = \frac{2(x^2 - 1) - x}{x(x^2 - 1)} = \frac{2x^2 - x - 2}{x(x^2 - 1)}$$

$$(c) \quad (fg)(x) = \left(\frac{2}{x}\right)\left(\frac{1}{x^2 - 1}\right) = \frac{2}{x(x^2 - 1)}$$

$$(d) \quad \left(\frac{f}{g}\right)(x) = \frac{2}{x} \div \frac{1}{x^2 - 1} = \frac{\frac{2}{x}}{\frac{1}{x^2 - 1}} = \left(\frac{2}{x}\right)(x^2 - 1) = \frac{2(x^2 - 1)}{x}$$

Domain: all real numbers x , $x \neq 0, \pm 1$

1.8.13.

$$(f+g)(2) = f(2) + g(2) \\ = (2+3) + (2^2 - 2) \\ = 7$$

1.8.14.

$$\begin{aligned}(f - g)(1) &= f(1) - g(1) \\ &= (1 + 3) - ((1)^2 - 2) \\ &= 5\end{aligned}$$

1.8.15.

$$\begin{aligned}(f - g)(3t) &= f(3t) - g(3t) \\ &= ((3t) + 3) - ((3t)^2 - 2) \\ &= 3t + 3 - (9t^2 - 2) \\ &= -9t^2 + 3t + 5\end{aligned}$$

1.8.16.

$$\begin{aligned}(f + g)(t - 2) &= f(t - 2) + g(t - 2) \\ &= ((t - 2) + 3) + ((t - 2)^2 - 2) \\ &= t + 1 + (t^2 - 4t + 4 - 2) \\ &= t^2 - 3t + 3\end{aligned}$$

1.8.17.

$$\begin{aligned}(fg)(6) &= f(6)g(6) \\ &= ((6) + 3)((6)^2 - 2) \\ &= (9)(34) \\ &= 306\end{aligned}$$

1.8.18.

$$\begin{aligned}(fg)(-6) &= f(-6)g(-6) \\ &= ((-6) + 3)((-6)^2 - 2) \\ &= (-3)(34) \\ &= -102\end{aligned}$$

1.8.19.

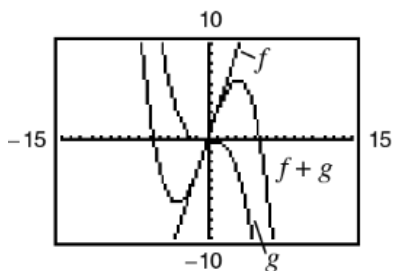
$$\begin{aligned}(f/g)(5) &= f(5)/g(5) \\ &= ((5) + 3)/((5)^2 - 2) \\ &= \frac{8}{23}\end{aligned}$$

1.8.20.

$$\begin{aligned}(f/g)(0) &= f(0)/g(0) \\ &= ((0)+3)/((0)^2-2) \\ &= -\frac{3}{2}\end{aligned}$$

1.8.21.

$$\begin{aligned}f(x) &= 3x, \quad g(x) = -\frac{x^3}{10} \\ (f+g)(x) &= 3x - \frac{x^3}{10}\end{aligned}$$

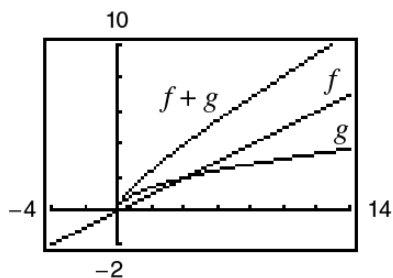


For $0 \leq x \leq 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

1.8.22.

$$\begin{aligned}f(x) &= \frac{x}{2}, \quad g(x) = \sqrt{x} \\ (f+g)(x) &= \frac{x}{2} + \sqrt{x}\end{aligned}$$

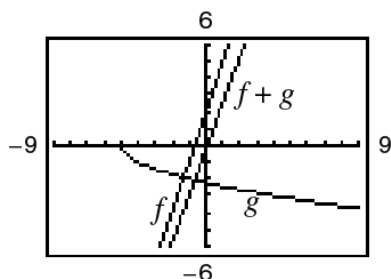


$g(x)$ contributes most to the magnitude of the sum for $0 \leq x \leq 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

1.8.23.

$$f(x) = 3x + 2, \quad g(x) = -\sqrt{x+5}$$

$$(f+g)(x) = 3x - \sqrt{x+5} + 2$$



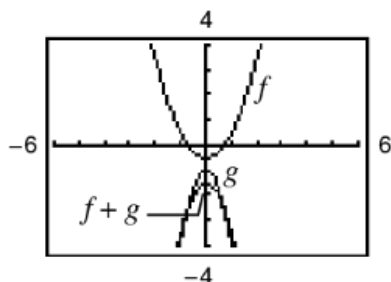
For $0 \leq x \leq 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $f(x)$ contributes most to the magnitude.

1.8.24.

$$f(x) = x^2 - \frac{1}{2}, \quad g(x) = -3x^2 - 1$$

$$(f+g)(x) = -2x^2 - \frac{3}{2}$$



For $0 \leq x \leq 2$, $g(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

1.8.25.

$$f(x) = x + 8, \quad g(x) = x - 3$$

$$(a) \quad (f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3) + 8 = x + 5$$

$$(b) \quad (g \circ f)(x) = g(f(x)) = g(x + 8) = (x + 8) - 3 = x + 5$$

$$(c) \quad (g \circ g)(x) = g(g(x)) = g(x - 3) = (x - 3) - 3 = x - 6$$

1.8.26.

$$f(x) = 3x, \quad g(x) = x^4$$

$$(a) \quad (f \circ g)(x) = f(g(x)) = f(x^4) = 3(x^4) = 3x^4$$

$$(b) \quad (g \circ f)(x) = g(f(x)) = g(3x) = (3x)^4 = 81x^4$$

$$(c) \quad (g \circ g)(x) = g(g(x)) = g(x^4) = (x^4)^4 = x^{16}$$

1.8.27.

$$f(x) = \sqrt[3]{x-1}, \quad g(x) = x^3 + 1$$

$$\begin{aligned} (a) \quad (f \circ g)(x) &= f(g(x)) \\ &= f(x^3 + 1) \\ &= \sqrt[3]{(x^3 + 1) - 1} \\ &= \sqrt[3]{x^3} = x \end{aligned}$$

$$\begin{aligned} (b) \quad (g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt[3]{x-1}) \\ &= (\sqrt[3]{x-1})^3 + 1 \\ &= (x-1) + 1 = x \end{aligned}$$

$$\begin{aligned} (c) \quad (g \circ g)(x) &= g(g(x)) \\ &= g(x^3 + 1) \\ &= (x^3 + 1)^3 + 1 \\ &= x^9 + 3x^6 + 3x^3 + 2 \end{aligned}$$

1.8.28.

$$f(x) = x^3, \quad g(x) = \frac{1}{x}$$

$$(a) \quad (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$$

$$(b) \quad (g \circ f)(x) = g(f(x)) = g(x^3) = \frac{1}{x^3}$$

$$(c) \quad (g \circ g)(x) = g(g(x)) = g\left(\frac{1}{x}\right) = x$$

1.8.29.

$$f(x) = \sqrt{x+4} \quad \text{Domain: } x \geq -4$$

$$g(x) = x^2 \quad \text{Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$$

Domain: all real numbers x

$$(b) (g \circ f)(x) = g(f(x)) \\ = g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x+4$$

Domain: $x \geq -4$

1.8.30.

$$f(x) = \sqrt[3]{x-5} \quad \text{Domain: all real numbers } x$$

$$g(x) = x^3 + 1 \quad \text{Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) \\ = f(x^3 + 1) \\ = \sqrt[3]{x^3 + 1 - 5} \\ = \sqrt[3]{x^3 - 4}$$

Domain: all real numbers x

$$(b) (g \circ f)(x) = g(f(x)) \\ = g(\sqrt[3]{x-5}) \\ = (\sqrt[3]{x-5})^3 + 1 \\ = x - 5 + 1 = x - 4$$

Domain: all real numbers x

1.8.31.

$$f(x) = |x| \quad \text{Domain: all real numbers } x$$

$$g(x) = x + 6 \quad \text{Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) = f(x+6) = |x+6|$$

Domain: all real numbers x

$$(b) (g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$$

Domain: all real numbers x

1.8.32.

$$f(x) = |x - 4| \text{ Domain: all real numbers } x$$

$$g(x) = 3 - x \text{ Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1|$$

Domain: all real numbers x

$$(b) (g \circ f)(x) = g(f(x)) = g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$$

Domain: all real numbers x

1.8.33.

$$f(x) = \frac{1}{x} \text{ Domain: all real numbers } x \text{ except } x = 0$$

$$g(x) = x + 3 \text{ Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) = f(x + 3) = \frac{1}{x + 3}$$

Domain: all real numbers x except $x = -3$

$$(b) (g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 3$$

Domain: all real numbers x except $x = 0$

1.8.34.

$$f(x) = \frac{3}{x^2 - 1} \text{ Domain: all real numbers } x \text{ except } x = \pm 1$$

$$g(x) = x + 1 \text{ Domain: all real numbers } x$$

$$(a) (f \circ g)(x) = f(g(x)) = f(x + 1) = \frac{3}{(x + 1)^2 - 1} = \frac{3}{x^2 + 2x + 1 - 1} = \frac{3}{x^2 + 2x}$$

Domain: all real numbers x except $x = 0$ and $x = -2$

$$(b) (g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x^2 - 1}\right) = \frac{3}{x^2 - 1} + 1 = \frac{3 + x^2 - 1}{x^2 - 1} = \frac{x^2 + 2}{x^2 - 1}$$

Domain: all real numbers x except $x = \pm 1$

1.8.35.

$$(a) (f + g)(3) = f(3) + g(3) = 2 + 1 = 3$$

$$(b) \left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$$

1.8.36.

$$(a) (f - g)(1) = f(1) - g(1) = 2 - 3 = -1$$

$$(b) (fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$$

1.8.37.

$$(a) (f \circ g)(2) = f(g(2)) = f(2) = 0$$

$$(b) (g \circ f)(2) = g(f(2)) = g(0) = 4$$

1.8.38.

$$(a) (f \circ g)(1) = f(g(1)) = f(3) = 2$$

$$(b) (g \circ f)(3) = g(f(3)) = g(2) = 2$$

1.8.39.

$$h(x) = (2x + 1)^2$$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

1.8.40.

$$h(x) = (1 - x)^3$$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

1.8.41.

$$h(x) = \sqrt[3]{x^2 - 4}$$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

1.8.42.

$$h(x) = \sqrt{9 - x}$$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

1.8.43.

$$h(x) = \frac{1}{x + 2}$$

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$.

1.8.44.

$$h(x) = \frac{4}{(5x+2)^2}$$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \frac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

1.8.45.

$$h(x) = \frac{-x^2 + 3}{4 - x^2}$$

One possibility: Let $f(x) = \frac{x+3}{4+x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

1.8.46.

$$h(x) = \frac{27x^3 + 6x}{10 - 27x^3}$$

One possibility: Let $g(x) = x^3$ and $f(x) = \frac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

1.8.47.

$$(a) \ c(t) = \frac{b(t) - d(t)}{p(t)} \times 100 = \text{percent change}$$

(b) $c(24)$ represents the percent change in the population due to births and deaths in the year 2024.

1.8.48.

$$(a) \ p(t) = d(t) + c(t)$$

(b) $p(24)$ represents the number of dogs and cats in 2024.

$$(c) \ h(t) = \frac{p(t)}{n(t)} = \frac{d(t) + c(t)}{n(t)}$$

$h(t)$ represents the number of dogs and cats per capita.

1.8.49.

$$(a) \ r(x) = \frac{x}{2}$$

$$(b) \ A(r) = \pi r^2$$

$$(c) (A \circ r)(x) = A(r(x)) = A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2$$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x .

1.8.50.

$$\begin{aligned} (a) N(T(t)) &= N(3t + 2) \\ &= 10(3t + 2)^2 - 20(3t + 2) + 600 \\ &= 10(9t^2 + 12t + 4) - 60t - 40 + 600 \\ &= 90t^2 + 60t + 600 \\ &= 30(3t^2 + 2t + 20), 0 \leq t \leq 6 \end{aligned}$$

This represents the number of bacteria in the food as a function of time.

(b) Use $t = 0.5$.

$$N(T(0.5)) = 30(3(0.5)^2 + 2(0.5) + 20) = 652.5$$

After half an hour, there will be about 653 bacteria.

$$\begin{aligned} (c) 30(3t^2 + 2t + 20) &= 1500 \\ 3t^2 + 2t + 20 &= 50 \\ 3t^2 + 2t - 30 &= 0 \end{aligned}$$

By the Quadratic Formula, $t \approx -3.513$ or 2.846 .

Choosing the positive value for t , you have $t \approx 2.846$ hours.

1.8.51.

False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$

1.8.52.

True. $(f \circ g)(c)$ is defined only when $g(c)$ is in the domain of f .

1.8.53.

(a) Answer not unique. *Sample answer:*

$$\begin{aligned} f(x) &= x + 3, g(x) = x + 2 \\ (f \circ g)(x) &= f(g(x)) = (x + 2) + 3 = x + 5 \\ (g \circ f)(x) &= g(f(x)) = (x + 3) + 2 = x + 5 \end{aligned}$$

(b) Answer not unique. *Sample answer:* $f(x) = x^2$, $g(x) = x^3$

$$(f \circ g)(x) = f(g(x)) = (x^3)^2 = x^6$$

$$(g \circ f)(x) = g(f(x)) = (x^2)^3 = x^6$$

1.8.54.

(a) $f(p)$: matches L_2 ; For example, an original price of $p = \$15.00$ corresponds to a sale price of $S = \$7.50$.

(b) $g(p)$: matches L_1 ; For example an original price of $p = \$20.00$ corresponds to a sale price of $S = \$15.00$.

(c) $(g \circ f)(p)$: matches L_4 ; This function represents applying a 50% discount to the original price p , then subtracting a \$5 discount.

(d) $(f \circ g)(p)$ matches L_3 ; This function represents subtracting a \$5 discount from the original price p , then applying a 50% discount.

1.8.55.

Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{because } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{because } f \text{ and } g \text{ are even} \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

1.8.56.

(a) $g(x) = \frac{1}{2}[f(x) + f(-x)]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$.

$$g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = \frac{1}{2}[f(x) + f(-x)] = g(x) \quad \checkmark$$

$$h(x) = \frac{1}{2}[f(x) - f(-x)]$$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$.

$$h(-x) = \frac{1}{2}[f(-x) - f(-(-x))] = \frac{1}{2}[f(-x) - f(x)] = -\frac{1}{2}[f(x) - f(-x)] = -h(x) \quad \checkmark$$

$$\begin{aligned} \text{(b)} \quad \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)] &= \frac{1}{2}[f(x) + f(-x) + f(x) - f(-x)] \\ &= \frac{1}{2}[2f(x)] \\ &= f(x) \end{aligned}$$

$$\text{(c)} \quad f(x) = x^2 - 2x + 1 = (x^2 + 1) + (-2x)$$

$$k(x) = \frac{1}{x+1} = -\frac{1}{(x+1)(x-1)} + \frac{x}{(x+1)(x-1)}$$

1.8.57.

Let $f(x)$ be an odd function, $g(x)$ be an even function, and define

$$h(x) = f(x)g(x). \text{ Then}$$

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)]g(x) \quad \text{because } f \text{ is odd and } g \text{ is even} \\ &= -f(x)g(x) \\ &= -h(x). \end{aligned}$$

So, h is odd and the product of an odd function and an even function is odd.

1.8.58.

The expression $3(2x+1)+2$ is found using $(g \circ f)(x)$ instead of $(f \circ g)(x)$.

$$\begin{aligned} \text{So, } (f \circ g)(x) &= 2(3x+2)+1 \\ &= 6x+5. \end{aligned}$$

1.8.59.

$$y^2 = x - 5$$

$$y^2 = (-x) - 5 \Rightarrow y^2 = -x - 5 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$(-y)^2 = x - 5 \Rightarrow y^2 = x - 5 \Rightarrow \text{x-axis symmetry}$$

$$(-y)^2 = (-x) - 5 \Rightarrow y^2 = -x - 5 \Rightarrow \text{No origin symmetry}$$

1.8.60.

$$2x + 5y = 17$$

$$2(-x) + 5y = 17 \Rightarrow -2x + 5y = 17 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$2x + 5(-y) = 17 \Rightarrow 2x - 5y = 17 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$2(-x) + 5(-y) = 17 \Rightarrow -2x - 5y = 17 \Rightarrow \text{No origin symmetry}$$

1.8.61.

$$y = x^2 + 1$$

$$y = (-x)^2 + 1 \Rightarrow y = x^2 + 1 \Rightarrow y\text{-axis symmetry}$$

$$-y = x^2 + 1 \Rightarrow y = -x^2 - 1 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1 \Rightarrow y = -x^2 - 1 \Rightarrow \text{No origin symmetry}$$

1.8.62.

$$x^2 + y^2 = 72$$

$$(-x)^2 + y^2 = 72 \Rightarrow x^2 + y^2 = 72 \Rightarrow y\text{-axis symmetry}$$

$$x^2 + (-y)^2 = 72 \Rightarrow x^2 + y^2 = 72 \Rightarrow x\text{-axis symmetry}$$

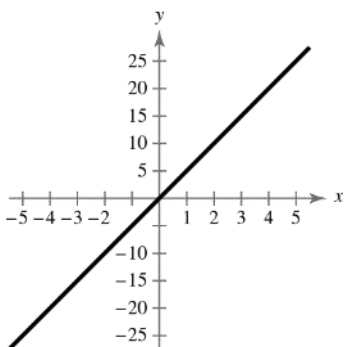
$$(-x)^2 + (-y)^2 = 72 \Rightarrow x^2 + y^2 = 72 \Rightarrow \text{Origin symmetry}$$

1.8.63.

$$y = 5x$$

$$f(x) = 5x$$

$$f(-x) = 5(-x) = -5x = -f(x) \text{ Odd function}$$

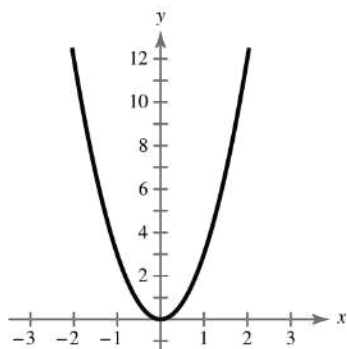


1.8.64.

$$y = 3x^2$$

$$f(x) = 3x^2$$

$$f(-x) = 3(-x)^2 = 3x^2 = f(x) \text{ Even function}$$



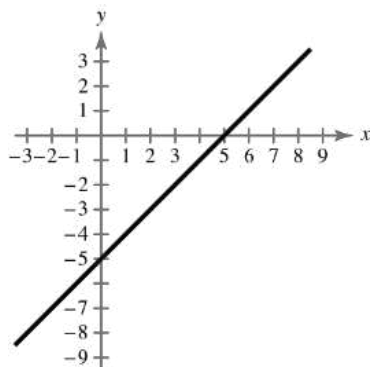
1.8.65.

$$y = x - 5$$

$$f(x) = x - 5$$

$$f(-x) = -x - 5$$

Neither even nor odd



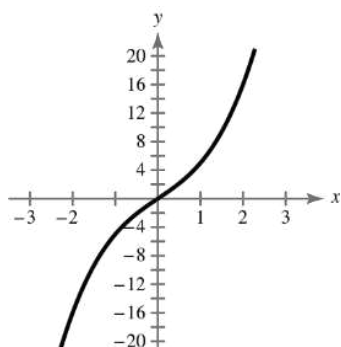
1.8.66.

$$y = x^3 + 4x$$

$$f(x) = x^3 + 4x$$

$$f(-x) = (-x)^3 + 4(-x) = -(x^3 + 4x) = -f(x)$$

Odd function



1.8.67.

$$2x + 3y = 5$$

$$3y = 5 - 2x$$

$$y = \frac{5}{3} - \frac{2}{3}x$$

1.8.68.

$$xy - 1 = 3y + x$$

$$xy - 3y = 1 + x$$

$$y(x - 3) = 1 + x$$

$$y = \frac{(1 + x)}{(x - 3)}$$

1.8.69.

$$x = \sqrt{y + 1}$$

$$x^2 = y + 1$$

$$y = x^2 - 1, \quad x \geq 0$$

1.8.70.

$$x = \frac{5 - y}{3y + 2}$$

$$3xy + 2x = 5 - y$$

$$3xy + y = 5 - 2x$$

$$y(3x + 1) = 5 - 2x$$

$$y = \frac{(5 - 2x)}{(3x + 1)}$$

1.8.71.

$$\begin{aligned} A &= \pi(r+2)^2 - \pi r^2 \\ &= \pi[(r+2)^2 - r^2] \\ &= \pi[r^2 + 4r + 4 - r^2] \\ &= \pi(4r + 4) \\ &= 4\pi(r + 1) \end{aligned}$$

1.8.72.

$$\begin{aligned} A &= \frac{1}{2} \left[\frac{5}{4}(x+3) \right] (x+3) - \frac{1}{2}(5)(4) \\ &= \frac{5}{8}(x+3)^2 - 10 \\ &= \frac{5}{8}(x^2 + 6x + 9) - 10 \end{aligned}$$

SECTION 1.9 INVERSE FUNCTIONS

1.9.1.

range; domain

1.9.2.

$$y = x$$

1.9.3.

To show that two function f and g are inverse functions, you must show that $f(g(x)) = x$ and $g(f(x)) = x$.

1.9.4.

If a function is one-to-one, no two x -values in the domain can correspond to the same y -value in the range. Therefore, a horizontal line can intersect the graph at most once.

1.9.5.

$$f(x) = 6x$$

$$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$$

$$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$$

1.9.6.

$$f(x) = \frac{1}{3}x$$

$$f^{-1}(x) = 3x$$

$$f(f^{-1}(x)) = f(3x) = \frac{1}{3}(3x) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$$

1.9.7.

$$f(x) = 3x + 1$$

$$f^{-1}(x) = \frac{x-1}{3}$$

$$f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\left(\frac{x-1}{3}\right) + 1 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x + 1) = \frac{(3x + 1) - 1}{3} = x$$

1.9.8.

$$f(x) = \frac{x-3}{2}$$

$$f^{-1}(x) = 2x + 3$$

$$f(f^{-1}(x)) = f(2x + 3) = \frac{(2x + 3) - 3}{2} = \frac{2x}{2} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3$$

$$= (x - 3) + 3 = x$$

1.9.9.

$$f(x) = x^3 + 1$$

$$f^{-1}(x) = \sqrt[3]{x-1}$$

$$f(f^{-1}(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = (x-1) + 1 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^3 + 1) = \sqrt[3]{(x^3 + 1) - 1} = \sqrt[3]{x^3} = x$$

1.9.10.

$$f(x) = \frac{x^5}{4}$$

$$f^{-1}(x) = \sqrt[5]{4x}$$

$$f(f^{-1}(x)) = f(\sqrt[5]{4x}) = \frac{(\sqrt[5]{4x})^5}{4} = \frac{4x}{4} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^5}{4}\right) = \sqrt[5]{4\left(\frac{x^5}{4}\right)} = \sqrt[5]{x^5} = x$$

1.9.11.

$$f(x) = x^2 - 4, \quad x \geq 0$$

$$f^{-1}(x) = \sqrt{x+4}$$

$$f(f^{-1}(x)) = f(\sqrt{x+4}) = (\sqrt{x+4})^2 - 4 = (x+4) - 4 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^2 - 4) = \sqrt{(x^2 - 4) + 4} = \sqrt{x^2} = x$$

1.9.12.

$$f(x) = x^2 + 2, \quad x \geq 0$$

$$f^{-1}(x) = \sqrt{x-2}$$

$$f(f^{-1}(x)) = f(\sqrt{x-2}) = (\sqrt{x-2})^2 + 2 = (x-2) + 2 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^2 + 2) = \sqrt{(x^2 + 2) - 2} = \sqrt{x^2} = x$$

1.9.13.

$$(f \circ g)(x) = f(g(x)) = f(4x + 9) = \frac{4x + 9 - 9}{4} = \frac{4x}{4} = x$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x-9}{4}\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

1.9.14.

$$f(g(x)) = f\left(-\frac{2x+8}{3}\right) = -\frac{3}{2}\left(-\frac{2x+8}{3}\right) - 4$$

$$= \frac{1}{2}(2(x+4)) - 4$$

$$= x + 4 - 4 = x$$

$$g(f(x)) = g\left(-\frac{3}{2}x - 4\right) = -\frac{2\left(-\frac{3}{2}x - 4\right) + 8}{3}$$

$$= -\frac{-3x - 8 + 8}{3}$$

$$= -\frac{-3x}{3}$$

$$= x$$

1.9.15.

$$f(g(x)) = f(\sqrt[3]{4x}) = \frac{(\sqrt[3]{4x})^3}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$$g(f(x)) = g\left(\frac{x^3}{4}\right) = \sqrt[3]{4\left(\frac{x^3}{4}\right)}$$

$$= \sqrt[3]{x^3}$$

$$= x$$

1.9.16.

$$(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x-5}) = (\sqrt[3]{x-5})^3 + 5 = x - 5 + 5 = x$$

$$(g \circ f)(x) = g(f(x)) = g(x^3 + 5) = \sqrt[3]{x^3 + 5 - 5} = \sqrt[3]{x^3} = x$$

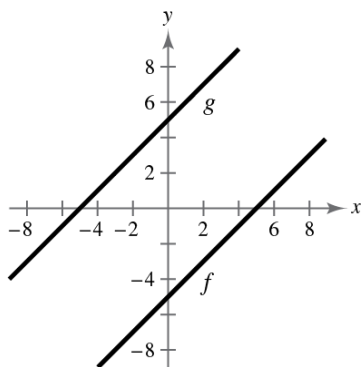
1.9.17.

$$f(x) = x - 5, \quad g(x) = x + 5$$

$$(a) \quad f(g(x)) = f(x + 5) = (x + 5) - 5 = x$$

$$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$$

(b)



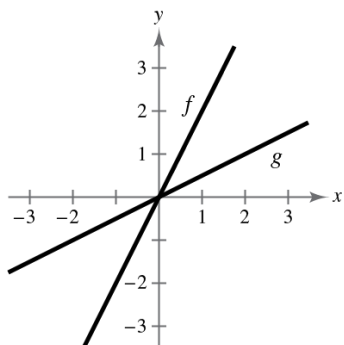
1.9.18.

$$f(x) = 2x, \quad g(x) = \frac{x}{2}$$

$$(a) \quad f(g(x)) = f\left(\frac{x}{2}\right) = 2\left(\frac{x}{2}\right) = x$$

$$g(f(x)) = g(2x) = \frac{2x}{2} = x$$

(b)



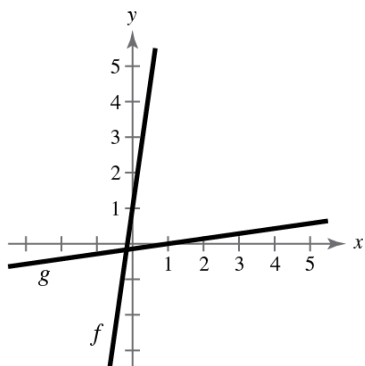
1.9.19.

$$f(x) = 7x + 1, \quad g(x) = \frac{x-1}{7}$$

$$(a) \quad f(g(x)) = f\left(\frac{x-1}{7}\right) = 7\left(\frac{x-1}{7}\right) + 1 = x$$

$$g(f(x)) = g(7x + 1) = \frac{(7x + 1) - 1}{7} = x$$

(b)



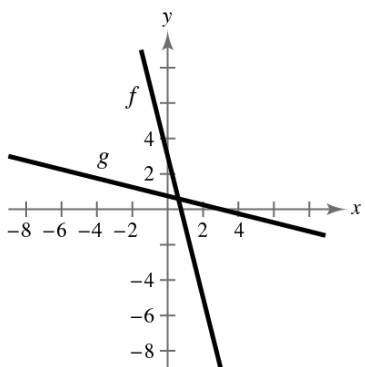
1.9.20.

$$f(x) = 3 - 4x, \quad g(x) = \frac{3-x}{4}$$

$$(a) \quad f(g(x)) = f\left(\frac{3-x}{4}\right) = 3 - 4\left(\frac{3-x}{4}\right) \\ = 3 - (3 - x) = x$$

$$g(f(x)) = g(3 - 4x) = \frac{3 - (3 - 4x)}{4} = \frac{4x}{4} \\ = x$$

(b)



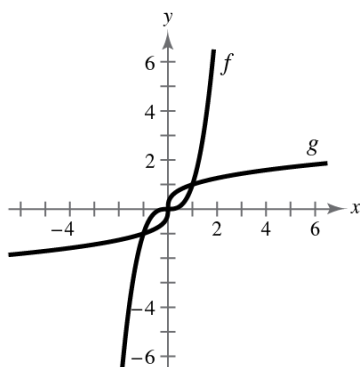
1.9.21.

$$f(x) = x^3, \quad g(x) = \sqrt[3]{x}$$

$$(a) \quad f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$$

$$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$$

(b)



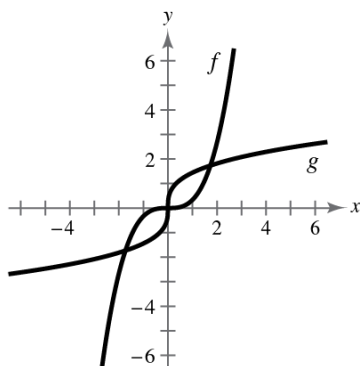
1.9.22.

$$f(x) = \frac{x^3}{3}, \quad g(x) = \sqrt[3]{3x}$$

$$(a) \quad f(g(x)) = f(\sqrt[3]{3x}) = \frac{(\sqrt[3]{3x})^3}{3} = \frac{3x}{3} = x$$

$$g(f(x)) = g\left(\frac{x^3}{3}\right) = \sqrt[3]{3\left(\frac{x^3}{3}\right)} = \sqrt[3]{x^3} = x$$

(b)



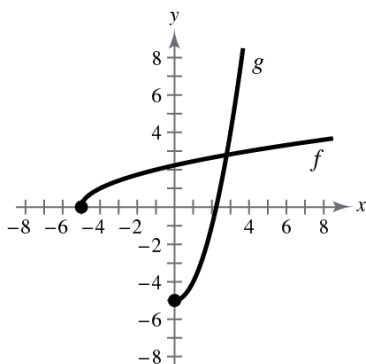
1.9.23.

$$f(x) = \sqrt{x+5}, \quad g(x) = x^2 - 5, \quad x \geq 0$$

$$\begin{aligned} \text{(a)} \quad f(g(x)) &= f(x^2 - 5), \quad x \geq 0 \\ &= \sqrt{(x^2 - 5) + 5} = x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(\sqrt{x+5}) \\ &= (\sqrt{x+5})^2 - 5 = x \end{aligned}$$

(b)



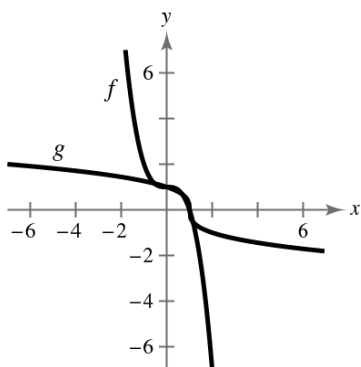
1.9.24.

$$f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1-x}$$

$$\begin{aligned} \text{(a)} \quad f(g(x)) &= f(\sqrt[3]{1-x}) = 1 - (\sqrt[3]{1-x})^3 \\ &= 1 - (1-x) = x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)} \\ &= \sqrt[3]{x^3} = x \end{aligned}$$

(b)



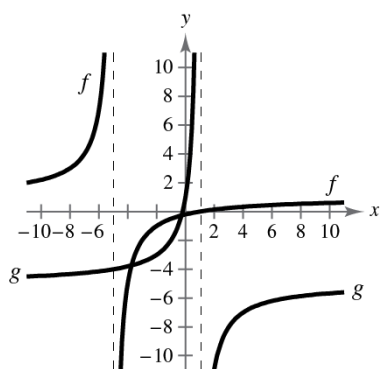
1.9.25.

$$f(x) = \frac{x-1}{x+5}, \quad g(x) = -\frac{5x+1}{x-1}$$

$$(a) \quad f(g(x)) = f\left(-\frac{5x+1}{x-1}\right) = \frac{\left(-\frac{5x+1}{x-1} - 1\right)}{\left(-\frac{5x+1}{x-1} + 5\right)} \cdot \frac{x-1}{x-1} = \frac{-(5x+1) - (x-1)}{-(5x+1) + 5(x-1)} = \frac{-6x}{-6} = x$$

$$g(f(x)) = g\left(\frac{x-1}{x+5}\right) = -\frac{\left[5\left(\frac{x-1}{x+5}\right) + 1\right]}{\left[\frac{x-1}{x+5} - 1\right]} \cdot \frac{x+5}{x+5} = -\frac{5(x-1) + (x+5)}{(x-1) - (x+5)} = -\frac{6x}{-6} = x$$

(b)

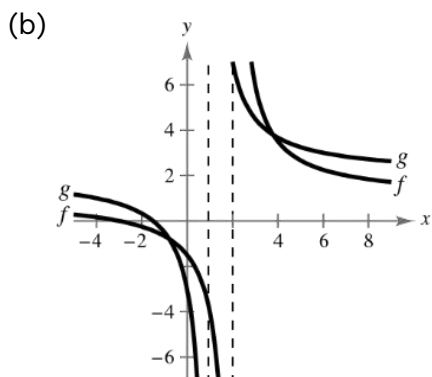


1.9.26.

$$f(x) = \frac{x+3}{x-2}, \quad g(x) = \frac{2x+3}{x-1}$$

$$(a) \quad f(g(x)) = f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1} + 3}{\frac{2x+3}{x-1} - 2} = \frac{\frac{2x+3+3x-3}{x-1}}{\frac{2x+3-2x+2}{x-1}} = \frac{5x}{5} = x$$

$$g(f(x)) = g\left(\frac{x+3}{x-2}\right) = \frac{2\left(\frac{x+3}{x-2}\right) + 3}{\frac{x+3}{x-2} - 1} = \frac{\frac{2x+6+3x-6}{x-2}}{\frac{x+3-x+2}{x-2}} = \frac{5x}{5} = x$$



1.9.27.

x	3	5	7	9	11	13
$f^{-1}(x)$	-1	0	1	2	3	4

1.9.28.

x	10	5	0	-5	-10	-15
$f^{-1}(x)$	-3	-2	-1	0	1	2

1.9.29.

Yes, because no horizontal line crosses the graph of f at more than one point, f has an inverse.

1.9.30.

No, because some horizontal lines intersect the graph of f twice, f does not have an inverse.

1.9.31.

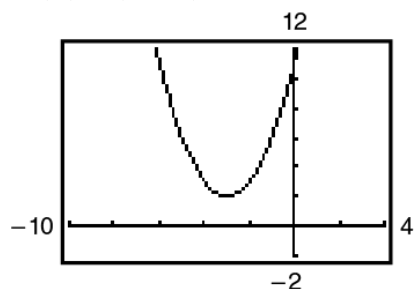
No, because some horizontal lines cross the graph of f twice, f does not have an inverse.

1.9.32.

Yes, because no horizontal lines intersect the graph of f at more than one point, f has an inverse.

1.9.33.

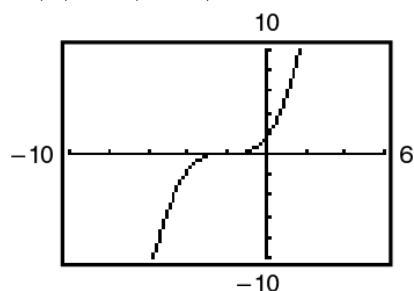
$$g(x) = (x + 3)^2 + 2$$



g does not pass the Horizontal Line Test, so g does not have an inverse.

1.9.34.

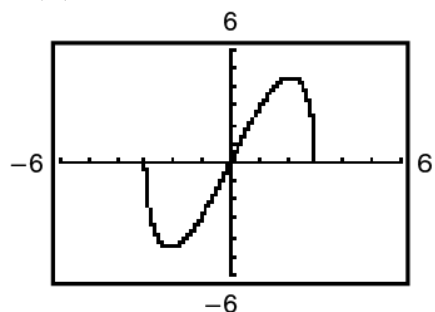
$$f(x) = \frac{1}{5}(x + 2)^3$$



f does pass the Horizontal Line Test, so f does have an inverse.

1.9.35.

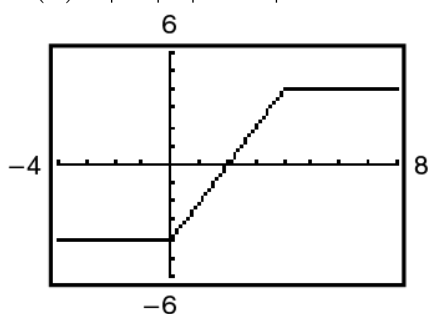
$$f(x) = x\sqrt{9 - x^2}$$



f does not pass the Horizontal Line Test, so f does not have an inverse.

1.9.36.

$$h(x) = |x| - |x - 4|$$



h does not pass the Horizontal Line Test, so h does not have an inverse.

1.9.37.

(a) $f(x) = x^5 - 2$

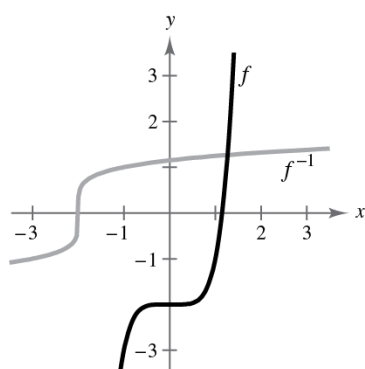
$$y = x^5 - 2$$

$$x = y^5 - 2$$

$$y = \sqrt[5]{x + 2}$$

$$f^{-1}(x) = \sqrt[5]{x + 2}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

1.9.38.

(a) $f(x) = x^3 + 8$

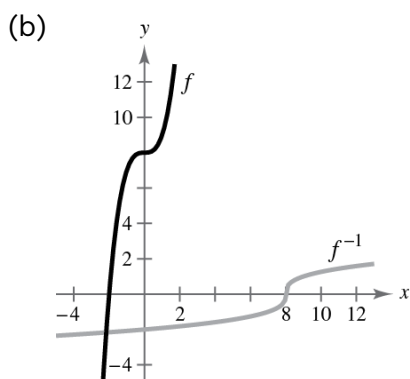
$$y = x^3 + 8$$

$$x = y^3 + 8$$

$$x - 8 = y^3$$

$$\sqrt[3]{x - 8} = y$$

$$f^{-1}(x) = \sqrt[3]{x - 8}$$



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

1.9.39.

(a) $f(x) = \sqrt{4 - x^2}, 0 \leq x \leq 2$

$$y = \sqrt{4 - x^2}$$

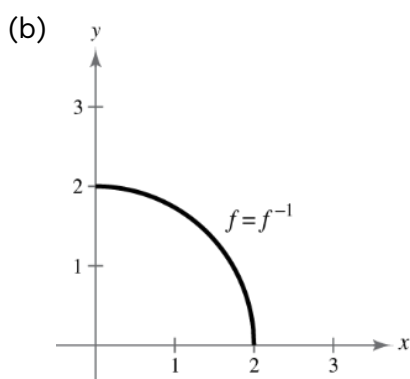
$$x = \sqrt{4 - y^2}$$

$$x^2 = 4 - y^2$$

$$y^2 = 4 - x^2$$

$$y = \sqrt{4 - x^2}$$

$$f^{-1}(x) = \sqrt{4 - x^2}, 0 \leq x \leq 2$$



(c) The graph of f^{-1} is the same as the graph of f .

(d) The domains and ranges of f and f^{-1} are all real numbers x such that $0 \leq x \leq 2$.

1.9.40.

(a) $f(x) = x^2 - 2, x \leq 0$

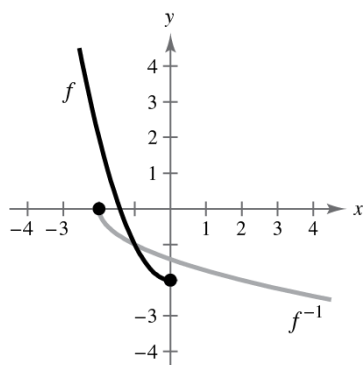
$$y = x^2 - 2$$

$$x = y^2 - 2$$

$$\pm \sqrt{x+2} = y$$

$$f^{-1}(x) = -\sqrt{x+2}$$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) $[-2, \infty)$ is the range of f and domain of f^{-1} .

$(-\infty, 0]$ is the domain of f and the range of f^{-1} .

1.9.41.

(a) $f(x) = \frac{4}{x}$

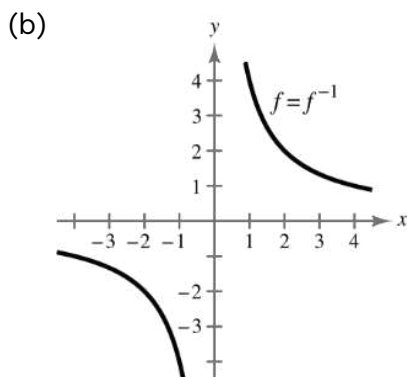
$$y = \frac{4}{x}$$

$$x = \frac{4}{y}$$

$$xy = 4$$

$$y = \frac{4}{x}$$

$$f^{-1}(x) = \frac{4}{x}$$

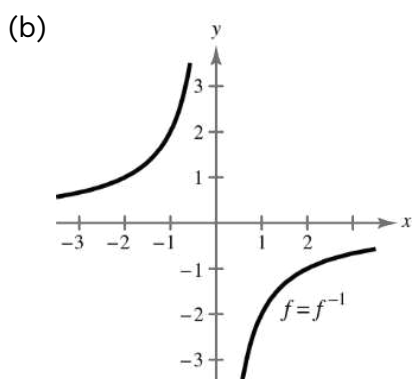


(c) The graph of f^{-1} is the same as the graph of f .

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

1.9.42.

(a) $f(x) = -\frac{2}{x}$
 $y = -\frac{2}{x}$
 $x = -\frac{2}{y}$
 $y = -\frac{2}{x}$
 $f^{-1}(x) = -\frac{2}{x}$

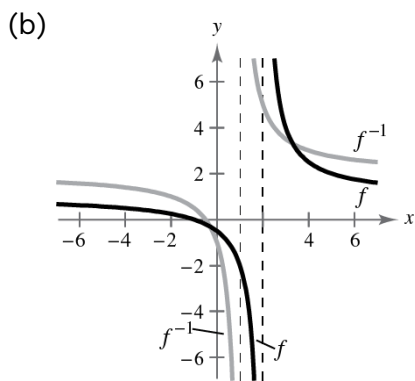


(c) The graphs are the same.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

1.9.43.

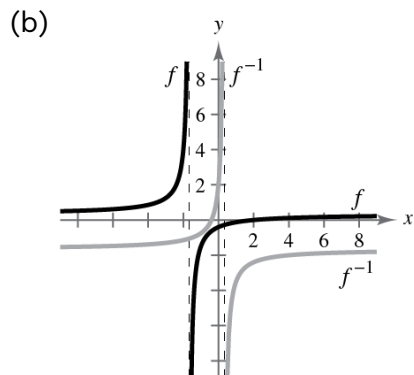
$$\begin{aligned}
 \text{(a)} \quad f(x) &= \frac{x+1}{x-2} \\
 y &= \frac{x+1}{x-2} \\
 x &= \frac{y+1}{y-2} \\
 x(y-2) &= y+1 \\
 xy - 2x &= y+1 \\
 xy - y &= 2x+1 \\
 y(x-1) &= 2x+1 \\
 y &= \frac{2x+1}{x-1} \\
 f^{-1}(x) &= \frac{2x+1}{x-1}
 \end{aligned}$$



- (c) The graph of f^{-1} is the reflection of graph of f in the line $y = x$.
- (d) The domain of f and the range of f^{-1} is all real numbers except 2.
The range of f and the domain of f^{-1} is all real numbers except 1.

1.9.44.

$$\begin{aligned}
 (a) \quad f(x) &= \frac{x-2}{3x+5} \\
 y &= \frac{x-2}{3x+5} \\
 x &= \frac{y-2}{3y+5} \\
 3xy + 5x - y + 2 &= 0 \\
 3xy - y &= -5x - 2 \\
 y(3x-1) &= -5x - 2 \\
 y &= \frac{-5x-2}{3x-1} \\
 y &= \frac{-5x-2}{3x-1} \\
 f^{-1}(x) &= -\frac{5x+2}{3x-1}
 \end{aligned}$$



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is all real numbers except $x = -\frac{5}{3}$.

The range of f and the domain of f^{-1} is all real numbers x except $x = \frac{1}{3}$.

1.9.45.

$$\begin{aligned}
 f(x) &= x^4 \\
 y &= x^4 \\
 x &= y^4 \\
 y &= \pm \sqrt[4]{x}
 \end{aligned}$$

This does not represent y as a function of x . f does not have an inverse.

1.9.46.

$$f(x) = \frac{1}{x^2}$$

$$y = \frac{1}{x^2}$$

$$x = \frac{1}{y^2}$$

$$y^2 = \frac{1}{x}$$

$$y = \pm \sqrt{\frac{1}{x}}$$

This does not represent y as a function of x . f does not have an inverse.

1.9.47.

$$g(x) = \frac{x+1}{6}$$

$$y = \frac{x+1}{6}$$

$$x = \frac{y+1}{6}$$

$$6x = y + 1$$

$$y = 6x - 1$$

This is a function of x , so g has an inverse.

$$g^{-1}(x) = 6x - 1$$

1.9.48.

$$f(x) = 3x + 5$$

$$y = 3x + 5$$

$$x = 3y + 5$$

$$x - 5 = 3y$$

$$\frac{x-5}{3} = y$$

This is a function of x , so f has an inverse.

$$f^{-1}(x) = \frac{x-5}{3}$$

1.9.49.

$$f(x) = \sqrt{2x+3} \Rightarrow x \geq -\frac{3}{2}, y \geq 0$$

$$y = \sqrt{2x+3}, x \geq -\frac{3}{2}, y \geq 0$$

$$x = \sqrt{2y+3}, y \geq -\frac{3}{2}, x \geq 0$$

$$x^2 = 2y+3, x \geq 0, y \geq -\frac{3}{2}$$

$$y = \frac{x^2-3}{2}, x \geq 0, y \geq -\frac{3}{2}$$

This is a function of x , so f has an inverse.

$$f^{-1}(x) = \frac{x^2-3}{2}, x \geq 0$$

1.9.50.

$$f(x) = \sqrt{x-2} \Rightarrow x \geq 2, y \geq 0$$

$$y = \sqrt{x-2}, x \geq 2, y \geq 0$$

$$x = \sqrt{y+2}, y \geq -2, x \geq 0$$

$$x^2 = y+2, x \geq 0, y \geq -2$$

$$x^2 + 2 = y, x \geq 0, y \geq -2$$

This is a function of x , so f has an inverse.

$$f^{-1}(x) = x^2 + 2, x \geq 0$$

1.9.51.

$$f(x) = \frac{6x+4}{4x+5}$$

$$y = \frac{6x+4}{4x+5}$$

$$x = \frac{6y+4}{4y+5}$$

$$x(4y+5) = 6y+4$$

$$4xy+5x = 6y+4$$

$$4xy-6y = -5x+4$$

$$y(4x-6) = -5x+4$$

$$y = \frac{-5x+4}{4x-6}$$

$$= \frac{5x-4}{6-4x}$$

This is a function of x , so f has an inverse.

$$f^{-1}(x) = \frac{5x - 4}{6 - 4x}$$

1.9.52.

The graph of f passes the Horizontal Line Test. So, you know f is one-to-one and has an inverse function.

$$f(x) = \frac{5x - 3}{2x + 5}$$

$$y = \frac{5x - 3}{2x + 5}$$

$$x = \frac{5y - 3}{2y + 5}$$

$$x(2y + 5) = 5y - 3$$

$$2xy + 5x = 5y - 3$$

$$2xy - 5y = -5x - 3$$

$$y(2x - 5) = -(5x + 3)$$

$$y = -\frac{5x + 3}{2x - 5}$$

$$f^{-1}(x) = -\frac{5x + 3}{2x - 5}$$

1.9.53.

$$f(x) = (x + 3)^2, \quad x \geq -3 \Rightarrow y \geq 0$$

$$y = (x + 3)^2, \quad x \geq -3, \quad y \geq 0$$

$$x = (y + 3)^2, \quad y \geq -3, \quad x \geq 0$$

$$\sqrt{x} = y + 3, \quad y \geq -3, \quad x \geq 0$$

$$y = \sqrt{x} - 3, \quad x \geq 0, \quad y \geq -3$$

This is a function of x , so f has an inverse.

$$f^{-1}(x) = \sqrt{x} - 3, \quad x \geq 0$$

1.9.54.

$$f(x) = |x - 2|, \quad x \leq 2 \Rightarrow y \geq 0$$

$$y = |x - 2|, \quad x \leq 2, \quad y \geq 0$$

$$x = |y - 2|, \quad y \leq 2, \quad x \geq 0$$

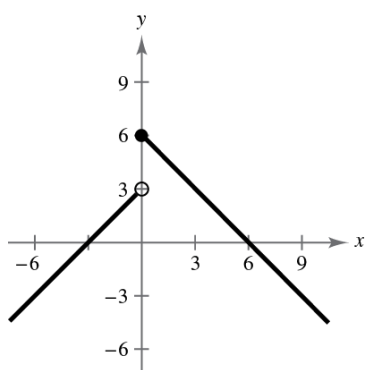
$$\begin{aligned} x &= y - 2 \quad \text{or} \quad -x = y - 2 \\ 2 + x &= y \quad \text{or} \quad 2 - x = y \end{aligned}$$

The portion that satisfies the conditions $y \leq 2$ and $x \geq 0$ is $2 - x = y$. This is a function of x , so f has an inverse.

$$f^{-1}(x) = 2 - x, x \geq 0$$

1.9.55.

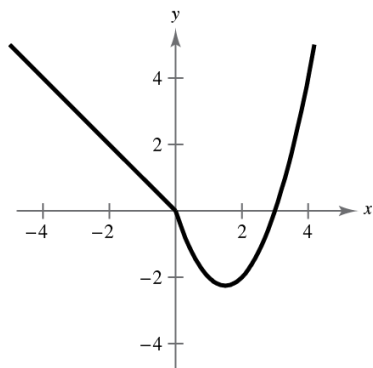
$$f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \geq 0 \end{cases}$$



This graph fails the Horizontal Line Test, so f does not have an inverse.

1.9.56.

$$f(x) = \begin{cases} -x, & x \leq 0 \\ x^2 - 3x, & x > 0 \end{cases}$$



The graph fails the Horizontal Line Test, so f does not have an inverse.

1.9.57.

(a) $y = 10 + 0.75x$

$x = 10 + 0.75y$

$x - 10 = 0.75y$

$\frac{x - 10}{0.75} = y$

So, $f^{-1}(x) = \frac{x - 10}{0.75}$.

x = hourly wage, y = number of units produced

(b) $y = \frac{24.25 - 10}{0.75} = 19$

So, 19 units are produced.

1.9.58.

(a) $y = 0.03x^2 + 245.50$, $0 < x < 100$

$\Rightarrow 245.50 < y < 545.50$

$x = 0.03y^2 + 245.50$

$x - 245.50 = 0.03y^2$

$\frac{x - 245.50}{0.03} = y^2$

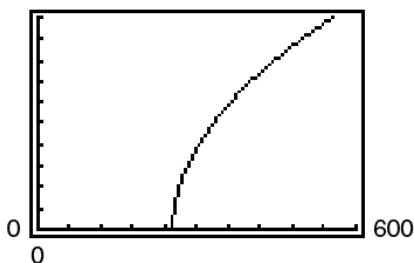
$\sqrt{\frac{x - 245.50}{0.03}} = y$, $245.50 < x < 545.50$

$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$

x = temperature in degrees Fahrenheit

y = percent load for a diesel engine

(b) 100



(c) $0.03x^2 + 245.50 \leq 500$

$0.03x^2 \leq 254.50$

$x^2 \leq 8483.33$

$x \leq 92.10$

Thus, $0 < x \leq 92.10$.

In Exercises 59–62, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \frac{x + 5}{2}$.

1.9.59.

$$\begin{aligned}(g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x - 4) \\ &= \frac{(x - 4) + 5}{2} \\ &= \frac{x + 1}{2}\end{aligned}$$

1.9.60.

$$\begin{aligned}(f^{-1} \circ g^{-1})(x) &= f^{-1}(g^{-1}(x)) \\ &= f^{-1}\left(\frac{x + 5}{2}\right) \\ &= \frac{x + 5}{2} - 4 \\ &= \frac{x + 5 - 8}{2} \\ &= \frac{x - 3}{2}\end{aligned}$$

1.9.61.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x - 5) \\ &= (2x - 5) + 4 \\ &= 2x - 1\end{aligned}$$

$$(f \circ g)^{-1}(x) = \frac{x + 1}{2}$$

Note: Comparing Exercises 59 and 61, $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

1.9.62.

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g(x+4) \\
 &= 2(x+4) - 5 \\
 &= 2x + 8 - 5 \\
 &= 2x + 3 \\
 y &= 2x + 3 \\
 x &= 2y + 3 \\
 x - 3 &= 2y \\
 \frac{x-3}{2} &= y \\
 (g \circ f)^{-1}(x) &= \frac{x-3}{2}
 \end{aligned}$$

In Exercises 63–68, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

1.9.63.

$$\begin{aligned}
 (f^{-1} \circ g^{-1})(1) &= f^{-1}(g^{-1}(1)) \\
 &= f^{-1}(\sqrt[3]{1}) \\
 &= 8(\sqrt[3]{1} + 3) = 32
 \end{aligned}$$

1.9.64.

$$\begin{aligned}
 (g^{-1} \circ f^{-1})(-3) &= g^{-1}(f^{-1}(-3)) \\
 &= g^{-1}(8(-3+3)) \\
 &= g^{-1}(0) = \sqrt[3]{0} = 0
 \end{aligned}$$

1.9.65.

$$\begin{aligned}
 (f^{-1} \circ f^{-1})(4) &= f^{-1}(f^{-1}(4)) \\
 &= f^{-1}(8[4+3]) \\
 &= 8[8(4+3)+3] \\
 &= 8[8(7)+3] \\
 &= 8(59) = 472
 \end{aligned}$$

1.9.66.

$$\begin{aligned}(g^{-1} \circ g^{-1})(-1) &= g^{-1}(g^{-1}(-1)) \\ &= g^{-1}(\sqrt[3]{-1}) \\ &= \sqrt[3]{\sqrt[3]{-1}} \\ &= \sqrt[3]{-1} \\ &= -1\end{aligned}$$

1.9.67.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x^3) = \frac{1}{8}x^3 - 3 \\ y &= \frac{1}{8}x^3 - 3 \\ x &= \frac{1}{8}y^3 - 3 \\ x + 3 &= \frac{1}{8}y^3 \\ 8(x + 3) &= y^3 \\ \sqrt[3]{8(x + 3)} &= y \\ (f \circ g)^{-1}(x) &= 2\sqrt[3]{x + 3}\end{aligned}$$

1.9.68.

$$\begin{aligned}g^{-1} \circ f^{-1} &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(8(x + 3)) \\ &= \sqrt[3]{8(x + 3)} \\ &= 2\sqrt[3]{x + 3}\end{aligned}$$

1.9.69.

False. $f(x) = x^2$ is even and does not have an inverse.

1.9.70.

True. If $f(x)$ has an inverse and it has a y -intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

1.9.71.

Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$\begin{aligned}(f \circ g)(x) = y &\Rightarrow f(g(x)) = y \\ g(x) &= f^{-1}(y) \\ x &= g^{-1}(f^{-1}(y)) \\ x &= (g^{-1} \circ f^{-1})(y).\end{aligned}$$

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

1.9.72.

Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$.

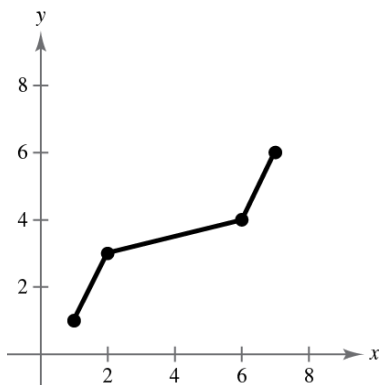
Letting (x, y) be any point on the graph of $f(x) \Rightarrow (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. So, $f^{-1}(x)$ is also an odd function.

1.9.73.

(a)

x	1	3	4	6
f	1	2	6	7

x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



(b)

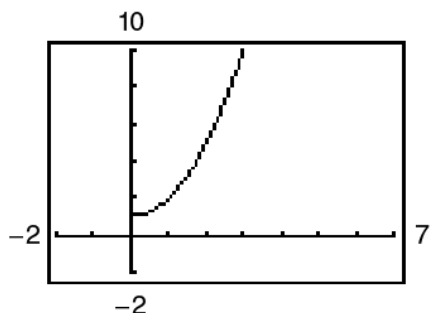
x	-4	-2	0	3
f	3	4	0	-1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

1.9.74.

- (a) $C(x)$ is represented by graph L_1 and $C^{-1}(x)$ is represented by graph L_2 .
- (b) $C(x)$ represents the cost of making x units of personalized T-shirts. $C^{-1}(x)$ represents the number of personalized T-shirts that can be made for a given cost.

1.9.75.



There is an inverse function $f^{-1}(x) = \sqrt{x-1}$ because the domain of f is equal to the range of f^{-1} and the range of f is equal to the domain of f^{-1} .

1.9.76.

x	-10	0	7	45
$f(f^{-1}(x))$	-10	0	7	45
$f^{-1}(f(x))$	-10	0	7	45

$f(x)$ and $f^{-1}(x)$ are inverses of each other.

1.9.77.

$$\begin{aligned}
 y &= -(x-5)^2 + 1 \\
 &= -(x^2 - 10x + 25) + 1 \\
 &= -(x^2 - 10x + 24) \\
 &= -(x-6)(x-4)
 \end{aligned}$$

Alternate solution:

$$\begin{aligned}
 y &= 1 - (x - 5)^2 \\
 &= [1 - (x - 5)][1 + (x - 5)] \\
 &= (6 - x)(x - 4) \\
 &= -(x - 6)(x - 4)
 \end{aligned}$$

1.9.78.

$$\begin{aligned}
 y &= 3(x - 4)^2 - 12 \\
 &= 3(x^2 - 8x + 16) - 12 \\
 &= 3(x^2 - 8x + 12) \\
 &= 3(x - 6)(x - 2)
 \end{aligned}$$

1.9.79.

$$\begin{aligned}
 y &= -\left(x - \frac{13}{2}\right)^2 + \frac{25}{4} \\
 &= \frac{25}{4} - \left(x - \frac{13}{2}\right)^2 \\
 &= \left[\frac{5}{2} + \left(x - \frac{13}{2}\right)\right]\left[\frac{5}{2} - \left(x - \frac{13}{2}\right)\right] \\
 &= (x - 4)(9 - x) \\
 &= -(x - 4)(x - 9)
 \end{aligned}$$

1.9.80.

$$\begin{aligned}
 y &= 3\left(x + \frac{1}{4}\right)^2 - \frac{363}{16} \\
 &= 3\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{363}{16} \\
 &= 3\left(x^2 + \frac{1}{2}x - \frac{120}{16}\right) \\
 &= 3\left(x^2 + \frac{1}{2}x - \frac{15}{2}\right) \\
 &= 3(x + 3)\left(x - \frac{5}{2}\right)
 \end{aligned}$$

1.9.81.

The function is increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$.

1.9.82.

The function is increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.

1.9.83.

The function is increasing on $(-\infty, -1)$ and $(1, \infty)$ and decreasing on $(-1, 1)$.

1.9.84.

The function is increasing on $(-2, -1)$ and $(0, \infty)$ and decreasing on $(-\infty, -2)$ and $(-1, 0)$.

1.9.85.

Relative maximum at $(2, 0)$

Relative minimum at $(0, -3)$

1.9.86.

Relative minimum at $(-2, -\frac{1}{4})$

No relative maxima

SECTION 1.10 MATHEMATICAL MODELING AND VARIATION

1.10.1.

sum of squared differences

1.10.2.

least squares regression

1.10.3.

correlation coefficient

1.10.4.

directly proportional

1.10.5.

You can say that z varies jointly as x and y , or z is jointly proportional to x and y .

1.10.6.

(a) $y = \frac{4}{x} \Rightarrow$ inverse variation

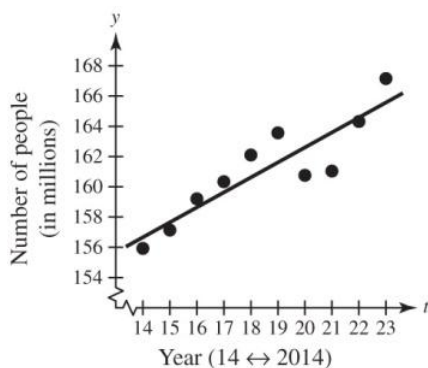
(b) $z = 12xy \Rightarrow$ joint variation

(c) $y = 3.5x \Rightarrow$ direct variation

(d) $z = \frac{6x}{y}$ combined variation

1.10.7.

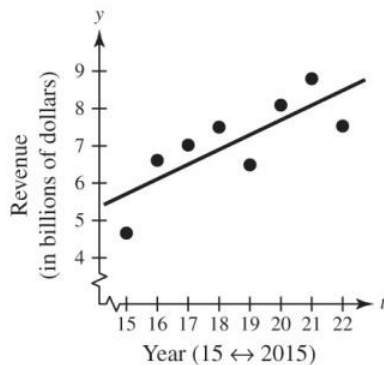
(a) Model: $y = 0.99t + 142.8$



(b) The model is a good fit for the data.

1.10.8.

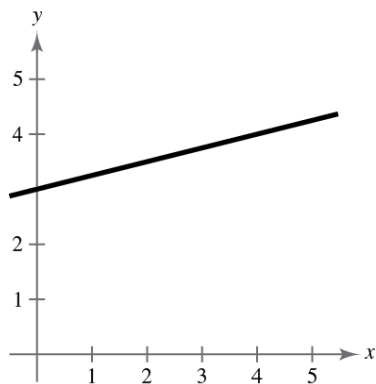
(a) Model: $y = 0.396t - 0.23$



(b) The model is not a good fit for the data.

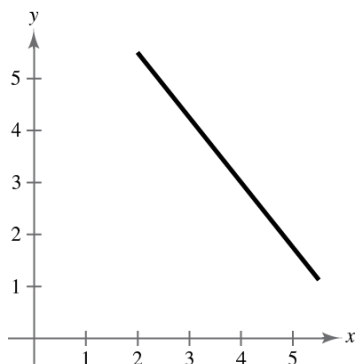
Sample answer: A cubic model may be a better fit for the data.

1.10.9.



Using the points $(0, 3)$ and $(4, 4)$, $y = \frac{1}{4}x + 3$.

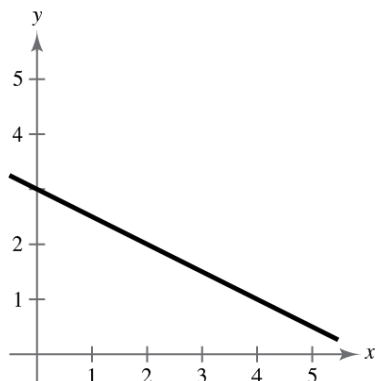
1.10.10.



The line appears to pass through $(2, 5.5)$ and $(6, 0.5)$, so its equation is

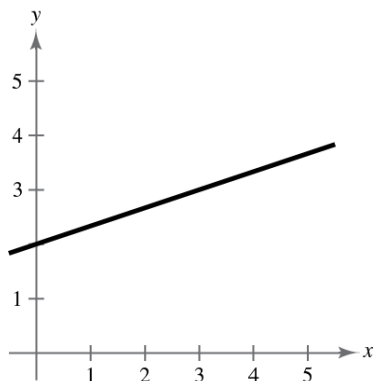
$$y = -\frac{5}{4}x + 8.$$

1.10.11.



Using the points $(2, 2)$ and $(4, 1)$, $y = -\frac{1}{2}x + 3$.

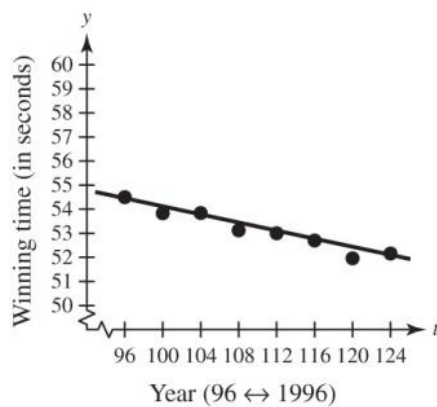
1.10.12.



The line appears to pass through $(0, 2)$ and $(3, 3)$ so its equation is $y = \frac{1}{3}x + 2$.

1.10.13.

(a) and (b)



Sample answer: Use the points $(96, 54.50)$ and $(124, 52.16)$.

$$m = \frac{52.16 - 54.50}{124 - 96} = \frac{-2.34}{28} \approx -0.084$$

$$y - 52.16 = -0.084(t - 124)$$

$$y - 52.16 = -0.084t + 10.416$$

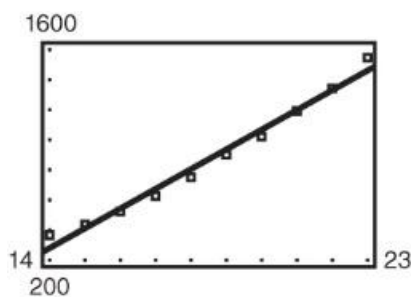
$$y = -0.084t + 62.576$$

(c) $y = -0.087t + 62.7$

(d) The models are similar.

1.10.14.

(a) and (c)



The model fits the data well.

(b) $y = 130.9t - 1552$

1.10.15.

$$y = kx$$

$$14 = k(2)$$

$$7 = k$$

$$y = 7x$$

1.10.16.

$$y = kx$$

$$12 = k(5)$$

$$\frac{12}{5} = k$$

$$y = \frac{12}{5}x$$

1.10.17.

$$y = kx$$

$$3 = k(-24)$$

$$-\frac{1}{8} = k$$

$$y = -\frac{1}{8}x$$

1.10.18.

$$y = kx$$

$$1 = k(5)$$

$$\frac{1}{5} = k$$

$$y = \frac{1}{5}x$$

1.10.19.

$$y = kx$$

$$8\pi = k(4)$$

$$2\pi = k$$

$$y = 2\pi x$$

1.10.20.

$$y = kx$$

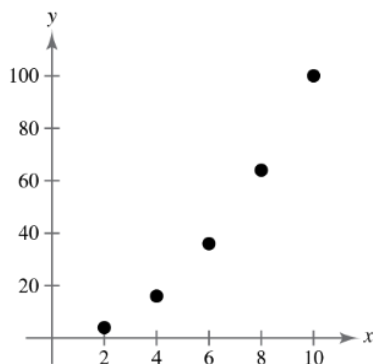
$$-1 = k(\pi)$$

$$-\frac{1}{\pi} = k$$

$$y = -\frac{1}{\pi}x$$

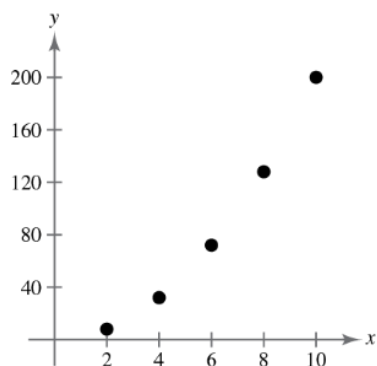
1.10.21.

x	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



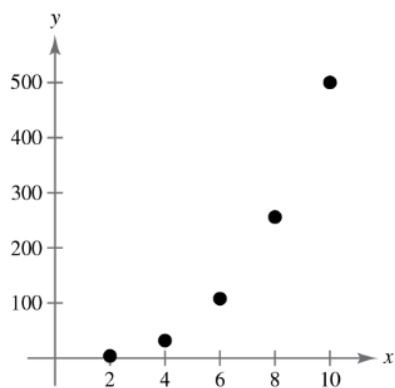
1.10.22.

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



1.10.23.

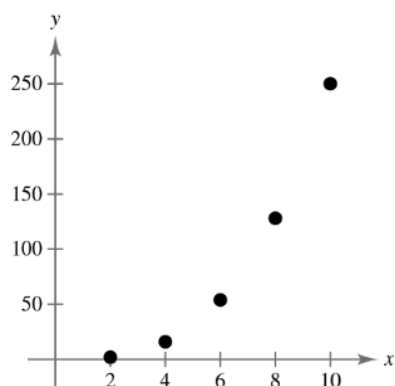
x	2	4	6	8	10
$y = \frac{1}{2}x^3$	4	32	108	256	500



1.10.24.

$$k = \frac{1}{4}, n = 3$$

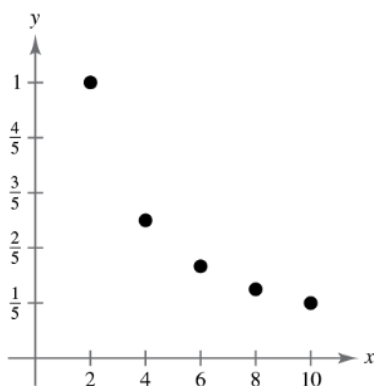
x	2	4	6	8	10
$y = \frac{1}{4}x^3$	2	16	54	128	250



1.10.25.

$$k = 2, n = 1$$

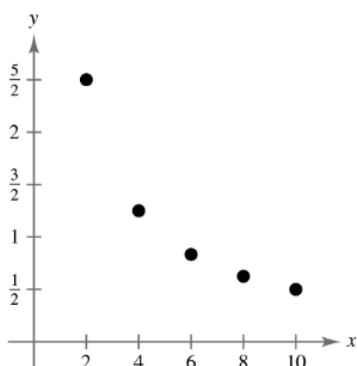
x	2	4	6	8	10
$y = \frac{2}{x}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$



1.10.26.

$$k = 5, n = 1$$

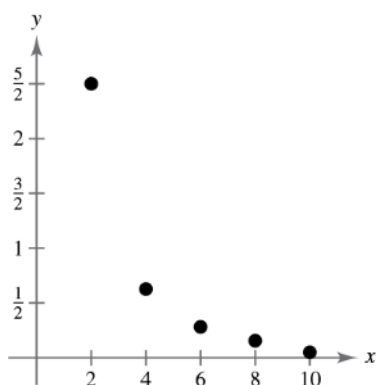
x	2	4	6	8	10
$y = \frac{5}{x}$	$\frac{5}{2}$	$\frac{5}{4}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{1}{2}$



1.10.27.

$$k = 10$$

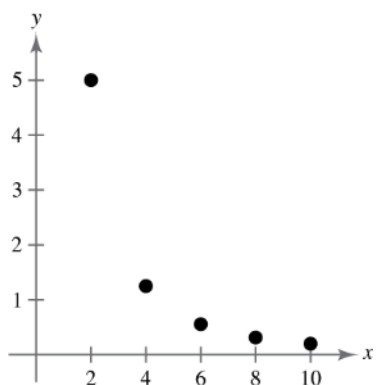
x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{2}$	$\frac{5}{8}$	$\frac{5}{18}$	$\frac{5}{32}$	$\frac{1}{10}$



1.10.28.

$$k = 20$$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	5	$\frac{5}{4}$	$\frac{5}{9}$	$\frac{5}{16}$	$\frac{1}{5}$



1.10.29.

$$y = \frac{k}{x}$$

$$1 = \frac{k}{5}$$

$$5 = k$$

$$y = \frac{5}{x}$$

This equation checks with the other points given in the table.

1.10.30.

$$y = kx$$

$$2 = k5$$

$$\frac{2}{5} = k$$

$$y = \frac{2}{5}x$$

This equation checks with the other points given in the table.

1.10.31.

$$y = \frac{k}{x}$$

$$24 = \frac{k}{5}$$

$$120 = k$$

$$y = \frac{120}{x}$$

This equation checks with the other points given in the table.

1.10.32.

$$y = kx$$

$$-7 = k(10)$$

$$-\frac{7}{10} = k$$

$$y = -\frac{7}{10}x$$

This equation checks with the other points given in the table.

1.10.33.

$$A = kr^2$$

1.10.34.

$$V = kt^3$$

1.10.35.

$$y = \frac{k}{x^2}$$

1.10.36.

$$h = \frac{k}{\sqrt{s}}$$

1.10.37.

$$F = \frac{kg}{r^2}$$

1.10.38.

$$z = kx^2y^3$$

1.10.39.

$$R = k(T - T_e)$$

1.10.40.

$$F = \frac{km_1m_2}{r^2}$$

1.10.41.

y is directly proportional to the square of x .

1.10.42.

t is inversely proportional to r .

1.10.43.

$$A = \frac{1}{2}bh$$

The area of a triangle is jointly proportional to its base and height.

1.10.44.

K is jointly proportional to m and the square of v .

1.10.45.

$$y = kx$$

$$54 = k(3)$$

$$18 = k$$

$$y = 18x$$

1.10.46.

$$A = kr^2$$

$$9\pi = k(3)^2$$

$$\pi = k$$

$$A = \pi r^2$$

1.10.47.

$$y = \frac{k}{x}$$

$$3 = \frac{k}{25}$$

$$75 = k$$

$$y = \frac{75}{x}$$

1.10.48.

$$y = \frac{k}{x^3}$$

$$7 = \frac{k}{2^3}$$

$$7 = \frac{k}{8}$$

$$56 = k$$

$$y = \frac{56}{x^3}$$

1.10.49.

$$z = kxy$$

$$64 = k(4)(8)$$

$$2 = k$$

$$z = 2xy$$

1.10.50.

$$F = krs^3$$

$$4158 = k(11)(3)^3$$

$$k = 14$$

$$F = 14rs^3$$

1.10.51.

$$\begin{aligned}
 P &= \frac{kx}{y^2} \\
 \frac{28}{3} &= \frac{k(42)}{9^2} \\
 \frac{28}{3} \cdot \frac{81}{42} &= k \\
 \frac{2 \cdot 27}{3} &= k \\
 18 &= k \\
 P &= \frac{18x}{y^2}
 \end{aligned}$$

1.10.52.

$$\begin{aligned}
 z &= \frac{kx^2}{y} \\
 6 &= \frac{k(6)^2}{4} \\
 \frac{24}{36} &= k \\
 \frac{2}{3} &= k \\
 z &= \frac{2/3 x^2}{y} = \frac{2x^2}{3y}
 \end{aligned}$$

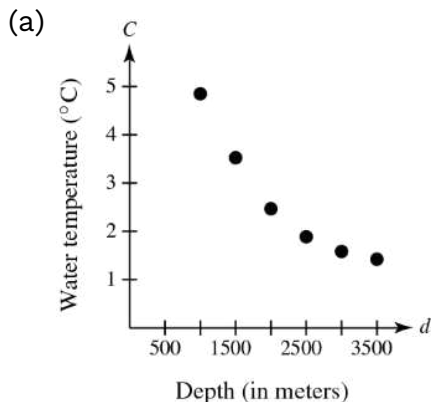
1.10.53.

$$\begin{aligned}
 I &= kP \\
 113.75 &= k(3250) \\
 0.035 &= k \\
 I &= 0.035P
 \end{aligned}$$

1.10.54.

$$\begin{aligned}
 I &= kP \\
 31.25 &= k(2500) \\
 k &= 0.0125 \\
 I &= 0.0125P
 \end{aligned}$$

1.10.55.



(b) The data shows an inverse variation model fits.

$$(c) \quad 4.85 = \frac{k_1}{1000} \quad 3.525 = \frac{k_2}{1500} \quad 2.468 = \frac{k_3}{2000} \quad 1.888 = \frac{k_4}{2500} \quad 1.583 = \frac{k_5}{3000}$$

$$1.422 = \frac{k_6}{3500}$$

$$4850.0 = k_1 \quad 5287.5 = k_2 \quad 4936.0 = k_3 \quad 4720.0 = k_4 \quad 4749.0 = k_5$$

$$4977.0 = k_6$$

$$\text{Mean: } k = \frac{4850 + 5287.5 + 4936 + 4720 + 4977}{6} \approx 4919.92, \text{ Model: } C = \frac{4919.92}{d}$$

$$(d) \quad C = \frac{4919.92}{d}$$

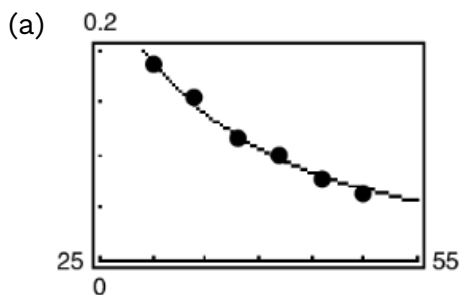
$$3 = \frac{4919.92}{d}$$

$$d = \frac{4919.92}{3} \approx 1639.97 \text{ meters}$$

The temperature is 3°C at approximately 1640 meters.

1.10.56.

$$y = \frac{171.33}{x^2}$$



$$(b) y = \frac{171.33}{(25)^2} \approx 0.2741 \text{ microwatt per centimeter squared}$$

1.10.57.

$$\begin{aligned} d &= kv^2 \\ 0.02 &= k \left(\frac{1}{4} \right)^2 \\ k &= 0.32 \\ d &= 0.32v^2 \\ 0.12 &= 0.32v^2 \\ v^2 &= \frac{0.12}{0.32} = \frac{3}{8} \\ v &= \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4} \approx 0.61 \text{ mi/hr} \end{aligned}$$

1.10.58.

$$\begin{aligned} W &= kmh \\ 2116.8 &= k(120)(1.8) \\ k &= \frac{2116.8}{(120)(1.8)} = 9.8 \\ W &= 9.8 \text{ mh} \end{aligned}$$

When $m = 100$ kilograms and $h = 1.5$ meters, we have
 $W = 9.8(100)(1.5) = 1470$ joules.

1.10.59.

$f_0 = k \frac{\sqrt{T_0}}{l_0 \sqrt{p}}$ where f_0 = original frequency, T_0 = original tension, l_0 = original length of string, and p = mass density

$$f_0 = 100$$

(a) The frequency of a string with four times the tension would double the frequency in order to maintain the direct proportion.

$$\text{Let } f_{\text{new}} = \frac{k \cdot \sqrt{T_{\text{new}}}}{l_{\text{new}} \cdot \sqrt{p}} \text{ and } T_{\text{new}} = 4T_0, l_{\text{new}} = l_0$$

$$f_{\text{new}} = \frac{k \cdot \sqrt{4T_0}}{l_0 \cdot \sqrt{p}} = 2 \cdot \frac{k \cdot \sqrt{T_0}}{l_0 \cdot \sqrt{p}} = 2 \cdot f_0 = 2 \cdot 100 = 200 \text{ Hz}$$

- (b) The frequency of a string with two times the length would half the frequency in order to maintain the inverse proportion.

$$\text{Let } f_{\text{new}} = \frac{k \cdot \sqrt{T_{\text{new}}}}{l_{\text{new}} \cdot \sqrt{\rho}} \text{ and } l_{\text{new}} = 2l_0, T_{\text{new}} = T_0$$

$$f_{\text{new}} = \frac{k \cdot \sqrt{T_0}}{2l_0 \cdot \sqrt{\rho}} = \frac{1}{2} \cdot \frac{k \cdot \sqrt{T_0}}{l_0 \cdot \sqrt{\rho}} = \frac{1}{2} f_0 = \frac{1}{2} \cdot 100 = 50 \text{ Hz}$$

- (c) The frequency of a string with four times the tension would double the frequency, and two times the length would half the frequency in order to maintain the inverse proportion. Therefore the frequency would remain the same,

$$\text{Let } f_{\text{new}} = \frac{k \cdot \sqrt{T_{\text{new}}}}{l_{\text{new}} \cdot \sqrt{\rho}} \text{ and } T_{\text{new}} = 4T_0, l_{\text{new}} = 2l_0$$

$$f_{\text{new}} = \frac{k \cdot \sqrt{4T_0}}{2l_0 \cdot \sqrt{\rho}} = \frac{2 \cdot k \cdot \sqrt{T_0}}{2 \cdot l_0 \cdot \sqrt{\rho}} = \frac{2}{2} f_0 = 1 \cdot 100 = 100 \text{ Hz}$$

1.10.60.

$\text{Load}_0 = \frac{k w_0 d_0^2}{l_0} = L_0$ where L_0 = original load, w_0 = width of beam, d_0 = depth of beam and l_0 = length of beam

$$(a) \text{ Load} = \frac{k(2w_0)d_0^2}{l_0} = 2 \cdot \frac{k w_0 (d_0)^2}{l_0} = 2 \cdot L_0$$

The safe load doubles.

$$(b) \text{ Load} = \frac{k w_0 \cdot (2d_0)^2}{l_0} = 4 \cdot \frac{k w_0 (d_0)^2}{l_0} = 4 \cdot L_0$$

The safe load is four times as great.

$$(c) \text{ Load} = \frac{k w_0 \cdot (d_0)^2}{\frac{1}{2} l_0} = 2 \cdot \frac{k w_0 (d_0)^2}{l_0} = 2 \cdot L_0$$

The safe load doubles.

$$(d) \text{ Load} = \frac{k \left(\frac{1}{2} w_0\right) \cdot (d_0)^2}{2 \cdot l_0} = \left(\frac{1}{2}\right) \cdot \frac{k w_0 (d_0)^2}{l_0} = \frac{1}{4} \cdot \frac{k w_0 (d_0)^2}{l_0} = \frac{1}{4} \cdot L_0$$

The safe load is one-fourth as great.

1.10.61.

True. If $y = k_1 x$ and $x = k_2 z$, then $y = k_1 (k_2 z) = (k_1 k_2) z$.

1.10.62.

False. If $y = \frac{k_1}{x}$ and $x = \frac{k_2}{z}$, then $y = \frac{k_1}{\frac{k_2}{z}} = \frac{k_1}{k_2} z$, which means y is directly proportional to z .

1.10.63.

False. In the equation $S = 4\pi r^2$, π is a constant, not a variable.

1.10.64.

- (a) The data shown could be represented by a linear model which would be a good approximation.
- (b) The points do not follow a linear pattern. A linear model would be a poor approximation. A quadratic model would be better.
- (c) The points do not follow a linear pattern. A linear model would be a poor approximation.
- (d) The data shown could be represented by a linear model which would be a good approximation.

1.10.65.

True. If $f(x) = (x - 2)p(x)$, then $f(2) = (2 - 2)p(2) = 0$.

1.10.66.

False. For example, the graph of $f(x) = -x^3$ rises to the left, but the leading coefficient is -1 .

1.10.67.

$$\begin{aligned} \frac{x-1}{5x-4} - \frac{2x}{5x-4} &= \frac{x-1-2x}{5x-4} \\ &= \frac{-x-1}{5x-4} \\ &= \frac{x+1}{4-5x} \end{aligned}$$

1.10.68.

$$\begin{aligned} \frac{3-x}{x+2} + \frac{x+8}{x+2} &= \frac{3-x+x+8}{x+2} \\ &= \frac{11}{x+2} \end{aligned}$$

1.10.69.

$$\begin{aligned}\frac{5}{x-1} + \frac{2x}{x+3} &= \frac{5(x+3) + 2x(x-1)}{(x-1)(x+3)} \\ &= \frac{2x^2 + 3x + 15}{(x-1)(x+3)}\end{aligned}$$

1.10.70.

$$\begin{aligned}\frac{3x}{x-2} - \frac{6x}{x^2-4} &= \frac{3x(x+2) - 6x}{(x-2)(x+2)} \\ &= \frac{3x^2}{(x-2)(x+2)}\end{aligned}$$

1.10.71.

$$\frac{x+3}{2x+1} \cdot \frac{4}{3(x+3)} = \frac{4}{3(2x+1)}, x \neq -3$$

1.10.72.

$$\begin{aligned}\frac{x^2}{x^2-1} \div \frac{2x}{x+1} &= \frac{x^2}{(x+1)(x-1)} \cdot \frac{x+1}{2x} \\ &= \frac{x}{2(x-1)}, x \neq 0, -1\end{aligned}$$

1.10.73.

$$x^3 + y = 8 \Rightarrow y = 8 - x^3 \text{ is a function of } x.$$

Domain: all real numbers x

1.10.74.

$$x = |y| \text{ is not a function of } x.$$

For example, for $x = 1$, $y = \pm 1$.

1.10.75.

$$1 - y^2 = x \text{ is not a function of } x.$$

For example, for $x = 0$, $y = \pm 1$.

1.10.76.

$xy = 3 \Rightarrow y = 3/x$ is a function of x .

Domain: all real numbers x except $x = 0$

1.10.77.

$y = \sqrt{x+5}$ is a function of x .

Domain: all real numbers x such that $x \geq -5$

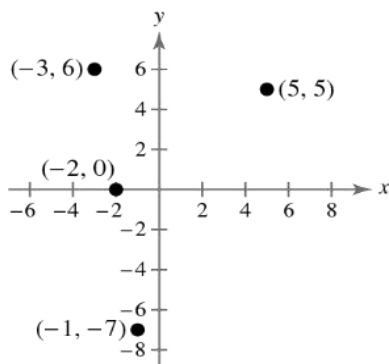
1.10.78.

$8x^2 + 2y^2 = 32$ is not a function of x .

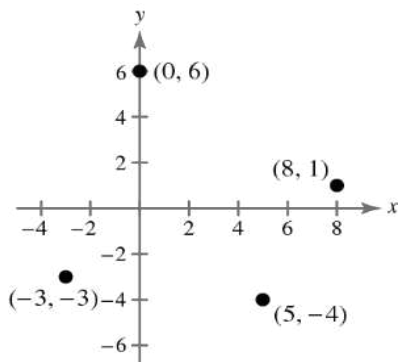
For example, for $x = 0$, $y = \pm 4$.

REVIEW EXERCISES FOR CHAPTER 1

1.R.1.



1.R.2.



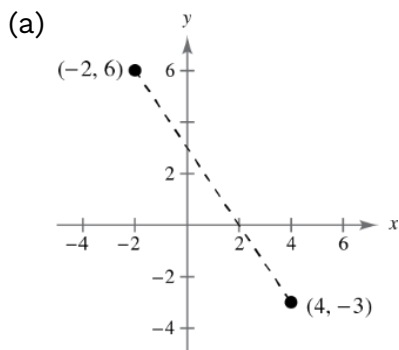
1.R.3.

$x > 0$ and $y = -2$ in Quadrant IV.

1.R.4.

$xy = 4$ means x and y have the same signs. This occurs in Quadrants I and III.

1.R.5.



$$\begin{aligned} \text{(b) } d &= \sqrt{(-2-4)^2 + (6+3)^2} \\ &= \sqrt{36+81} = \sqrt{117} = 3\sqrt{13} \end{aligned}$$

$$\text{(c) Midpoint: } \left(\frac{-2+4}{2}, \frac{6-3}{2} \right) = \left(1, \frac{3}{2} \right)$$

1.R.6.

$$(4-4, 8-8) = (0, 0)$$

$$(6-4, 8-8) = (2, 0)$$

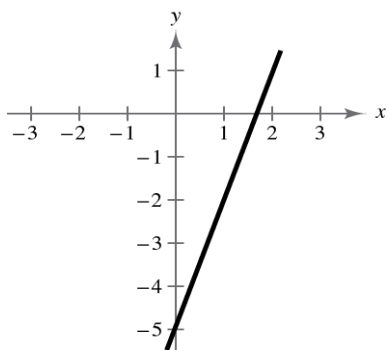
$$(4-4, 3-8) = (0, -5)$$

$$(6-4, 3-8) = (2, -5)$$

1.R.7.

$$y = 3x - 5$$

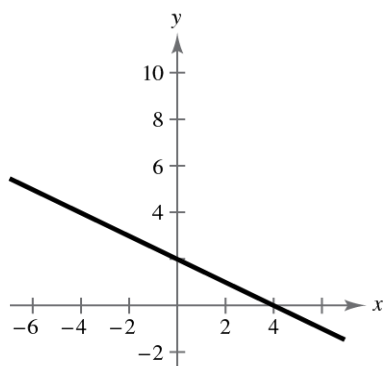
x	-2	-1	0	1	2
y	-11	-8	-5	-2	1



1.R.8.

$$y = -\frac{1}{2}x + 2$$

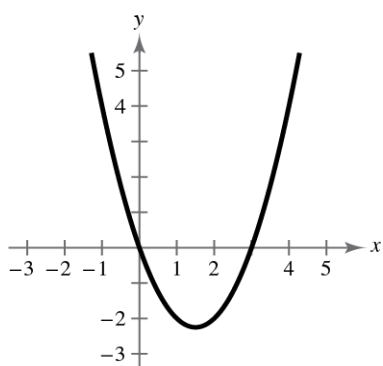
x	-4	-2	0	2	4
y	4	3	2	1	0



1.R.9.

$$y = x^2 - 3x$$

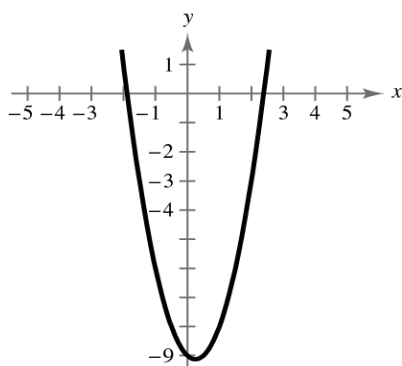
x	-1	0	1	2	3	4
y	4	0	-2	-2	0	4



1.R.10.

$$y = 2x^2 - x - 9$$

x	-2	-1	0	1	2	3
y	1	-6	-9	-8	1	6



1.R.11.

$$y = 2x + 7$$

x-intercept: Let $y = 0$.

$$0 = 2x + 7$$

$$x = -\frac{7}{2}$$

$$\left(-\frac{7}{2}, 0\right)$$

y-intercept: Let $x = 0$.

$$y = 2(0) + 7$$

$$y = 7$$

$$(0, 7)$$

1.R.12.

x-intercept: Let $y = 0$.

$$y = |x + 1| - 3$$

$$0 = |x + 1| - 3$$

For $x + 1 > 0$, $0 = x + 1 - 3$, or $-2 = x$.

For $x + 1 < 0$, $0 = -(x + 1) - 3$, or $-4 = x$.

$$(2, 0), (-4, 0)$$

y-intercept: Let $x = 0$.

$$y = |x + 1| - 3$$

$$y = |0 + 1| - 3 \text{ or } y = -2$$

$$(0, -2)$$

1.R.13.

$$y = (x - 3)^2 - 4$$

$$\text{x-intercepts: } 0 = (x - 3)^2 - 4 \Rightarrow (x - 3)^2 = 4$$

$$\Rightarrow x - 3 = \pm 2$$

$$\Rightarrow x = 3 \pm 2$$

$$\Rightarrow x = 5 \text{ or } x = 1$$

$$(5, 0), (1, 0)$$

$$y\text{-intercept: } y = (0 - 3)^2 - 4$$

$$y = 9 - 4$$

$$y = 5$$

$$(0, 5)$$

1.R.14.

$$y = x\sqrt{4 - x^2}$$

$$x\text{-intercepts: } 0 = x\sqrt{4 - x^2}$$

$$x = 0 \quad \sqrt{4 - x^2} = 0$$

$$4 - x^2 = 0$$

$$x = \pm 2$$

$$(0, 0), (-2, 0), (2, 0)$$

$$y\text{-intercept: } y = 0 \cdot \sqrt{4 - 0} = 0$$

$$(0, 0)$$

1.R.15.

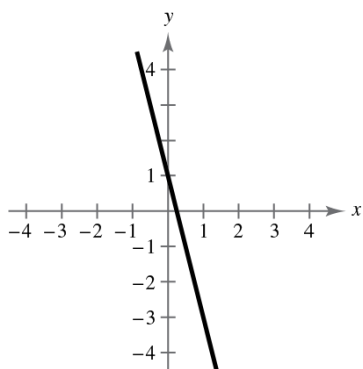
$$y = -4x + 1$$

$$\text{Intercepts: } \left(\frac{1}{4}, 0\right), (0, 1)$$

$$y = -4(-x) + 1 \Rightarrow y = 4x + 1 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = -4x + 1 \Rightarrow y = 4x - 1 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = -4(-x) + 1 \Rightarrow y = -4x - 1 \Rightarrow \text{No origin symmetry}$$



1.R.16.

$$y = 3x - 7$$

$$\text{x-intercept: } 0 = 3x - 7$$

$$7 = 3x$$

$$\frac{7}{3} = x$$

$$\left(\frac{7}{3}, 0\right)$$

$$\text{y-intercept: } y = 3(0) - 7$$

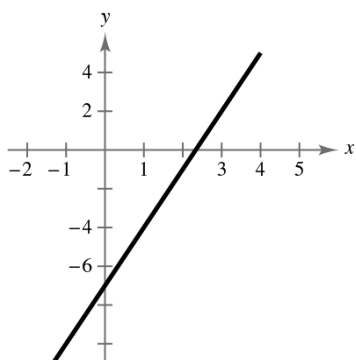
$$y = -7$$

$$(0, -7)$$

$$y = 3(-x) - 7 \Rightarrow y = -3x - 7 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = 3(x) - 7 \Rightarrow y = -3x + 7 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = 3(-x) - 7 \Rightarrow y = 3x + 7 \Rightarrow \text{No origin symmetry}$$



1.R.17.

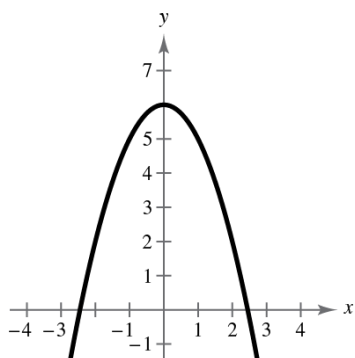
$$y = 6 - x^2$$

$$\text{Intercepts: } (\pm\sqrt{6}, 0), (0, 6)$$

$$y = 6 - (-x^2) \Rightarrow y = 6 - x^2 \Rightarrow \text{y-axis symmetry}$$

$$-y = 6 - x^2 \Rightarrow y = -6 + x^2 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = 6 - (-x)^2 \Rightarrow y = -6 + x^2 \Rightarrow \text{No origin symmetry}$$



1.R.18.

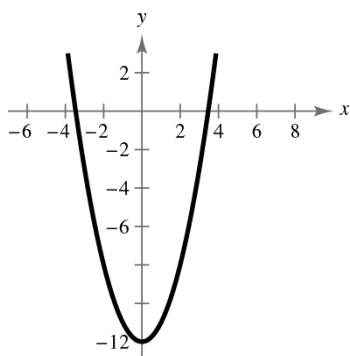
$$y = x^2 - 12$$

$$\text{Intercepts: } (\pm 2\sqrt{3}, 0), (0, -12)$$

$$y = (-x)^2 - 12 \Rightarrow y = x^2 - 12 \Rightarrow y\text{-axis symmetry}$$

$$-y = x^2 - 12 \Rightarrow y = -x^2 + 12 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = (-x)^2 - 12 \Rightarrow y = -x^2 + 12 \Rightarrow \text{No origin symmetry}$$



1.R.19.

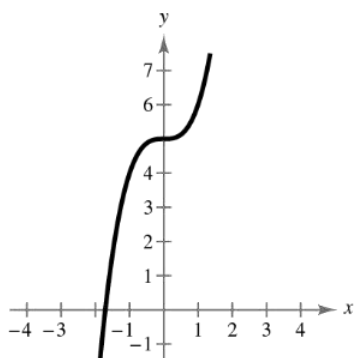
$$y = x^3 + 5$$

$$\text{Intercepts: } (\sqrt[3]{-5}, 0), (0, 5)$$

$$y = (-x)^3 + 5 \Rightarrow y = -x^3 + 5 \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = x^3 + 5 \Rightarrow y = -x^3 - 5 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = (-x)^3 + 5 \Rightarrow y = x^3 - 5 \Rightarrow \text{No origin symmetry}$$



1.R.20.

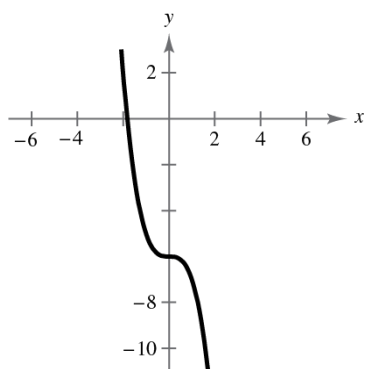
$$y = -6 - x^3$$

Intercepts: $(\sqrt[3]{-6}, 0), (0, -6)$

$$y = -6 - (-x)^3 \Rightarrow y = -6 + x \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = -6 - x^3 \Rightarrow y = 6 + x^3 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = -6 - (-x)^3 \Rightarrow y = 6 - x^3 \Rightarrow \text{No origin symmetry}$$



1.R.21.

$$y = \sqrt{x+5}$$

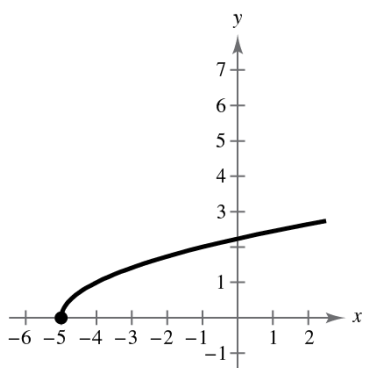
Domain: $[-5, \infty)$

Intercepts: $(-5, 0), (0, \sqrt{5})$

$$y = \sqrt{-x+5} \Rightarrow \text{No } y\text{-axis symmetry}$$

$$-y = \sqrt{x+5} \Rightarrow y = -\sqrt{x+5} \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = \sqrt{-x+5} \Rightarrow y = -\sqrt{-x+5} \Rightarrow \text{No origin symmetry}$$



1.R.22.

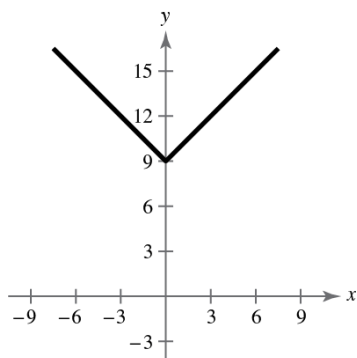
$$y = |x| + 9$$

Intercept: $(0, 9)$

$$y = |-x| + 9 \Rightarrow y = |x| + 9 \Rightarrow y\text{-axis symmetry}$$

$$-y = |x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow \text{No } x\text{-axis symmetry}$$

$$-y = |-x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow \text{No origin symmetry}$$

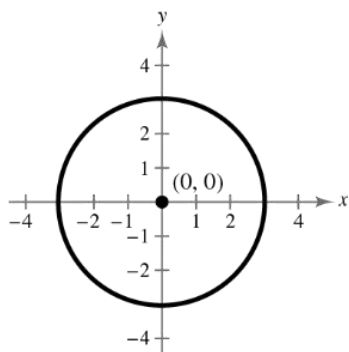


1.R.23.

$$x^2 + y^2 = 9$$

Center: $(0, 0)$

Radius: 3

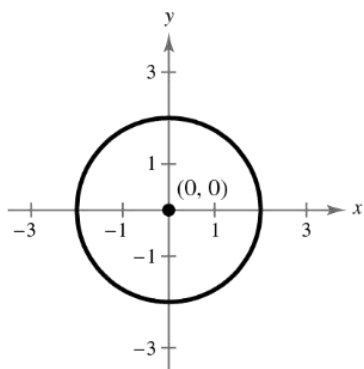


1.R.24.

$$x^2 + y^2 = 4$$

Center: $(0, 0)$

Radius: 2



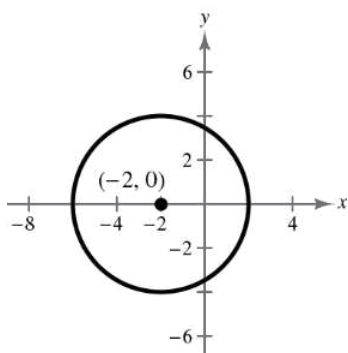
1.R.25.

$$(x + 2)^2 + y^2 = 16$$

$$(x - (-2))^2 + (y - 0)^2 = 4^2$$

Center: $(-2, 0)$

Radius: 4

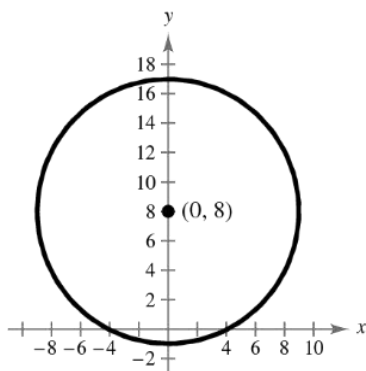


1.R.26.

$$x^2 + (y - 8)^2 = 81$$

Center: $(0, 8)$

Radius: 9



1.R.27.

Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

$$\text{Center: } \left(\frac{0+4}{2}, \frac{0+(-6)}{2} \right) = (2, -3)$$

$$\text{Radius: } r = \sqrt{(2-0)^2 + (-3-0)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\text{Standard form: } (x-2)^2 + (y-(-3))^2 = (\sqrt{13})^2$$

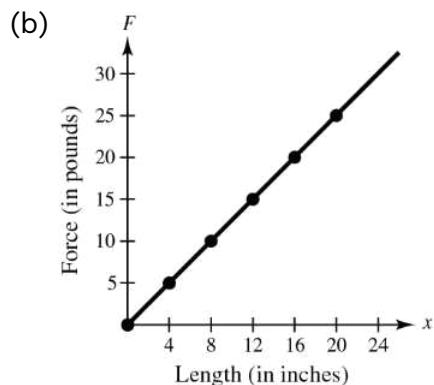
$$(x-2)^2 + (y+3)^2 = 13$$

1.R.28.

$$F = \frac{5}{4}x, \quad 0 \leq x \leq 20$$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25



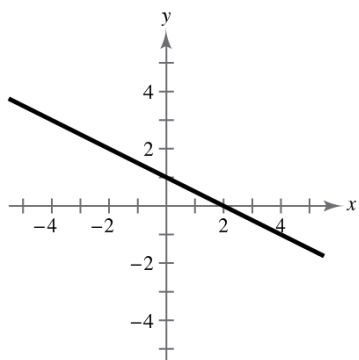
(c) When $x = 10$, $F = \frac{50}{4} = 12.5$ pounds.

1.R.29.

$$y = -\frac{1}{2}x + 1$$

$$\text{Slope: } m = -\frac{1}{2}$$

$$y\text{-intercept: } (0, 1)$$



1.R.30.

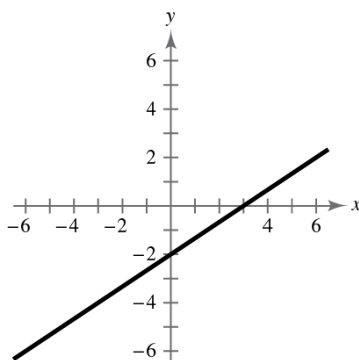
$$2x - 3y = 6$$

$$-3y = -2x + 6$$

$$y = \frac{2}{3}x - 2$$

$$\text{Slope: } m = \frac{2}{3}$$

$$y\text{-intercept: } (0, -2)$$

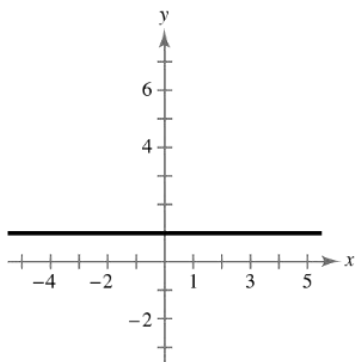


1.R.31.

$$y = 1$$

Slope: $m = 0$

y -intercept: $(0, 1)$

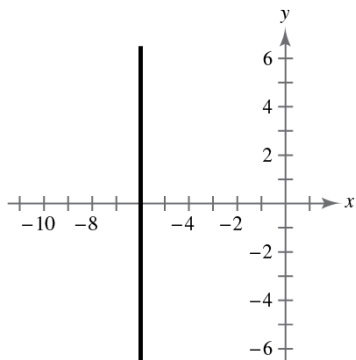


1.R.32.

$$x = -6$$

Slope is undefined.

y -intercept: none



1.R.33.

$$(5, -2), (-1, 4)$$

$$m = \frac{4 - (-2)}{-1 - 5} = \frac{6}{-6} = -1$$

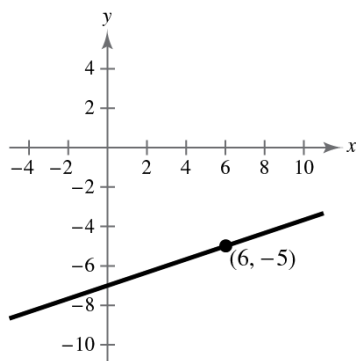
1.R.34.

$$(-1, 6), (3, -2)$$

$$m = \frac{-2 - 6}{3 - (-1)} = \frac{-8}{4} = -2$$

1.R.35.

$$\begin{aligned}(6, -5), m &= \frac{1}{3} \\ y - (-5) &= \frac{1}{3}(x - 6) \\ y + 5 &= \frac{1}{3}x - 2 \\ y &= \frac{1}{3}x - 7\end{aligned}$$

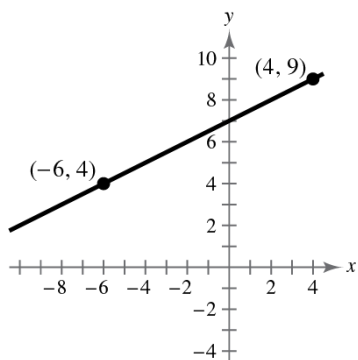


1.R.36.

$$\begin{aligned}(-4, -2), m &= -\frac{3}{4} \\ y - (-2) &= -\frac{3}{4}(x - (-4)) \\ y + 2 &= -\frac{3}{4}(x + 4) \\ y + 2 &= -\frac{3}{4}x - 3 \\ y &= -\frac{3}{4}x - 5\end{aligned}$$

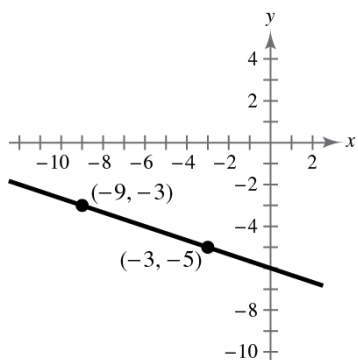
1.R.37.

$$\begin{aligned}(-6, 4), (4, 9) \\ m &= \frac{9 - 4}{4 - (-6)} = \frac{5}{10} = \frac{1}{2} \\ y - 4 &= \frac{1}{2}(x - (-6)) \\ y - 4 &= \frac{1}{2}(x + 6) \\ y - 4 &= \frac{1}{2}x + 3 \\ y &= \frac{1}{2}x + 7\end{aligned}$$



1.R.38.

$$\begin{aligned} &(-9, -3), (-3, -5) \\ m &= \frac{-5 - (-3)}{-3 - (-9)} = \frac{-2}{6} = -\frac{1}{3} \\ y - (-3) &= -\frac{1}{3}(x - (-9)) \\ y + 3 &= -\frac{1}{3}(x + 9) \\ y + 3 &= -\frac{1}{3}x - 3 \\ y &= -\frac{1}{3}x - 6 \end{aligned}$$



1.R.39.

$$\begin{aligned} \text{Point: } &(3, -2) \\ 5x - 4y &= 8 \\ y &= \frac{5}{4}x - 2 \\ \text{(a) Parallel slope: } m &= \frac{5}{4} \end{aligned}$$

$$\begin{aligned}y - (-2) &= \frac{5}{4}(x - 3) \\y + 2 &= \frac{5}{4}x - \frac{15}{4} \\y &= \frac{5}{4}x - \frac{23}{4}\end{aligned}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$\begin{aligned}y - (-2) &= -\frac{4}{5}(x - 3) \\y + 2 &= -\frac{4}{5}x + \frac{12}{5} \\y &= -\frac{4}{5}x + \frac{2}{5}\end{aligned}$$

1.R.40.

Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$

$$y = \frac{5}{3} - \frac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$\begin{aligned}y - 3 &= -\frac{2}{3}(x + 8) \\3y - 9 &= -2x - 16 \\3y &= -2x - 7 \\y &= -\frac{2}{3}x - \frac{7}{3}\end{aligned}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$\begin{aligned}y - 3 &= \frac{3}{2}(x + 8) \\2y - 6 &= 3x + 24 \\2y &= 3x + 30 \\y &= \frac{3}{2}x + 15\end{aligned}$$

1.R.41.

Verbal Model: Sale price = (List price) – (Discount)

Labels: Sale price = S

List price = L

Discount = 20% of $L = 0.2L$

Equation: $S = L - 0.2L$

$S = 0.8L$

1.R.42.

Verbal Model:

Amount earned = (starting fee) + (per page rate) · (number of pages)

Labels: Hourly wage = A

Starting fee = 50

Per page rate = 2.50

Number of pages = p

Equation: $A = 2.5p + 50$

1.R.43.

$$16x - y^4 = 0$$

$$y^4 = 16x$$

$$y = \pm 2\sqrt[4]{x}$$

No, y is not a function of x . Some x -values correspond to two y -values.

1.R.44.

$$2x - y - 3 = 0$$

$$2x - 3 = y$$

Yes, the equation represents y as a function of x .

1.R.45.

$$y = \sqrt{1-x}$$

Yes, the equation represents y as a function of x . Each x -value, $x \leq 1$, corresponds to only one y -value.

1.R.46.

$$|y| = x + 2 \text{ corresponds to } y = x + 2 \text{ or } -y = x + 2.$$

No, y is not a function of x . Some x -values correspond to two y -values.

1.R.47.

$$f(x) = x^2 + 1$$

$$(a) f(2) = (2)^2 + 1 = 5$$

$$(b) f(-4) = (-4)^2 + 1 = 17$$

$$(c) f(t^2) = (t^2)^2 + 1 = t^4 + 1$$

$$(d) f(t+1) = (t+1)^2 + 1 \\ = t^2 + 2t + 2$$

1.R.48.

$$h(x) = |x - 2|$$

$$(a) \ h(-4) = |-4 - 2| = |-6| = 6$$

$$(b) \ h(-2) = |-2 - 2| = |-4| = 4$$

$$(c) \ h(0) = |0 - 2| = |-2| = 2$$

$$(d) \ h(-x + 2) = |-x + 2 - 2| = |-x| = |x|$$

1.R.49.

$$f(x) = \sqrt{25 - x^2}$$

$$\text{Domain: } 25 - x^2 \geq 0$$

$$(5 + x)(5 - x) \geq 0$$

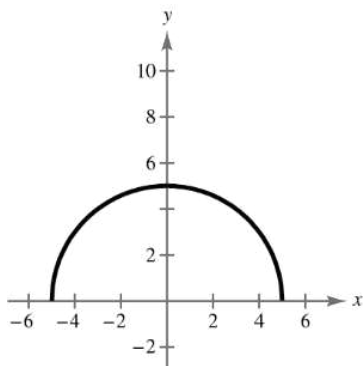
$$\text{Critical numbers: } x = \pm 5$$

$$\text{Test intervals: } (-\infty, -5), (-5, 5), (5, \infty)$$

$$\text{Test: Is } 25 - x^2 \geq 0?$$

$$\text{Solution set: } -5 \leq x \leq 5$$

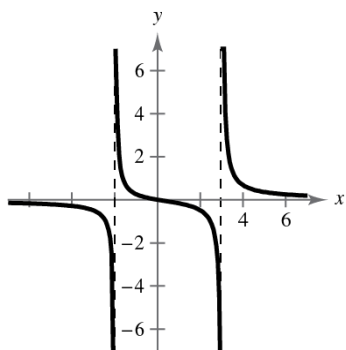
$$\text{Domain: all real numbers } x \text{ such that } -5 \leq x \leq 5, \text{ or } [-5, 5]$$



1.R.50.

$$\begin{aligned} h(x) &= \frac{x}{x^2 - x - 6} \\ &= \frac{x}{(x + 2)(x - 3)} \end{aligned}$$

$$\text{Domain: All real numbers } x \text{ except } x = -2, 3$$



1.R.51.

$$v(t) = -32t + 48$$

$$v(1) = 16 \text{ feet per second}$$

1.R.52.

$$0 = -32t + 48$$

$$t = \frac{48}{32} = 1.5 \text{ seconds}$$

1.R.53.

$$f(x) = 2x^2 + 3x - 1$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\left[2(x+h)^2 + 3(x+h) - 1\right] - (2x^2 + 3x - 1)}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h} \\ &= \frac{h(4x + 2h + 3)}{h} \\ &= 4x + 2h + 3, \quad h \neq 0 \end{aligned}$$

1.R.54.

$$\begin{aligned}
 f(x) &= x^3 - 5x^2 + x \\
 f(x+h) &= (x+h)^3 - 5(x+h)^2 + (x+h) \\
 &= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h \\
 \frac{f(x+h) - f(x)}{h} &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h} \\
 &= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h} \\
 &= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h} \\
 &= 3x^2 + 3xh + h^2 - 10x - 5h + 1, h \neq 0
 \end{aligned}$$

1.R.55.

$$y = (x-3)^2$$

A vertical line intersects the graph no more than once, so y is a function of x .

1.R.56.

$$x = -|4 - y|$$

A vertical line intersects the graph more than once, so y is *not* a function of x .

1.R.57.

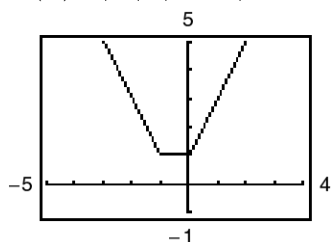
$$\begin{aligned}
 f(x) &= 3x^2 - 16x + 21 \\
 3x^2 - 16x + 21 &= 0 \\
 (3x-7)(x-3) &= 0 \\
 3x-7 &= 0 \quad \text{or} \quad x-3 = 0 \\
 x &= \frac{7}{3} \quad \text{or} \quad x = 3
 \end{aligned}$$

1.R.58.

$$\begin{aligned}
 f(x) &= 5x^2 + 4x - 1 \\
 5x^2 + 4x - 1 &= 0 \\
 (5x-1)(x+1) &= 0 \\
 5x-1 &= 0 \Rightarrow x = \frac{1}{5} \\
 x+1 &= 0 \Rightarrow x = -1
 \end{aligned}$$

1.R.59.

$$f(x) = |x| + |x + 1|$$



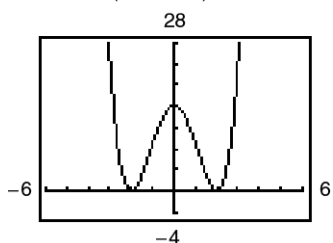
f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

f is constant on $(-1, 0)$.

1.R.60.

$$f(x) = (x^2 - 4)^2$$



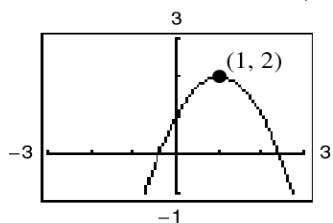
f is increasing on $(-2, 0)$ and $(2, \infty)$.

f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

1.R.61.

$$f(x) = -x^2 + 2x + 1$$

Relative maximum: $(1, 2)$

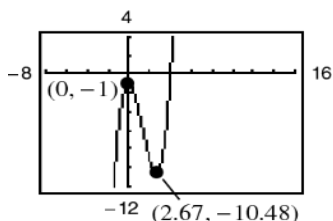


1.R.62.

$$f(x) = x^3 - 4x^2 - 1$$

Relative minimum: $(2.67, -10.48)$

Relative maximum: $(0, -1)$



1.R.63.

$$f(x) = -x^2 + 8x - 4$$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4$$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

1.R.64.

$$f(x) = x^3 + 2x + 1$$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{34 - 4}{2} = \frac{30}{2} = 15$$

The average rate of change of f from $x_1 = 1$ to $x_2 = 3$ is 15.

1.R.65.

$$f(x) = x^4 - 20x^2$$

$$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$$

The function is even, so the graph has y -axis symmetry.

1.R.66.

$$f(x) = 2x\sqrt{x^2 + 3}$$

$$\begin{aligned} f(-x) &= 2(-x)\sqrt{(-x)^2 + 3} \\ &= -2x\sqrt{x^2 + 3} \\ &= -f(x) \end{aligned}$$

The function is odd, so the graph has origin symmetry.

1.R.67.

(a) $f(2) = -6$, $f(-1) = 3$

Points: $(2, -6)$, $(-1, 3)$

$$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$$

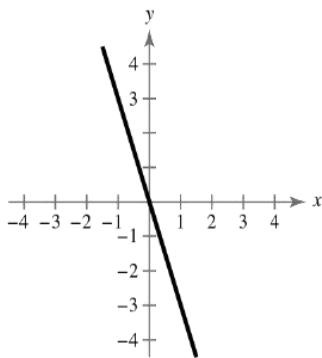
$$y - (-6) = -3(x - 2)$$

$$y + 6 = -3x + 6$$

$$y = -3x$$

$$f(x) = -3x$$

(b)



1.R.68.

(a) $f(0) = -5$, $f(4) = -8$

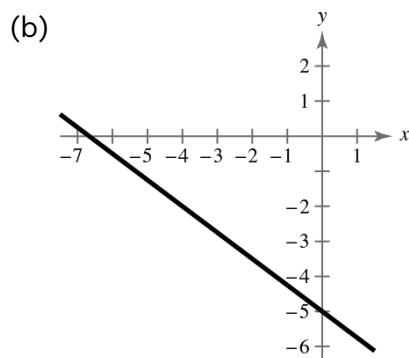
Points: $(0, -5)$, $(4, -8)$

$$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$$

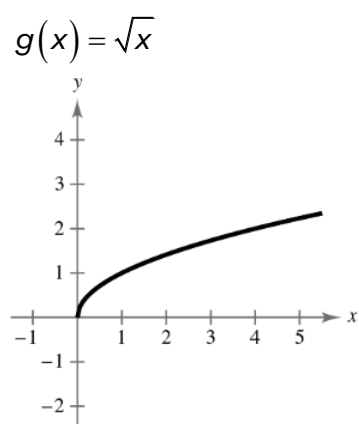
$$y - (-5) = -\frac{3}{4}(x - 0)$$

$$y = -\frac{3}{4}x - 5$$

$$f(x) = -\frac{3}{4}x - 5$$

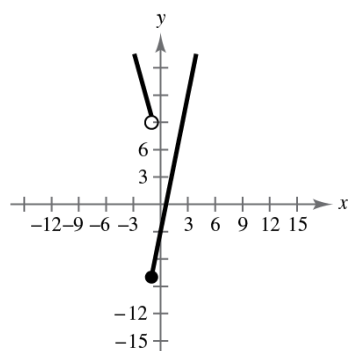


1.R.69.



1.R.70.

$$f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$$

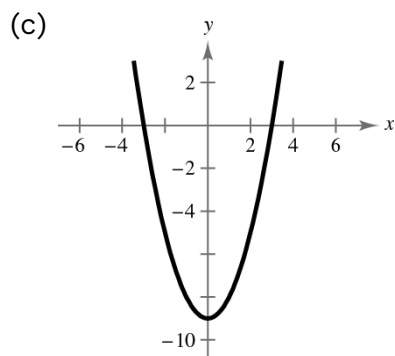


1.R.71.

(a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward



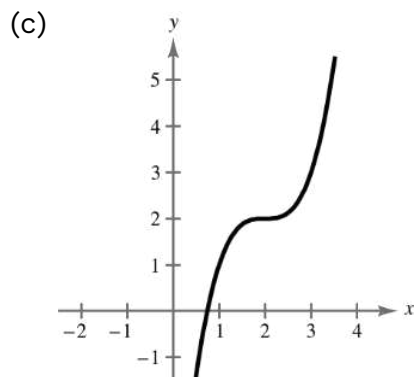
(d) $h(x) = f(x - 2) + 2$

1.R.72.

(a) $f(x) = x^3$

(b) $h(x) = (x - 2)^3 + 2$

Horizontal shift 2 units to the right; vertical shift 2 units upward



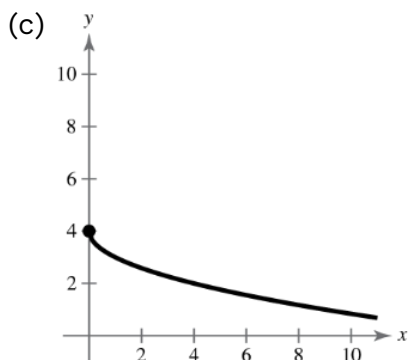
(d) $h(x) = f(x - 2) + 2$

1.R.73.

(a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x} + 4$

Reflection in the x-axis and a vertical shift 4 units upward



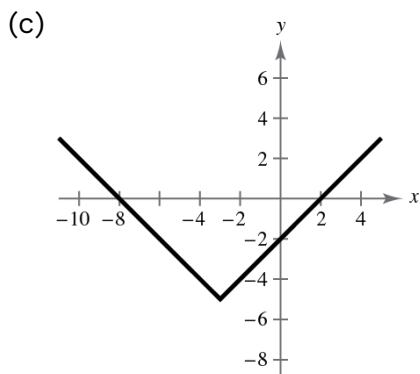
(d) $h(x) = -f(x) + 4$

1.R.74.

(a) $f(x) = |x|$

(b) $h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left and a vertical shift 5 units downward



(d) $h(x) = f(x + 3) - 5$

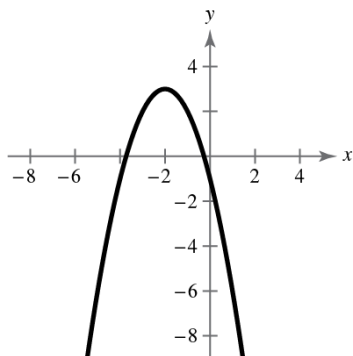
1.R.75.

(a) $f(x) = x^2$

(b) $h(x) = -(x + 2)^2 + 3$

Reflection in the x -axis, a horizontal shift 2 units to the left, and a vertical shift 3 units upward

(c)



(d) $h(x) = -f(x + 2) + 3$

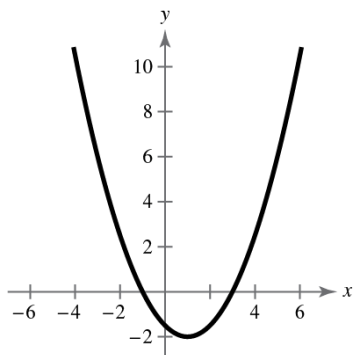
1.R.76.

(a) $f(x) = x^2$

(b) $h(x) = \frac{1}{2}(x - 1)^2 - 2$

Horizontal shift one unit to the right, vertical shrink, and a vertical shift 2 units downward

(c)



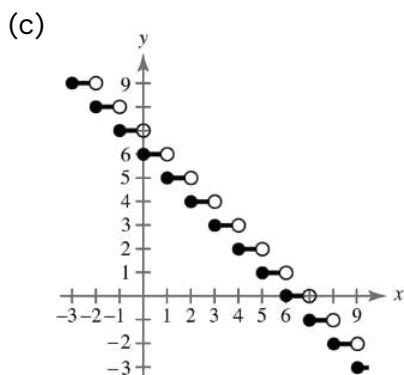
(d) $h(x) = \frac{1}{2}f(x - 1) - 2$

1.R.77.

(a) $f(x) = \lceil x \rceil$

(b) $h(x) = -\lceil x \rceil + 6$

Reflection in the x-axis and a vertical shift 6 units upward



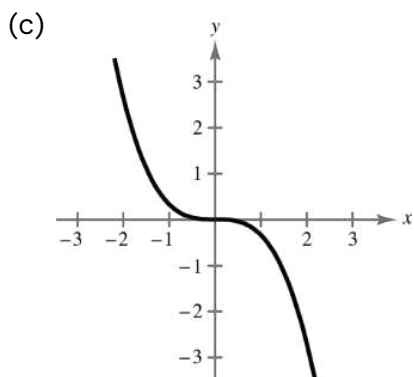
(d) $h(x) = -f(x) + 6$

1.R.78.

(a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x -axis and a vertical shrink
(each y -value is multiplied by $\frac{1}{3}$)



(d) $h(x) = -\frac{1}{3}f(x)$

1.R.79.

$f(x) = x^2 + 3$, $g(x) = 2x - 1$

(a) $(f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^2 + 3}{2x - 1}$, Domain: $x \neq \frac{1}{2}$

1.R.80.

$$f(x) = x^2 - 4, \quad g(x) = \sqrt{3-x}$$

$$(a) (f+g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3-x}$$

$$(b) (f-g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3-x}$$

$$(c) (fg)(x) = f(x)g(x) = (x^2 - 4)(\sqrt{3-x})$$

$$(d) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{\sqrt{3-x}}, \text{ Domain: } x < 3$$

1.R.81.

$$f(x) = \frac{1}{3}x - 3, \quad g(x) = 3x + 1$$

The domains of f and g are all real numbers.

$$\begin{aligned} (a) (f \circ g)(x) &= f(g(x)) \\ &= f(3x + 1) \\ &= \frac{1}{3}(3x + 1) - 3 \\ &= x + \frac{1}{3} - 3 \\ &= x - \frac{8}{3} \end{aligned}$$

Domain: all real numbers

$$\begin{aligned} (b) (g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{1}{3}x - 3\right) \\ &= 3\left(\frac{1}{3}x - 3\right) + 1 \\ &= x - 9 + 1 \\ &= x - 8 \end{aligned}$$

Domain: all real numbers

1.R.82.

$$f(x) = x^3 - 4, \quad g(x) = \sqrt[3]{x+7}$$

The domains of f and g are all real numbers.

$$\begin{aligned} (a) (f \circ g)(x) &= f(g(x)) \\ &= \left(\sqrt[3]{x+7}\right)^3 - 4 \\ &= x + 7 - 4 \\ &= x + 3 \end{aligned}$$

Domain: all real numbers

$$\begin{aligned} \text{(b)} \quad (g \circ f)(x) &= g(f(x)) \\ &= \sqrt[3]{(x^3 - 4)} + 7 \\ &= \sqrt[3]{x^3 + 3} \end{aligned}$$

Domain: all real numbers

In Exercises 83 and 84, use the following functions: $f(x) = x - 100$, $g(x) = 0.95x$.

1.R.83.

$(f \circ g)(x) = f(0.95x) = 0.95x - 100$ represents the sale price if first the 5% discount is applied and then the \$100 rebate.

1.R.84.

$(g \circ f)(x) = g(x - 100) = 0.95(x - 100) = 0.95x - 95$ represents the sale price if first the \$100 rebate is applied and then the 5% discount.

1.R.85.

$$\begin{aligned} f(x) &= 3x + 8 \\ y &= 3x + 8 \\ x &= 3y + 8 \\ x - 8 &= 3y \\ y &= \frac{x - 8}{3} \\ y &= \frac{1}{3}(x - 8) \end{aligned}$$

So, $f^{-1}(x) = \frac{1}{3}(x - 8) = \frac{x - 8}{3}$.

$$f(f^{-1}(x)) = f\left(\frac{1}{3}(x - 8)\right) = 3\left(\frac{1}{3}(x - 8)\right) + 8 = x - 8 + 8 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x + 8) = \frac{1}{3}(3x + 8 - 8) = \frac{1}{3}(3x) = x$$

1.R.86.

$$\begin{aligned} f(x) &= \frac{x - 4}{5} \\ y &= \frac{x - 4}{5} \\ x &= \frac{y - 4}{5} \\ 5x &= y - 4 \\ y &= 5x + 4 \end{aligned}$$

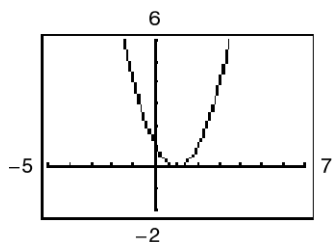
So, $f^{-1}(x) = 5x + 4$.

$$f(f^{-1}(x)) = f(5x + 4) = \frac{5x + 4 - 4}{5} = \frac{5x}{5} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-4}{5}\right) = 5\left(\frac{x-4}{5}\right) + 4 = x - 4 + 4 = x$$

1.R.87.

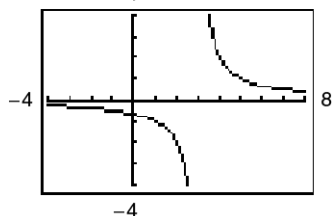
$$f(x) = (x-1)^2$$



The function is not one-to-one. Some horizontal lines intersect the graph more than once.

1.R.88.

$$h(t) = \frac{2}{t-3}$$



The function is one-to-one. Horizontal lines intersect the graph at most once.

1.R.89.

(a) $f(x) = \frac{1}{2}x - 3$

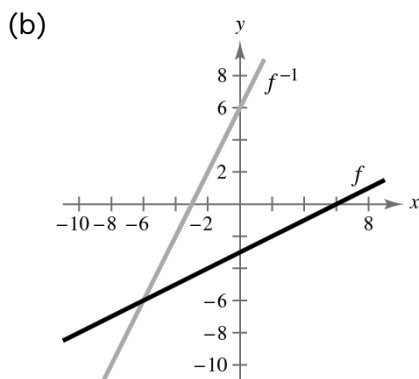
$$y = \frac{1}{2}x - 3$$

$$x = \frac{1}{2}y - 3$$

$$x + 3 = \frac{1}{2}y$$

$$2(x + 3) = y$$

$$f^{-1}(x) = 2x + 6$$



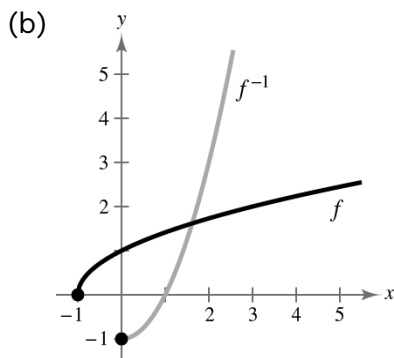
(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

1.R.90.

(a) $f(x) = \sqrt{x+1}$
 $y = \sqrt{x+1}$
 $x = \sqrt{y+1}$
 $x^2 = y+1$
 $x^2 - 1 = y$
 $f^{-1}(x) = x^2 - 1, \quad x \geq 0$

Note: The inverse must have a restricted domain.



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is $[-1, \infty)$.

The range of f and the domain of f^{-1} is $[0, \infty)$.

1.R.91.

$f(x) = 2(x-4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x-4)^2$, $x > 4$ and $y > 0$.

$$y = 2(x-4)^2$$

$$x = 2(y-4)^2, \quad x > 0, \quad y > 4$$

$$\frac{x}{2} = (y-4)^2$$

$$\sqrt{\frac{x}{2}} = y - 4$$

$$\sqrt{\frac{x}{2}} + 4 = y$$

$$f^{-1}(x) = \sqrt{\frac{x}{2}} + 4, \quad x > 0$$

1.R.92.

$f(x) = |x-2|$ is increasing on $(2, \infty)$.

Let $f(x) = x-2$, $x > 2$, $y > 0$.

$$y = x - 2$$

$$x = y + 2, \quad x > 0, \quad y > 2$$

$$x + 2 = y, \quad x > 0, \quad y > 2$$

$$f^{-1}(x) = x + 2, \quad x > 0$$

1.R.93.

$$T = \frac{k}{r}$$

$$3 = \frac{k}{65}$$

$$k = 3(65) = 195$$

$$T = \frac{195}{r}$$

When $r = 80$ mph, $T = \frac{195}{80} = 2.4375$ hours ≈ 2 hours, 26 minutes.

1.R.94.

$$d = \frac{kb}{p}$$

When $p = 4$ and $b = 25,000$, $d = 1250$.

$$1250 = \frac{k(25,000)}{4} \Rightarrow k = \frac{1}{5} = 0.2$$

When $p = 5.50$,

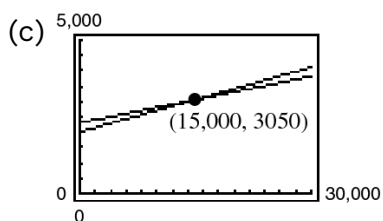
$$d = \frac{(0.2)(25,000)}{5.50} \approx 909 \text{ pastries.}$$

PROBLEM SOLVING FOR CHAPTER 1

1.PS.1.

(a) $W_1 = 0.07S + 2000$

(b) $W_2 = 0.05S + 2300$



Point of intersection: $(15,000, 3050)$

Both jobs pay the same, \$3050, if you sell \$15,000 per month.

- (d) No. If you think you can sell \$20,000 per month, keep your current job with the higher commission rate.

For sales over \$15,000 it pays more than the other job.

1.PS.2.

Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L), (6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5), (M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$$

1.PS.3.

- (a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are even} \\ &= h(x) \end{aligned}$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are odd} \\ &= -h(x) \end{aligned}$$

So, $h(x)$ is also odd. (If $f(x) \neq g(x)$)

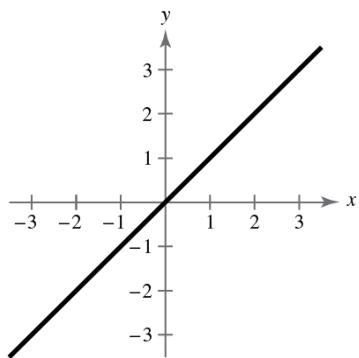
(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \text{ because } f \text{ is odd and } g \text{ is even} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

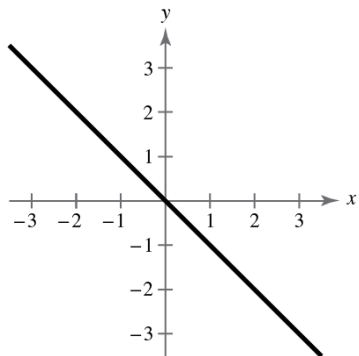
So, $h(x)$ is neither odd nor even.

1.PS.4.

$$f(x) = x$$



$$g(x) = -x$$



$$(f \circ f)(x) = x \text{ and } (g \circ g)(x) = x$$

These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true. General formula: $y = -x + c$

1.PS.5.

$$f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$

$$\begin{aligned} f(-x) &= a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 \\ &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x) \end{aligned}$$

So, $f(x)$ is even.

1.PS.6.

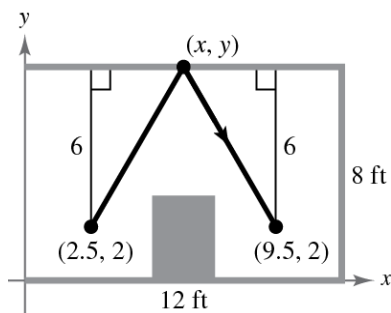
It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$.

The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$.

The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$.

The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \leq x \leq 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \leq 9.5 \end{cases}$$



1.PS.7.

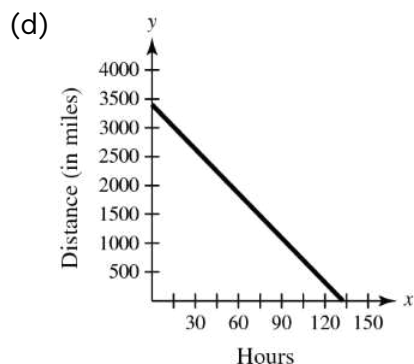
- (a) April 11: 10 hours
- April 12: 24 hours
- April 13: 24 hours
- April 14: $23\frac{2}{3}$ hours
- Total: $81\frac{2}{3}$ hours

$$(b) \text{ Speed} = \frac{\text{distance}}{\text{time}} = \frac{2100}{81\frac{2}{3}} = \frac{180}{7} = 25\frac{5}{7} \text{ mph}$$

$$(c) D = -\frac{180}{7}t + 3400$$

$$\text{Domain: } 0 \leq t \leq \frac{1190}{9}$$

$$\text{Range: } 0 \leq D \leq 3400$$



1.PS.8.

$$(a) \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(1)}{2 - 1} = \frac{1 - 0}{1} = 1$$

$$(b) \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{0.75 - 0}{0.5} = 1.5$$

$$(c) \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.25) - f(1)}{1.25 - 1} = \frac{0.4375 - 0}{0.25} = 1.75$$

$$(d) \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.125) - f(1)}{1.125 - 1} = \frac{0.234375 - 0}{0.125} = 1.875$$

$$(e) \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1.0625) - f(1)}{1.0625 - 1} = \frac{0.12109375 - 0}{0.0625} = 1.9375$$

(f) Yes, the average rate of change appears to be approaching 2.

(g) a. $(1, 0), (2, 1), m = 1, y = x - 1$

b. $(1, 0), (1.5, 0.75), m = \frac{0.75}{0.5} = 1.5, y = 1.5x - 1.5$

c. $(1, 0), (1.25, 0.4375), m = \frac{0.4375}{0.25} = 1.75, y = 1.75x - 1.75$

d. $(1, 0), (1.125, 0.234375), m = \frac{0.234375}{0.125} = 1.875, y = 1.875x - 1.875$

$$\text{e. } (1, 0), (1.0625, 0.12109375), m = \frac{0.12109375}{0.0625} = 1.9375, \\ y = 1.9375x - 1.9375$$

$$\text{(h) } (1, f(1)) = (1, 0), m \rightarrow 2, y = 2(x - 1), y = 2x - 2$$

1.PS.9.

(a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

$$\text{(a) } (f \circ g)(x) = f(x + 6) = 4(x + 6) = 4x + 24$$

$$\text{(b) } (f \circ g)^{-1}(x) = \frac{x - 24}{4} = \frac{1}{4}x - 6$$

$$\text{(c) } f^{-1}(x) = \frac{1}{4}x \\ g^{-1}(x) = x - 6$$

$$\text{(d) } (g^{-1} \circ f^{-1})(x) = g^{-1}\left(\frac{1}{4}x\right) = \frac{1}{4}x - 6$$

$$f(x) = x^3 + 1 \text{ and } g(x) = 2x$$

$$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$$

$$(f \circ g)^{-1}(x) = \sqrt[3]{\frac{x - 1}{8}} = \frac{1}{2} \sqrt[3]{x - 1}$$

$$f^{-1}(x) = \sqrt[3]{x - 1}$$

$$g^{-1}(x) = \frac{1}{2}x$$

$$\text{(e) } (g^{-1} \circ f^{-1})(x) = g^{-1}(\sqrt[3]{x - 1}) = \frac{1}{2} \sqrt[3]{x - 1}$$

(f) Answers will vary.

$$\text{(g) Conjecture: } (f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$$

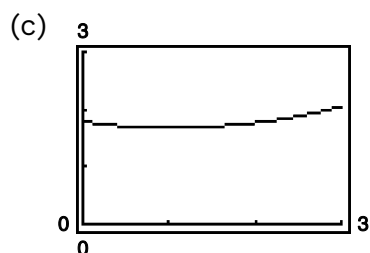
1.PS.10.

(a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$.

The total time is

$$\begin{aligned} T(x) &= \frac{\sqrt{4+x^2}}{2} + \frac{\sqrt{1+(3-x)^2}}{4} \\ &= \frac{1}{2}\sqrt{4+x^2} + \frac{1}{4}\sqrt{x^2-6x+10}. \end{aligned}$$

(b) Domain of $T(x)$: $0 \leq x \leq 3$

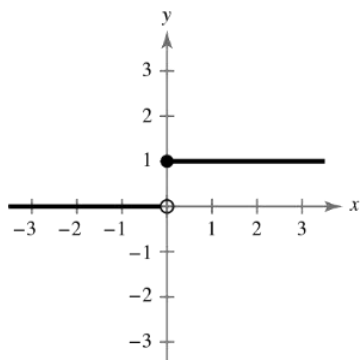


(d) $T(x)$ is a minimum when $x = 1$.

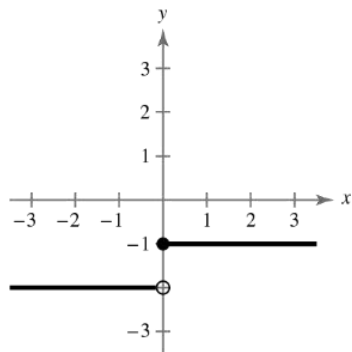
(e) Answers will vary. *Sample answer:* To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way. The distance $x = 1$ yields a time of 1.68 hours.

1.PS.11.

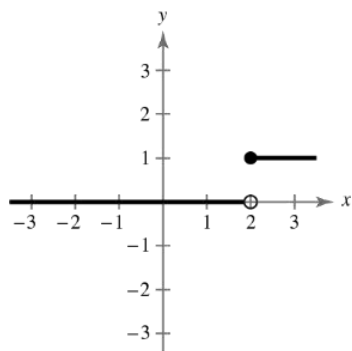
$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



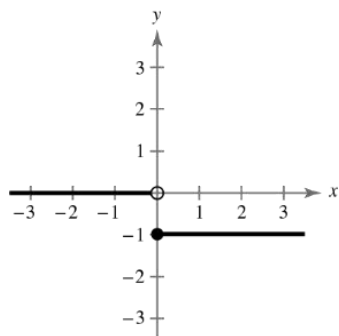
(a) $H(x) - 2$



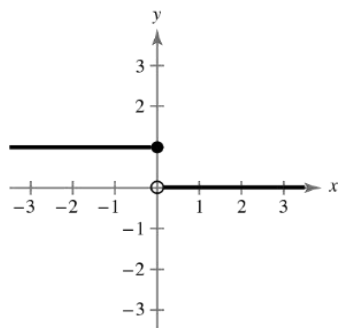
(b) $H(x - 2)$



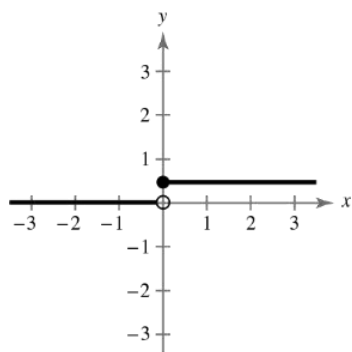
(c) $-H(x)$



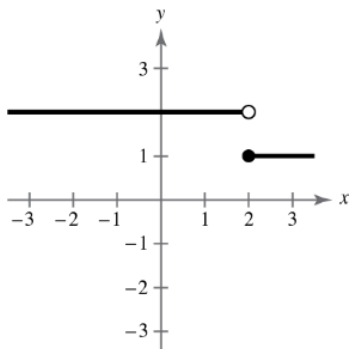
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x-2)+2$



1.PS.12.

$$f(x) = y = \frac{1}{1-x}$$

(a) Domain: all real numbers x except $x = 1$

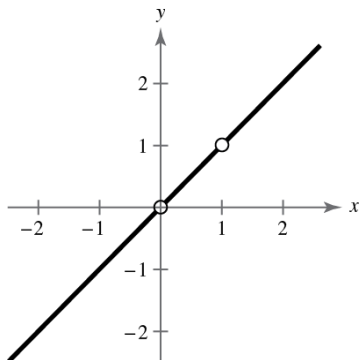
Range: all real numbers y except $y = 0$

$$\begin{aligned} \text{(b) } f(f(x)) &= f\left(\frac{1}{1-x}\right) \\ &= \frac{1}{1-\left(\frac{1}{1-x}\right)} = \frac{1}{\frac{1-x-1}{1-x}} \\ &= \frac{1-x}{-x} = \frac{x-1}{x} \end{aligned}$$

Domain: all real numbers x except $x = 0$ and $x = 1$

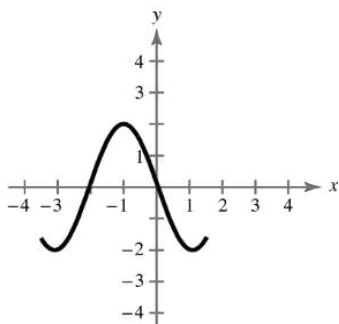
$$\text{(c) } f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\left(\frac{x-1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$$

The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.

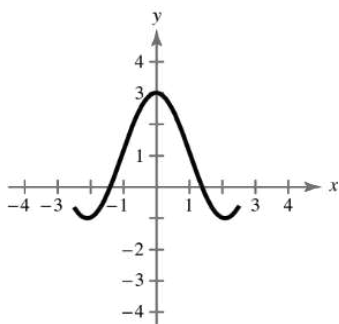


1.PS.13.

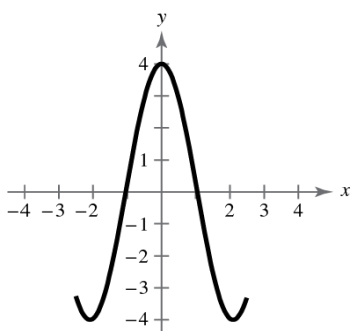
(a) $f(x+1)$



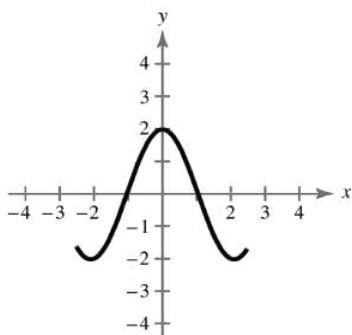
(b) $f(x) + 1$



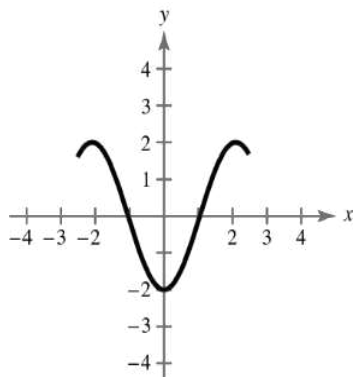
(c) $2f(x)$



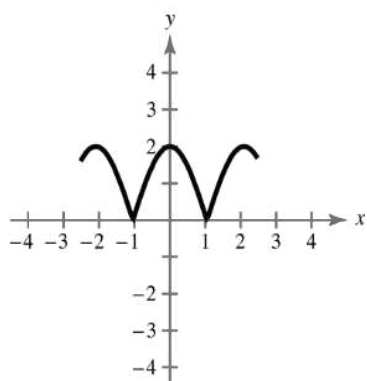
(d) $f(-x)$



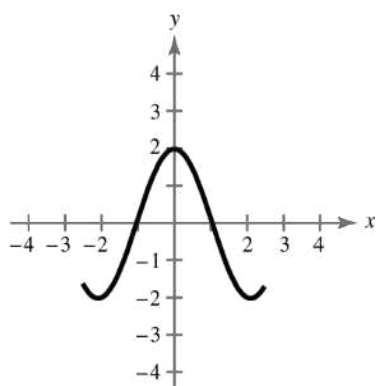
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



1.PS.14.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$ f^{-1}(x) $
-4	$ f^{-1}(-4) = 2 = 2$
-3	$ f^{-1}(-3) = 1 = 1$
0	$ f^{-1}(0) = -1 = 1$
4	$ f^{-1}(4) = -3 = 3$